Answers to WJEC GCSE Mastering Mathematics Higher

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2 Number Strand 2 • Unit 7 • Answers

Practising skills (pages 4–5)

1 **a** 7.8×10^5
b 1.7×10^{-2}
c 8.7×10^3
d 8.4×10^3
e 4×10^{-3}
f 3.1×10^{-2}

2 **a** 1.18×10^6
b 7.66×10^5
c 5.32×10^6
d 2.6×10^5
e 6.74×10^6
f -3.88×10^6

3 **a** 6×10^{12}
b 8×10^8
c 1×10^8
d 9×10^2
e 1×10^{-1}
f 6.3×10^{-9}

4 **a** 3×10^2
b 2×10 or 2×10^1, although first is better.
c 3×10^4
d 1.5×10^4
e 5×10
f 2.5×10^{-3}

5 **a** 5.99×10^5 to 6.01×10^4
b 6.01×10^{-3} to 5.99×10^{-2}
c 7.11×10^2 to 7.09×10^3
d 7.09×10^{-6} to 7.11×10^{-7}

6 If the numbers multiplied together give a result greater than 10, then will need to add an additional 1 to the index.

Developing fluency (pages 5–6)

1 **a** 3.208×10^2
b 3.2×10^2
c 1.28×10^2
d 8.01×10^2

2 9×10^{15}

3 **a** Jupiter, Saturn, Earth, Mercury

b 3.17×10^{2}

c 5.76×10^{3}

d 1.82×10

4 **a** 2.33×10^{18}

b 1.07×10

c 2.33×10^{18} (NB this is the same as a)

5 7×10^{21}

Problem solving (pages 6–7)

1 Yes, total mass = 5.424×10^{5} kg

2 **a** 6.25×10^{12} m

b 13.6 cm

3 **a** 1×10^{-4} m

b 1.783×10^{-2} kg

4 **a** 18 seconds

b 1 km. The time taken for the light to travel this distance is assumed to be zero.

5 Australia has the greatest area of land/person. UK has the least.

Country	Area (in km²)	Population	Area (in km²) per person
Australia	7.7×10^{6}	2.2×10^{7}	0.350
Brazil	8.5×10^{6}	2.0×10^{8}	0.043
China	9.6×10^{6}	1.4×10^{9}	0.007
Germany	3.6×10^{5}	8.3×10^{7}	0.0043
UK	2.4×10^{5}	6.4×10^{7}	0.0038
USA	9.8×10^{6}	3.2×10^{8}	0.031

6 327.06 (330 – 2.94) metres

Reviewing skills (page 8)

1 **a** 9.29×10^{4}

b **i** 84 800

ii 8400

iii 300

c $84\,800 + 8400 - 300 = 92900 = 9.29 \times 10^{4}$

2 **a** 9×10^{12}

b 3.2×10^{-7}

c 5.3×10^{6}

d 2×10^{-3}

e 1.52×10^{2}

f 3.8×10^{3}

3 **c** 3.004×10^{-26} kg

Practising skills (pages 10–11)

1

$1\frac{1}{9}$	0.111 111 11…	$0.\dot{1}$	$0.\overline{1}$	0.[1]
$\frac{3}{11}$	0.272 727…	$0.\dot{2}\dot{7}$	$0.\overline{27}$	0.[27]
$\frac{7}{90}$	0.077 777….	$0.0\dot{7}$	$0.0\overline{7}$	0.0[7]
$\frac{9}{11}$	0.818 181 81…	$0.\dot{8}\dot{1}$	$0.\overline{81}$	0.[81]
$\frac{2}{7}$	0.285 714 285 7…	$0.\dot{2}85\,71\dot{4}$	$0.\overline{285\,714}$	0.[285 714]

2 **a** terminating
b recurring
c terminating
d terminating
e recurring
f recurring
g terminating
h recurring
i terminating

3 **a** $\frac{8}{9}$
b $\frac{2}{9}$
c $\frac{8}{11}$

4 **a** $\frac{110105}{999999}$
b $\frac{5}{9} = 0.55555...$
d $\frac{4}{7} = 0.571\,428\,57\ldots$
e $\frac{7}{44} = 0.159\,090\,909\,0\ldots$

5 **a** $\frac{12}{99}$
b $\frac{3}{90}$
c $\frac{7}{999}$
d $\frac{1}{900}$
e $\frac{901}{900}$

Developing fluency (pages 11–12)

1 **a** 0.857 142 : 8571

b It is the same 6 digits in the same order, starting at a different point each time.

2 True. $10\times = 4.9999$ and $100\times = 49.999\,99$. So, written as a fraction $0.499\,999\,9$ is $\frac{45}{90} = \frac{1}{2}$.

3 None terminate except for 2 and 5.

4 **a** $\frac{1}{5} = 0.2$ and all other fractions are multiples of this terminating decimal.

b 2, 4, 5, 8, 10, 16, 20, 25

c 2 and 5

d 200, 250, 256

Problem solving (pages 12–13)

1 **a** **ii** has the greatest percentage error.

b Rounding $\frac{1}{3}$ to 0.3 has the greatest percentage error.

2 **a** 3.[142 857]

b 0.04%

c 0.000 008%

d After 14 decimal places it terminates, so it is not a recurring decimal.

3 **a** 6, 0.6, 0.06, 0.006

b $6\frac{2}{3}$

c 6, –0.6, 0.06, –0.006

d $5\frac{5}{11}$

4 **a** **i** $p = 7$

ii $p = 3$ (etc.)

b It is 6.

c No, there are two sets of digits.

d The length of the repeating pattern is a factor of $p - 1$.

5 **a** $0.\dot{1}$ is equivalent to $\frac{1}{9}$

b $0.1\dot{2}$ is equivalent to $\frac{11}{90}$

This can be worked out from scratch, or can be considered like this:

$0.111\,111\,111\,1\ldots$

$+\,0.011\,111\,111\,1\ldots$

$0.122\,222\,222\,2\ldots$

This is $\frac{1}{9} + \frac{1}{90} = \frac{10}{90} + \frac{1}{90} = \frac{11}{90}$.

c $0.12\dot{3}$ is equivalent to $\frac{111}{900}$

d $0.123\dot{4}$ is equivalent to $\frac{1111}{9000}$

e $0.1234\dot{5}$ is equivalent to $\frac{11111}{90000}$

If the pattern is continued we will get to $\frac{111111111}{900000000}$. There are 9 ones in the numerator of this one, so they total 9. That means the numerator is divisible by 9, so it is not a recurring decimal (and is equivalent to 0.123 456 79).

Reviewing skills (page 13)

1 a $\frac{29}{90}$

b $\frac{32}{99}$

c $\frac{302}{999}$

2 a $0.[2]$

b $0.[1]$

c $0.0[2]$

d $\frac{6}{45} = \frac{1}{45} + \frac{1}{9} = 0.1[3]$

Practising skills (page 16)

1 **a** 2
b 4
c 1
d 2
e 3
f 3
g 4
h 5

2 **a** 30
b 50
c 400
d 900
e 7000
f 20 000
g 20
h 60
i 0.9
j 0.7
k 0.02
l 0.02

3 **a** 870
b 920
c 620
d 710
e 700
f 3300
g 5100
h 19 000
i 73 000
j 8000
k 0.64
l 0.60

4 **a** 400 000
b 380 000
c 384 000
d 384 000
e 384 030

5 **a** 8
b 8.0
c 8.00
d 8.000
e 8.0000

6 **a** 0.008
b 0.0081
c 0.008 11
d 0.008 106
e 0.008 106 0

7 **a** 20
b 0.60
c 71000
d 4
e 6.51
f 27.00

Developing fluency (page 17)

1

	Number	Round to 1 significant figure	Round to 2 significant figures
a	742	700	740
b	628	600	630
c	199	200	200
d	4521	5000	4500
e	3419	3000	3400
f	8926	9000	8900
g	8974	9000	9000
h	36 294	40 000	36 000
i	0.2583	0.3	0.26
j	0.079 61	0.08	0.080
k	0.000 3972	0.0004	0.000 40
l	0.001 023	0.001	0.0010

2 **a** 20
b 10
c 51
d 0.048
e 17 600 000
f 100
g 300
h 1.01
i 677

3 **a** 0.2
b 0.48

4 **a** true
b true
c false
d false

5 **a** Ada and Cain, Ben and Dewi
b Ada, Ben and Cain
c Ben and Cain

Problem solving (page 18)

1 Ami. Ifor has rounded down instead of up, Milly thinks that the zero is not significant and Iwan has changed the size of the number.

2 **a** $C = 24\,\text{cm}$

b $C = 24.8\,\text{cm}$

c $C = 25.12\,\text{cm}$

d $C = 25.136\,\text{cm}$

3 **a** £590

b £591.80

c Real value is £587.475. Therefore, to 1 significant figure, **a** is more accurate. It is also easier.

4 **a** 0.004 cm

b Measure the height of a number of the same book stacked in a pile or use a more accurate measuring device.

Reviewing skills (page 19)

1 **a** 1
b 0.01
c 1000
d 1 000 000

2 **a** 6.4
b 20
c 0.0052
d 0.010

3 **a** 0.3068
c 515 300
d 2.0

4 **a** 30 000

Practising skills (page 21)

1 **a** 79.5, 80.5
b 75, 85
c 299.5, 300.5
d 250, 350

2 **a** 4999.5 m, 5005.5 m.
b 4995 m, 5005 m
c 4950 m, 5050 m
d 4500 m, 5500 m

3 **a** 595 m, 605 m
b 597.5 m, 602.5 m
c 550, 650
d 575, 625

4 $5.5 \leqslant m < 6.5$

5 $8.5 \leqslant l < 9.5$

6 **a** 2.5, 3.5
b 55, 65
c 0.35, 0.45
d 0.065, 0.075

7 **a** 23.5, 24.5
b 355, 365
c 0.825, 0.835
d 0.0185, 0.0195

8

	Number	Lower limit	Upper limit
a	4	3.5	4.5
b	70	65	75
c	600	595	605
d	0.3	0.25	0.35
e	0.06	0.055	0.065
f	80 km	75 km	85 km
g	68 mg	67.5 mg	68.5 mg
h	0.032	0.0315	0.0325

Developing fluency (page 22)

1 1475 ml $\leqslant V <$ 1525 ml

2 52.5 km

3 182.25 cm^2 $\leqslant A <$ 210.25 cm^2

4 No. The minimum in one bag is 245 g. If all 3 bags are at the lower limit she will only have 735 g.

5 225 m $\leqslant p <$ 255 m, 2812.5 m^2 $\leqslant A <$ 3612.5 m^2

6 **iii**, all the others are to the nearest 5, **iii** is to the nearest 10.

7 **a** $n = 680 +/- 5$

b $675 \leqslant n < 685$

c 685 and 675

d 10

Problem solving (page 23)

1 6 (Max = 652 members)

2 No, only 9 per page (5 horizontally and 4 vertically).

3 **a** If each person's mass is given to the nearest kg, their total mass could be 652 kg.

b 80 kg each

4 48

Reviewing skills (page 24)

1 **a** 39.5, 40.5 ml

b 35, 45 ml

c 650, 750 kg

d 695, 705 kg

2 **c** 625 and 675

3 315 g $\leqslant m <$ 325 g

4 **a** 850 cm, 950 cm

b 1500 km, 2500 km

c 0.15 g, 0.25 g

d 0.0045 m, 0.0055 m

5 **a** 7050 m, 7150 m

b 48.5 cm, 49.5 cm

c 515 mm, 525 mm

d 0.00275 km, 0.002 85 km

6 The upper bound of a side is 20.5 cm. If each side is the upper limit the perimeter will be 82 cm and the ribbon would be too short.

Practising skills (page 27)

1 **a** 3.5
b 3.55
c 3.555
d 7.5415
e 12.000 05

2 **a** 6.5 kg and 7.5 kg
b 6.5 m and 7.5 m
c 6.5 litres and 7.5 litres

3 **a** 52.5 cm and 53.5 cm
b 52.95 cm and 53.05 cm
c 5.25 cm and 5.35 cm

4 **a** 14.5 kg and 15.5 kg
b 15.65 kg and 15.75 kg
c 15.355 kg and 15.365 kg

5 149.9875 kg and 150.0125 kg

Developing fluency (page 28)

1 0.55 cm

2 1384.25 cm^2 and 1304.25 cm^2

3 **a** 193.375 and 390.625
b 245.700 (245.699 875) and 258.400 (258.400 125)

4 **a** 5048.125 m^2 and 4987.125 m^2
b It is correct to 2 significant figures

5 **a** 61.7π and 61.9π
b Yes. Using the true area we get the true radius as 30.902 cm (to 3 d.p.) which is within her bounds.

6 **a** 6 375 km and 6 385 km
b 40 118 km and 40 055 km (to the nearest km)

7 The upper bound implies that the value is less that that upper bound, not less than or equal to.

Problem solving (pages 29–30)

1 Each of the boxes has a height of 25 cm to the nearest centimetre. This means that the upper bound for the height is 25.5 cm.

$4 \times 25.5\,\text{cm} = 102\,\text{cm} = 1.02\,\text{m}$. This is 2 cm more than the height available and so the boxes may not fit.

2 The piece of wood is 80 cm long to the nearest 2 cm and so the longest the shelf can be is 81 cm. Therefore the maximum length the space can be is 1 cm.

3 **a** 8.5 cm and 9.5 cm, 5.5 cm and 6.5 cm, 18.5 cm and 19.5 cm

b The answer is **ii** - she can possibly do this. If each measurement is at the lowest bound then the total volume of the 8 cartons is 6 919 cm^3, which is less than the 7 000 cm^3 required. Using the highest bounds however gives enough volume, so it may be possible

4 $205.56\,\text{cm} \leqslant A < 211.69\,\text{cm}$ (to 2 decimal places)

5 **a** 0.5 / (25.5/60/60) = 70.59 mph = 71 mph to 1 d.p.

b Lower bound of Viv's speed = $\frac{0.495}{25.75} \times 3600 = 69.2$ mph

6 $8.5075\,\text{m/s} \leqslant \text{speed} \leqslant 8.5867\,\text{m/s}$

7 **a** upper bound = 53.1, lower bound = 48.7

b 50 (1 significant figure)

8 Malcolm is definitely wrong. Lower bound = 325.1 ml, upper bound = 339 ml

Reviewing skills (page 30)

1 Yes, the upper bound of the combined masses is 452 kg and the lower bound of lift's limit is 475 kg.

2 166 CDs

3 Upper bound 490.25 cm^2. Lower bound 446.25 cm^2.

Practising skills (page 34)

1 £40
2 £15
3 £30
4 **a** £115
b £23
5 **a** 70 cm
b 128 m
c 450 km
d 5 m
e 850 km
6 **a** £230
b £173.91
c £226.09; no because compound interest is not just adding two percentages together.

Developing fluency (pages 34–35)

1 **a** £70
b £120
c £60
d £20
2 **a** £80
b £18
c £175
d £1250
3 900 ml
4 40 hours
5 £9400
6 £180 000
7 £44
8 319 pages
9 3640 miles
10 £140

Problem solving (pages 36–37)

1 £12.50 each
2 £7200
3 €50
4 He bid £800.
5 Total cut is 7.1%.
6 460
7 12 years
8 10% lower than the average.

Reviewing skills (page 37)

1 **a** £6
2 **a** 140 cm on his 13th birthday
b He grew 7 cm.
3 **a** 240
b 36
4 £22

Practising skills (pages 39–40)

1 **a** 6600
b 7260
c 7986
d 33.1%

2 **a** £7986
b 33.1%
c The increase is compound and is therefore being applied to an increasingly large amount.

3 **a** 4000
b 3200
c 2560
d 51.2% of original value

4 **a** £2560
b 48.8% decrease
c 2 more years

5

% change	Decimal multiplier
20% increase	1.2
60% decrease	0.4
12% increase	1.12
12% decrease	0.88
150% increase	1.5

6 **a** 300, 180, 162
b 19%
c 162
d They are the same calculation expressed as a percentage and a decimal multiplier.

Developing fluency (pages 40–41)

1 **a** £30
b £530
c £31.80

2 **a** **i** £100
ii £1000
iii £52.50
b **i** £102.50
ii £1221.02
iii £54.36

3 **a** 40%
b 16%
c 6.4%
d 2.56%
e 1.024%

4 **a** **i** £2400

ii £600

b **i** £2400 × 0.8 = £1920

ii 7 more years (9 years from purchase)

5 358

6 **a** £92 220

b £97 753.20

c 2005

7 **a** 69.57%

b $(1 \times 0.93)^{10} = 0.484 \times 100 = 48.4\%$

c 2.66%

d 0.07%

8 **a** £690.79, meaning £110.79 of interest is paid.

b £2203.52, meaning £553.52 is paid in interest.

c £68 146.10, meaning £44 146.10 is paid in interest.

9 15 km and 12 km

Problem solving (pages 41–43)

1 They are both wrong, the reduction equates to 44%.

2 Written to nearest thousand:

a 37 870 000

b 33 367 000

3 Decreased by 3.2%.

4 Yes, £4 485 000 in two years.

5 The Friendly Bank gives £13 more interest.

6 The decrease was applied to a higher price so the reduced price was slightly higher than the original.

7 17.6 m^3

8 Andy (£1610.95) better than Tina (£1607.73).

9 No, it has increased by 23.2%.

10 **a**

Year	Principal	Interest	Amount at end of year
1	£800	£64	£864
2	£864	£69.12	£933.12
3	£933.12	£74.65	£1007.77

b $1007.77 = 800 \times (1+\frac{8}{100})^3$

c It applies the interest as a compound rate.

d **i** £1292.07

ii £757.49

iii £119.03

11 Gross annual percentage rates don't take into account compound interest whereas AER percentage rates do. This means that the quarterly AER will include not only the interest on the original investment but also interest on any interest paid in the previous quarter(s). That is why the quarterly AER will always be higher than the gross annual percentage rate.

Reviewing skills (page 43)

1 **a** Simple interest = £5280, compound interest = £5948.07

b Simple interest = £28 800, compound interest = £50 540.36

2 **a** £6800

b £5508

3 No, after 3 months his weight will be 17.33 stones.

Practising skills (pages 45–46)

1 **a** 1.3
b 1.03
c 0.97
d 0.67

2 **a** 70% increase
b 7% increase
c 7% decrease
d 97.7% decrease

3 **a** 5
b It is doubling.
c 10 hours

4 **a** £490.89
b After 11 years

5 **a** £2000
b 5%
c £2954.91

6 **a** 1×32^{54}
b 5×34^{12}

Developing fluency (pages 46–47)

1

£100 invested for 3 years at 6%	100×1.06^3
£600 invested for 10 years at 3%	600×1.03^{10}
£300 invested for 6 years at 10%	300×1.1^6
£500 invested for 3 years at 7%	500×1.07^3
£700 invested for 5 years at 3%	700×1.03^5
£300 invested for 7 years at 5%	300×1.05^7

2 **a** £645.94
b The salesman is incorrect – the price difference is nearly 7%.

3 **a** After 23 years.
b It won't impact the half-life at all.

4 **a** Interest rate is 6%.
b £4700

5 Job B will be more profitable in the long run (if you stay more than 4 years) as the percentage is of a greater number each year.

6 £238

7 5 years

Problem solving (pages 47–48)

1 £5898.41

2 **a** 20

b An increase from 34 to 1324 is an increase of 3894%.

3 2011

4 Yes, 1620 mice at £1.50 = £2430.

5 **a** 52

b 488

6 9 years

7 Growth4Now as Oldtown is 2.17% AER.

8 Interest is compounded.

9 0.4%

10 **a** Silver

b £27.67

Reviewing skills (page 49)

1 **a** 488.7 cm^3

b 14 hours

2 **a** £2163.20

b 18 years

3 **a** About 249 fish.

b 18 years

Practising skills (page 53)

1 **a** 24p
b 72p

2 **a** 15p
b £1.20

3 **a** £6
b £138

4 **a** **i** 35p
ii 30p
iii Dan's discounts
b **i** £1.30
ii £0.96
ii Dan's discounts
c **i** £3.72
ii £3.84
iii Bev's Bargains

5

Ingredient	Quantity for 5 people	Quantity for 1 person	Quantity for 8 people
Minced beef	900 g	180 g	1440 g
Stock	480 ml	96 ml	768 ml
Onion	2	$\frac{2}{5}$	$3\frac{1}{5}$
Tin of tomatoes	1	$\frac{1}{5}$	$1\frac{3}{5}$
Potatoes	700 g	140 g	1120 g
Worcestershire sauce	40 ml	8 ml	64 ml

Developing fluency (pages 54–55)

1 £112

2 £6.80

3 £5.66

4 **a** 6 kg for £14.70

b 150 ml for £24

c 60 g for £12.06

5 No. He will charge £14.60

6 The maximum Roland can buy is 400 × 153.24 = 61 296 Krona. Since the bank only has 10 Krona notes, this means that they will give him 61 290 Krona. 61 290 ÷ 153.24 = 399.96, therefore it will cost Roland 399.96€.

7 Offer 1 (buy two 40 ml and get one 40 ml free) is the best value, as it is £32.40 for 120 ml, that is £27 for 100 ml.

Offer 2 gives £27.75 for 100 ml.

The large bottle of perfume costs £30 for 100 ml.

8 **a** **i** She bought 660 Zloty.

ii It cost her £149.32

b 660 − 84.40 = 575.60 Zloty left. The shop bought back 575 Zloty, giving her £127.49 in return.

9 £65.60

10 2 hours 6 minutes

11 19 hours

12 22 hours

Problem solving (pages 56–58)

1 **a** £537.50

b 6 hours

2 13 (brown sugar is the limiting ingredient)

3 No; small = 7.4p per chocolate, large = 6.9p per chocolate, medium = 6.8p per chocolate

4 Harvey's company at 50p/mile (Albert's is 49.5p/mile)

5 Yes, she can do it for £14.56

6 Using these exchange rates, £1 is worth 1.27 Euros.

7 41 cents

8 Not enough flour, she needs 168 g; she has enough milk as she needs 1.704 litres

9 £8.50 in 2p coins, £42 in 5p coins

10 **a** In 2014 the room cost 135 Euros per night. To calculate how much it cost in 2015 we multiply the price by 1.11 (111%): 135 × 1.11 = 149.85 Euros per night.

Now we must find out how much the room cost per night in pounds in 2014: $\frac{135\ Euros}{1.12} = £120.54$

We also need to know how much the room cost per night in 2015: $\frac{149.85\ Euros}{1.32} = £113.52$

Finally we work out the difference in prices for the 10 nights: (10 × 120.54) – (10 × 113.52) = 70.20, i.e. the room cost £70.20 *less* in 2015 than it did in 2014 for the 10 nights. The reason for this is because although the room cost more in Euros, the exchange rate was better in 2015.

b The exchange rate in 2016 is £1 = 1.22 Euros. The cost of the room in 2016 is 149.85 × 1.02 = 152.847. The total cost of the room for 10 days is therefore 10 × 152.847 = 1528.47 Euros. We need to find this in pounds: $\frac{1528.47}{1.22} = 1252.84$

Therefore the Davies family need to budget £1252.84 for their trip in 2016.

11 1 hour 50 minutes

Reviewing skills (page 59)

1 a £2.34

b £21.06

2 a 2 m for £8.10 is better value, as 2 m for £8.10 is £4.05 per m; 60 cm for £2.40 is £4.15 per m

b 600 g for £5.40 is £9/kg so is better value as 750 g for £7.20 is £9.60/kg

c 2 litres for £22.12 is £11.06/l so is better value as 800 ml for £8.92 is £11.15/l

3 a 300 ml for £2.16 is 72p per 100 ml; 400 ml for £2.72 is 68p per 100 ml; 500 ml for £3.45 is 69p per 100 ml

b 400 ml

Number Strand 6 • Unit 4 • Answers

Practising skills (pages 62–63)

1 a

x	1	3	2.5	1.75
y	4	12	10	7

b $y = 4x$

c i proportional

ii constant of proportionality

2 a $m = kn$

b $y = kx$

c $A = kB$

d $P = kQ$

e $T = kd$

f $C = kd$

3 a i 20

ii 30

iii 50

iv 5

b $k = 5$

c $y = 5x$

4 a i 24

ii 36

iii 3

iv 1

b $k = 3$

c $D = 3w$

5 a $V = kn$

b $k = 0.5$

c $V = 1$

6 a $y = kx$

b $y = 50$

c $x = 1.2$

7 a $C = kd$

b $C = 15$

c $d = 10$

8 a $P = 27$ when $L = 7$

b $A = 4B$

c $M = 8$ when $N = 4$

Developing fluency (pages 63–64)

1 **a** true
b true
c false

2 $e = 6$

3 Both are correct. If you times both sides of Bobby's formula by 2, you arrive at Blaise's formula.

$x = \frac{1}{2}y$, (×2)
$2x = y$
$y = 2x$

4

x	8	12	36
V	10	15	45

5 Error is when $n = 14$, $T = 60$. When $n = 14$, T should equal 63.

6 Yes, h and t are proportional. When you divide each value for h by its value for t you get a constant of 1.5. Therefore $h = 1.5t$.

Problem solving (pages 64–65)

1 **a** $p = 250n$
b 14

2 **a** moving at a constant speed
b 20 = speed in km/h
c 25 km

3 **a** $c = 3.3d$
b 3.3
c 99 cm

4 $L = \frac{A}{15}$, £34

5 2500

Reviewing skills (page 65)

1 **a** **i** 70
ii 105
iii 140
b $k = 7$
c $t = 7d$

2 **a** $y = kx$
b $y = 42$
c $x = 5$

3

s	70	210	35	560	7
t	1	3	0.5	8	$\frac{1}{10}$

Number Strand 6 • Unit 5 • Answers

Practising skills (pages 68–69)

1 **a** Proof $3 = \frac{12}{4}$

b Proof (1, 12) $12 = \frac{12}{1}$

(2, 6) $6 = \frac{12}{2}$

(3, 4) $4 = \frac{12}{3}$

(6, 2) $2 = \frac{12}{6}$

(12, 1) $1 = \frac{12}{12}$

c **i** true

ii false

iii false

iv true

d **i** false

ii true

iii true

iv true

2 **a** $y = kx$

b $y = \frac{k}{m}$

c $T = kd$

d $M = \frac{k}{t}$

e $W = \frac{k}{x}$

f $C = kd$

3 **a** $A = kd$ and $Ct = k$ and $\frac{w}{r} = k$

b $M = \frac{k}{n}$ and $Ct = k$

4 **a** $y = \frac{k}{x}$

b $k = 10$

c **i** $y = 5$

ii $y = 1$

5 **a** $c = \frac{k}{d}$

b $k = 25$

c **i** $c = 6.25$

ii $c = 100$

iii $c = 0.25$

iv $c = 2500$

d **i** $d = 1$

ii $d = 625$

iii $d = 250$

iv $d = 2.5$

6 **a** $M = \frac{24}{t}$

b $M = 2.4$

c $t = 4$

7 **a** $E = \frac{36}{h}$

b $E = 3$

c $h = 0.24$

Developing fluency (pages 69–70)

1 **a** True

b False

c False

d True

e True

2 **a** $y = \frac{36}{x}$

b $y = 9$

c Yes, when $x = 6$, $y = 6$

3 **a** £12 000

b £$w = \frac{£24\,000}{n}$

c No effect

4 **a** $I = \frac{12}{R}$

b 2

c 120

5 **a**

m	6	24	16
n	8	2	3

b $m = \frac{48}{n}$

c m is inversely proportional to n

6 $C = 400$ and $m = 0.6$

7 Levi is correct. $h = 0.4d$

8 $y = 2$

Problem solving (pages 70–71)

1 a For example:

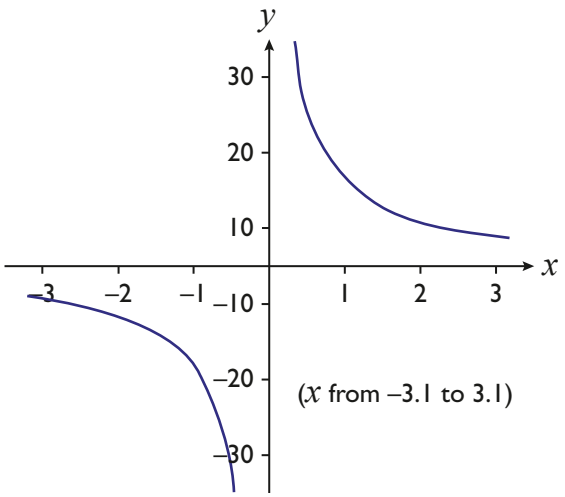
l	1	2	4	5	10	20	25	50	100
w	100	50	25	20	10	5	4	2	1

b 40 metres

2 a

x	1	2	2.5	4	5	10
y	10	5	4	2.5	2	1

b



c i 20

ii 100

d The graph is a curve and gets closer and closer to each axis, but does not ever actually touch the axes. These are called asymptotes.

e Yes

3 3100

4 a The amount of money £27 000 available.

b $g = \frac{27000}{n}$, where g is the grant and n is the number of people.

c £60.00

5 a $v = \frac{210}{t}$, or $vt = 210$

b v is inversely proportional to t since the product of each pair is a constant.

c The journey is of 210 miles.

d 60 hours

Reviewing skills (page 72)

1 a $E = \frac{K}{T}$

b $2.5 \times 4 = 10$

c $E = 5$

d $T = 0.5$

2 a $v = \frac{240}{t}$

b $v = 80$

c $t = 4.8$

3 a $v = \frac{300}{t}$

b i 150 hours

ii 400 hours

c 1.5 km per hour

Number Strand 6 • Unit 6 • Answers

Practising skills (pages 75–76)

1 **a** y is proportional to x
b y is proportional to x squared
c y is proportional to x cubed
d y is proportional to the square root of x
e y is inversely proportional to x squared
f y is inversely proportional to x cubed
g y is inversely proportional to the cube root of x

2 **a** **i** times by 4
ii times by 9
iii times by 100
b **i** $\frac{1}{4}$
ii $\frac{1}{9}$
iii $\frac{1}{100}$

3 **a** $72\,\text{cm}^3$
b $200\,\text{cm}^3$
c $98\,\text{cm}^3$
d $2x^2\,\text{cm}^3$
e Sarah is not correct. It is proportional to the square of its side.

4 **a** 0.02
b 0.0002
c 0.000002
d 0.2
e 20 000
f 2 000 000
g 20

5 **a** $y = \frac{5}{4}x$
b $y = \frac{5}{16}x^2$
c $y = \frac{5}{64}x^3$
d $y = \frac{5}{2}\sqrt{x}$
e $y = \frac{80}{x^2}$
f $y = \frac{320}{x^3}$
g $y = \frac{10}{\sqrt{x}}$

6 **a** $k = 0.031$ (to 2 s.f.)
b 4.5
c 5.66

7 **a** $k = 0.0039$ (to 2 s.f.)
b 6.75
c **i** $y = 0.0039x^3$
ii $x = \sqrt[3]{\frac{y}{0.039}}$

8 **a** $k = 0.67$ (to 2 s.f.)
b **i** $y = 0.67\sqrt{x}$
ii $x = \left(\frac{y}{0.67}\right)^2$
c **i** y is proportional to the square root of x
ii x is proportional to the square of y

Developing fluency (pages 76–77)

1 **a** $s = \frac{108}{t^3}$
b 0.864
c 2.78

2 **a** The equation is $y = \frac{2}{x^2}$
b $p = 8$

3 **a** $T = 2 \times \sqrt{L}$
b 5.66 seconds
c 16 m

4 **a** $s = 4\pi r^2$
b 145.3 cm^2
c 20 mm

5 **a** $W = 10e$
b 4
c $W = 0.8$

6 **a** $F = \frac{1.8 \times 10^{20}}{d^2}$
b 3.7×10^6 newtons

Problem solving (pages 77–78)

1 **a** $£k = n \times £p$; *or* $n = \frac{£k}{£p}$
b n is inversely proportional to p
c £15
d £50

2 **a** $w = \frac{1.1}{\sqrt{A}}$
b 20.9%

3 **a** $P = \frac{12}{\sqrt{h}}$
b 0.15 atmospheres

4 **a** $S = \frac{8000}{f^2}$

b Shutter speed increases from 125 to 500 = 375

5 **a** $P = 1.5625s^3$

b 5.04 m/s

c 8

Reviewing skills (page 78)

1 **a** $y = 1.6$

b $y = 0.256$

c $y = 0.040\,96$

d $y = 4$

e $y = 390.625$

f $y = 2\,441.4$

g $y = 25$

2 **a** $d = 5t^2$

b 720 m

c 8 s

Practising skills (pages 82–83)

1 a

Index form	In full	Ordinary number
2^5	$2 \times 2 \times 2 \times 2 \times 2$	32
2^4	$2 \times 2 \times 2 \times 2$	16
2^3	$2 \times 2 \times 2$	8
2^2	2×2	4
2^1	2	2
2^0	1	1
2^{-1}	$\frac{1}{2}$	0.5
2^{-2}	$\frac{1}{(2 \times 2)}$	0.25
2^{-3}	$\frac{1}{(2 \times 2 \times 2)}$	0.125

b i 128

ii $\frac{1}{2^4} = \frac{1}{16} = 0.0625$

2 a

Index form	In full	Ordinary number	In words
10^3	$10 \times 10 \times 10$	1000	One thousand
10^2	10×10	100	One hundred
10^1	10	10	Ten
10^0	1	1	One
10^{-1}	$\frac{1}{10}$	0.1	One tenth
10^{-2}	$\frac{1}{(10 \times 10)}$	0.01	One hundredth
10^{-3}	$\frac{1}{(10 \times 10 \times 10)}$	0.001	One thousandth

b 1 000 000, one million

c 0.000 001, one millionth

3 a i $3^2 \times 3^3$

$3 \times 3 \times 3 \times 3 \times 3$

3^5

check: 9×27

243

$3^5 = 243$

ii $5^4 \times 5^2$

$5 \times 5 \times 5 \times 5 \times 5 \times 5$

5^6

check 625×25

15 625

$5^6 = 15\,625$

iii $10^3 \times 10^4$

$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

10^7

check $1000 \times 10\,000$

10 000 000

$107 = 10\,000\,000$

b $a^m \times a^n = a^{(m+n)}$

4 a 2^6, 64

b 3^6, 729

c 5^3, 125

d 5^3, 125

5 a i $3^6 \div 3^4$

3^2

check $729 \div 81 = 9$

$3^2 = 9$

ii $5^4 \div 5^3$

$5^1 = 5$

check $625 \div 125 = 5$

iii $10^5 \times 10^2$

10^3

check $100\,000 \div 100 = 1000$

$10^3 = 1000$

b $\frac{a^m}{a^n} = a^{(m-n)}$

6 a 2^3, 8

b 3^1, 3

c 10^3, 1000

d 5^3, 125

7 a i $(2^4)^3$

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

2^{12}

check $16 \times 16 \times 16 = 4096$

$2^{12} = 4096$

ii $(3^2)^5$

$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

3^{10}

check $9 \times 9 \times 9 \times 9 \times 9 = 59\,049$

$3^{10} = 59\,049$

iii $(10^3)^2$

$10 \times 10 \times 10 \times 10 \times 10 \times 10$

10^6

check $1000 \times 1000 = 1\,000\,000$

$10^6 = 1000\,000$

b $(a^m)^n = a^{(mn)}$

8 a 2^{10}, 1024

b 2^{10}, 1024

c 10^{12}, 1 000 000 000 000 (one billion)

d 10^{12}, 1 000 000 000 000 (one billion)

Developing fluency (pages 83–84)

1 a 2^1

b 3^2

c same

d 2^5

e 2^6

f 1^{-2}

g 2^{-3}

h same

2 a 1 and 9, 2 and 8, 3 and 7, 4 and 6, 5 and 5; 5 pairs

b There are infinitely many pairs, as long as they add up to 10.

3 a 3^{13}

b 2^{12}

c 10^6

d 5^6

4 a 6^9

b 6^9

c 6^{22}

d 6^9

5 a 3^5

b 3^{21}

c 3^5

d 3^5

6 **a** 0
b 2^9
c 2^6
d 3^{-5}

7 i, a, g and h, b, c and d, e, f, j

8 Marged is correct. Each value is larger than the previous one, but her estimated values are too low.

Problem solving (page 84)

1 **a** Various combinations, check students' answers.
b 11
c no

Reviewing skills (page 84)

1 **a** 5^1
b 5^1
c 5^1

2 **a** 10^{-2}
b 10^{-1}
c 10^{-2}

3 **a** 2^2
b 7^1
c 10^0

4 $23^2 \times 23^{-2} = 1 = 23^0$

Number Strand 7 • Unit 7 • Answers

Practising skills (page 87)

1 Missing number is $7^{\frac{1}{3}}$

$3^{\frac{1}{2}}$	$\sqrt{3}$
$3^{\frac{1}{7}}$	$\sqrt[7]{3}$
$7^{\frac{1}{2}}$	$\sqrt[2]{7}$
$2^{\frac{1}{3}}$	$\sqrt[3]{2}$
$7^{\frac{1}{3}}$	$\sqrt[3]{7}$
$2^{\frac{1}{7}}$	$\sqrt[7]{2}$

2 **a** $\sqrt{8}$
b $\sqrt{11}$
c $\sqrt{107}$
d $\sqrt[9]{3}$
e $\sqrt[12]{113}$
f $\sqrt[18]{18}$

3 **a** $19^{\frac{1}{2}}$
b $29^{\frac{1}{2}}$
c $15^{\frac{1}{3}}$
d $3^{\frac{1}{3}}$
e $3^{\frac{1}{8}}$
f $7^{\frac{1}{13}}$

4 **a** 8, 3 and 2
b 2, 2 and 196
c 16, 4, 16

5 **a** 8
b 2
c 10
d 5
e 11
f 15

Developing fluency (page 88)

1 Missing number is $7^{\frac{2}{3}}$.

$3^{\frac{7}{2}}$	$\sqrt{3^7}$
$3^{\frac{2}{7}}$	$\sqrt[7]{3^2}$
$7^{\frac{3}{2}}$	$\sqrt[2]{7^3}$
$2^{\frac{7}{3}}$	$\sqrt[3]{2^7}$
$7^{\frac{2}{3}}$	$\sqrt[3]{7^2}$
$2^{\frac{3}{7}}$	$\sqrt[7]{2^3}$

2 **a** $\sqrt[2]{4^3} = \sqrt[2]{64} = 8$

b $\sqrt[3]{8^2} = \sqrt[3]{64} = 4$

c $\sqrt[4]{625} = 5$

d $\sqrt[2]{36^3} = \sqrt[2]{46656} = 216$

e $\sqrt[9]{1^7} = \sqrt[9]{1} = 1$

f $\sqrt[5]{32^2} = \sqrt[5]{1024} = 4$

3 **a** $75^{\frac{1}{2}}$

b $11^{\frac{3}{2}}$

c $15^{\frac{4}{3}}$

d $3^{\frac{8}{3}}$

e $3^{\frac{3}{8}}$

f $7^{\frac{14}{13}}$

4 **a** $8^{\frac{2}{3}} = \sqrt[3]{8^2} = 4$

b $196^{\frac{3}{6}} = \sqrt[6]{196^3} = 14$

c $4^{\frac{3}{2}} = \sqrt[2]{4^3} = 2^3 = 8$

5 **a** $4^{\frac{5}{3}}$

b $4^{\frac{3}{5}}$

6 **a** $4^{\frac{1}{2}}$ or $4^{\frac{3}{6}}$

b $9^{\frac{1}{2}}$ or $9^{\frac{2}{4}}$ or $9^{\frac{3}{6}}$ or $9^{\frac{4}{8}}$

c $2^{\frac{6}{3}}$ or $2^{\frac{8}{4}}$

Problem solving (pages 89–90)

1 **c, a, e, d, b**

2 **a** $b = b^{\frac{b}{b}} = b$

b $a = 2, b = 4$ or $a = 4, b = 2$

3 **a** side $= \sqrt[3]{V} = V^{\frac{1}{3}}$

cube $SA = 6$ face $SA = 6 \times \text{side}^2 = 6 \times V^{\frac{2}{3}}$

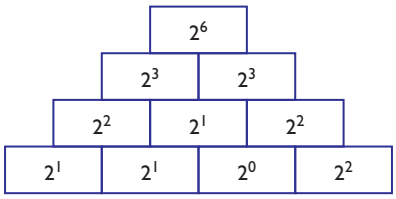
b $V = \sqrt{\left(\frac{A}{6}\right)^3}$

4 **a** 1 year

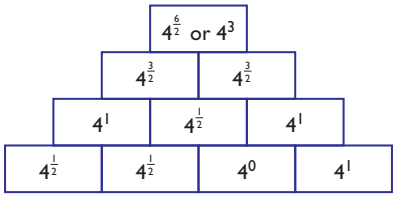
b 11.8 years

c 109 million kilometres

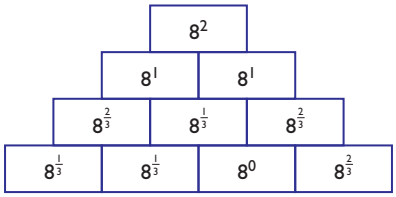
5 powers of 2



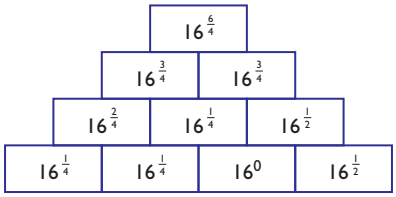
powers of 4



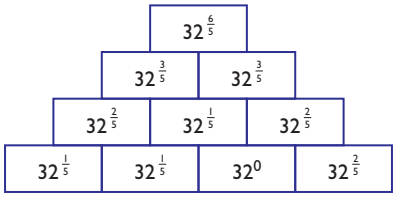
powers of 8



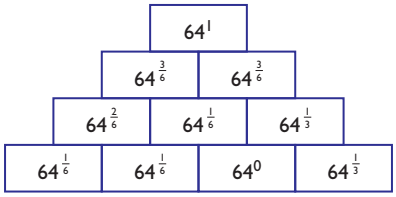
powers of 16



powers of 32



powers of 64



Reviewing skills (page 90)

1

$5^{\frac{1}{2}}$	$3^{\frac{5}{2}}$	$5^{\frac{2}{3}}$	$3^{\frac{2}{5}}$	$3^{\frac{1}{2}}$	$2^{\frac{2}{3}}$	$2^{\frac{5}{3}}$	$5^{\frac{3}{2}}$	$2^{\frac{1}{2}}$
$\sqrt{5}$	$\sqrt{3^5}$	$\sqrt[3]{5^2}$	$\sqrt[5]{3^2}$	$\sqrt{3}$	$\sqrt[3]{2^2}$	$\sqrt[3]{2^5}$	$\sqrt{5^3}$	$\sqrt{2}$

2 **a** 4

b 27

c 4

3 **a** $8^{\frac{1}{3}} = 2$

b $25^{\frac{3}{2}} = \left(\sqrt{25}\right)^3 = 125$

c $32^{\frac{3}{5}} = 8$

Number Strand 7 • Unit 8 • Answers

Practising skills (page 92)

1 a i Note that **i** is not required as it has already been worked out in question.

ii $3^2 \times 5$

iii $5^2 \times 5$

iv $4^2 \times 3$

v $2^2 \times 14$

vi $5^2 \times 3$

b i $2\sqrt{10}$

ii $3\sqrt{5}$

iii $5\sqrt{5}$

iv $4\sqrt{3}$

v $2\sqrt{14}$

vi $5\sqrt{3}$

2 a $3\sqrt{2}$

b $6\sqrt{5}$

c $30\sqrt{2}$

d 10

e $10\sqrt{10}$

f $60\sqrt{5}$

3 a $\frac{5\sqrt{3}}{3}$

b $\frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$

c $\frac{\sqrt{2}}{2}$

d $\frac{8\sqrt{3}}{3}$

e $\frac{12\sqrt{10}}{10} = \frac{6\sqrt{10}}{5}$

f $\frac{5\sqrt{13}}{13}$

4 a $\frac{3}{\sqrt{40}} = \frac{3\sqrt{10}}{20}$

b $\frac{2}{\sqrt{40}} = \frac{\sqrt{10}}{10}$

c $\frac{5}{\sqrt{40}} = \frac{\sqrt{10}}{4}$

d $\frac{5}{\sqrt{400}} = \frac{1}{4}$

e $\frac{\sqrt{5}}{\sqrt{40}} = \frac{\sqrt{2}}{4}$

f $\frac{\sqrt{5}}{\sqrt{4000}} = \frac{\sqrt{5}}{\sqrt{400 \times 10}} = \frac{\sqrt{2}}{40}$

5 **a** $\sqrt{5}$

b $\sqrt{5}$

c $10\sqrt{5}$

d $\frac{4}{5}$

e $\frac{2\sqrt{3}}{5}$

f $\frac{4}{5}$

Developing fluency (pages 93–94)

1 **a** 22

b $28+10\sqrt{3}$

c $8+2\sqrt{15}$

d same question as part **c** $8+2\sqrt{15}$

e 1

f $81+4\sqrt{410}$

2 **a** $5\sqrt{10}$

b $20\sqrt{2}$

c $20\sqrt{5}$

3 **a** $8\sqrt{5}$ cm

b $20\,\text{cm}^2$

4 **a** $x\times y\neq\sqrt{x}\times\sqrt{y}$. Sometimes true, only true when the numbers are 1.

b $x+y\neq\sqrt{x}+\sqrt{y}$. Sometimes true, only true when the numbers are 1.

c $x-y\neq\sqrt{x}-\sqrt{y}$. Sometimes true, only true when the numbers are 1.

d $x\div y\neq\sqrt{x}\div\sqrt{y}$. Sometimes true, only true when the numbers are 1.

5 Always true. Break the $\sqrt{x^3}$ into a multiple as $\sqrt{x\times x^2}$ which is $x\sqrt{x}$.

6 $2\sqrt{3}$, $\sqrt{15}-\sqrt{3}$ and $\sqrt{30-6\sqrt{5}}$

7 **a** $\sqrt{2\frac{2}{3}}=\sqrt{\frac{8}{3}}=\sqrt{4\times\frac{2}{3}}=2\sqrt{\frac{2}{3}}$

b $\sqrt{3\frac{3}{8}}=\sqrt{\frac{27}{8}}=\sqrt{9\times\frac{3}{8}}=3\sqrt{\frac{3}{8}}$

c $\sqrt{a\frac{a}{a^2-1}}=\sqrt{\frac{a(a^2-1)+a}{a^2-1}}=\sqrt{\frac{a^3}{a^2-1}}=\sqrt{a^2\times\frac{a}{a^2-1}}=a\sqrt{\frac{a}{a^2-1}}$

Problem solving (pages 94–95)

1 a $4\,\text{cm}^2$

b $h^2 = \left(\sqrt{5}+1\right)^2 + \left(\sqrt{5}-1\right)^2 = 12$

$h = \sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$ cm

2 a $4\sqrt{5} + 20$ cm

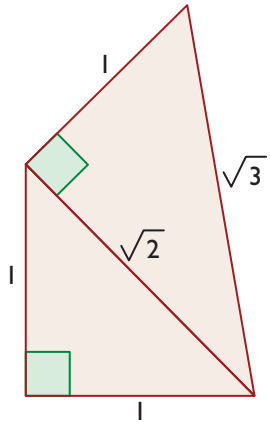
b $8\,\text{cm}^2$

3 a 1

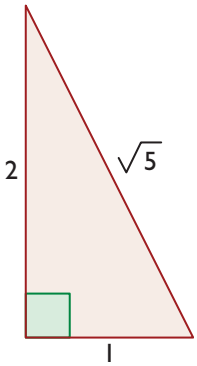
b $\sqrt{5} - 1 = \sqrt{4+1} - 1$, which is >1

c It is smaller than 1.

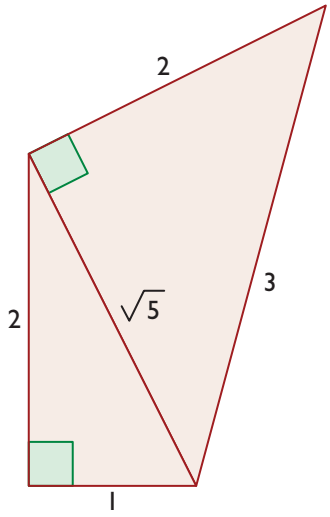
4 a In this diagram $\left(\sqrt{2}\right)^2 + 1^2 = 2 + 1 = 3$, so the hypotenuse is $\sqrt{3}$.



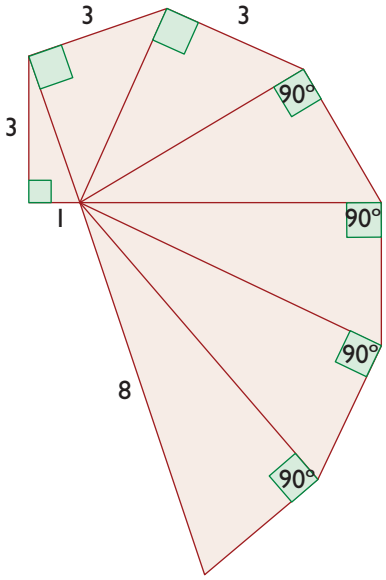
b In the first diagram $1^2 + 2^2 = 1 + 4 = 5$, so the first hypotenuse is $\sqrt{5}$.



In the second diagram $\sqrt{5}^2 + 2^2 = 5 + 4 = 9$, so the hypotenuse is $\sqrt{9} = 3$, which is not a surd.



c In the third scenario, after seven triangles you get to an integer.



Reviewing skills (page 96)

1 a $3\sqrt{3}$

b $\sqrt{82}$

c $2\sqrt{3}$

2 a $\frac{7\sqrt{3}}{3}$

b $\frac{3\sqrt{35}}{7}$

c $\frac{9\sqrt{14}}{7}$

3 a 59

b $69+16\sqrt{5}$

c $-1-\sqrt{2}$

Practising skills (page 100)

1 **a** 5^3
b 2^5
c 9^{14}
d 7^{10}

2 **a** f^2
b g^4
c d^6
d a^{24}

3 **i** $x \times x \times x \times x \times x = x^5$
ii $x \times x \times x \times x \times x = x^5$
iii $\frac{x \times x \times x}{x \times x} = x$
iv $\frac{x \times x}{x \times x \times x} = x^{-1}$
v $x \times x \times x \times x \times x \times x = x^6$
vi $x \times x \times x \times x \times x \times x = x^6$

4 **a** $20a^5$
b $2b$
c $6c^{14}$
d $30d^8$
e $2e$
f $8f^5g^8$
g $24m^4p^5$
h $2s^3$

5 **a** $3a^2$
b $4c^6$
c $5d^5$
d $3f^5$

6 a i

×	x	4
x	x^2	$4x$
2	$2x$	8

ii $x^2 + 6x + 8$

b i $= x^2 + 8x + 15$

×	x	5
x	x^2	$5x$
3	$3x$	15

ii $= x^2 + 2x - 24$

×	x	−4
x	x^2	$-4x$
6	$6x$	−24

iii $x^2 - 12x + 35$

×	x	−7
x	x^2	$-7x$
−5	$-5x$	35

Developing fluency (pages 101–102)

1 a middle row x^8 x^7; top x^{15}

b bottom row a^5 a^4; middle row a^9

2 $n^{12} \div n^4 = n^8$; $n^6 \div n^3 = n^3$; $n^4 \times n^2 = n^6$; $n^3 \div n = n^2$; $n^3 \times n^2 = n^5$; $(n^5)^2 = n^{10}$

3 a $x^2 + 11x + 18$

b $x^2 + 2x - 15$

c $x^2 - 3x - 4$

d $x^2 - 7x + 10$

4 a $x^2 + 14x + 48$

b $x^2 + x - 12$

5 a Tim is correct, Harry has not multiplied each term in one bracket by each term in the other bracket.

b i $a^2 + 6a + 9$

ii $b^2 + 12b + 36$

iii $c^2 - 8c + 16$

6 a $x^2 + 15x + 50$

b top row x^2; $10x$; bottom row $5x$; 50

c They are equal – the small rectangles represent the stages of expanding the $(x + 5)(x + 10)$ bracket.

7 a i $4x^2 + 8x$

ii $2x^2 + 4x$

b the black area is $2x$ times $(x + 2) = 2x^2 + 4x$ as well.

Problem solving (pages 102–103)

1 $w(w + 20) = w^2 + 20w$

2 $a^2 + 8a$

3 $\frac{3}{2}b^2 + 12b$

4 $-4s \times s^2 = 60s^3$

5 **a** $n = 8$

b $n = 6$

c $n = 3$

6 **a** $2x^2 + 8x + 8$

b red is $0.5(2x + 4)(2x + 4) = 2x^2 + 8x + 8$

Reviewing skills (page 103)

1 **a** a^{14}

b b^6

c $c^0 = 1$

d d^{18}

2 **a** $12a^7$

b $4b^5$

c 1

d $5g^8$

3 **a** $a^2 + 11a + 30$

b $b^2 + 4b - 21$

c $c^2 - 4c - 5$

d $d^2 - 8d + 15$

4 **a** $x^2 + 8x - 48$

b Area of red rectangle outside blue square: $12x - 48$.

Area of red rectangle inside blue square: $x^2 - 4x$

Area of blue square outside red rectangle: $4x$

c Total area: $x^2 + 12x - 48$

Practising skills (page 107)

1 **a** 12
b 45
c 40
d 3

2 **a** 20
b 67
c -17
d -1.8

3 **a** 24
b 45
c 24
d 9

4 **a** 90
b 200
c -100
d 200

5 **a** $x = y + 8$
b $x = \frac{y}{3}$
c $x = 5y$
d $x = \frac{(y-1)}{2}$

6 **a** $x = y - 4$
b $x = \frac{(y+3)}{4}$
c $b = \frac{a}{6}$
d $t = \frac{p}{m}$

7 **a** 24
b 106
c 49
d 54

8 **a** $\frac{1}{3}$
b $9\frac{1}{2}$
c $4\frac{1}{2}$
d $9\frac{1}{3}$

9 **a** $-1\frac{1}{2}$
b 6
c $-4\frac{3}{4}$
d $3\frac{1}{5}$

Developing fluency (pages 107–108)

1 a i 27

ii 32

iii 26

b $\frac{points - d}{3} = w$

c 5

2 a i 13

ii 14

iii 21

b $\frac{c - 2p}{1.5} = m$

c 3 miles

3 a $C = 35h + 18$

b i 88

ii 228

iii 368

c $h = \frac{(c - 18)}{35}$

d i 4

ii 12

iii 27

4 a i $452.389\,\text{cm}^2$

ii $1809.557\,\text{m}^2$

iii $2\,010\,619.3\,\text{km}^2$

b $514\,718\,540.4\,\text{km}^2$

c $r = \sqrt{\frac{s}{4\pi}}$

d 2.00 cm

5 a i 0

ii 35

iii 100

iv −40

b $F = \frac{9c}{5} + 32$

c i 32

ii 95

iii 572

iv −40

6 a 512

b i $M = \frac{hp}{4t^2}$

ii $t = \sqrt{\frac{hp}{4M}}$

iii $h = \frac{4Mt^2}{P}$

c i 50

ii 20

iii 7

7 a $x = \frac{(y + 6)}{5}$ **b** $x = \left(\frac{y}{5}\right) + 6$ **c** $p = \frac{T}{4m}$ **d** $p = \frac{(T - m^2)}{4r}$

Problem solving (pages 109–110)

1 **a** £165

b $h = \frac{c - 15}{30}$

c 1 hour 15 minutes

2 **a** £104

b $d = \frac{c - 20}{12}$

c 15 days

3 **a** £49.50

b $c = \frac{100(B - 15) - 5t}{10}$

c 80 calls

4 **a** $m = dv$

b 252 g/cm^3

5 **a** $h = \frac{v}{\pi r^2}$

b 10.4 cm

6 **a** 32 cm^2

b $h = \frac{2A}{(a + b)}$

c 5 cm

d $a = \left(\frac{2A}{h}\right) - b$

e 4.0 cm

7 **a** 10π cm^2

b $l = \frac{4A}{\pi h}$

c 10 cm

Reviewing skills (page 110)

1 **a** $e = d - f$

b $t = \frac{(s - \mathrm{b})}{\mathrm{a}}$

c $g = fh$

d $w = \frac{(v + 3)}{4}$

e $r = \frac{\mathrm{C}}{2\pi}$

f $r = \sqrt{\frac{A}{\pi}}$

g $r = \frac{100P}{Mt}$

h $t = \sqrt{\frac{S}{3}}$

2 **a** **i** 10

ii 0

iii –20

iv 30

b $a = \frac{(v - u)}{t}$ **c** $a = \frac{15}{3} = 3$

3 **d** $t = S^2\sqrt{3}$

Algebra Strand 1 • Unit 11 • Answers

Practising skills (page 113)

1 **a** a term
b coefficient
c an expression
d a term and an expression
e an equation
f a formula
g an identity

2 $2n$, when you multiply any (whole) number by 2 you get an even number.

3 $0 = x^2 - 6x + 5$ is the equation because it is only true when $x = 1$ or 5; $(x - 5)(x - 1) = x^2 - 6x + 5$ is the identity because it is always true. ($H = x^2 - 6x + 5$ is the formula)

4 **a** $x + 5 + 3x - 6 = x + 3x + 5 - 6 = 4x - 1$
b $x(x + 1) - x - 1 = x^2 + x - x - 1 = x^2 - 1$
c $4(x - 3) + 3(x + 7) = 4x - 12 + 3x + 21 = 7x + 9$.

5 **a** $2n$, is an even number
b n^2 is a square number
c $7n$ is a multiple of 7
d $2n - 1$ is an odd number
e $4n - n$ is a multiple of 3

6 1, 9, 15, 99

Developing fluency (pages 114–115)

1 **a** $n + 1$ is one more than n so will only be odd if n is even
b $2n - 1$ is one less than a multiple of 2 so always odd
c $3n + 5$ is 5 more than a multiple of 3 so sometimes odd
d $2n + 1$ is one more than a multiple of 2 so always odd
e $6n$ is a multiple of 6 so always even

2 **a** Multiples of 5
b In the sequence 9, 15, 21, ...

3 **a** **i** Sometimes true
ii Always true, addition is commutative
iii Never true – 5 cannot equal 9
iv Always true, halving is equivalent to dividing by 2
v Sometimes true, when $x = 0$
vi Always true, dividing by a half is equivalent to multiplying by 2
b The ones which are always true as they are the same regardless of the value of x.

4 **a** Both equal $3x - 16$
b $(x + 1)(x - 1) = x^2 + x - x - 1 = x^2 - 1$
c Both equal $-6x^2 + 13x + 5$

5 **a** $n + n + 1 = 2n + 1$ which is always odd

b $n + n + 1 + n + 2 + n + 3 + n + 4 + = 5n + 10 = 5(n + 2)$ which is always a multiple of 5

c $n + n + 1 + n + 2 + n + 3 = 4n + 6 = 2(n + 3)$ which is always a multiple of 2

6 n, $2n$, $2n + 5$ and 8, 16 and 21

7 **a** **i** BC: $y - a$; CL: $x - b$

ii $a + b + y - a + x - b = x + y$

b **i** QR: $x - p$; RL: $y - q$

ii $p + q + x - p + y - q = x + y$

c Answers from part **ii** of **a** and **b** both equal $x + y$.

d It is the same length as Hank and Dwight's journeys.

Problem solving (pages 115–116)

1 **a** **i** 2, 4, 3 and 5, 4 : $3^2 + 4^2 = 5^2$

ii 6, 36, 35 and 37, 12 : $12^2 + 35^2 = 37^2$

iii 10, 100, 99 and 101, 20 : $20^2 + 99^2 = 101^2$

iv 1000, 1000 000, 999 999 and 1000 001, 2000 : $2000^2 + 999\,999^2 = 1000\,001^2$

b $2n$, $4n^2$, $4n^2 - 1$ and $4n^2 + 1$, $4n$: $(4n)^2 + (4n^2 - 1)^2 = (4n^2 + 1)^2$

expand

$16n^2 + (4n^2 - 1)(4^2 - 1) = (4n^2 + 1)(4n^2 + 1)$

$16n^2 + 16n^4 - 8n^2 + 1 = 16n^4 + 8n^2 + 1$

are equal

2 **a**

	$3x + 15$	**$4x - 2$**	**F**	**$3(x + 5)$**	**$7x - 5$**	**$3(x + 2) + 9$**
$3x + 15$	*identity*	*equation*	*formula*	*identity*	Equation $x = 5$	*identity*
$4x - 2$	*equation*	*identity*	*formula*	*equation*	*equation*	*equation*
F	*formula*	*formula*	*identity*	*formula*	*formula*	*formula*
$3(x + 5)$	*identity*	*equation*	*formula*	*identity*	*equation*	*identity*
$7x - 5$	*equation*	*equation*	*formula*	*equation*	*identity*	*equation*
$3(x + 2) + 9$	*identity*	*equation*	*formula*	*identity*	*equation*	*identity*

b The table is symmetrical.

c An expression never involves an equals sign so cannot be in this form of table.

Reviewing skills (page 116)

1 **a** and **b**

i equation

$x = 4$

ii identity

$2x + 14 + 3x + 6 = 5x + 20$

iii identity

$x^2 - 8x - 1 = (x - 2)(x - 6) - 13 = x^2 - 8x + 12 - 13$

iv identity $x^2 - 2x - x^2 + 1 = 1 - 2x = x^2 - 2x + 1 - x^2$

2 **a** False, negative values of n will make left side less than right side.

b True. Definition of identity.

c True, the last digit squared for any number is never 2, therefore no square number is ever ending in 2.

d True. It's an identity.

Practising skills (page 119)

1 **a** a
b a^5
c a^{-6}
d a
e $a^{(-5)}$
f a^{-6}

2 **a** $p^{2\frac{1}{2}}$
b $p^{2\frac{1}{2}}$
c $p^{2\frac{1}{2}}$
d $p^{2\frac{1}{2}}$
e $p^{2\frac{1}{2}}$

3 **a** cd^{-1}
b c^3d^5
c cd

4 **a** $\frac{1}{4}$
b $\frac{1}{10}$
c 2
d 20
e 1

5 **a** $\frac{1}{y}$ or y^{-1}
b $\frac{1}{y^2}$ or y^{-2}
c y
d $\frac{1}{y^{\frac{1}{2}}}$ or $y^{-\frac{1}{2}}$
e $y^{\frac{2}{3}}$

6 a^2b

7 ab^2c^4

8 **a** $30u^5w^3$
b $\frac{8u^{3.5}}{w^4}$
c $\frac{a}{2}$
d $\frac{ac^2}{3b}$

Developing fluency (pages 119–120)

1 **a** a^2

b b^{-1}

c c^5

2 **a** $\frac{1}{3}$

b $-\frac{1}{2}$

c -1

3 **a** Always true, squares are always positive numbers, therefore $-p$ cannot itself be a square and cannot have a square root.

b Never true. Cubes can be negative numbers if the starting number is negative.

4 $p^{\frac{1}{3}}$; $p \div p^{\frac{2}{3}}$

5 **a** false

b true

c true

d true

e false

f true

6 **a** $\frac{1}{-1}$

b $\frac{1}{a^2c}$ or $a^{-2}c^{-1}$

c $\frac{y}{x}$

d $p^{-1}q$

e a^{-1} or $\frac{1}{a}$

7 **a** $5a^{\frac{5}{3}}b^{-\frac{1}{2}}$

b $\frac{3}{2}a^{\frac{7}{30}}b^{-4}$

c $4p^{-3}$

d There is an error in the Student book.

8 $6a^3 \times (3bc)^{-1} \times a^{-2}$ and $\frac{c^3 \times 6\sqrt{a^4b^2}}{3c \times ab^2 \times c^3}$

Problem solving (pages 120–121)

1 **a** $\frac{5}{3}$

b $-\frac{11}{2}$

c -4

2 **a** $a^2 \times a^{-2} = 1$

b $\sqrt[3]{a} \times a^6 = a^2$

c $\left(\sqrt{a}\right)^2 = a$

d $\sqrt[3]{a} \div \sqrt{a} = a^{-\frac{1}{6}}$

3 **a** xy

b $8y$

c $\frac{x}{y^2}$

4 **a** $a-1$

b $b^{-\frac{1}{2}}+b^{\frac{3}{2}}$

c $c^{-\frac{1}{2}}+c^{\frac{3}{2}}+c^{\frac{5}{2}}$

Reviewing skills (page 121)

1 **a** $24uwy$

b $4u^2w^4y^6$

c $\frac{c}{4}$

d $a^{-2}b^{-3}c^{-3}$

2 **a** a^2bc^4

b $a^5b^4c^3$

c $a^{\frac{2}{3}}bc^2$

3 **a** $\frac{3a^3}{b^{\frac{3}{2}}}$

b $\frac{2b^{\frac{7}{6}}}{a^{\frac{7}{2}}}$

4 There is an error in the student book. x^2 should be on the bottom. Then, the answer is **b**.

Practising skills (page 125)

1 **a** $x^2 + 10x + 21$
b $x^2 - 10x + 21$
c $x^2 + 4x - 21$
d $x^2 - 4x - 21$
e $x^2 - 9$
f $x^2 - 49$

2 **a** $6x^2 + 23x + 20$
b $6x^2 - 7x - 20$
c $6x^2 - 23x + 20$
d $6x^2 + 7x - 20$
e $9x^2 - 16$
f $4x^2 - 25$

3 **a** $x^2 + 3x + 2$
b $-x^2 + 3x - 2$
c $2x^2 - 7x + 3$
d $3x^2 + 5x - 2$

4 **a** $x^2 + y^2 + 2xy$
b $x^2 - y^2$
c $x^2 + y^2 - 2xy$

5 **a** $16x^2 - 2$
b $16x^2 + 8\sqrt{2}x + 2$
c $2x^2 - 8\sqrt{2}x + 16$
d $2x^2 - 16$

6 **a** yes
b no
c yes
d no

7 $\frac{x^2y}{x^3y}, \frac{2}{2x}$

8 **a** $\frac{1}{a}$
b $\frac{3a}{2}$
c $\frac{4 + c}{c^2}$
d $\sqrt{3}d$

9 a $\frac{4x-3y}{12}$

b $\frac{x+y}{xy}$

c $\frac{2y-3x}{xy}$

d $\frac{x^2+y}{xy}$

10 a $x = 12$

b $x = 6$

Developing fluency (pages 126–127)

1 a $8x^2 + 20x + 8$

b $8x^2 - 20x + 8$

c $8x^2 - 12x - 8$

d $8x^2 + 12x - 8$

2 $4x - 1$

3 $\left(1+\frac{20}{100}\right)\left(1-\frac{20}{100}\right)=1-\left(\frac{20}{100}\right)^2 < 1$ so it is not equivalent to no change.

4 a no

b yes

c yes

d no

5 a Only true when x or $y = 0$.

b Always. They are equivalent expressions.

c Sometimes. They are not equivalent expressions but $x = 1$ is a solution to the equation.

d Always. They are equivalent expressions.

6 a $\frac{w-1}{w}$

b $x + 1$

c y

d $\frac{z^2-1}{z+2}$

7 a $\frac{w^2+5w-3}{(w+2)(w-1)}$

b $\frac{4x^2+8x}{(x-3)(x+1)}$

c $\frac{x^2+y^2+3y-x}{(x-1)(y+3)}$

d $\frac{2(y+2z)}{(y-4)(z+2)}$

8 a $x = \frac{1}{3}$

b $x = -\frac{3}{4}$

c $x = -\frac{5}{3}$

d $x = \mp\sqrt{3}$

9 **a** **D**
b **C**
c **E**
d **A**
e **B**

10 $2x^3 + 3x^2 + 5x + 2$

11 $625x^4 - 81$; it is the difference of two squares each time and so terms cancel out.

12 **a** $3x$, $2x$, x
b $3x$, $2x$, 6 and 4, $2x$, x and $3x$, -3, x
c $3x$, -3, 6 and 4, $2x$, 6 and 4, -3, x
d 4, -3, 6

Problem solving (pages 127–128)

1 **a** $(x + 6)(2x - 1) = 2x(x + 4)$
b $x = 2$; 3 cm by 8 cm and 4 cm by 6 cm

2 $(n + 1)^2 - n^2 \equiv 2n + 1$

3 **a** **i** $\frac{x+2}{4}$
ii $\frac{4}{x+3}$
b $x = -\frac{2}{3}$

4 **a** For example, use 16, the $P = 17$, $Q = 15$ and the third side is 8.
b $(n^2 + 1)^2 \equiv (n^2 - 1)^2 + 4n^2$

Reviewing skills (page 128)

1 **a** $2y^2 - y - 1$
b $y^2 - 4$
c $8y$

2 **a** $\frac{5m+13}{(m+2)(m+3)}$
b $\frac{9+4(m+2)}{3(m+2)}$
c $\frac{x^2(y^2 - y)}{3}$

3 **a** $x = \frac{2}{7}$
b $x = -4$

Practising skills (pages 130–131)

1 a $x = \frac{ac}{y}$

b $x = \frac{a}{cy}$

c $x = acy$

d $x = \frac{ay}{c}$

2 a $x = b(a - c)$

b $x = \sqrt{y} - a$

c $x = \sqrt{qy - 2p}$

d $x = y - \frac{p}{q}$

3
$$5x - 2y = 8xy - 3$$
$$5x - 8xy = 2y - 3$$
$$x(5 - 8y) = 2y - 3$$
$$x = \frac{2y - 3}{5 - 8y}$$

4 a $\frac{b - ab}{1 - a}$

b $\frac{a - 5y}{2 - w}$

c $\frac{y + py}{3 - p}$

5 a i $a = \frac{2A}{h} - b$

ii $b = \frac{2A}{h} - a$

iii $h = \frac{2A}{(a + b)}$

b They are very similar – the term $\frac{2A}{h}$ is the same.

6 a $p = \frac{1}{2}(cy + q)$

b $p = \frac{q - cy}{3y}$

c $p = \frac{q}{y + x}$

7 a $x = \frac{4y + 3}{3y + 2}$

b $x = \frac{3y - 1}{y + 5}$

c $x = \frac{7 - 7y}{2y + 3}$

Developing fluency (pages 131–132)

1 a $t = \sqrt{\frac{2s}{a}}$

b $a = \sqrt{c^2 - b^2}$

c $h = \frac{2A}{b}$

2 a $r = \sqrt[3]{\frac{3V}{4\pi}}$

b 3 cm

3 a i $u = \sqrt{v^2 - 2as}$

ii $a = \frac{v^2 - u^2}{2s}$

iii $s = \frac{v^2 - u^2}{2a}$

b i $u = 10$

ii $a = -10$

iii $s = 3.2$

4 a i $x = \frac{t+2}{2-t}$

ii $x = -5$

b i $t = \frac{2x-2}{x+1}$

ii $t = 3$

5 a $a = \frac{2(s - ut)}{t^2}$

b $a = -1.6$

6 a $f = \frac{uv}{u + v}$

b $f = 1.2$

7 a i $x \rightarrow$ add $a \rightarrow$ square $\rightarrow$ reciprocal $\rightarrow$ multiply by $b \rightarrow y$

ii $y \rightarrow$ divide by $b \rightarrow$ reciprocal $\rightarrow$ square root $\rightarrow$ subtract $a \rightarrow x$

b i $x \rightarrow$ add $b \rightarrow$ square root $\rightarrow$ reciprocal $\rightarrow$ multiply by $a \rightarrow y$

ii $y \rightarrow$ divide by $a \rightarrow$ reciprocal $\rightarrow$ square $\rightarrow$ subtract $b \rightarrow x$

8 a $p = x^2 - \frac{y}{q}$

b $p = x - \frac{a^2}{b^2}$

c $p = \sqrt{\frac{xb - y}{x}}$

d $p = \left(\frac{a}{b} - x\right)^2$

9 a $x = \frac{w^2y + y}{1 - w^2}$

b $y = \frac{x - w^2x}{1 + w^2}$

c $p = \frac{q^2}{q - 6}$

Problem solving (pages 132–133)

1 $l = 24.8\,\text{cm}$

2 **a** $4\pi r^2 k = 2\pi r(r+h)$

$2kr = r + h$

$2kr - r = h;\ h = (2k-1)r$

b $4\pi r^2 h = \frac{8}{3}\pi r^3$

$\frac{3}{2}h = r$

so: $\frac{3}{2}h(2k-1) = h$

$2k - 1 = \frac{2}{3}$

$k = \frac{5}{6}$

3 **a** $n = \frac{5a + 8b}{2b - a}$

b Solve equation, $10 = \frac{5a + 8b}{2b - a}$ to give $15a = 12b$

4 **a** $h = \frac{3V}{\pi(R^2 + r^2 + Rr)}$

b 17.05 cm

5 **a** $a = \frac{bx}{\sqrt{b^2 - y^2}}$

b $b^2 - y^2 > 0$ or $y^2 < b^2$

6 **a** $c = -(ax^2 + bx)$

b −4.125

Reviewing skills (page 134)

1 **a** $\frac{1 + 5t}{1 - 2t}$

b $x = \frac{5y - 2}{3y + 1}$

c $x = \frac{2}{y + 5} - 3$

2 **a** $y = \frac{\sqrt{a - x}}{c}$

b $a = c^2y^2 + x$

c $x = a - c^2y^2$

3 **a** $L = ma\left(\frac{\pi}{2T}\right)^2$

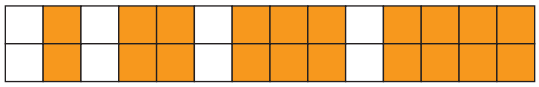
b $g = \left(\frac{R^2 + h^2}{h}\right)\left(\frac{2\pi}{T}\right)^2$

4 The answer is **c**

Practising skills (page 138)

1 a i even numbers

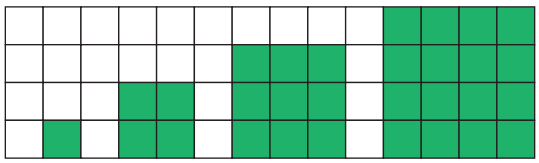
ii



iii $2n$

b i square numbers

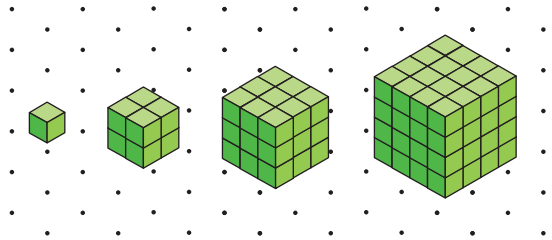
ii



iii n^2

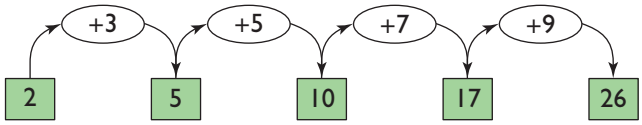
c i cube numbers

ii

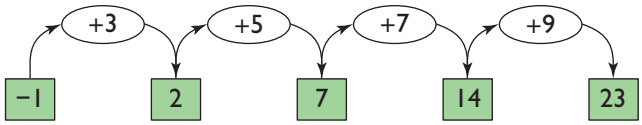


iii n^3

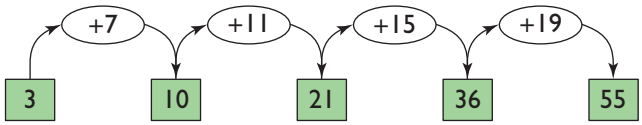
2 a



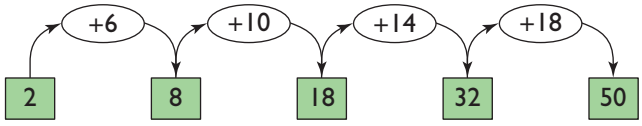
b



c



d



3 a 11, 14, 19, 26, 35

b 0, 7, 26, 63, 124

c 1, 3, 6, 10, 15 [triangular numbers]

4 **a** n^2

b $n^2 + 1$

c $n^2 - 1$

d $2n^2$

5 **a** $n^2 + 10$

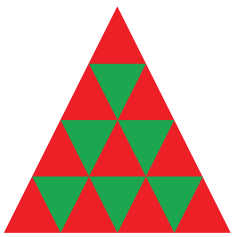
b n^3

c $n^3 + 5$

d $2n^3$

Developing fluency (pages 139–141)

1 a



b

Pattern number	1	2	3	4	5
Number of red triangles	1	3	6	10	15
Number of green triangles	0	1	3	6	10
Total number of triangles, T	1	4	9	16	25

c Number of red triangles and number of green triangles are triangular numbers
Total number of triangles are square numbers

d 100

i 55

ii 45

e $T = n^2$

2 a i Double the previous term

ii the difference between one term and the next is 1 more than the difference between the previous 2 terms.

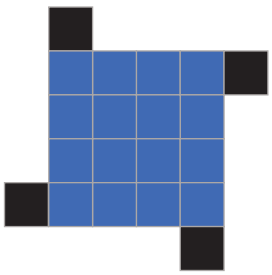
b i 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16 384, 32 768, 65 536

ii 1, 2, 4, 7, 11, 16, 22, 29, 37, 46, 56, 67, 79, 92, 106, 121

c 2, 4 and 16

d 4, 16, 64, 256, 1024, 4096, 16 384, 65 536

3 a



b

Pattern number	1	2	3	4	5
Number of black tiles	4	4	**4**	**4**	**4**
Number of blue tiles	1	4	**9**	**16**	**25**
Total number of tiles, T	5	8	**13**	**20**	**29**

c 104

d 15th pattern

e $T = n^2 + 4$

f i No, because 400 is a square number, and each number in the sequence is 4 more than a square number.

ii 19th pattern; 35 tiles left over

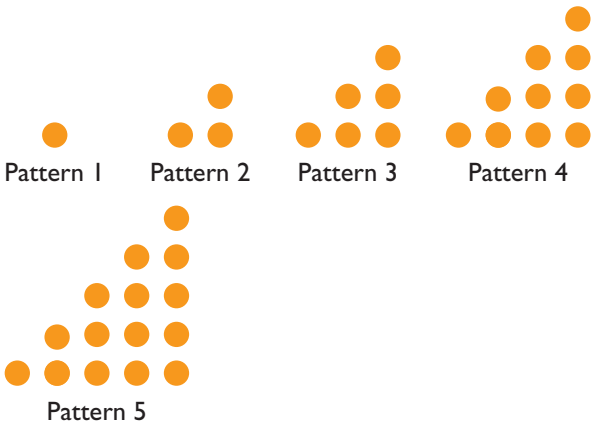
4 a i 3

ii 5

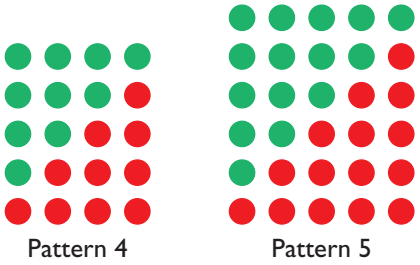
iii 7

b 9
c 100
d n^2
e Pattern 7
f 10 000

5 a 2, 6, 12, 20, 30
b i nth term $= \frac{1}{2}n(n+1)$
ii Triangular numbers
iii

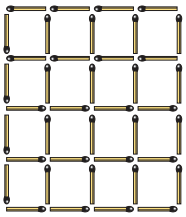


iv 820
c i



ii $n(n+1)$
iii Number of red circles $= \frac{1}{2}n(n+1)$
Number of green circles $= \frac{1}{2}n(n+1)$
iv The number of red and green circles are triangular numbers.

6 a



b

Pattern number	1	2	3	4	5
Number of matches, M	4	12	24	40	60

c 144
d i 1, 3, 6, 10, 15
ii Number of matches = 4 × triangular numbers
iii 840
iv $M = 2n(n+1)$

Problem solving (pages 141–142)

1 **a** 21, 34, 55, 89, 144

b 2, 3, 5, 13, 89

c $8 = 2^3$, $144 = 2^4 \times 3^2$

2 144

3 **a** **i** 3

ii 5

iii 8

b Fibonacci

c For example, there are two ways to make rectangles using 6 dominoes. We can use all of the ones we made using 5 dominoes and can put a single vertical domino on the front:

There will be 8 of these.

Alternatively, we can use all of the ones we made using 4 dominoes and can put two horizontal dominoes on the front:

There will be 5 of these.

To work out the next one, therefore, we need to add the previous two numbers:

5 + 8 = 13

It is only putting one vertical or two horizontal dominoes in front of the previous numbers as three dominoes horizontally is too high

Reviewing skills (page 142)

1 a 6, 9, 14, 21, 30

b 1, 7, 17, 31, 49

c 4, 11, 30, 67, 128

d 0, 2, 6, 12, 20

2 a $n^2 + 2$

b $n^2 - 3$

c $n^3 + 1$

d $3n^2$

3 a

Pattern number	1	2	3	4	5	n
Number of blue squares	1	4	9	16	25	n^2
Number of red squares	2	6	12	20	30	$n^2 + n$
Total number of squares	3	10	21	36	55	$2n^2 + n$

b n^2

c 110

d $n^2 + n$

e $500 = n^2 + n$; $n^2 + n - 500 = 0$. Cannot solve for n whole number so no pattern will have 500 squares

f $T = 2n^2 + n = n(2n + 1)$. A composite number

Practising skills (pages 145–146)

1 **a** 5, 20, 45, 80, 125

b 6, 9, 14, 21, 30

c 1, 7, 17, 31, 49

d 7, 16, 31, 52, 79

e 5, 35, 85, 155, 245

2 **a** = ii; **b** = iii; **c** = iv; **d** = i

3 **a** 16, n^2

b 17, $n^2 + 1$

c 18, $2n^2$

d 35, $2n^2 + 3$

4 **a**

Term	$n^2 + 3$	1st difference	2nd difference
1	4		
2	7	3	
3	12	5	2
4	19	7	2
5	28	9	2

b **i**

Term	$n^2 - 1$	1st difference	2nd difference
1	0		
2	3	3	
3	8	5	2
4	15	7	2
5	24	9	2

ii

Term	$n^2 + 2$	1st difference	2nd difference
1	3		
2	6	3	
3	11	5	2
4	18	7	2
5	27	9	2

iii

Term	$2n^2 - 3$	1st difference	2nd difference
1	−1		
2	5	6	
3	15	10	4
4	29	14	4
5	47	18	4

iv

Term	$5n^2$	1st difference	2nd difference
1	5		
2	20	15	
3	45	25	10
4	80	35	10
5	125	45	10

v

Terms	$3n^2 + 5$	1st difference	2nd difference
1	8		
2	17	9	
3	32	15	6
4	53	21	6
5	80	27	6

c no

5 **a** = iii; **b** = v; **c** = i; **d** = ii; **e** = iv

6 **a** and **b**

i $n^2 + 4$

ii no – linear

iii $3n^2$

iv no – cubic

v $2n^2 + 3$

Developing fluency (pages 146–147)

1 **a** A: 12, B: 36, C: 35, D: 48, E: 47

b A: $2n$, B: n^2

c **i** Each term in C is one less than the corresponding term in B.

ii $n^2 - 1$

d **i** Sequence D = sequence A + Sequence B

ii $n^2 + 2n$

e There is an error in the Student book. The first two values in the table should be 2 and 7.

i A and C

ii $n^2 + 2n - 1$

2 **a** Drawing of a 5 × 5 pattern.

b

Pattern	1	2	3	4	5
Number of tiles	4	9	16	25	36

c Pattern 9

d $(n + 1)^2$

e **i** 50

ii 2551

iii $(n + 1)^2 - n$ or $n^2 + n + 1$

3 **a** 64

b n^2

c 10

d Because the square numbers go odd-even-odd-even... and so on. The difference between an odd and an even number is always odd.

4 **a** 15
b 231
c 15
d 5050
e **i** No because the 17th stack needs 153 tins and the 18th stack needs 171 tins.
ii 78 and 91

5 **a** **i** 6
ii 12
iii 8
b 26

Cube size	2	3	4	5	10	n
1 sticker	0	6	24	54	384	$6(n-2)^2$
2 stickers	0	12	24	36	96	$12(n-2)$
3 stickers	8	8	8	8	8	8
Total number of stickered cubes	8	26	56	98	488	$6(n-2)^2 + 12(n-2) + 8 = 6n^2 - 12n + 8$

Problem solving (page 148)

1 **a** 42
b $n(n+1)$
c $2(n+1)$
d between 50 and 51

2 **a**

Points	Number of lines at each point	Total number of lines
2	1	1
3	2	3
4	3	6
5	4	10
6	5	15
7	6	21
8	7	28
n	$n-1$	$\frac{n(n-1)}{2}$

b 630 lines

3 33 and 24

4 **a** 35
b $n-3$
c $\frac{1}{2}n(n-3)$. It is a half of what was written since each diagonal is connected to 2 vertices.
d 16

Reviewing skills (page 149)

1 a 70, $10n + 10$

b 48, $3n^2$

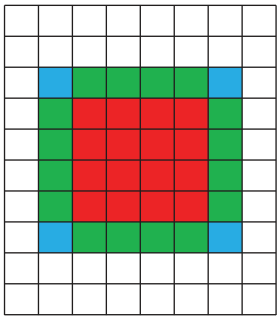
c There is an error on the Student Book. This is not a sequence.

d 177, $11n^2 + 1$

2 a

Term	$n^2 + 5$	1st difference	2nd difference
1	6		
2	9	3	
3	14	5	2
4	21	7	2
5	30	9	2

3 a



b

Pattern number	1	2	3	4	5
Number of green squares	4	8	12	16	20
Number of blue squares	4	4	4	4	4
Number of red squares	1	4	9	16	25
Total number of squares, S	9	16	25	36	49

c i 40

ii 4

iii 100

iv 144

d i $4n$

ii 4

iii n^2

e $S = n^2 + 4n + 4$ or $S = (n + 2)^2$

Practising skills (pages 152–153)

1 **a** $t_n = n^2 - 2n + 4$
b $t_n = n^2 - 2n + 1$
c $t_n = n^2 - 2n + 7$
d $t_n = 2n^2 - 4n + 8$
e $t_n = 2n^2 - 4n + 3$

2 **a** aC
b bA
c cB
d dD

3 **a** $U_n = n^2 + n - 4$
b $U_n = n^2 + 2n - 5$
c $U_n = n^2 + 4n + 3$
d $U_n = n^2 - n + 1$

4 **a** $U_n = 2n^2 + 3n - 5$
b $U_n = \frac{1}{2}n^2 - 2n + 3$
c $U_n = 3n^2 - 5n + 2$
d $U_n = 6 - 3n - n^2$

Developing fluency (page 153)

1 **a** 11, 13, 17, 23, 31
b No, the 11th term will not be prime. If $n = 11$, $n^2 - n + 11$ is divisible by 11.

2 $a = 1.5$, $b = 4$

3 $a = 3$, $b = 1$, $c = 19$

4 **a** 41, 43, 47, 53, 61
b 41st term is not prime

5 $t_n = 1.5n^2 + 1.5n$

Problem solving (pages 154–155)

1 **a** (teams, matches), (2, 1), (3, 3), (4, 6)
b 28
c $\frac{1}{2}(n(n-1))$
d 380

2 **a** 24
b $(n + 1)^2 - 1$
c Pattern 12
3 **a** 14 for shape 2, 27 for shape 3
b 65
c $2n^2 + 3n$
d $2n^2 + 6n$
4 77 (This is $n = 5$)

Reviewing skills (page 155)

1 **a** $U_n = n^2 - 2n - 3$
b $U_n = 14 - n - 2n^2$
2 $a = 3$, $b = -4$, $c = 5$, $d = 9$, $e = 60$

Practising skills (pages 159–162)

1 a i A C
ii A B
b i $x = 6$
ii $y = 3$

2 a i (2, 1)
ii (3, 1)
iii (3, 5)
b i 4
ii 1
c 4

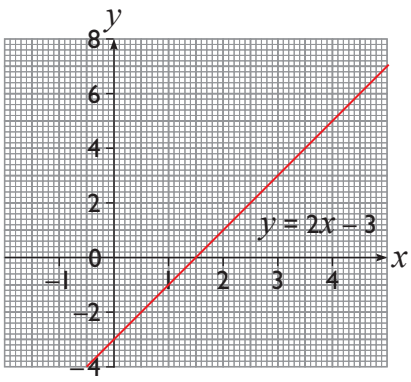
3 a For example, the start and end points of each line are:
i (3, 0) and (6, 3)
ii (2, 0) and (5, 6)
iii (0, 0) and (3, 6)
iv (0, 2) and (4, 6)
b i 1
ii 2
iii 2
iv 1
c i and **iv**
ii and **iii** The gradients are the same.

4 a 3; $y = 3x - 1$
b 1; $y = x + 1$
c 0; $y = 0$
d 4; $y = 4x + 2$
e −1; $y = -x + 2$
f −2; $y = -2x + 5$
g −3; $y = -3x - 3$
h −5; $y = -5x + 8$

5 a For example the start and end points of the lines are:
i (0, 0) and (2, 6)
ii (2, 0) and (3.2, 6)
iii (0, 6) and (6, 0)
iv (0, 3) and (6, 0)
b i 3
ii 5
iii −1
iv −0.5
c Negative gradient

6 a For example (0, −3) and (2, 1)

b



c i 2

ii −3

d Part **i** refers to the multiple of x, which is 2.

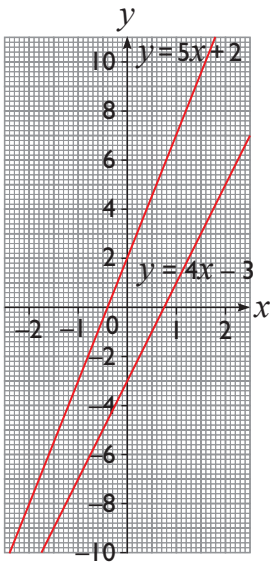
Part **ii** refers to the constant subtraction, −3.

7 a

x	−2	−1	0	1	2
$4x$	−8	−4	0	4	8
-3	−3	−3	−3	−3	−3
$y = 4x - 3$	−11	−7	−3	1	5

x	−2	−1	0	1	2
$5x$	−10	−5	0	5	10
$+2$	+2	+2	+2	+2	+2
$y = 5x + 2$	−8	−3	+2	7	12

b



c Gradient of $y = 5x + 2$ is 5; gradient of $y = 4x - 3$ is 4

d Intercept of $y = 5x + 2$ is 2; intercept of $y = 4x - 3$ is −3

e You can see the answers in the equations

f Gradient = 8, intercept = −5

8 a A:

i (0, 8) (4, 0)

ii −2

iii 8

B:

i (4, 8) (1, 0)

ii 2.666

iii −8/3

C:

i (0, 3) (5, 8)

ii 1

iii 3

D:

i (0, 5) (5, 0)

ii −1

iii 5

E:

i (0, 1) (8, 1)

ii 0

iii 1

b i No line matches

ii D

iii A

iv C

v E

Line B is $y = \frac{8}{3}x - \frac{8}{3}$

Developing fluency (pages 162–163)

1 B, C, A

2 **i** C

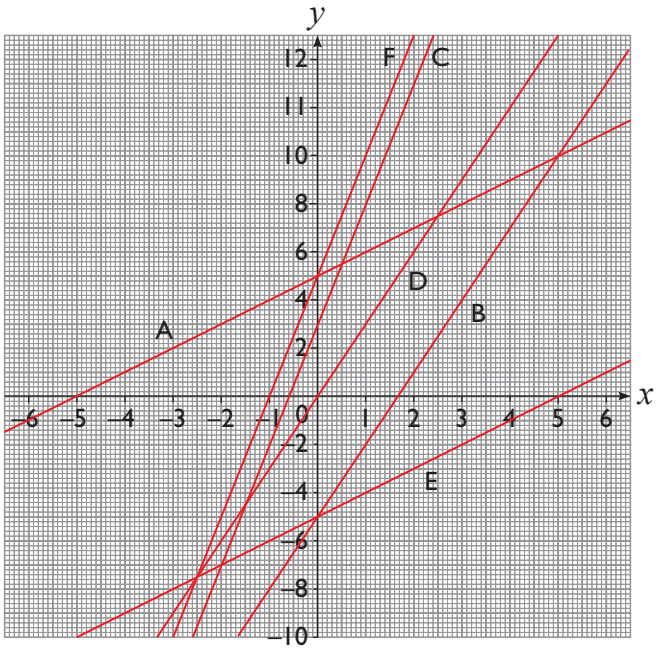
ii A, F

iii A, B

iv E

v E

3 **a**



b A and E; B and D; C and F

c A and F; B and E

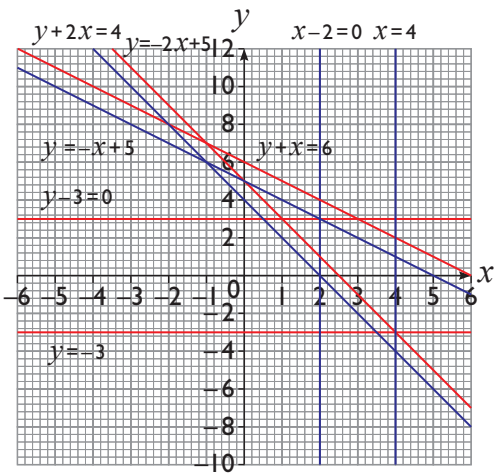
4 **a** $y = 3x$

b $y = 2x + 3$

c $y = 4x + 1$

d $y = x - 2$

5 **a**



b **i** $(x = 4)$ and **viii** $(x - 2 = 0)$;
ii $(y = -3)$ and **vii** $(y - 3 = 0)$;
iii $(y + x = 6)$ and **vi** $(y = -x + 5)$;
iv $(2x + y = 4)$ and **v** $(y = -2x + 5)$

c From their gradients (you have to rearrange some equations to $y = mx + c$ format to find gradient, m)

6 **a** $C = 0.05m + 20$

b £80

c 1600 miles

Problem solving (pages 164–165)

1 a $C = 5d + 10$

b Cost to hire is 5 pounds per day plus £10

c i It is a constant rate per day (i.e. doesn't get more expensive per day with every day hired)

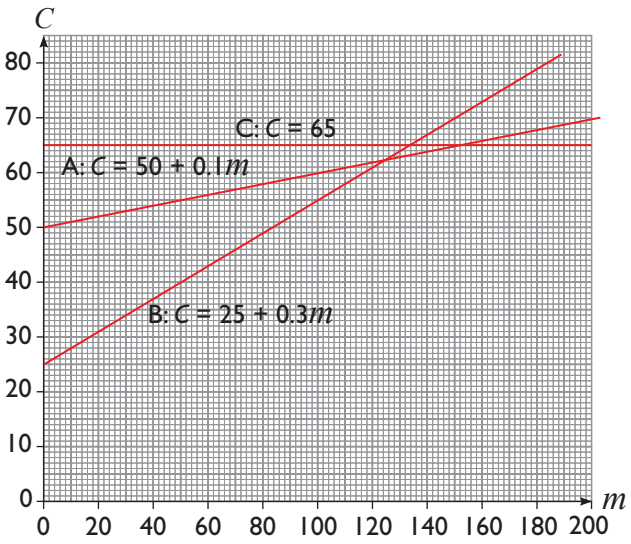
ii There is a base cost of £10

2 a B and E; C and D are parallel

b B and C meet at (0, 3)

3 a B: $C = 25 + 0.3m$; C: $C = 65$

b



c Company C

4 a i 2

ii −3

b $y = 2x - 3$

c No, as $12 \neq 2(8) - 3$

d $y = 2x + 1$

Reviewing skills (pages 165–166)

1 A: $y = 4x + 3$
B: $y = 8x + 2$
C: $y = 2x - 2$
D: $y = -5x + 4$
E: $y = -x - 5$
F: $y = 6x - 3$

2 **a** $x = -4$
b $y = 3x + 1$
c $y = -x + 4$
d $y = -\frac{1}{2}x + 3$
e $y = 3$

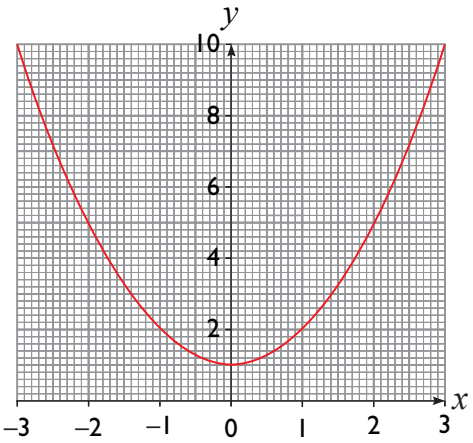
3 **a** $C = 40 + 30h$
b **i** £40
ii £30 per hour
c 7 hours

Practising skills (pages 170–171)

1 a

x	−3	−2	−1	0	1	2	3
x^2	9	4	1	0	1	4	9
$+1$	1	1	1	1	1	1	1
$y = x^2 + 1$	10	5	2	1	2	5	10

b



c $x = 0$

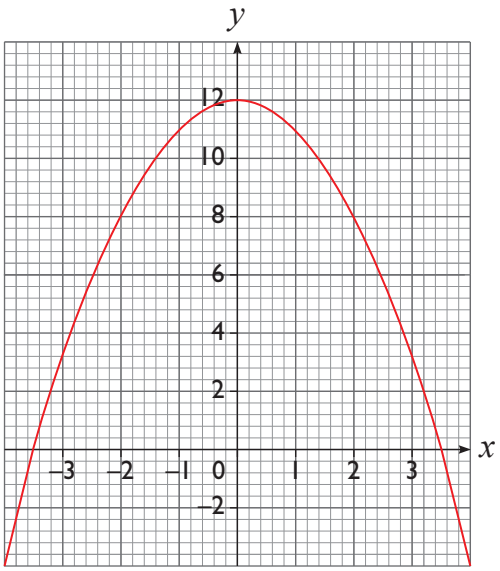
d (0, 1)

e −1.2 and +1.2

2 a

x	−4	−3	−2	−1	0	1	2	3	4
12	12	12	12	12	12	12	12	12	12
$-x^2$	−16	−9	−4	−1	0	−1	−4	−9	−16
$y = 12 - x^2$	−4	3	8	11	12	11	8	3	−4

b



c $x = 0$

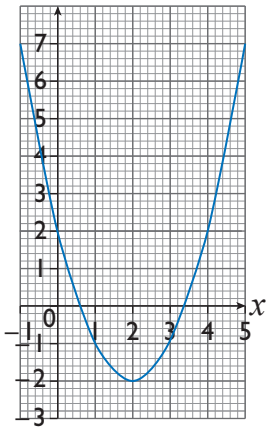
d (0, 12)

e −3.5 and 3.5

3 a

x	−1	0	1	2	3	4	5
x^2	1	0	1	4	9	16	25
$-4x$	4	0	−4	−8	−12	−16	−20
+2	+2	+2	+2	+2	+2	+2	+2
$y = x^2 - 4x + 2$	7	2	−1	−2	−1	2	7

b



c $x = 2$

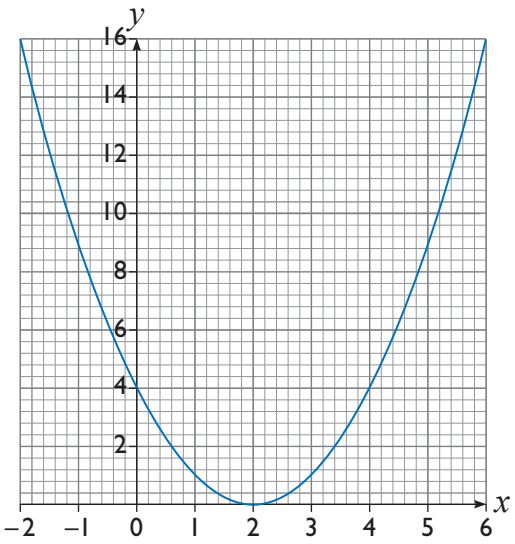
d (2, −2)

e 1 and 3

4 a

x	−2	−1	0	1	2	3	4	5	6
$x - 2$	−4	−3	−2	−1	0	1	2	3	4
$y = (x - 2)^2$	16	9	4	1	0	1	4	9	16

b



c $x = 2$

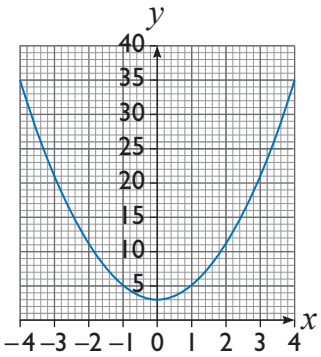
d (2, 0)

e $x = 4.65, -0.65$

5 a

x	−4	−3	−2	−1	0	1	2	3	4
$2x^2$	32	18	8	2	0	2	8	18	32
$+3$	+3	3	3	3	3	3	3	3	3
$y = 2x^2 + 3$	35	21	11	5	3	5	11	21	35

b

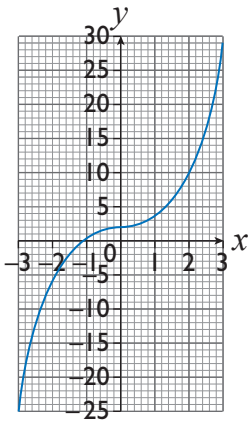


c Symmetry $x = 0$, intercept $y = 3$, min point (0, 3)

6 a

x	−3	−2	−1	0	1	2	3
x^3	−27	−8	−1	0	1	8	27
$+2$	+2	2	2	2	2	2	2
$y = x^3 + 2$	−25	−6	1	2	3	10	29

b

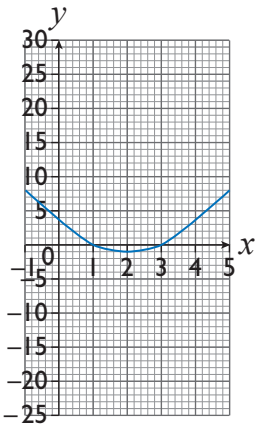


c Crosses x at −1.26 and crosses y at 2

d Rotational symmetry order 2 about (0, 2)

Developing fluency (pages 171–172)

1 a

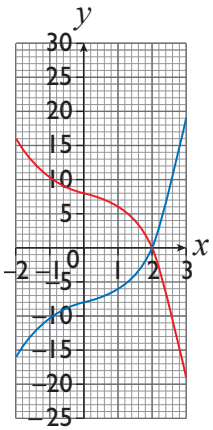


b $x = 1, 3$

c $x = -0.6, 4.6$

d The minimum point is $(2, -1)$, the curve does not go below $y = -1$

2 a



b $x = 2$

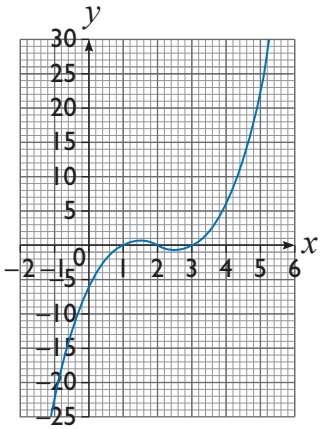
c when $x > 2$

d mirror images about the line $y = 0$

3 a

x	0	1	2	3	4
x^3	0	1	8	27	64
$-6x^2$	0	−6	−24	−54	−96
$11x$	0	11	22	33	44
-6	−6	−6	−6	−6	−6
$y = x^3 - 6x^2 + 11x - 6$	−6	0	0	0	6

b

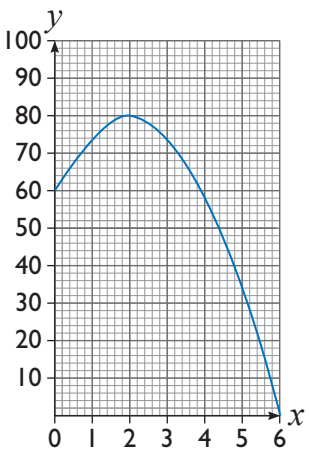


c $x = 1, 2, 3$

d One value; $x = 3.2$

e $x = 3.8$

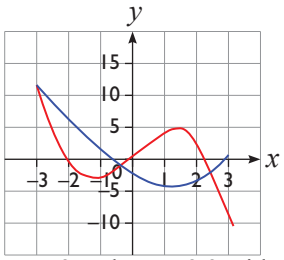
4 a



b 80 m

c 6 seconds

5 a



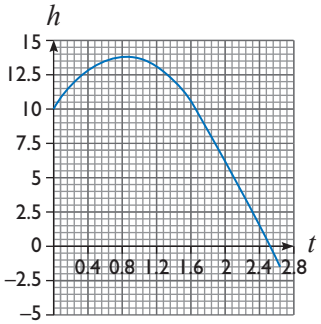
b $x = -3$ and $x = +2.3$ with a crossover at -0.4 AO3, 1b

6 $h = 10 + 8t - 5t^2$

a

t	0	0.5	1	1.5	2	2.5
10	10	10	10	10	10	10
+ 8t	0	4	8	12	16	20
− 5t^2	0	−1.25	−5	−11.25	−20	−31.25
$h = 10 + 8t - 5t^2$	10	12.75	13	10.75	6	−1.25

b



c 13.2 m

d 1.6 seconds

e i After 2.4 seconds

ii The pebble is under the sea, so its motion won't be modelled by the same equation; graph doesn't have any meaning after 2.4 s

Problem solving (pages 173–174)

1 a $-4 < x < 1$

b $-1 < x < 4$

c $-2 < x < 2$

d (1.5, −6.25)

e (−1.5, 6.25)

2 a $x < -2$ and $0 < x < 2$

b $x < -2$ and $0 < x < 2$

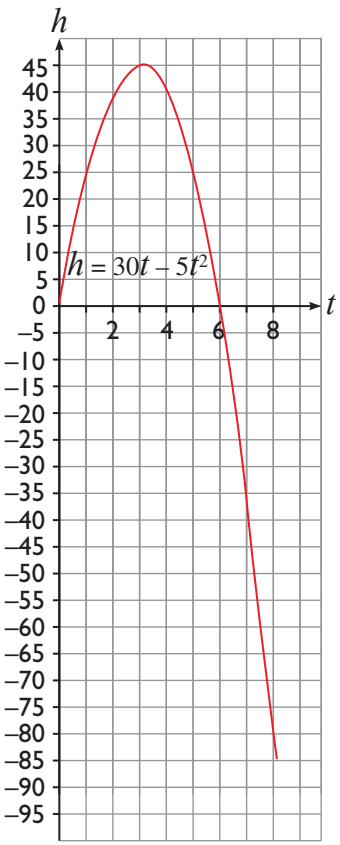
c $x < -2$ and $0 < x < 2$

d They are reflections of each other in the x-axis.

e Rotational symmetry order 2 about the origin.

3 a check − put values of h and t from table into equation

b

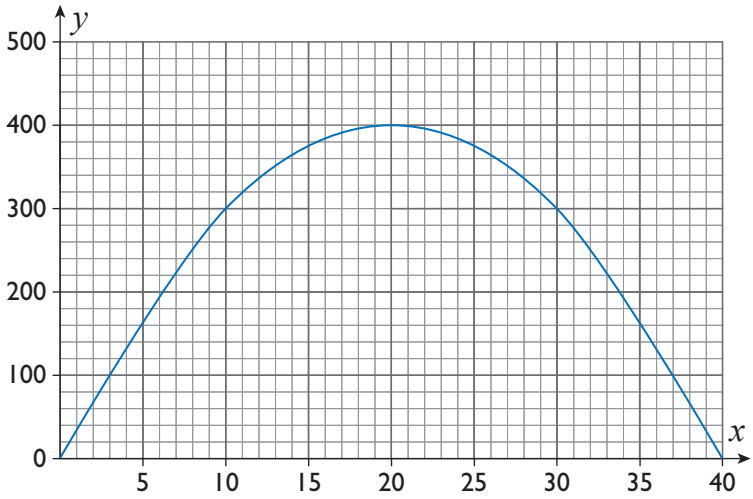


c 4 seconds

d 80 m

4 a Length x, width is $\frac{1}{2}(80 - 2x)$ so $A = x(40 - x)$

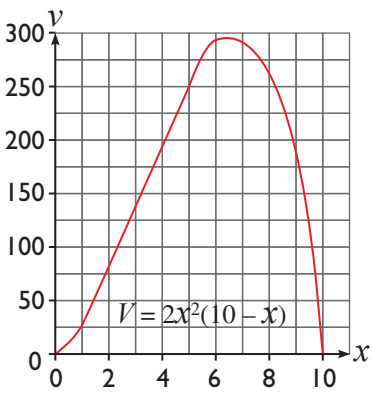
b



c 15 cm (or 25 cm)

d when $x = 20$, curve is maximum. $4 \times 20 = 80$ = perimeter, which is square

5 a



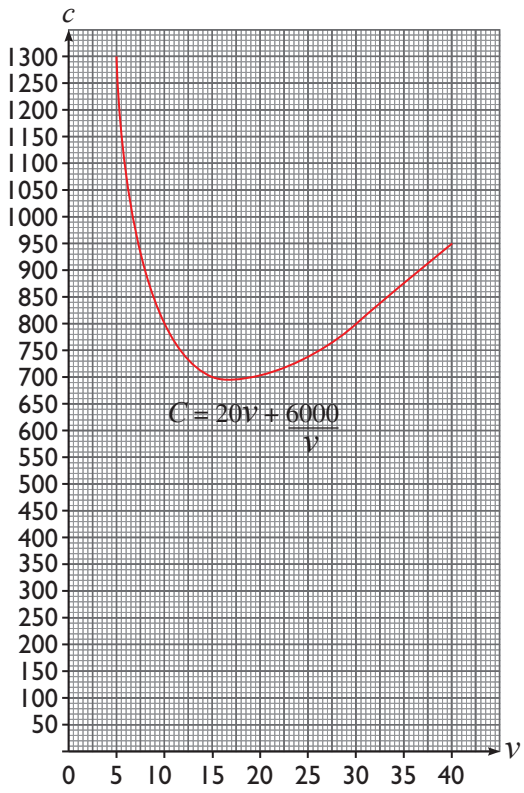
b 4.1 cm

c approx. 292 cm^3 × 5 = 1460 cm^3

6 a

v	5	10	15	20	25	30	35	40
$20v$	100	200	300	400	500	600	700	800
$\frac{6000}{v}$	1200	600	400	300	240	200	171.4	150
$C = 20v + \frac{6000}{v}$	1300	800	700	700	740	800	871.4	950

b



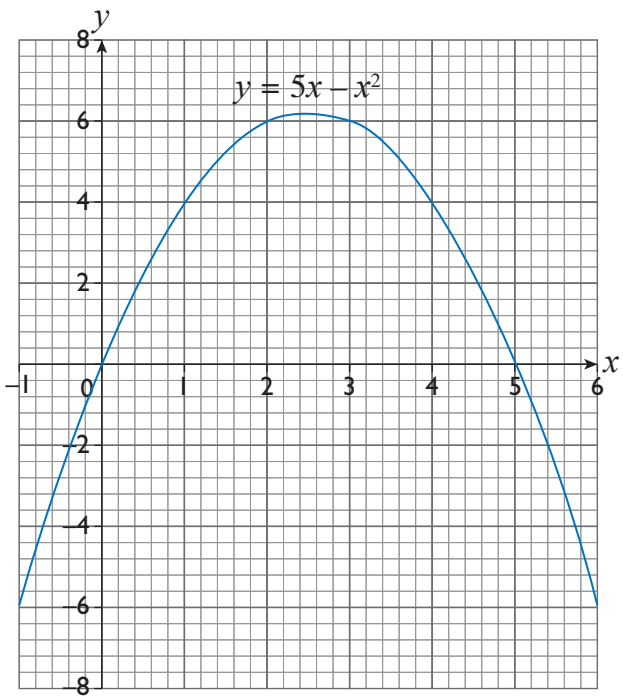
c 17.3 km/h

Reviewing skills (page 175)

1 a

x	−1	0	1	2	3	4	5	6
$5x$	−5	0	5	10	15	20	25	30
$-x^2$	−1	0	−1	−4	−9	−16	−25	−36
$y = 5x - x^2$	−6	0	4	6	6	4	0	−6

b



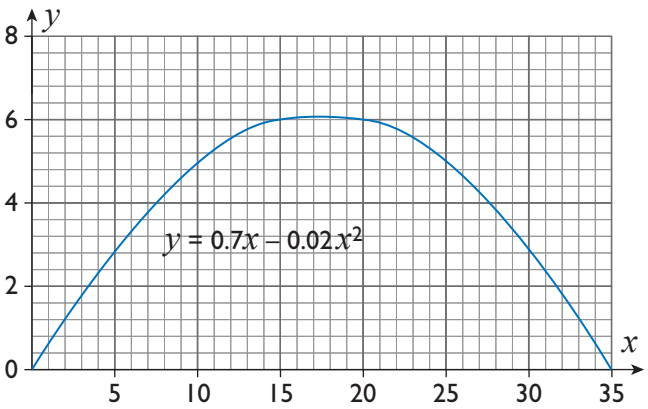
c Symmetrical about $x = 2.5$; maximum point (2.5, 6.25), crosses x at 0 and 5

d $x = 0, 5$

2 a

x	0	10	15	20	25	35
$0.7x$	0	7	10.5	14	17.5	24.5
$-0.02x^2$	0	−2	−4.5	−8	−12.5	−24.5
$y = 0.7x - 0.02x^2$	0	5	6	6	5	0

b

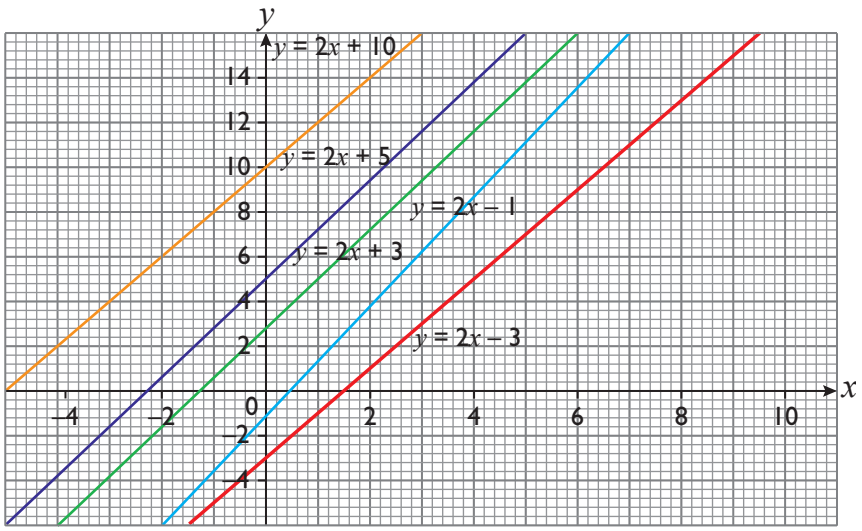


c 30 m and closer

Practising skills (page 179)

1 **a** and **f**, **b** and **d**, **c** and **g**; **e** is the odd one out. **h** Anything of the form $y = -5x + c$ is parallel to **e**.

2 **a**



b **i** 3
ii 10
iii −1
iv −3
v 5

c **i** $y = 2x + 3$
ii $y = 2x + 10$
iii $y = 2x - 1$
iv $y = 2x - 3$
v $y = 2x + 5$

3 **a** 3
b Substituting values for x and y into the equation gives $11 = 6 + k$.
c $k = 5$; $y = 3x + 5$
d (0, 5)

4 **a** $y = 3x - 2$
b $y = 7x - 9$
c $y = 2x - 19$
d $y = \frac{1}{2}x + 1$
e $y = -\frac{1}{2}x + 3$

5 **a** $\frac{9-1}{4-2} = 4$
b $y = 4x - 7$
c Check: $9 = 4(4) - 7$

6 i a 1.5

b $y = 1.5x - 0.5$

c check: $1 = 1.5 - 0.5$, $7 = 1.5 \times 5 - 0.5$

ii a 1

b $y = x + 2$

c check: $3 = 1 + 2$, $7 = 5 + 2$

iii a 1.5

b $y = 1.5x - 0.5$

c check: $4 = 1.5 \times 3 - 0.5$, $7 = 1.5 \times 5 - 0.5$

iv a $\frac{4}{7}$

b $y = \frac{4}{7}x + 4\frac{1}{7}$

c check: $3 = -\frac{8}{7} + 4\frac{1}{7}$, $7 = \frac{20}{7} + 4\frac{1}{7}$

v a $-\frac{2}{3}$

b $y = -\frac{2}{3}x + 10\frac{1}{3}$

c check: $9 = -\frac{4}{3} + 10\frac{1}{3}$, $7 = -\frac{10}{3} + 10\frac{1}{3}$

Developing fluency (pages 180–181)

1 a $y = 2x + 4$ and $y = 3x + 5$

b (0, 4) and (0, 5)

c i blue

ii blue

iii blue

iv neither

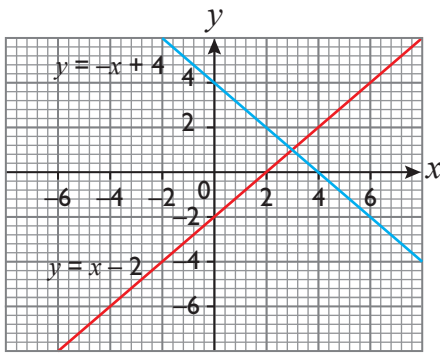
v blue

2

	Gradient is 3	Gradient is −7
y-Intercept is 6	$y = 3x + 6$	$y = -7x + 6$
y-Intercept is −5	$y = 3x - 5$	$y = -7x - 5$

3 a l: $y = -x + 4$, m: $y = x - 2$

b



c 90 degrees; they are perpendicular

4 **a** $y = x + 1$

b $y = x + 2$

c $y = x + 4$

d $y = x + 4$

e $y = x + 2$

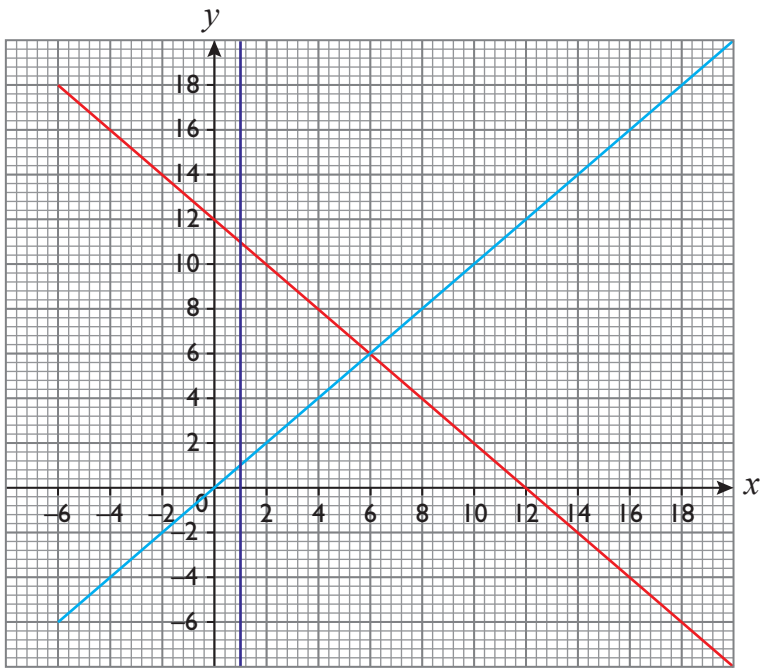
f $y = x + 1$

a is same as f; b as e; c as d

5 Always negative. If a > b, then point B always has a greater y value, but lower x value than point A – the line between them has a negative gradient. If b > a the reverse applies, but the line between them still has a negative gradient.

6 **a** $y = x$; $y = -x + 12$; $x = 1$

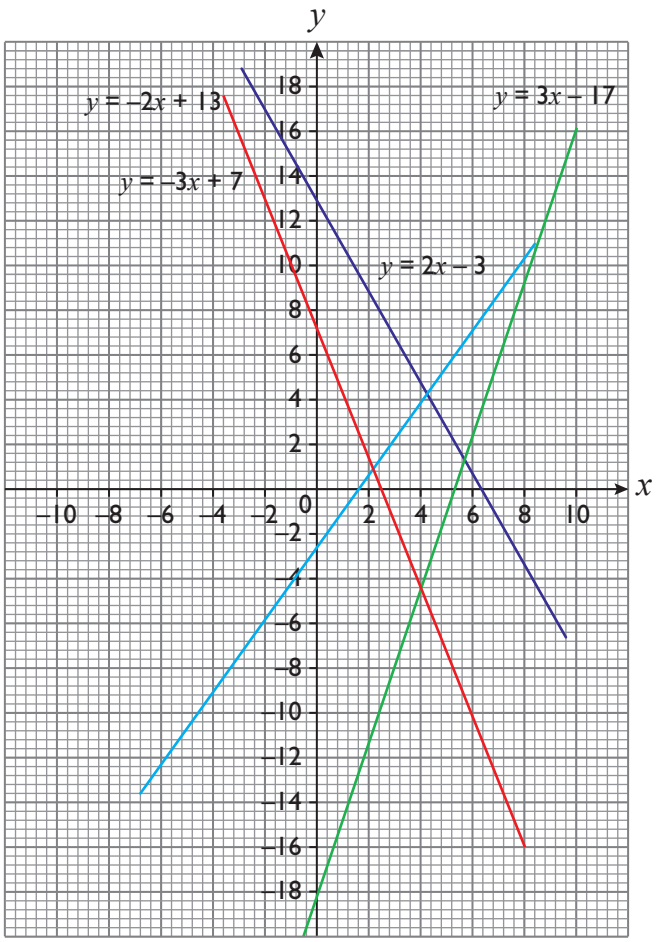
b



c isosceles

d 25 units^2

7 a



b A(4, −5), B(2, 1), C(4, 5), D(6, 1)

c A kite

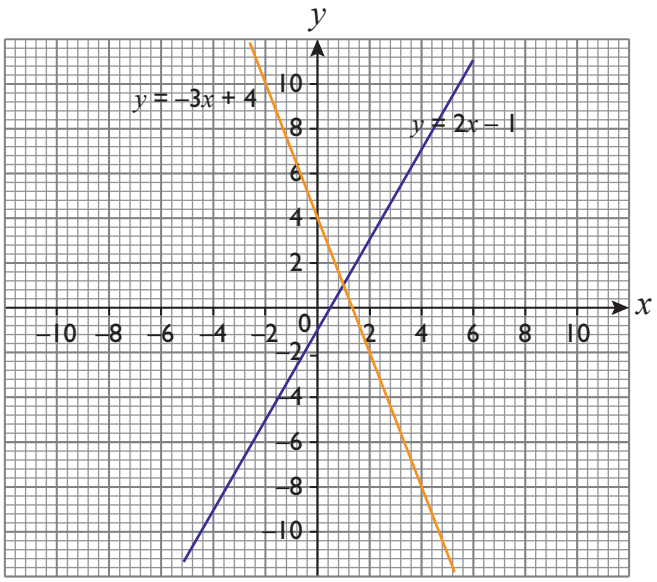
d (4, 1)

e AC: $x = 4$; BD: $y = 1$

Problem solving (pages 181–182)

1 a $l : y = 2x - 1$; $m: y = -3x + 4$

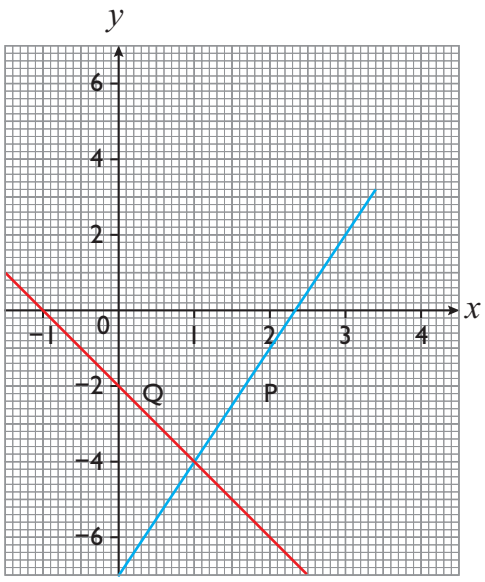
b



c (1, 1)

2 a p: $y = 3x - 7$; q: $y = -2x - 2$

b



c (1, −4)

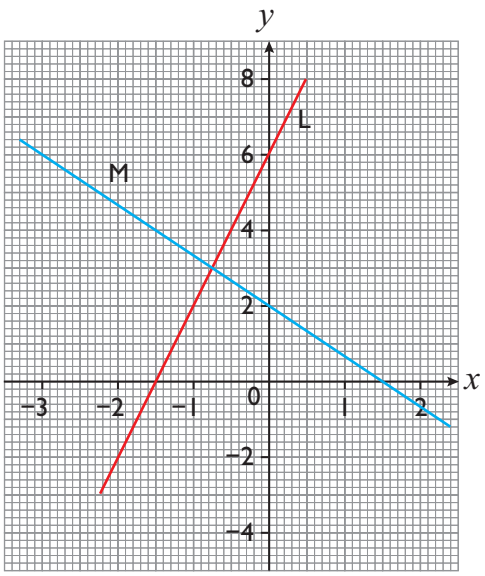
d $-4 = -4(1)$ so true

3 a r: $y = x + 5$, s: $x + y = 1$

b (−2, 3)

c $x + x + 5 = 1$ $x = -2$. $Y = 3$. They are the same.

4 a

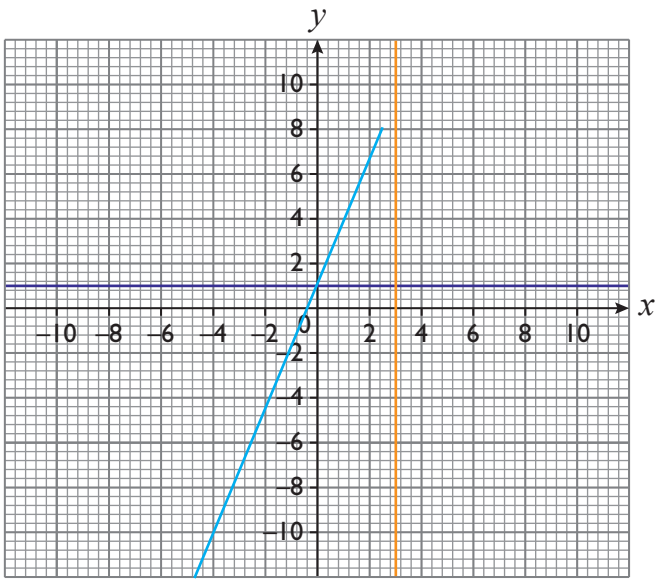


b l: $y = 4x + 6$; m: $4x + 3y = 6$

c (−0.75, 3)

d $4x + 3(4x + 6) = 4x + 12x + 18 = 6$ $16x = -12$ $x = -0.75$

5 a



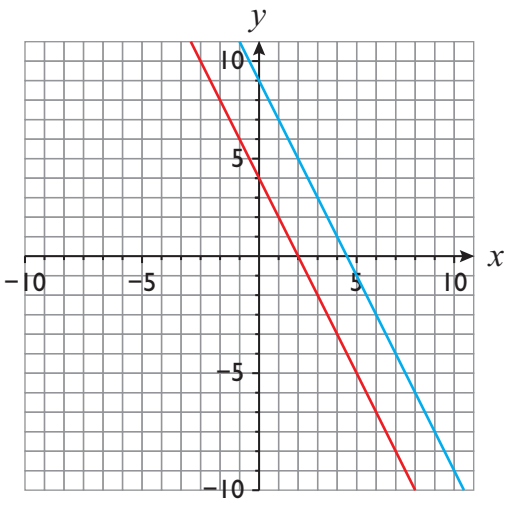
b (3, 1)

c $y = 3x - 8$

d 27 square units

6 a Both lines have gradient −2

b



c $y = \frac{1}{2}x + 4$

d $y + 2x = 9$, $x = 2$, $y = 5$ so $5 + 2 \times 2 = 9$

$y = \frac{1}{2}x + 4$, $x = 2$, $y = 5$ so $\frac{1}{2} \times 2 + 4 = 5$

e 5 square units

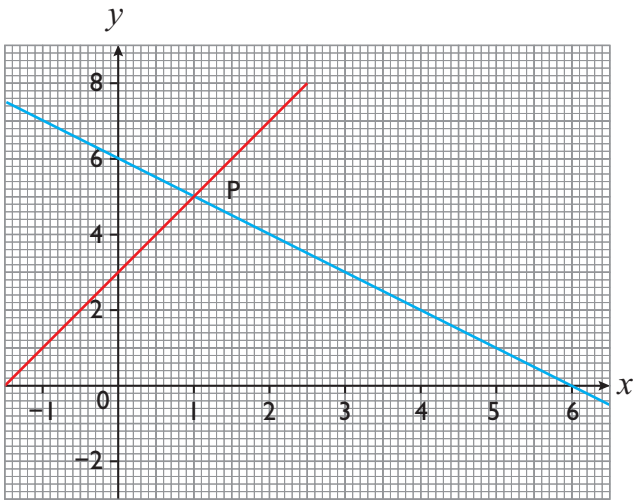
Reviewing skills (page 182)

1 a $y = 2x + 5$

b $y = -x + 7$

c $y = 3x - 1$

2 a



b (1, 5)

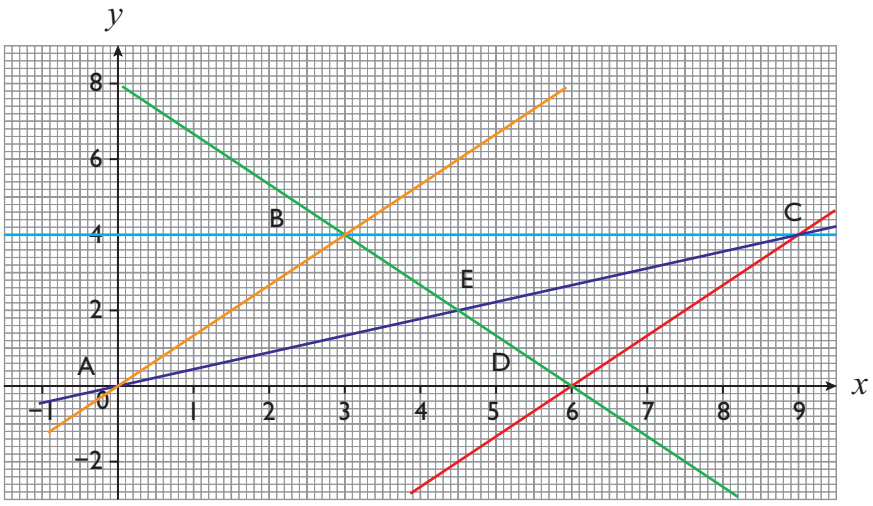
c Derive the equation of the line as $y = -2x + 7$. Insert coordinates P to prove.

3 a Gradient of both AB and CD is $\frac{4}{3}$, therefore they are parallel. Gradient of AD and BC is 0 therefore they are parallel. Gradient AC is $\frac{4}{9}$; gradient BD is $-\frac{4}{3}$ so the diagonals are not perpendicular and therefore ABCD is not a rhombus.

b AC: $y = \left(\frac{4}{9}\right)x$; BD $y = -\frac{4}{3}x + 8$

c Substitute the coordinates into both equations

d



4 b $y = 2x - 5$

Practising skills (pages 185–187)

1 a A: $-\frac{4}{3}$; B: $\frac{3}{8}$; C: $\frac{1}{4}$; D: -2; E: $-\frac{8}{3}$; F: -4; G: $\frac{3}{4}$; H: $\frac{1}{2}$

b A and G; B and E; C and F; D and H

2 a A: 3; B: $-\frac{1}{2}$; C: 3; D: -5; E: 1; F: $-\frac{1}{3}$; G: 2; H: -1

b i A and C

ii B and G; A and F; C and F; E and H

3 a Line A **i** 3 **ii** $(0, -1)$ **iii** $y = 3x - 1$

Line B **i** -3 **ii** $(0, 2)$ **iii** $y = -3x + 2$

Line C **i** $\frac{1}{3}$ **ii** $(0, -1)$ **iii** $y = \frac{1}{3}x - 1$

Line D **i** $-\frac{1}{3}$ **ii** $(0, 3)$ **iii** $y = -\frac{1}{3}x + 3$

b A and D; B and C

4 a i $2y = x + 2$ and $y = \frac{1}{2}x - 3$;

ii $y = 2(2x + 1)$ and $x + 4y = 1$; $y - 2x + 4 = 0$ and $2y + x - 7 = 0$.

iii $y = 3x - 2$ and $2x + 5y + 10 = 0$

b $3y = x + 9$

5 a Gradient of red line $= \frac{1}{2}$, gradient of green line $= \frac{1}{2}$; so lines are parallel.

$-2 \times \frac{1}{2} = -1$ so lines are perpendicular to blue line.

b Red line: $y = \frac{1}{2}x + 2$, green line: $y = \frac{1}{2}x - 1$

6 a $y = 5x + 1$

b $y = -\frac{1}{5}x + 1$ (Note: There is an error in the Student book. 'z-intercept' should say 'y-intercept'.)

Developing fluency (pages 188–189)

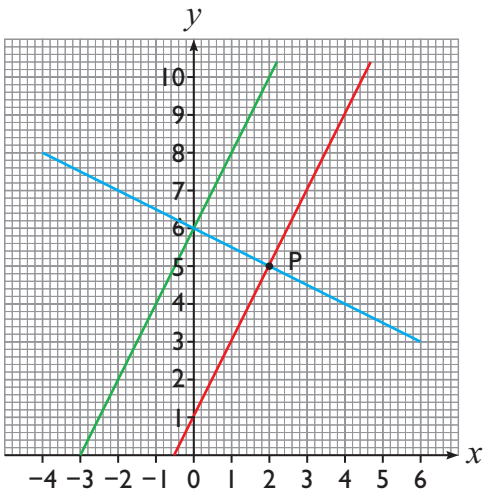
1 a $y = 2x$

b $y = -\frac{1}{2}x + 5$

c (10, 0)

d 20 square units

2 a, b, c, e



c $y = -\frac{1}{2}x + 6$

d (0, 6)

e $y = 2x + 6$

f $y = -\frac{1}{2}x + \frac{7}{2}$ or $y = -\frac{1}{2}x + \frac{17}{2}$

3 a $y = \frac{1}{5}x + 2$

b $y = -5x + 28$

c (0, 28)

d 65 square units

4 a i $y = 2x - 1$ and $y = -\frac{1}{2}x + \frac{3}{2}$

ii $y = 3x - 2$ and $y = -\frac{1}{3}x + \frac{4}{3}$

iii $y = 4x - 3$ and $y = -\frac{1}{4}x + \frac{5}{4}$

b $y = mx - (m-1)$ and $y = -\frac{1}{m}x + \frac{m+1}{m}$

5 $y = \frac{4}{3}x + \frac{1}{3}$ and $y = -\frac{3}{4}x + 10\frac{3}{4}$

6 $y = 2x - 10$

7 a Pupils' own drawings.

b (2, −2)

c Q: $y = -3x + 4$; R: $y = -\frac{x}{2} - 1$

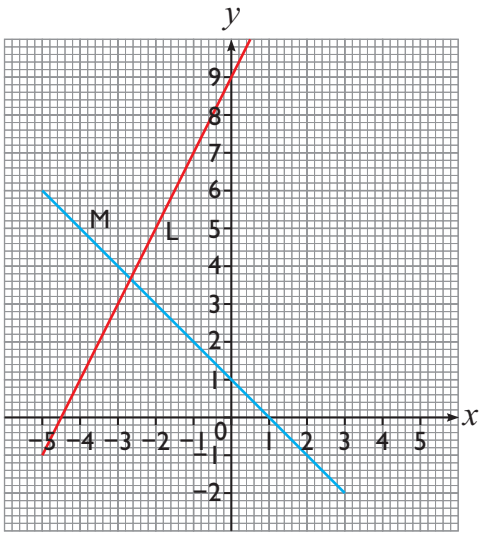
d $Q: y = -3(2) + 4 = -2$; $R: y = -\frac{2}{2} - 1 = -2$

Problem solving (pages 190–191)

1 a Equations are: $y = \frac{1}{2}x + 2$; $y = \frac{1}{2}x - \frac{1}{2}$; $y = -2x + 2$; $y = -2x - 8$

b Area is 10 square units.

2 a



b $\left(-\frac{8}{3}, \frac{11}{3}\right)$

c L: $y = 2(\frac{8}{3}) + 9 = \frac{11}{3}$; M $y = -(-\frac{8}{3}) + 1 = \frac{11}{3}$

3 a $y = \frac{12 - 4x}{3}$

b P (0, 4); PQ gradient $= -\frac{4}{3}$

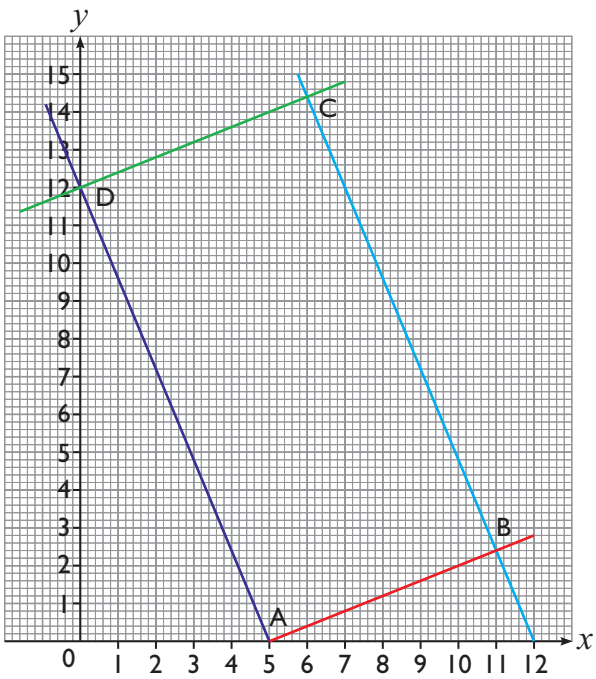
c Q (3, 0). PQ = 5 units

d PS is 5 units, base 4 units, height 3 units. S is at point (0 + 4, 4 + 3) = (4, 7)

4 (0.8, −0.6)

5 Pupils' own drawings.

6 a Pupils' own drawings.



b (11, 2.5)

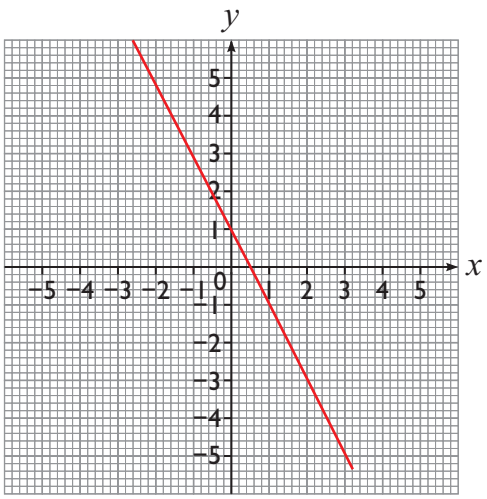
c $y = -\frac{12}{5}x + 28.9$

d $y = \frac{5}{12}x - \frac{25}{12}$

e 84.5

Reviewing skills (page 191)

1 a



b i $y = \frac{1}{2}x - 3$

ii $y = \frac{1}{2}x + \frac{1}{2}$

2 $y = -\frac{1}{5}x + 5$

Algebra Strand 3 • Unit 7 • Answers

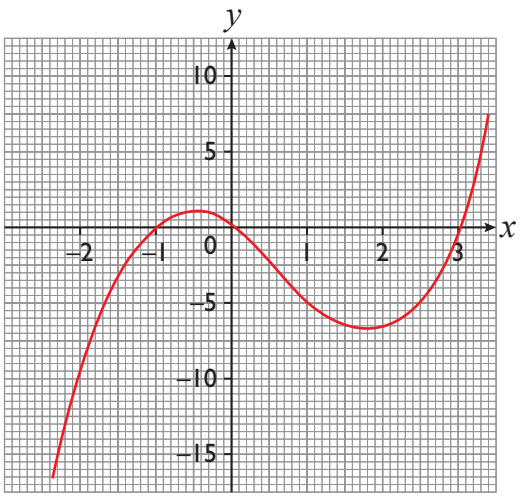
Practising skills (page 196)

1 B: $2y - 3x = 4$
2 I: $y = -x^2$
3 E: $2y - 3x + 4 = 0$
4 M: $y = 1 - \frac{1}{x}$
5 L: $y = -\frac{1}{x}$
6 F: $y = x^2$
7 O: $y = -x^2 - 3x + 4$
8 C: $y = x^3 + x^2 - 2x$
9 J: $y = -x^3$
10 G: $y + 3x - 2 = 0$
11 N: $y = \frac{2}{x}$
12 H: $y = x^2 - 3x - 4$
13 K: There is an error in the Student book. These do not match.
14 D: $y = x^3$
15 A: $y = \frac{1}{x} + x$

Developing fluency (pages 196–198)

1 a −2, −1, 0, 1, 2, 3, 4
−10, 0, 0, −4, −6, 0, 20

b

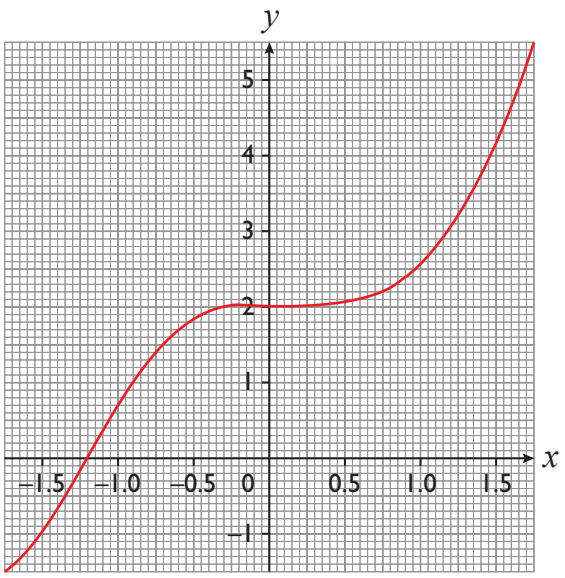


2 a Negative
3 All

4 a

x	−3	−2	−1	0	1	2	3
x^3	−27	−8	−1	0	1	8	27
+2	+2	+2	+2	+2	+2	+2	+2
$y = x^3 + 2$	−25	−6	1	2	3	10	29

b



c $x = -1.25$ $y = 2$

d Not symmetrical

5 a 2

B 2

c 3

d 3

6 a ii

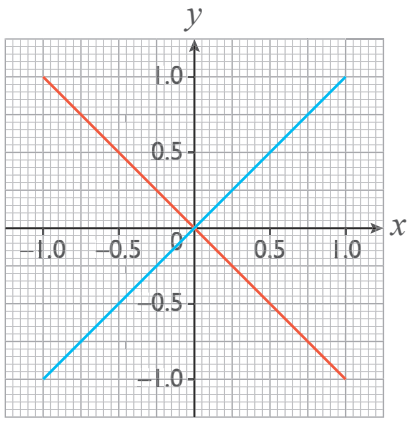
b i

c iii

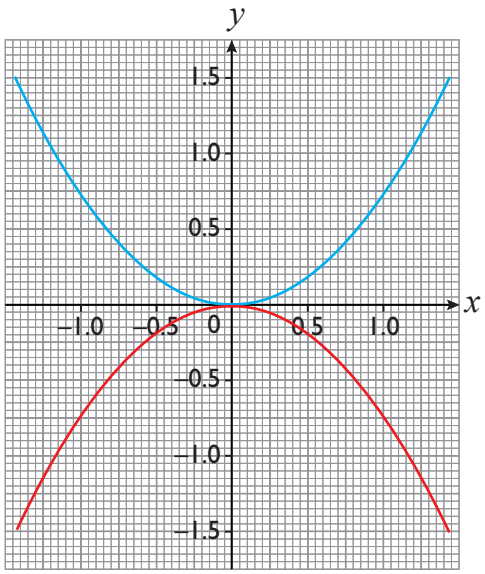
e Graph of $y = x^3$

x	−3	−2	−1	0	1	2	3	4
x^3	−27	−8	−1	0	1	8	27	64

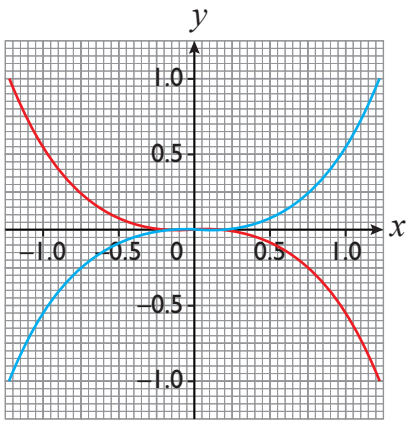
7 a i



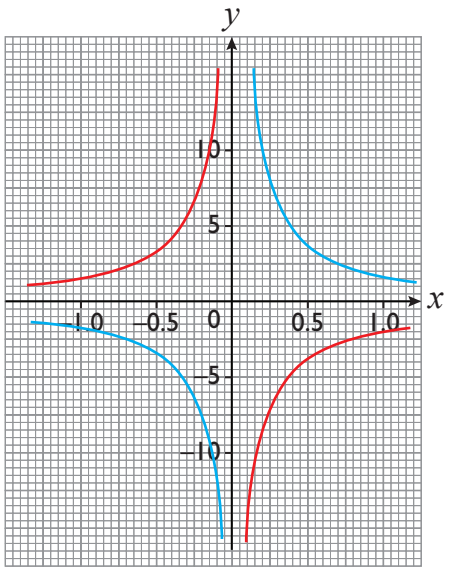
ii



iii



iv



b Reflect in the x axis.

8 a i $-\frac{1}{4}$

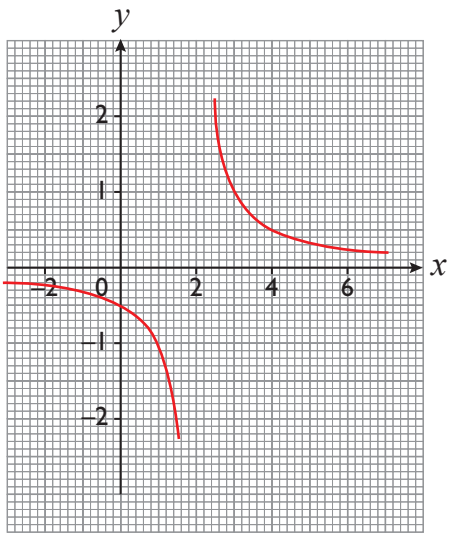
ii $-\frac{1}{2}$

iii -1

iv $\frac{1}{2}$

b Goes to infinity

c



d Find $x = \frac{8}{3}$

Problem solving (pages 198–199)

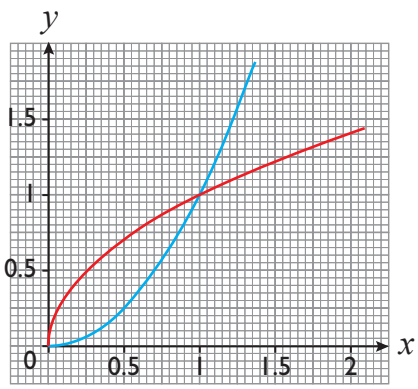
1 i $x - \frac{1}{x}$

ii $1 - x^3$

iii $\frac{4}{x}$

iv $x^3 + x$

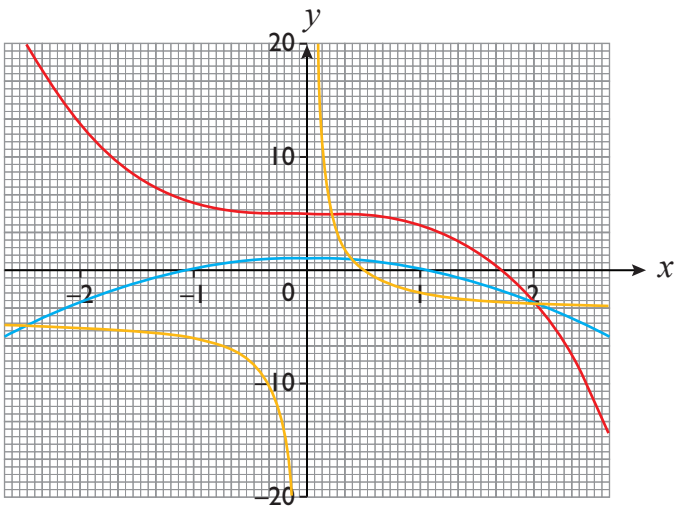
2 a and **b**



c $y = x$

3 a 1, 5, 4

b



c $y = -3$

4 a $x < -2$ $0 < x < 2$

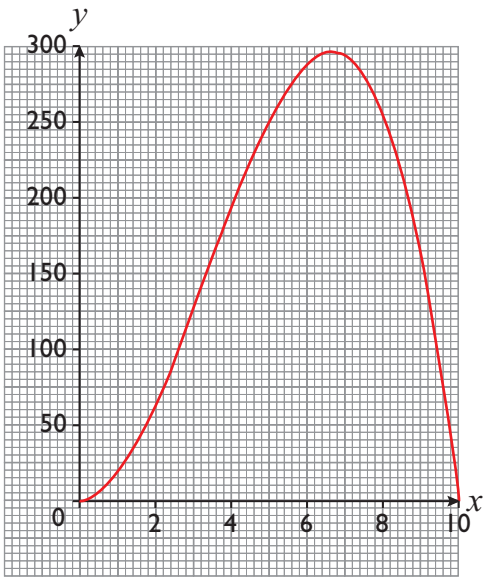
b $x < -2$ $0 < x < 2$

c $x < -2$ $0 < x < 2$

d Reflection in line $y = 0$

e Rotational symmetry around (0,0) of order 2.

5 a



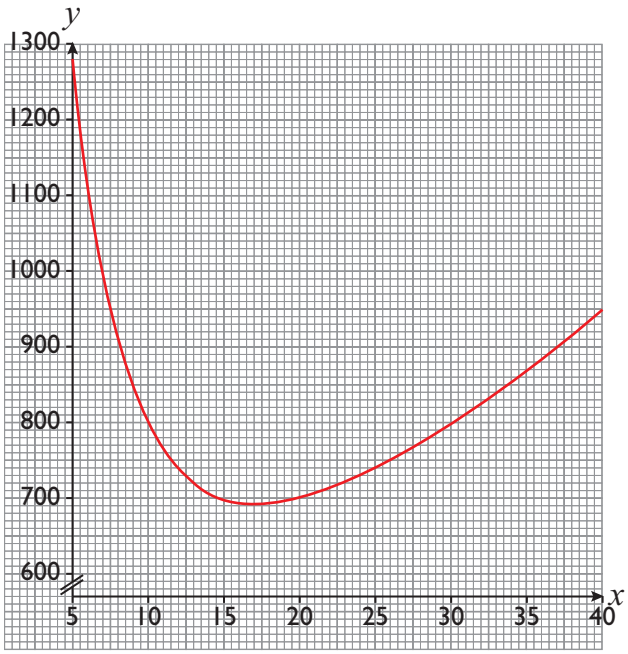
b About 4 or 9.

c About 290.

6 a

v	5	10	15	20	25	30	35	40
c	1300	800	700	700	740	800	871	950

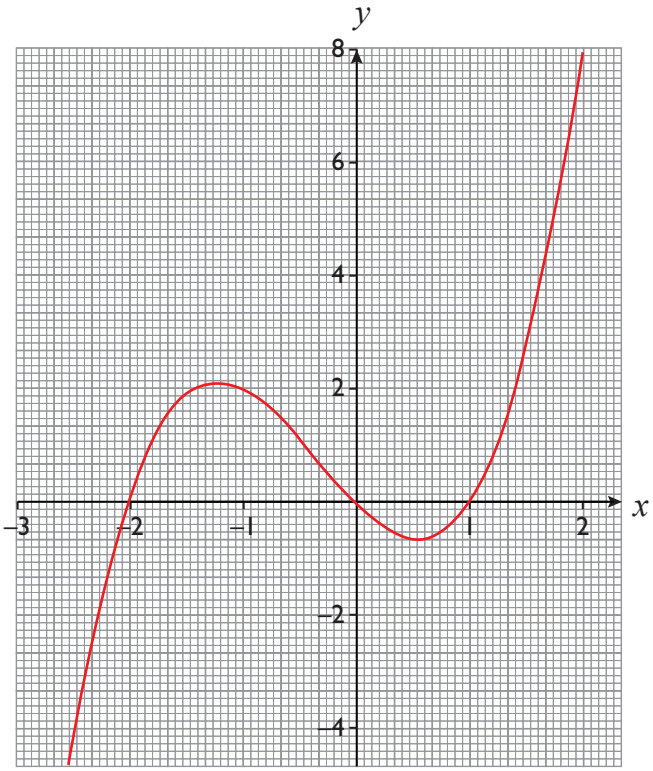
b



c About 17.

Reviewing skills (page 200)

1 $x = -2, 0, 1$

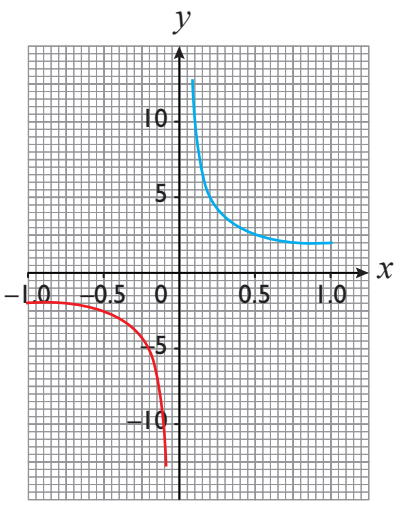


2 a

x	−5	−4	−3	−2	−1	−0.5	−0.2	0.2	0.5	1	2	3	4	5
$\frac{1}{x}$	−0.2	−0.25	−0.33	−0.5	−1	−2	−5	5	2	1	0.5	0.33	0.25	0.2
y	−5.2	−4.25	−3.33	−2.5	−2	−2.5	−5.2	5.2	2.5	2	2.5	3.33	4.25	5.2

b $\frac{1}{0}$ isn't defined

c



d About 0.4.

3 a, b, and d See student sketches

c They are reflections in the x axis.

4 The answer is **d**.

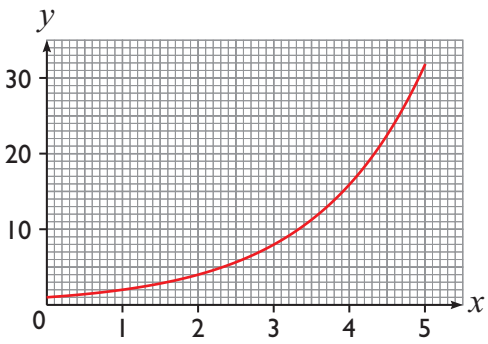
Practising skills (pages 205–206)

1 **A** and **D**

2 **a**

x	0	1	2	3	4	5
2^x	1	2	4	8	16	32

b



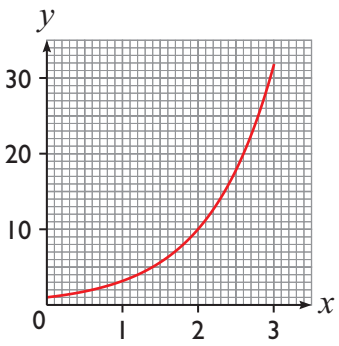
c The value of y increases more rapidly as x increases.

d $y = 4.6$

3 **a**

x	0	0.5	1	1.5	2	2.5	3
3^x	1	1.7	3	5.2	9	15.6	27

b

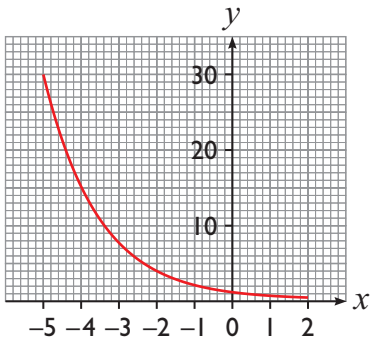


c $x = 2.3$

4 a

x	−5	−4	−3	−2	−1	0	1	2
2^{-x}	32	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

b



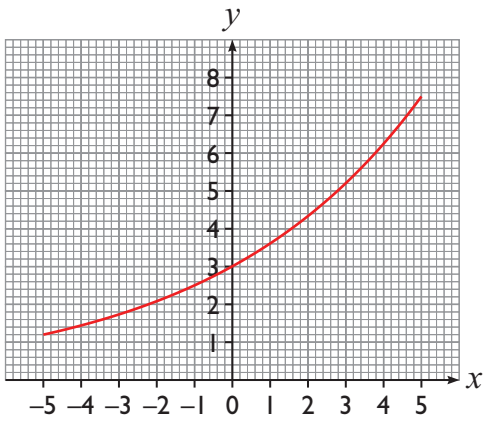
c The graph decreases rapidly as x increases. The graph gets closer to the x-axis, but never crosses it.

d $x = -3.3$

5 a

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
3×1.2^x	1.2	1.5	1.7	2.1	2.5	3	3.6	4.3	5.2	6.2	7.5

b



c (0, 3)

d $(0, a)$

6 a (0, 1)

b i 3.5

ii $y = 3.5^x$

7 a i 1

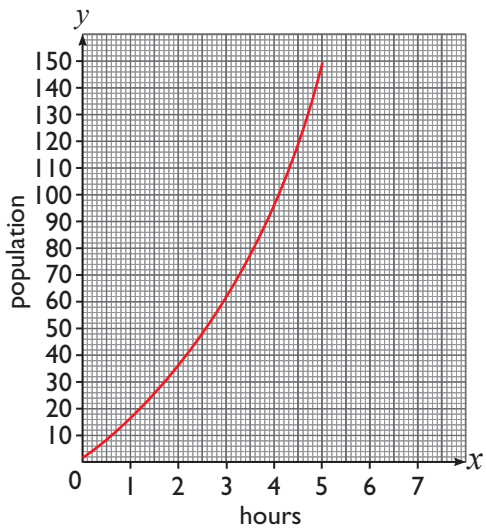
ii $a = 5$

b i $b = 2$

ii $y = 5 \times 2^x$

Developing fluency (pages 207–209)

1 a



b About 23.

c 5 hours, 40 minutes.

2 a i Account A: £535, account B: £630

ii Account A: £572.45, account B: £661.50

b $y = 600 \times 1.05^x$

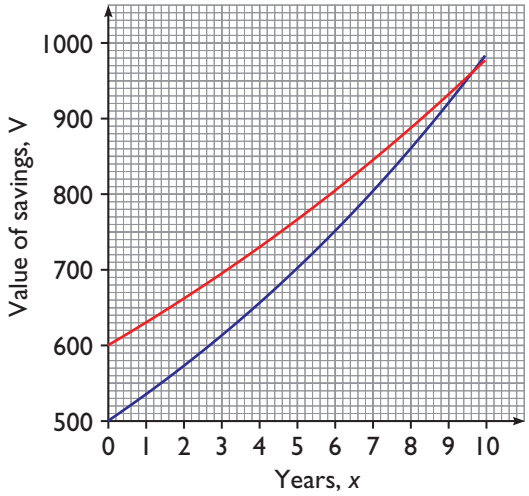
c i

x	0	1	2	3	4	5	6	7	8	9	10
Account A	500	535	572.45	612.52	655.40	701.28	750.37	802.89	859.09	919.23	983.58

ii

x	0	1	2	3	4	5	6	7	8	9	10
Account B	600	630	661.50	694.58	729.30	765.77	804.06	844.26	886.47	930.80	977.34

d

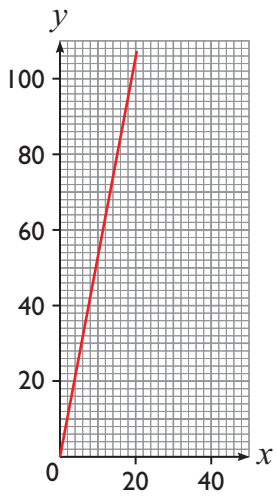


e Approximately 9 years and 8 months. Both accounts will have around £961.

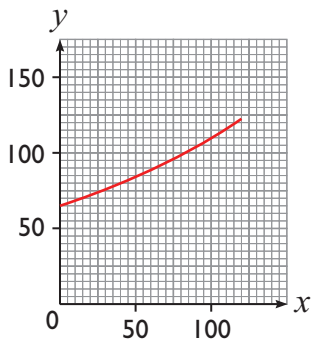
3 **a** 5.3 million

b To increase by 9.73%, you need 109.73% so multiply by 1.0973.

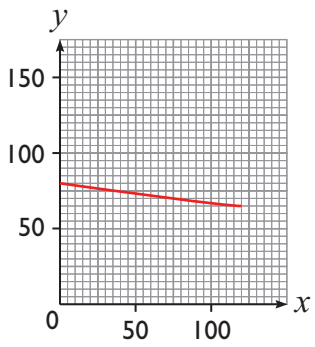
c Graph with equation $y = 4.8 \times 1.0973^x$



d Graph with equation $y = 64 \times 1.0054^x$



e Graph with equation $y = 80 \times 0.9982^x$



4 $y = 3 \times 5^x$

5 7000 gallons

6 14.1%

Problem solving (pages 210–212)

1 a i £1400

ii 18 000

b 0.6; $V = 18\,000 \times 0.6^x$

2 a i 400

ii 1600

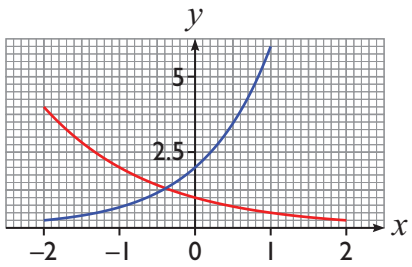
b $N_0 = 400$, $a = 2$, $N = 400 \times 2^t$

c 6400

3 a

x	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
$y = 2 \times 3^x$	0.22	0.38	0.67	1.15	2	3.46	6	10.4	18

b, c

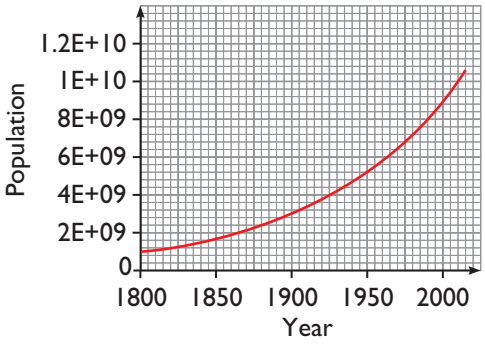


d $x = -0.4$

4 a $y = 5 + 2^x$

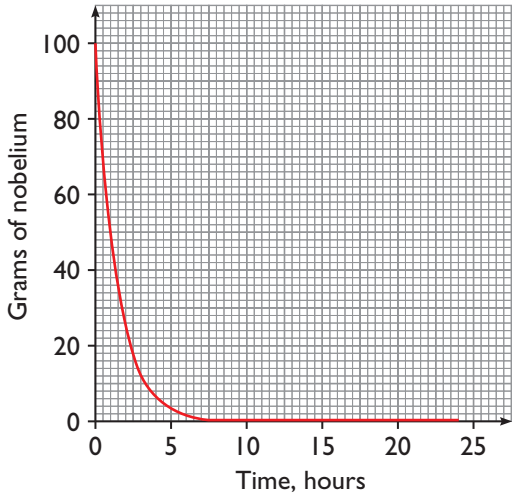
b $g = 37$

5 a

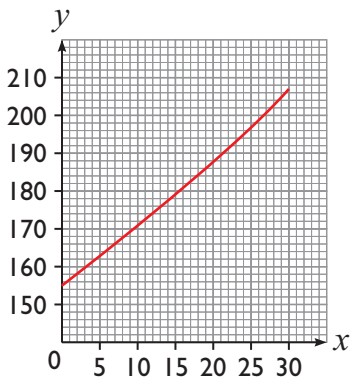


b Aneesa's growth of 1.1% is too big – population would reach 7 billion in 1978 with this growth.

6



7 a



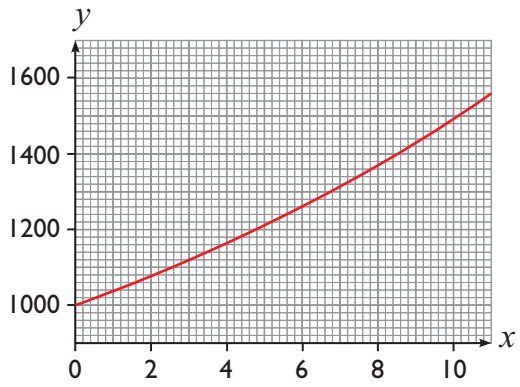
b £19.15

c Pupil's own thoughts. No, it is a risky loan and he will end up paying over £50 more than he borrowed; Yes, he needs the money now and can afford the extra £50 on his pay day.

Reviewing skills (page 212)

1 a 1000×1.06^x

b Graph of $y = 1000 \times 1.06^x$



c 4.5 years

Practising skills (pages 216–217)

1 a The most obvious answer is also controversial. 45° is arguably the same in both systems, although as a bearing it should be written with an initial zero: 045°. 225° is certainly the same in both systems. Adding multiples of 360° to each of these answers will also work, although it is usual for bearings to be lower than 360°.

b Sin 10° (the height at 10°) is approximately 0.17; sin 20° is approximately 0.34; sin 30° is 0.5; sin 40° is approximately 0.64.

c It increases the most near the start. From 0° to 10° and from 10° to 20° it increases a lot, but then it increases by less. By the time it gets to 70° and 80° it is barely increasing at all.

d The height drops very slowly at first but this drop increases more quickly as the angle gets close to 180°. It is the reverse of how it increased, this is because of the symmetry in the diagram.

e Here the sine of 100° is the same as that of 80° because of the symmetry in the diagram. Similarly, sin 110° is equal to sin 70°, etc. Sin 190° is the negative version of sin 10° and this continues for the angles that lead back round to 360°.

f The sine of the angle is the height of the line. Because the radius of the circle is 1, the highest the height can be is 1 and the lowest is −1.

g This gives you a sine wave.

Developing fluency (pages 217–220)

1 a i 90°

ii 270°

iii 180°

b i 0°

ii 180°

iii 90°

c A

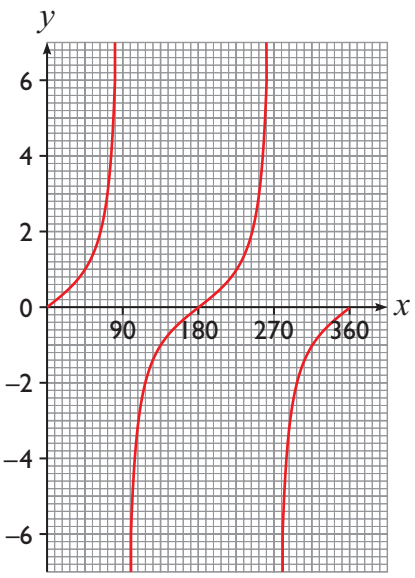
d C

e B and D

2 a

x	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
$\tan x$	0	0.176	0.364	0.577	0.839	1.191	1.732	2.747	5.671	

b



c i around 26° and 206°

ii around 386°

d Both tan 90° and tan 270° are undefined − the graph of $y = \tan x$ does not cross the lines $x = 90°$ and $x = 270°$. Just before $x = 90°$ and $x = 270°$, $\tan x$ is close to infinity, and just after these values $\tan x$ is close to negative infinity.

3 a −360°, −180°, 0°, 180° and 360°

b −90° and 270°

c −150°, −30°, 210° and 330°

4 Possible answers include:

a i Both graphs are the same shape: $y = \sin x$ is a translation of $y = \cos x$ by the vector $\begin{pmatrix} 90 \\ 0 \end{pmatrix}$. Both graphs have y-values between − 1 and 1. Both graphs repeat every 360°.

ii $y = \sin x$ has rotational symmetry about the origin and $y = \cos x$ has the y-axis as a line of symmetry.

b i Both graphs have rotational symmetry about the origin.

ii $y = \tan x$ repeats every 180° but $\sin x$ repeats every 360°. The y-values for $y = \tan x$ can be any value but for $y = \sin x$ the y values lie between −1 and 1.

5 a −270°, 90°, …

b −360°, 0°, 360°, …

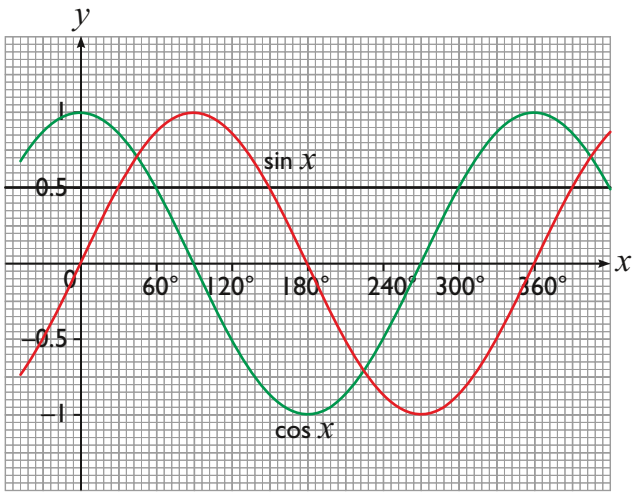
c −285°, −135°, 45°, 225°, …

d −360°, −180°, 0°, 180°, 360°, …

e −270°, −90°, 90°, 270°, …

f −360°, −180°, 0°, 180°, 360°, …

6



sin 30° = 0.5 = cos 60°

7 A, 90° at the centre of the circle would never meet the tangent to the circle.

8 **a** $x = 405°$ is the only solution.

b 1125°

9 **a** max. 3; $x = 45°, 225°$; min. −3; $x = 135°, 315°$

b **i** 4

ii $x = 15°, 75°, 195°, 255°$

Problem solving (pages 221–222)

1 **a** $\frac{\sqrt{3}}{3}$

b $-\sqrt{3}$

c $\frac{\sqrt{3}}{3}$

d $-\frac{\sqrt{3}}{3}$

2 **a** 80 m

b 240 m

c −10 m; the roller-coaster goes underground

3 **a** 36.9°

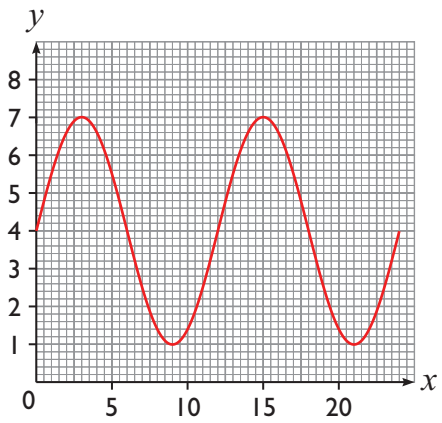
b −323.1, −216.9, 36.9. 143.1

4 **a** $\frac{1}{2}$

b $4 \times \cos(5 \times 12°) = 4 \times \cos 60° = 2 = \cos 60° = \frac{1}{2}$

c 12°, 60°, 84°, 132°, 156°, 204°, 228°, 276°, 300°, 348°

5 a



b 1.4 m

c $t = 3$ and $t = 15$, so time is 3 a.m. and 3 p.m.

d Leave at 1.23 a.m. and return at 4.36 p.m.

Reviewing skills (page 222)

1 a False

b True

c False

d True

e True

f False

g True

h False

i True

2 a −90°, 270°

b −180°, 180°

c −225°, −45°, 135°, 315°

Practising skills (pages 225–226)

1 **a** 3.6

b Robert knew the solution was between 3.5 and 3.6, and once he worked out that 3.55 was too small, it had to be closer to 3.6.

2 When $x = 4.65$, $x^3 + 2x = 109.844\,625$, which is too small. So the answer is closer to 4.7. Jenny is correct.

3 **a** When $x = 2$, $x^3 + 2x = 12$, which is less than 20. When $x = 3$, $x^3 + 2x = 33$ which is more than 20.

b

x	$x^3 + 2x$	Comment
2	12	Too small
3	33	Too large
2.5	20.625	Too large
2.4	18.624	Too small
2.45	19.606 125	Too small

$x = 2.5$ (to 1 d.p.)

4

x	$x^3 + 6x$	Comment
2	20	Too small
3	45	Too large
2.3	25.967	Too large
2.2	23.848	Too small
2.25	24.890 625	Too small

$x = 2.3$ (to 1 d.p.)

5 **a** $8^2 = 64$ and $9^2 = 81$ and 70 lies between them.

b $8.3^2 = 68.89$ and $8.4^2 = 70.56$, so he is correct.

c $8.35^2 = 69.7225$ so the root of 70 lies between 8.35 and 8.4, so the square root of 70 = 8.4 (to 1 d.p.)

Developing fluency (pages 226–227)

1

x	x^3	x^2	$x^3 + x^2$	Comment
4	64	16	80	Too small
5	125	25	150	Too large
4.3	79.507	18.49	97.997	Too small
4.4	85.184	19.36	104.544	Too large
4.35	82.312 87	18.9225	101.235 375	Too large

The solution is $x = 4.3$ (to 1 d.p.)

(Note: There is an error in the student book. The table shown in the student book should match the one above.)

2 $x = 4.2$ (to 1 d.p.)

3 $x = 3.9$

4 3.3

5 **a** $x = 1.3$

b When $x = -1$, $x^3 - 3x^2 + x = -5$. Too small. When $x = 0$, $x^3 - 3x^2 + x = 0$. Too large. So there is a solution between -1 and 0.

When $x = 2$, $x^3 - 3x^2 + x = -2$. Too small. When $x = 3$, $x^3 - 3x^2 + x = 3$. Too large. So there is a solution between 2 and 3.

c $x = -0.5$

Problem solving (page 227)

1 $x + 2 = 6.3$ cm

2 $x = -0.8$, $x = 1.5$ and 4.3

3 $x = 0.2$ and $x = 4.8$

Reviewing skills (page 227)

1 **a** When $x = 1$, $x^3 + x = 2$ which is less than 9. When $x = 2$, $x^3 + x = 10$ which is more than 9.

b

x	$x^3 + x$	Comment
1	2	Too small
2	10	Too large
1.9	8.759	Too small
1.95	9.364 875	Too large

$x = 1.9$ (to 1 d.p.)

2 $x = 4.8$

Practising skills (pages 230–231)

1 **a** $<$
b $>$
c $>$
d $<$
e $>$
f $<$
g $>$
h $<$

2 **a** $x > 5$
b $x \geq 5$
c $x \leq 5$
d $x \geq 5$
e $x < -5$
f $x \geq -5$
g $x \geq 5$
h $x \leq 5$

3 **a** $x \geq -2$
b $x \leq 5$
c $-3 < x \leq 7$
d $-6 \leq x < 2$
e $-6 < x < -1$
f $-1 \leq x \leq 1$

4 **a**

b

c

d

5 **a** $2 < 14$ – TRUE
b **i** $-9 < 3$ – TRUE
ii $-6 < 30$ – TRUE
iii $6 < -30$ – FALSE
iv $-1 < 5$ – TRUE
v $1 < -5$ – FALSE

6 **a** $x < 3$
b $x > -3$
c $x < 5$
d $x \geq -6$
e $x > 4$
f $x < 9$

7 **a** 3, 4, 5, 6
b −2
c 1, 2, 3, 4
d −5, −4 , −3, −2, −1, 0, 1

Developing fluency (pages 231–232)

1 **a** £7 ≤ pay ≤ £11
b age ≥ 18
c age > 64
d 100 < number < 150 (allow equals as well)
e number ≤ 16
f 12 ≤ age ≤ 17

2 **a** $6 < \sqrt{38} < 7$
b $9 < \sqrt{90} < 10$
c $14 < \sqrt{199} < 15$
d $2 < \sqrt{5} < 3$

3 **a** brother: $2x$; sister; $x + 10$
b $4x < 30$
c he could be 7
d brother 14; sister 17
e no

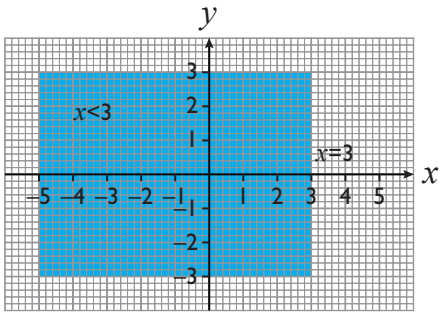
4 **a** $6w$
b $18 \leq 6w < 42$
c $18 \leq 2w^2 < 98$

5 **a** $x \geq 4$
b $x < 1$
c $x < 25.5$
d $x \geq 12$

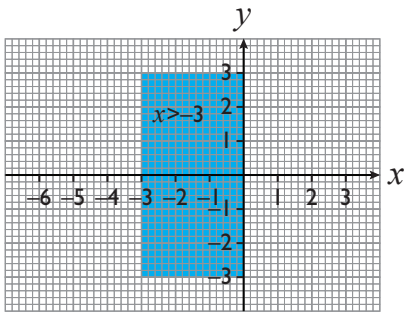
6 **a** $x < 3$
b $x > -3$
c $x < -3$
d $x > 3$

7 Wendy, because if you divide by a negative number you reverse the sign.

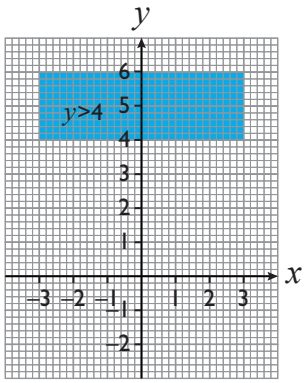
8 a i and **ii**



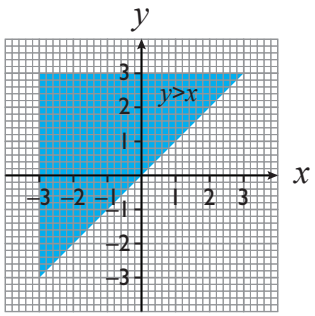
b i



ii



iii



9 a $x < -5$

b $x > 7$

c $x \geqslant = -1.5$

d $x \leqslant = -1$

e $x > 4$

f $x > 6$

Problem solving (pages 233–234)

1 **a** $2n - 7 < 5$

b 1, 2, 3, 4, 5

2 **a** $2(n - 8) > 11$

b 14, 15, 16, 17, 18, 19

3 **a** Sue is x, Ben is $2x$, Ceri is $x + 4$; so $4x + 4 < 28$

b Sue could be 3, 4, 5 years old

c Ben 6, 8, 10 years old; Ceri 7, 8, 9 years old

4 **a** Narinder is x, Rashmi is $x + 2$, Bhavinder is $2(x + 2)$ so $4x + 6 < 42$

b $x < 9$

c 10 years

5 **a** $4x + 10 > 21$; $4x + 10 < 33$

b Areas 24 to 50 cm^2

6 **a** $0.5m + 80 < 1m$

b $m > 160$

7 **a** **i** $x = 5$

ii $y = 1$

iii $y = 4$

b $y \leq x + 1, x \leq 5, y \geq 1, y \leq 4$

c $0 \leq l \leq \sqrt{34}$

8 **a** They reduce to $x < 7$, $x < 2$, $x > -1$, $x > 3$, B and D cannot both be satisfied.

b A, B and C can be true, if x is 0 or 1.

c A, B, C; A, C, D

Reviewing skills (page 234)

1 **a** $x < 4$

b $x > 11$

c $x \leq 2$

d $x \geq 2$

2 **a** $x > 1$

b $x > -2$

c $x > 2$

d $x < 5$

3 **a** Abbi is a; Bobbi is $2a$; Cathy is $a - 5$; $4a - 5 < 35$

b $a < 10$

c 18 years

Practising skills (page 237)

1 **a** $x = 6, y = 3$
b $x = 12, y = 4$
c $x = 3, y = 2$
d $x = -1, y = 2$

2 **a** $x = -2, y = -4$
b $x = 7, y = 3$
c $x = -2, y = -1$
d $x = 7, y = 5$

3 **a** $x = 3, y = 1$
b $x = -3, y = -3$
c $x = 4, y = 10$
d $x = 1, y = 9$

4 **a** $x = 5, y = 1$
b $x = 5, y = 1$
c $x = 5, y = 1$
d $x = 5, y = 1$

5 **a** $x = 8, y = 2$
b $x = -6, y = -2$
c $x = 4, y = 3$
d $x = 2, y = 4$

Developing fluency (pages 237–238)

1 **a** $x = 2y, x + y = 15$
b $x = 10, y = 5$

2 $x + y = 6, x - y = 1, x = 3.5, y = 2.5$

3 **a** $h = 2p, 3h + 4p = 35$
b $h = £7, p = £3.50$
c £24.50

4 **a** $r = m + 2, 3r + 5m = 62$
b $r = £9, m = £7$

5 $s - c = 7, 4s + 7c = 523$, stalls cost £52, circle cost £45

6 **a** $x + y = 42; 40y + 16x = 1080$
b adults $(y) = 17$
children $(x) = 25$

7 **a** Rearrange second equation to $x = y + 7$, then $x = 9, y = 2$
b Rearrange second equation to $x = 2y + 7$ then $x = 1, y = -3$

8 a $x = 17 - 3y$

b $5(17 - 3y) - 2y = 0$

$85 - 17y = 0$

$y = 5$

$x = 2$

$2 + 3(5) = 17$ TRUE

$5(2) - 2(5) = 0$ TRUE

Problem solving (pages 238–239)

1 a $m + n = 27$

$m + 15 = n$, or $n + 15 = m$

b 21 and 6 in either combination

2 a $X + Y = 12$

b $Y = 12 - X$

c In XY, X is the 'tens'. In YX, Y is the 'tens'. Thus the numbers are $(10X + Y)$ and $(10Y + X)$. The difference between the two numbers is 18 so $(10X + Y) - (10Y + X) = 18$.

d $9X - 9Y = 18$

Substitute for Y: $18X - 108 = 18$

$18X = 126$

$X = 7$; $Y = 5$

e XY is 75

3 a $c = 5 + a$

b $15a + 10c = 550$

c $a = 20$, $c = 25$

4 a $l - s = 100$, $6l = 10s$

b $s = 150$, $l = 250$

c 1250 g or 1.25 kg

5 Form equations, solves to find masses of cans as 240 g and 100 g.

Works out 90p can is the better buy.

Reviewing skills (page 240)

1 a $y = 3, x = 9$

b $x = 4, y = 8$

c $x = -5, y = -2$

d $x = -3, y = -16$

2 a $x = -4, y = -2$

b $x = 4, y = 8$

c $x = 4, y = -1$

d $x = 1, y = -1$

3 a $s = 3f$; $40f + 80s = 560$

b $f = 2$; $s = 6$

c 8

Practising skills (pages 243–244)

1 **a** $-4x$
b x
c $-2x$
d $5x$
e 0
f -6
g 45
h -8

2 **a** $x = 2, y = -1$
b $x = 5, y = 3$
c $x = -3, y = -1$
d $x = 2, y = -3$
e $x = -3, y = -2$
f $x = 1, y = 4$

3 **a** $x = 4, y = 1$
b $x = 5, y = -1$
c $x = 2, y = 6$
d $x = 1, y = -4$
e $x = 2, y = -2$
f $x = 5, y = -3$

4 **a** $x = 3, y = -2$
b $x = 1, y = -5$
c $x = 4, y = 5$
d $x = -3, y = 2$
e $x = 4, y = 3$
f $x = 6, y = -1$

5 **a** because the coefficients are different
b equation 2 by $\times 2$
c $x = 2, y = 3$

6 **a** $x = 4, y = -3$
b $x = 5, y = -2$
c $x = -3, y = 5$
d $x = 8, y = -1$
e $x = 1, y = -6$
f $x = -4, y = -5$

Developing fluency (pages 244–245)

1 a eq. 1 × 2 and eq. 2 × 3 to eliminate x, or eq. 1 × 3 and eq.2 × 4 to eliminate y.

b $x = 5, y = 2$

2 a $a = 3, b = 2$

b $x = 6, y = 1$

c $p = 5, q = -4$

d $x = 5, y = -1$

e $x = 3, y = -4$

f $m = 7, n = 4$

3 a $5a + 3b = 118$, $8a - 2b = 114$

b $a = 17, b = 11$

c Area rectangle $= (3(17) - 4(11)) \times (17 + 3(11)) = 350$

Area triangle $= \frac{1}{2} \times 2(17) \times (17 + 2(11)) = 663$

Triangle is bigger in area

4 a $5s + 3p = 99$, $2s + 4p = 62$

b $s = 15, p = 8$

c 86 km

5 a $2a + 3c = 24$, $3a + 5c = 38$

b $a = 6, c = 4$

c £138

6 a (Change the signs), $-x + 4y = 6$

(Add) $4x = 24$

b By changing the signs you are doing the equivalent of multiplying by −1. Adding negative values is the same as subtracting.

c $x = 6, y = 3$

Problem solving (page 246)

1 a $4b + a = 150$

$2b + a = 100$

b Banana is 25p and apple is 50p

c 575p

2 a $2w + 2l = 32$

$w + 2l = 26$

b $l = 10$ and $w = 6$

3 a $2d + 3c = 315$

$d + 2c = 175$

b £2275

4 a $4m + 6s = 54$

$45m + 60s = 570$

b 6 mountain bikes and 5 sports bikes

5 a $10c + 40t = 7.30$

$5c + 50t = 7.25$

b $c = 0.25$ $t = 0.12$

c £4.25

6 a $x + y + (360 - 3x) = 180$

b $x + 2y = 180$ (as the triangle is isosceles)

c $x + y + (360 - 3x) = 180$, so $180 + y = 2x$

Substituting $x + 2y = 180$, ($x = 180 - 2y$) we have $180 + y = 2(180 - 2y)$

$180 + y = 360 - 4y$, $5y = 180$, $y = 36$ so $x = 108$

as $108 = 3 \times 36$, $x = 3y$

d As $x = 3y$ and $x + 2y = 180$, then $3y + 2y = 180$, $5y = 180$, $y = 36$

Reviewing skills (page 247)

1 a $x = 2, y = -1$

b $x = 4, y = 2$

c $x = 3, y = -3$

2 a $x = 12, y = 5$

b $x = 4, y = -3$

c $x = 5, y = -2$

3 a $x = 2, y = 0$

b $x = 5, y = 3$

c $x = 8, y = 3$

4 a $4p + 3a = 75$

$3p + a = 45$

b A peach tree is £12 and an apple tree is £9

c £99

Practising skills (pages 249–250)

1 a (3, 1)

b $x = 3, y = 1$

c $3 + 1 = 4, 1 = 3 - 2$

2 a (1, 1)

b i $x = 1, y = 1$

ii $16x - 12 = 3x + 1, x = 1, y = 1$

3 a m is $y = x$, e is $3y = 4x - 1$

b (1, 1)

c i $x = 1, y = 1$

ii $3x = 4x - 1$ or $3y = 4y - 1, x = 1, y = 1$

4 a m then e

b (1, 2)

c $3y = x + 5$ and $x + 2y = 5$

d Proof by substitution

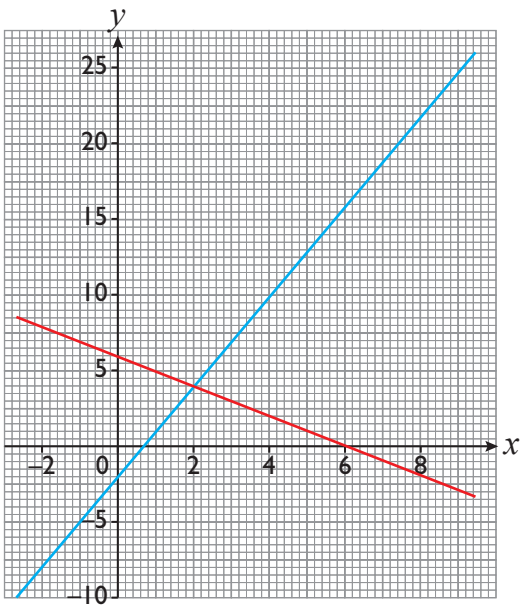
5 a

x	0	1	2	3
$3x$	0	3	6	9
-2	-2	-2	-2	-2
$y = 3x - 2$	-2	1	4	7

b

x	0	1	2	3
6	6	6	6	6
$-x$	0	-1	-2	-3
$y = 6 - x$	6	5	4	3

c

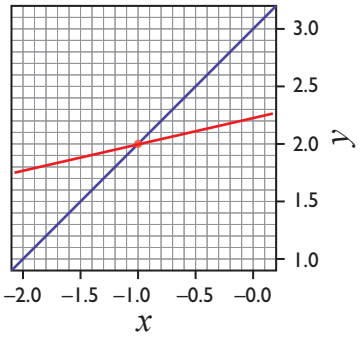


d (2, 4)

e $x = 2, y = 4$

Substitution method: $6 - x = 3x - 2$

6 a



b $(-1, 2)$

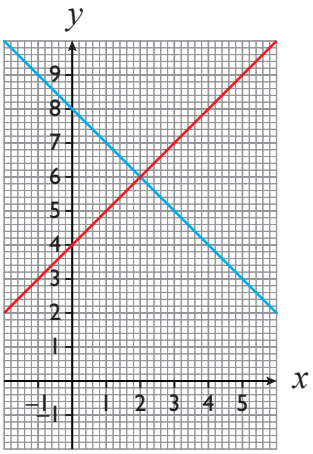
7 a

x	0	1	2	3
$y = 8 - x$	8	7	6	5

b

x	0	1	2	3
$y = x + 4$	4	5	6	7

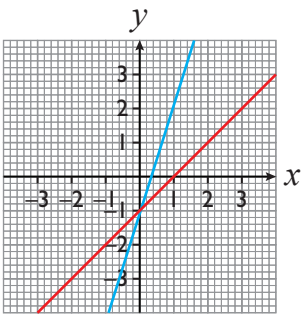
c



d $(2, 6)$ $x = 2, y = 6$

e $x = 2, y = 6$ via substitution, $x + 4 = 8 - x$

8 a



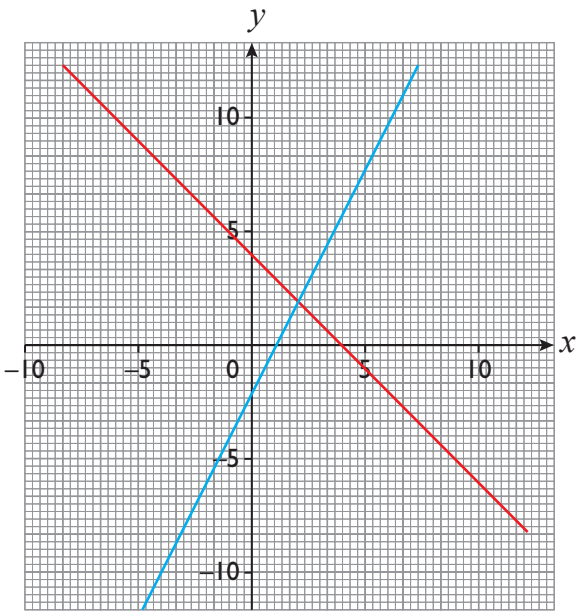
b $(0, -1)$, $x = 0, y = -1$

c $x - 1 = 3x - 1, x = 0, y = -1$

Developing fluency (pages 251–252)

1 a $4x = 2x^2 \quad 4 = 2x \quad x = 2 \quad y = 8$

b

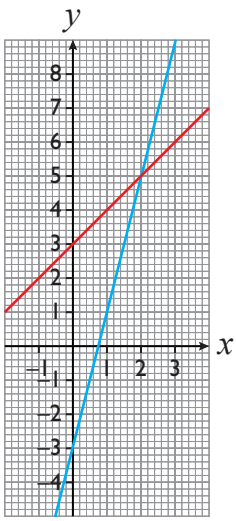


c check graph

2 a $x + 3 = 4x - 3$

$x = 2, y = 5$

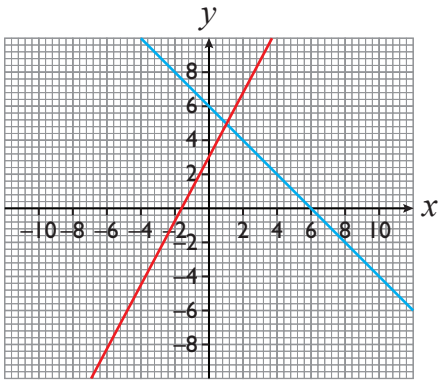
b



c check graph

d $5 = 8 - 3, 5 = 2 + 3$

3 a (1, 5)

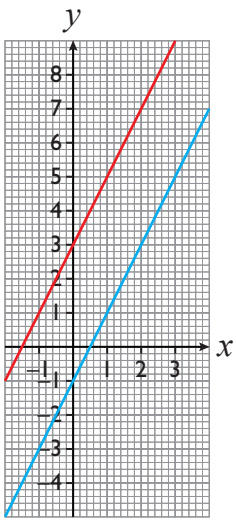


b 5 = 3 + 3, 1 + 5 = 6

4 a There is no solution, $2x + 3$ can never be equal to $2x - 1$.

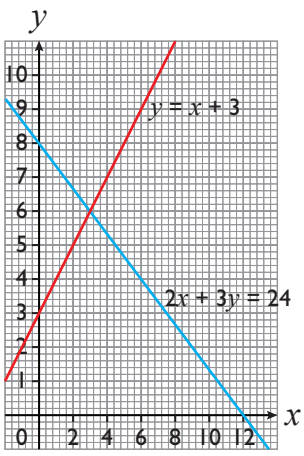
b The coefficients of x are the same, the coefficients of y are the same.

c



d The lines are parallel so there is no solution, they will never intersect.

5 a



b $x = 3$ and $y = 6$

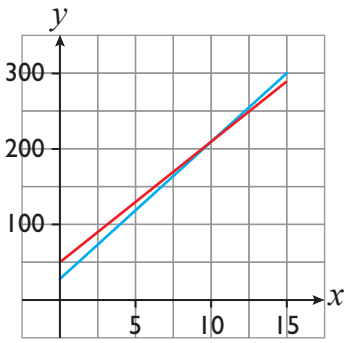
c Students' own check.

d 40.5 square units

6 a i $C = 30 + 18u$

ii $C = 50 + 16u$

b

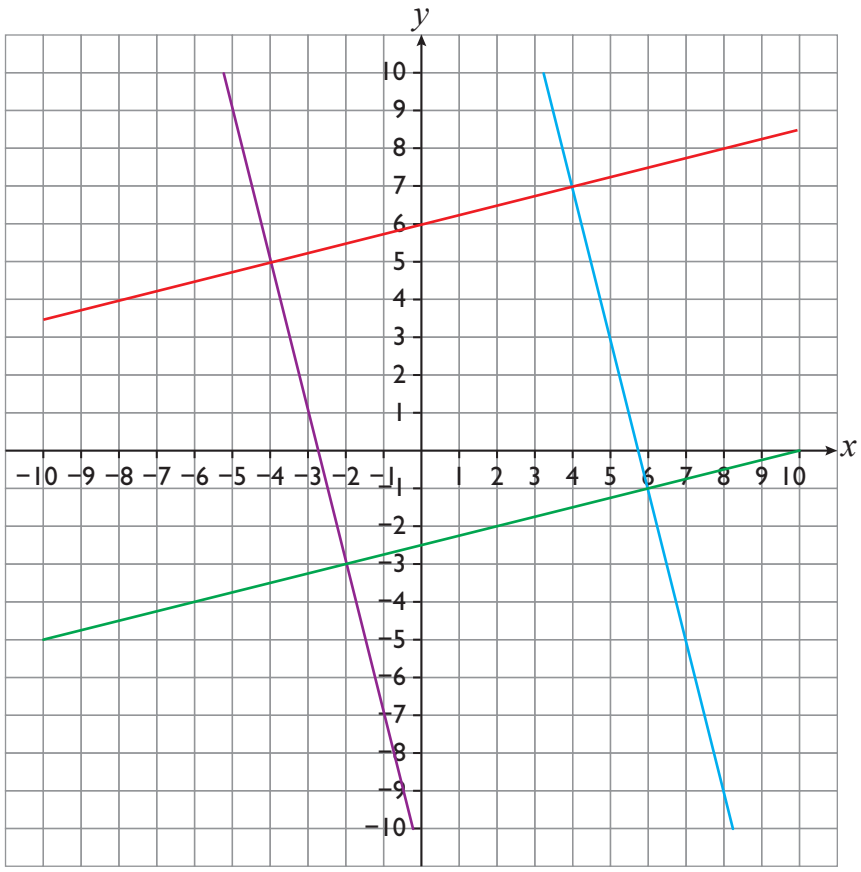


c i $u = 10$; £2.10

ii Students' own check.

d Sparkle

7 a



b (−4, 5), (4, 7),(6, −1) and (−2, −3)

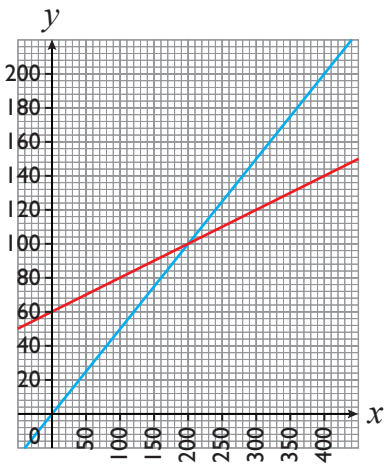
c Students' own check.

d Square

Problem solving (pages 252–254)

1 a $C = 0.5m$, $C = 60 + 0.2m$

b



c Cars 2 go

2 a

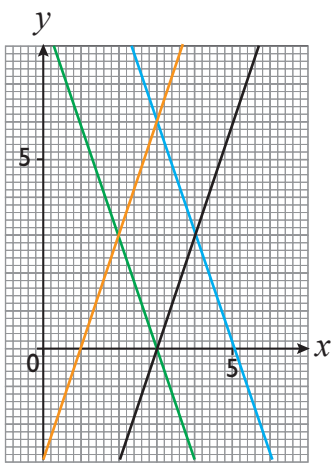
x	0	1	2	3	4	5
$3x$	0	3	6	9	12	15
-3	−3	−3	−3	−3	−3	−3
$y = 3x - 3$	-3	0	3	6	9	12

x	0	1	2	3	4	5
$3x$	0	3	6	9	12	15
-9	−9	−9	−9	−9	−9	−9
$y = 3x - 9$	−9	−6	−3	0	3	6

x	0	1	2	3	4	5
$-3x$	0	−3	−6	−9	−12	−15
$+15$	+15	+15	+15	+15	+15	+15
$y = -3x + 15$	15	12	9	6	3	0

x	0	1	2	3	4	5
$-3x$	0	−3	−6	−9	−12	−15
$+9$	+9	+9	+9	+9	+9	+9
$y = -3x + 9$	9	6	3	0	−3	−6

b

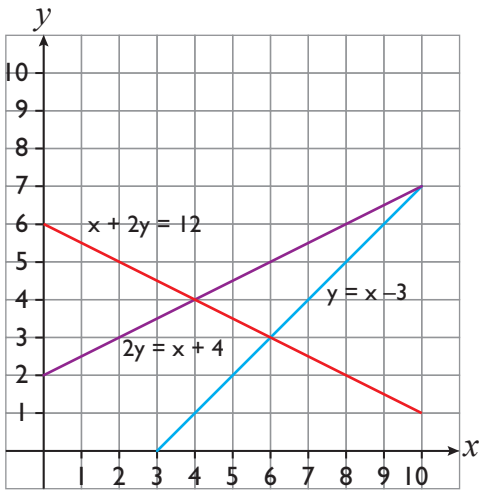


c P: (3, 0), Q: (2, 3), R: (3, 6), S: (4, 3)

d Students' check their answer.

e rhombus

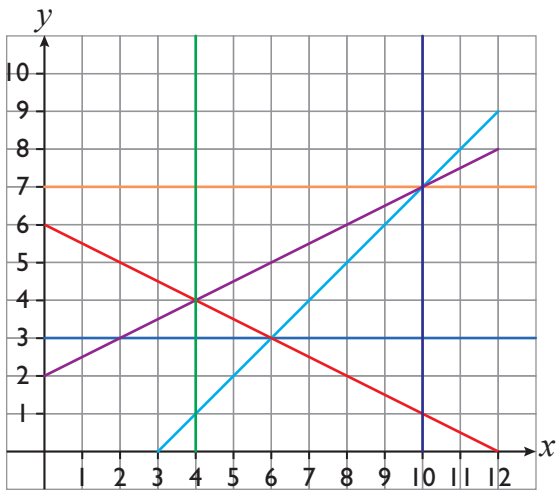
3 a



b i (4, 4), (6, 3), (10, 7)

ii A: $x = 4$ and $y = 4$; B: $x = 6$ and $y = 3$; C: $x = 10$ and $y = 7$

c i



ii $x = 4$, $x = 10$, $y = 3$, $y = 7$

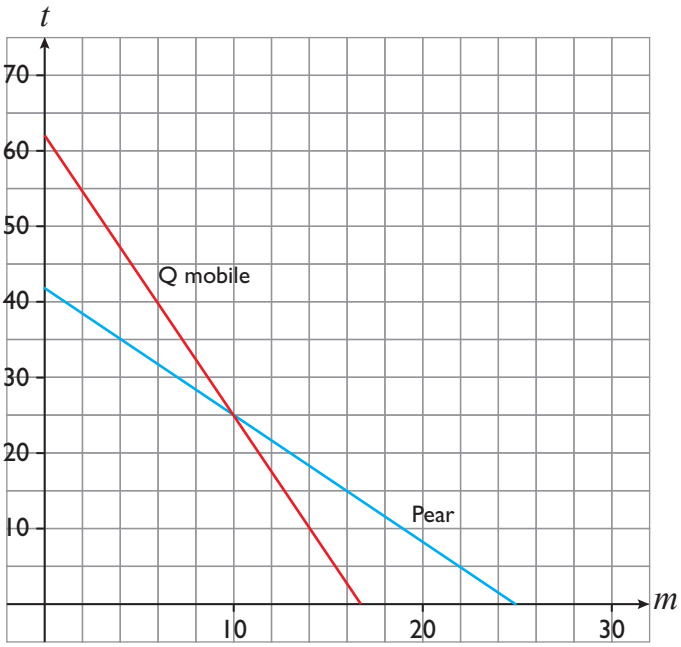
iii 24 square units

d 6 square units

4 a i $30m + 8t = 500$

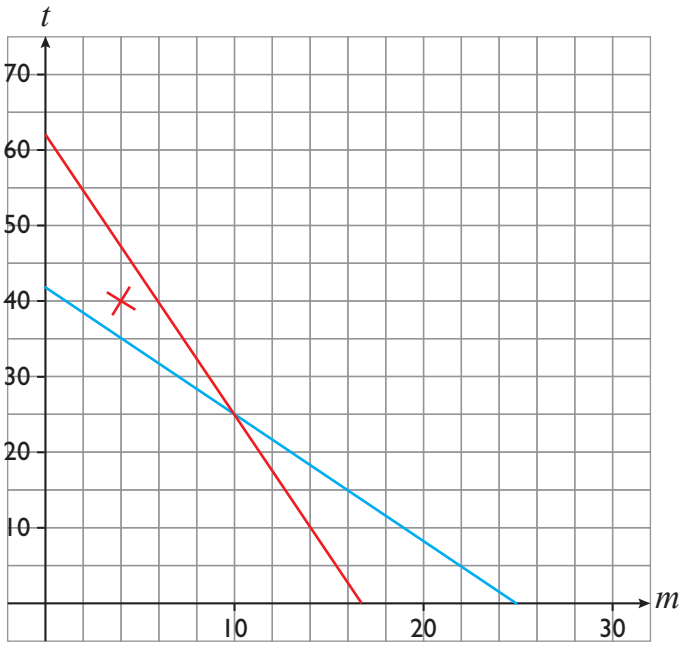
ii $20m + 12t = 500$

b



c 10 minutes of calls, 25 texts

d i



ii Daisy would have to top up 60p.

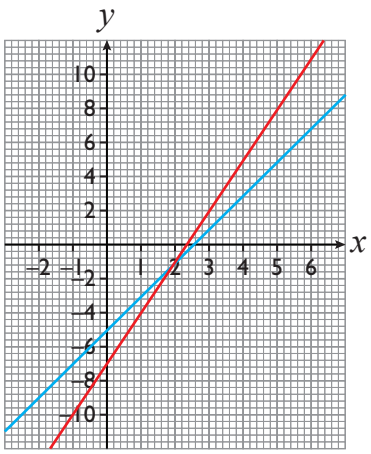
iii Chloe would have 60p credit left.

5 (0.5, 2) (−1, −1.5) (please note that the answers have been rounded)

6 (−1, 2) (2.5, −1) (please note that the answers have been rounded)

Reviewing skills (page 254)

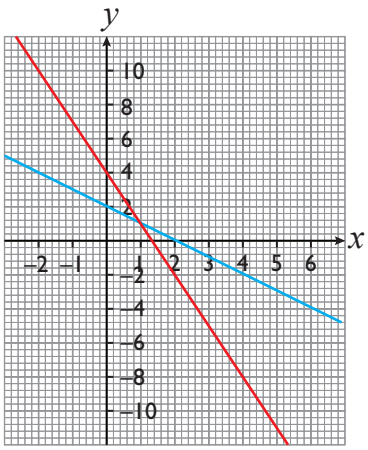
1 a



b (2, −1)

c $-1 = 4 - 5$; $-1 = 6 - 7$

2 a



$x = -1$, $y = 1$.

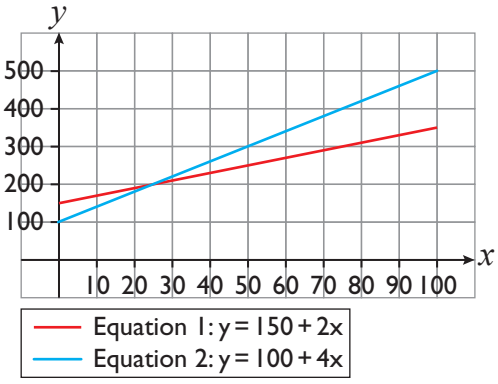
b $1 = -3 + 4$; $1 = -1 + 2$

c Students' check their answer.

3 a i $C = 150 + 2m$

ii $C = 100 + 4m$

b



c i 25 miles; £200

ii Students' own check.

d i Gwilym's Coaches

ii £110

Practising skills (pages 257–258)

1 (5, 9)

2 (−1, 4)

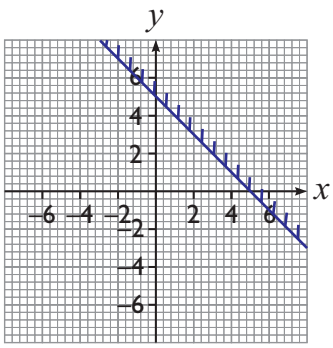
3 **a** $y < x$

b $x + y \geqslant 1$

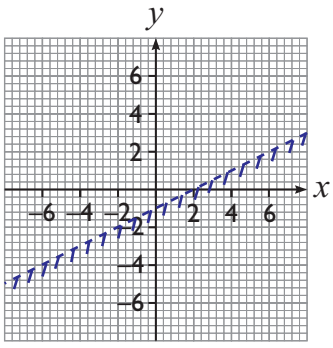
c $y > 2x - 3$

d $y < 5 - 2x$

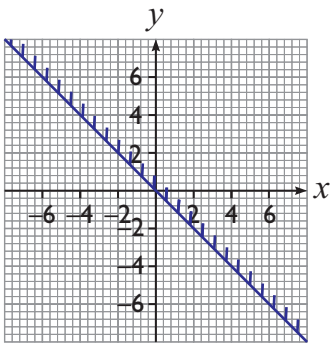
4 **a**



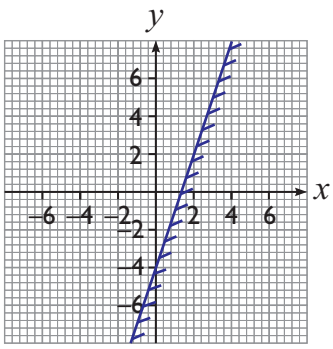
b



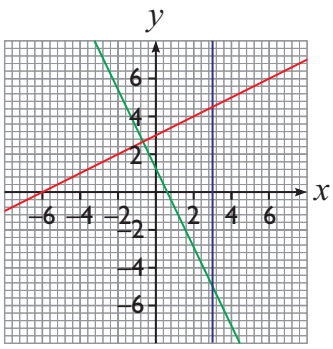
c



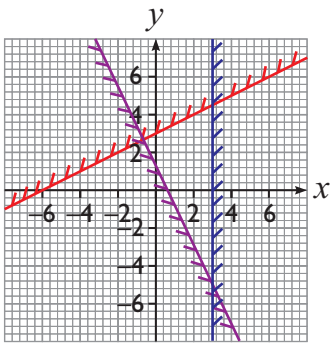
d



5 a



b



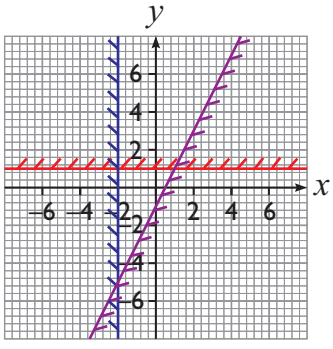
6 $y \leqslant 5, y + 3x + 4 > 0, y > 3x - 1$

Developing fluency (pages 259–260)

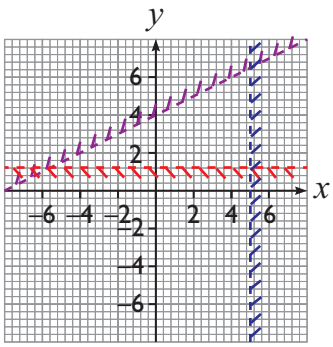
1 c is correct.

2 b is correct.

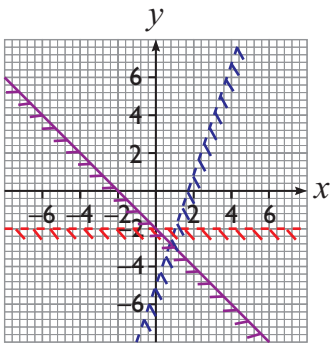
3 a



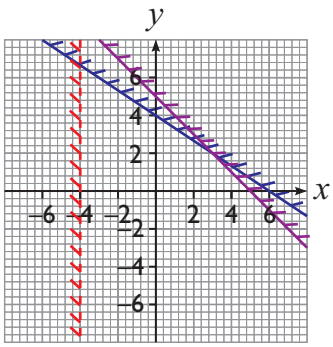
b



c



d



4 **a** $\{(x, y) : x < y - 2, x + 2y \leqslant 6, x \geqslant -2\}$

b $\{(x, y) : x \geqslant -4, 3y < 2x - 1, x - 2y \leqslant 6\}$

c $\{(x, y) : x \leqslant 5 - 2y, 2x < y, 3x + 2y > -6\}$

d $\{(x, y) : x > 2y - 3, 3x - 1 \leqslant y, 2x + 3y > -6\}$

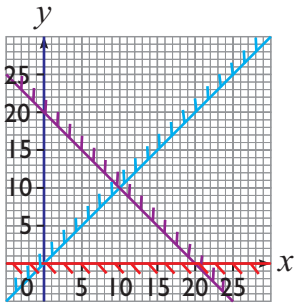
5 (1, 0), (−1, 0), (−1, 1), (−1, 2), (0, 0), (0, 1)

6 **a** $x + y$ is the total number of chocolates and is less than or equal to 20

b The number of chocolates cannot be negative, $y \geqslant 0$ also.

c $y < x$

d



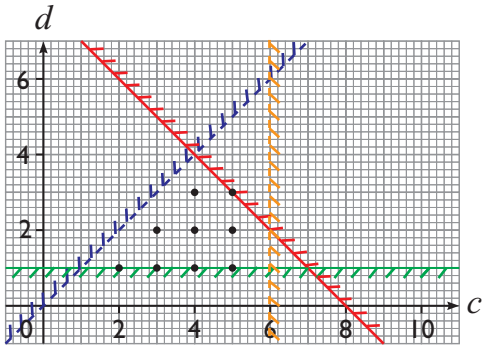
7 a i $c + d \leqslant 8$

ii $c > d$

iii $d \geqslant 1$

iv $c < 6$

b i



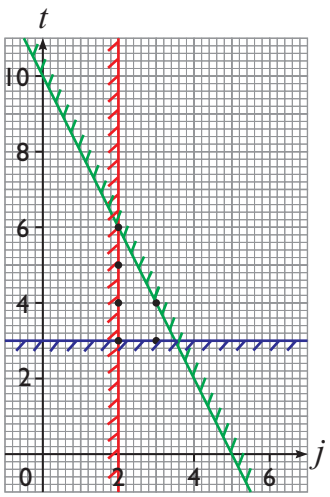
ii 9

8 a i $j \geqslant 2$

ii $t \geqslant 3$

iii $40j + 20t \leqslant 200$

b i

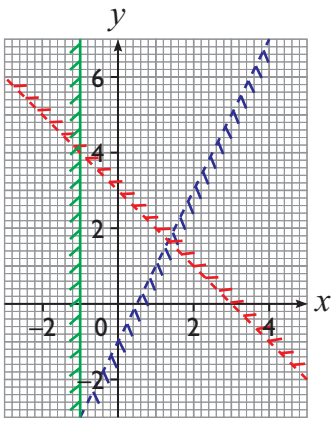


ii 6

Problem solving (pages 260–261)

1 $y \geqslant -1, x \geqslant -1, 3y \leqslant 2x + 3, 2y + 3x \leqslant 6$

2 a

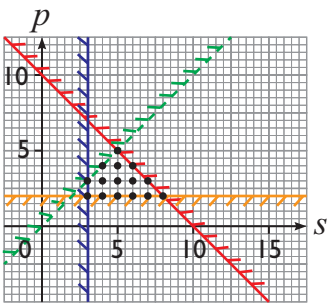


b No

c $y \geqslant 2x - 1$

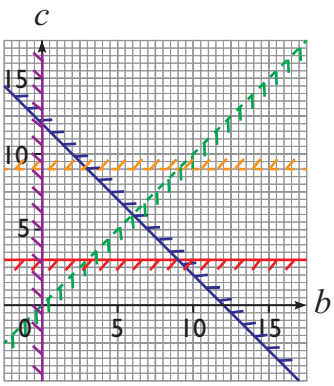
3 a $s + p \leqslant 10, p \geqslant 2, s \geqslant 3, s > p$

b



4 a $b \geqslant 0, 3 \leqslant c < 9, c > b, b + c \leqslant 12$

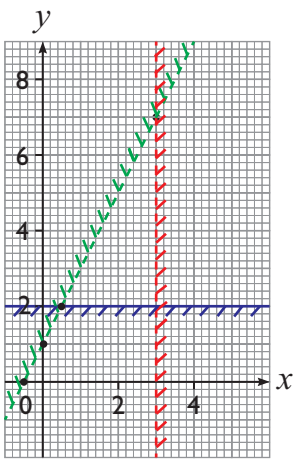
b



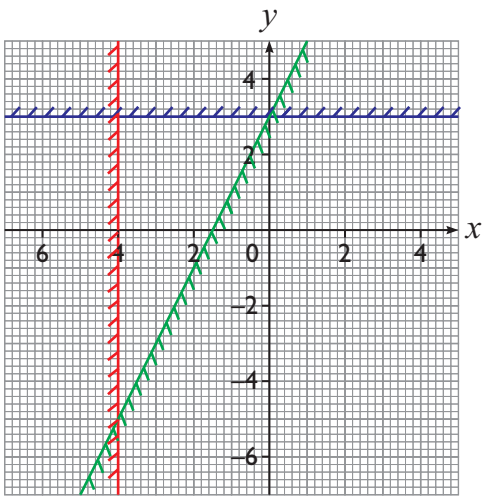
c 29

Reviewing skills (page 262)

1



2



3 $\{x < 3, y \geqslant -1, y \leqslant 4, y < 2x + 1\}$

Practising skills (page 265)

1 **a** $3x(x-5)-2(x-1)=3x^2-15x-2x+2=3x^2-17x+2$

b $\frac{1}{n}+\frac{1}{2n}=\frac{2}{2n}+\frac{1}{2n}=\frac{3}{2n}$

c $(2n+1)^3-(2n-1)^3=8n^3+12n^2+6n+1-(8n^3-12n^2+6n-1)=24n^2+2$

d $\frac{x^2-4x-5}{x^2+7x+6}=\frac{(x-5)(x+1)}{(x+6)(x+1)}=\frac{(x-5)}{(x+6)}$

2 $2n$, $4n$ $6n+4$

3 **a** Let a be one number, $(a+1)$ be the next number: $a+(a+1)=2a+1$.
$2a+1$ cannot divide by 2 with a whole number result.

b $2a+1$ is any odd number. $(2a+1)(2a+1)=4a^2+4a+1$. Always odd since $4a^2$ is even (divisible by 2), $4a$ is even (divisible by 2), +1 (not divisible by 2), so gives an odd number.

c $(2a+1)(2a)=4a^2+2a$. Always even, as terms divide by 2.

4 First odd number is $2a+1$, other odd number is $2a+1+2a+1+1=4a+3$ so
$(4a+3)-(2a+1)=2a+2$, divisible by 2.

5 $(n+1)(n+1)(n+1)-(n-1)(n-1)(n-1)=n^3+3n^2+6n+1-(n^3-3n^2-6n+1)=6n^2+12n+2$, which is divisible by 2.

6 When $n=1$; $n^2+2n+4=7$, prime.
When $n=2$; $n^2+2n+4=12$, not prime.

7 $2a$ is any even number: $2a\times 2a=4a^2$ which is divisible by 2, so even.

8 $2a+1$ is any odd number: $(2a+1)(2a+1)=4a^2+4a+1$. Always odd since $4a^2$ is even (divisible by 2), $4a$ is even (divisible by 2), +1 (not divisible by 2), so gives an odd number.

Developing fluency (pages 265–266)

1 **a** Always even, all terms divisible by 2.

b Always odd, even term plus 1 makes odd term.

c Sometimes odd. When n is even, $5n-1$ is odd. When n is odd, $5n-1$ is even.

d Always odd. An always even term $(4n)$ plus 3 makes an odd term.

2 First number (a) squared= a^2. Second number $(a+1)$ squared= $(a+1)^2=a^2+2a+1$.
$a^2+2a+1-a^2=2a+1$, which is always odd.

3 **a** First number (a), second number $(a+1)$, third number $(a+2)$.
$a+a+1+a+2=3a+3$, which is divisible by 3.

b First even number $(2a)$, second number $(2a+2)$, third number $(2a+4)$.
$2a+2a+2+2a+4=6a+6$, which is divisible by 6.

4 For example:

a Square: 1, 2; 11, 12. $1 \times 12 = 12$; $2 \times 11 = 22$. $22 - 12 = 10$

b Square: 23, 24; 33, 34. $23 \times 34 = 782$. $24 \times 33 = 792$. $792 - 782 = 10$

c First number (a), second number ($a + 1$), third number ($a + 10$), forth number ($a + 11$).
First multiplication: $a \times (a + 11) = a^2 + 11a$
Second multiplication: $(a + 1)(a + 10) = a^2 + 11a + 10$
Difference: $a^2 + 11a + 10 - (a^2 + 11a) = 10$, for any value of a

5 For example:

a Square from 1 to 5 and 41 to 45. $1 \times 45 = 45$; $5 \times 41 = 205$.
$205 - 45 = 160$

b Square 56 to 60 and 96 to 100. $56 \times 100 = 5600$. $60 \times 96 = 5760$.
$5760 - 5600 = 160$

c First number (a), second number ($a + 4$), third number ($a + 40$), fourth number ($a + 44$).
First multiplication: $a \times (a + 44) = a^2 + 44a$
Second multiplication: $(a + 4)(a + 40) = a^2 + 44a + 160$
Difference: $a^2 + 44a + 160 - (a^2 + 44a) = 160$, for any value of a

6 For example: $321 - 123 = 198$. $198 = 2 \times 99$.
Number abc. a is hundreds, b is tens, c is units. Reversed number abc. c is hundreds, b is tens, a is units.
$100c + 10b + a - (100a + 10b + c) = 99c - 99a$, which is divisible by 99.

7 **a** **i** $Area = c^2 + 4\left(\frac{1}{2} \times b \times a\right) = 2ab + c^2$

ii $Area = (a + b)^2 = a^2 + 2ab + b^2$

b **i** $a^2 + 2ab + b^2 = 2ab + c^2$
$a^2 + b^2 = c^2$

ii Pythagoras

8 Areas A + B + C = 15
Areas A + B + C + D = x^2
Area D = $x^2 - 15 = 1 \times (x^2 - 15)$
Long side of D = $x^2 - 15$
Sides of A are $x - 1$ and $x^2 - 15$
Therefore area A = $(x - 1)(x^2 - 15) = x^3 - x^2 - 15x + 15$
Area A + Area D = 5, so $x^3 - x^2 - 15x + 15 + x^2 - 15 = 5$
$x^3 = 15x + 5$

9 Area trapezium = $\frac{a+b}{2}h$
$= \frac{(x+1)+(x+5)}{2} \times 2x = \frac{2x+6}{2} \times 2x = 2x(x+3) = 2x^2 + 6x = 13.5$
$2x^2 + 6x - 13.5 = 0$
$x = -4.5,\ 1.5$
Length of slope $y^2 + (2x)^2 + (x + 5 - x - 1)^2 = 4x^2 + 16$
$y = \sqrt{4x^2 + 16}$, when $x = 1.5$, $y = 5$.
Perimeter = 1.5 + 1 + 5 + 1.5 + 5 + 2(1.5) = 17 cm

Problem solving (page 267)

1 $a + a + 1 + a + 2 + a + 3 = 4a + 6$, which is divisible by 2.

2 $(a + 2)^2 - a^2 + = 4a + 4$, is double $a + a + 2 = 2a + 2$.

3 $3(a+1)^2 - 3a^2 = 3(a^2 + 2a + 1) - 3a^2 = 6a + 3$, is divisible by 3, not divisible by 2.

4 $(a)^3 + (a+1)^3 + (a+2)^3 = 3a^3 + 9a^2 + 18a + 9$, is divisible by 3.

5 C is guilty.

A cannot be the murdered person because in that case 'E is innocent' would be false and there would be 2 guilty people. So A and E are innocent.

If B is guilty, the statement 'One of A, C and E committed the murder' is false. But if B is innocent it is true. So one of B, A, C and E is guilty. However, A and E are known to be innocent. So either B or C is guilty and therefore D is innocent.

C's statement gives no useful information.

D is known to be innocent so either B or C is guilty but this is already known.

E is known to be innocent so E's statement 'B is innocent' is true.

So the murderer must be C.

Reviewing skills (page 267)

1 a i $2n$

ii $2n + 1$

b i Odd number 1 is $2n + 1$, odd number 2 is $4n + 1$. $4n + 1 - 2n + 1 = 2n$. Even.

ii $2n(2n + 1) = 4n^2 + 2n$, which is divisible by 2, so always even.

2 a $= 25n^2 + 10n + 1 - (25n^2 - 10n + 1) = 20n$, which is divisible by 4.

b $= a^2n^2 + 2an + 1 - (a^2n^2 - 2an + 1) = 4an$, which is divisible by 4.

3 a $a + a + 1 = 2a + 1$

b $(a+1)^2 - a^2 = a^2 + 2a + 1 - a^2 = 2a + 1$

Practising skills (pages 271–272)

1 **a** $x^2 + 3x$
b $x^2 - x$
c $x^2 - 25$
d $x^2 + x - 6$
e $x^2 - 3x - 40$
f $x^2 - 5x + 4$
g $x^2 - 4$

2 **a** added 7; multiplied 12
b added 8; multiplied 12
c added 5; multiplied 4
d added 4; multiplied −5
e added 6; multiplied 9
f added 6; multiplied −16
g added −7; multiplied 12
h added 0; multiplied −4

3 **a** 4 and 2
b 4 and −2
c −4 and 2
d −4 and −2
e 8 and 1
f 8 and −1
g −8 and 1
h −8 and −1

4 **a** −3 and −2
b 3 and 2
c −3 and 2
d 3 and −2
e 6 and 1
f 6 and −1
g −6 and −1
h −6 and 1

5 **a** $2(a + 5) + 7(a + 5) = \boxed{9}(a + 5)$
b $12(b - 6) - 3(b - 6) = \boxed{9}(b - 6)$
c $8(c + 2) + (c + 2) = \boxed{9}(c + 2)$
d $-2(d - 7) + (d - 7) = \boxed{-1}(d - 7)$

6 **a** $x(x + 5) + 7(x + 5) = (\boxed{x} + \boxed{7})(x + 5)$
b $x(x - 6) - 3(x - 6) = (\boxed{x} - \boxed{3})(x - 6)$
c $x(x + 2) + (x + 2) = (\boxed{x} + \boxed{1})(x + 2)$
d $-x(x - 7) + (x - 7) = (\boxed{-}x + \boxed{1})(x - 7)$

7 a $x^2 + 2x + \boxed{4}x + 8$

$x(x + 2) + \boxed{4}(x + 2)$

$\left(x + \boxed{4}\right)(x + 2)$

b $x^2 - 3x - \boxed{5}x + 15$

$x(x - 3) - \boxed{5}\left(x - \boxed{3}\right)$

$\left(x - \boxed{5}\right)\left(x - \boxed{3}\right)$

c $x^2 + 4x - \boxed{6}x - 24$

$x\left(x + \boxed{4}\right) - \boxed{6}\left(x + \boxed{4}\right)$

$\left(x - \boxed{6}\right)\left(x + \boxed{4}\right)$

d $x^2 + \boxed{2}x - 1x - 2$

$x\left(x + \boxed{2}\right) - 1\left(x + \boxed{2}\right)$

$\left(x - \boxed{1}\right)\left(x + \boxed{2}\right)$

8 a $(x + 3)(x + 5)$

b $(x - 4)(x - 5)$

c $(x + 4)(x + 4)$

d $(x + 7)(x - 3)$

e $(x + 2)(x - 3)$

f $(x + 16)(x + 1)$

g $(x + 1)(x - 16)$

h $(x - 10)(x + 2)$

9 a $x(x + 8)$

b $(x - 10)(x + 10)$

c $x(x - 16)$

d $(x - 4)(x + 4)$

e $(x + 1)(x + 6)$

f $(x - 1)(x + 1)$

g $(x - 12)(x + 12)$

h $(x - \sqrt{5})(x + \sqrt{5})$

Developing fluency (pages 273–274)

1 a PKXM = ac, KSNX = ad, MQLX = bc and XLRN = bd, assuming X is the middle apex

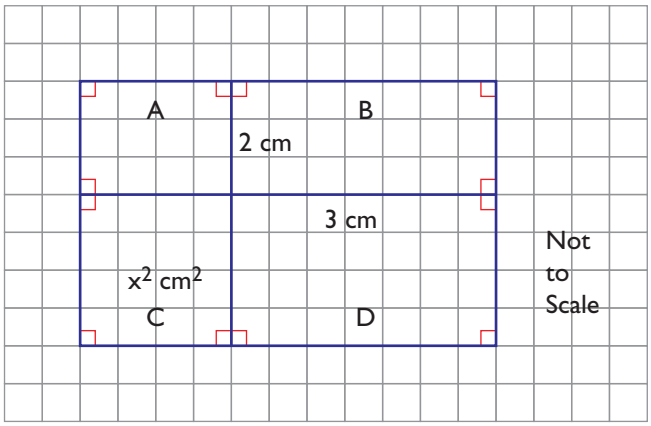
b i PMNS

ii MQRN

iii PQRS

c $(a + b)(c + d)$

2 a



A is $x \times 2$ cm^2

B is 3×2 cm^2

C is $x \times x$ cm^2

D is $x \times 3$ cm^2

b $(x + 3)$ and $(x + 2)$

c Proof: Area is $x^2 + 3x + 2x + 6$ which equals $x^2 + 5x + 6 = (x + 3)(x + 2)$

d Proof: $(x + 3)(x + 2) = x^2 + 3x + 2x + 6$

3 a

$2x$	8
x^2	$4x$

b Sides are $x + 2$ and $x + 4$

perimeter $= 2[(x + 2) + (x + 4)] = 2[2x + 6] = 4x + 12$

area $= x^2 + 2x + 4x + 8 = x^2 + 6x + 8$

c Proof: Area is $x^2 + 2x + 4x + 8 = x^2 + 6x + 8 = (x + 2)(x + 4)$

d Proof: $x^2 + 6x + 8 = x^2 + 2x + 4x + 8 = (x + 2)(x + 4)$

4 a Figure 1: a^2; Figure 2: $a(a - b) + b(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$; Figure 3: $a(a - b) + b(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$ OR $(a + b)(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$.

b $(a + b)(a - b)$

c Proof: Area Figure 2 equals area Figure 3. $a^2 - b^2 = (a + b)(a - b)$

d Proof: Expand $(a + b)(a - b)$ to get $a^2 - ab + ab - b^2 = a^2 - b^2$

5 a i $x^2 + 11x + 10$

ii $x^2 + 7x + 10$

b $4x$

c Proof: $(2 + 10)(2 + 1) - (2 + 2)(2 + 5) = 36 - 28 = 8 = 4(2)$

6 a i $x^2 + 10x + 21$

ii $x^2 + 9x + 14$

b $x + 7$

c Factorise with $(x + 7)$: $(x + 7)((x + 3) - (x + 2)) = (x + 7)(x + 3 - x - 2) = (x + 7)(1) = (x + 7)$

Problem solving (page 274)

1 $64^2 - 36^2 = (64 + 36)(64 - 36) = 2800$ cm^2

2 Let the integer be n, then $n^2 - 4 = (n + 2)(n - 2)$, and if $n > 3$, n – 2 must be at least 2, which means the product is a composite number (with two different factors), it cannot be prime.

Reviewing skills (page 274)

1 **a** $(x-4)(x-2)$

b $(x-3)(x+4)$

c $(x-4)(x+3)$

d $(x-2)(x+5)$

e $(x-5)(x+2)$

f $(x-4)(x-4)$

g $(x-7)(x+7)$

h $(x-\sqrt{3})(x+\sqrt{3})$

2 Proof: $x^2+8x+7-(x^2+8x+12)+x^2-4=x^2-9=(x+3)(x-3)$

Practising skills (page 277)

1 **a** $x = -4$ or $x = -1$
b $x = -7$ or $x = 3$
c $x = 1$ or $x = 1$
d $x = -2$ or $x = -1$

2 **a** $(x - 3)(x - 4)$; $x = 3$ or $x = 4$
b $(x + 1)(x - 2)$; $x = -1$ and $x = 2$
c $(x - 3)(x + 5)$; $x = 3$ and $x = -5$
d $(x + 5)(x + 1)$; $x = -5$ and $x = -1$
e $(x + 2)(x - 2)$; $x = -2$ and $x = 2$
f $x(x - 7)$; $x = 0$ and $x = 7$
g $(x + 4)(x - 3)$; $x = 3$ and $x = -4$
h $(x + 3)(x - 3)$; $x = -3$ and $x = 3$

3 **a** $x^2 + 4x + 3 = 0$; $(x + 3)(x + 1) = 0$; $x = -1$ and $x = -3$
b $x^2 + 2x - 8 = 0$; $(x + 4)(x - 2) = 0$; $x = 2$ and $x = -4$
c $x^2 + x = 0$; $x(x + 1) = 0$; $x = 0$ and $x = -1$
d $x^2 - 3x - 4 = 0$; $(x - 4)(x + 1) = 0$; $x = 4$ and $x = -1$
e $x^2 - 49 = 0$; $(x + 7)(x - 7) = 0$; $x = -7$ and $x = 7$

4 **a** $(x + 8)(x - 8) = 0$; $x = -8$ and $x = 8$
b $x^2 - 9x - 36 = 0$; $(x - 12)(x + 3) = 0$; $x = -3$ and $x = 12$
c $x = 2$ and $x = -8$
d $(x + 2)(x - 1) = 0$; $x = -2$ and $x = 1$
e $x(x - 9) = 0$; $x = 0$ and $x = 9$

5 **a** $x^2 - 8x + 15 = 0$
b $x^2 + 8x + 15 = 0$
c $x^2 + 2x - 48 = 0$
d $x^2 - 1 = 0$
e $x^2 + x - 90 = 0$
f $x^2 + 6x = 0$

Developing fluency (pages 277–278)

1 **a** $x(x + 4)$
b $x(x + 4) = 45$; $x^2 + 4x - 45 = 0$
c $(x + 9)(x - 5) = 0$; $x = -9$ or 5
d Taking positive value for x, 5 cm and 9 cm

2 **a** $x(x - 2) = 48$; $x^2 - 2x - 48 = 0$, $x = 8$ and -6
b Taking positive value for x, 8 cm by 6 cm

3 $\frac{1}{2}x(x + 6) = 8$; $x^2 + 6x - 16 = 0$; $x = 2$ and -8. Taking positive value for x, 2 cm base and 8 cm height.

4 $\frac{1}{2}x(x-2+x+6)=35$; $x^2+2x-35=0$; $x=5$ and -7. Taking positive value for x, 11 cm base, 3 cm for the parallel side and 5 cm height.

Problem solving (page 278)

1 **a** length $=(x^2-64)\div(x-8)=(x+8)$

b $x^2-64=36$

$x^2-100=0$

$(x+10)(x-10)=0$

$x=10, x=-10$

Taking positive value for x, length = 18; width = 2

c $x-8=\frac{x+8}{2}$

$2(x-8)=(x+8)$

$2x-16=x+8$

$x=24$

d Area of square = 225cm^2 which gives $x=17$; therefore perimeter = 68cm

2 **a** $5t(t-9)=0$; $t=0$ or $t=9$ seconds

b $5t(t-9)=100$; $t=4$ or $t=5$ seconds

3 $(x+6)(x-5)=26$; $x^2+x-30=26$; $x^2+x-56=0$; $(x+8)(x-7)$. Taking positive value for x, $x=7$ so length is 13 cm and width is 2 cm.

4 $(x-5)^2=4$, so $x-5=\pm2$; x is 3 or 7

5 **a** $(x+5)^2-25=24$; $x^2+10x-24=0$; $(x+12)(x-2)=$ so $x=2$. Then $x+5=7$, the length is 7 cm.

b The other value of x is -12 and you can't have a negative length.

Reviewing skills (page 279)

1 **a** $x=-2, -3$

b $x=2, 4$

c $x=2, -1$

d $x=4, -7$

e $x=-1, -4$

f $x=2, -2$

g $x=5, -3$

h $x=\sqrt{10}, -\sqrt{10}$

2 $x^2-3x-54=0$; $x=9$ and -6; Taking positive value for x, the rectangle is 9 cm by 6 cm.

Practising skills (pages 283–285)

1

Pair of numbers	Product	Sum
3 and 4	12	7
−5 and 2	−10	3
−2 and 3	−6	1
−3 and −2	6	−5
−2 and 8	−16	6
−4 and 6	−24	2

2 **a** $= 2x^2 - 12x + 5x = 30$
$= 2x(x - 6) + 5(x - 6)$
$= (2x + 5)(x - 6)$

b $= 5x^2 + 9x - 5x - 9$
$= x(5x + 9) - 1(5x + 9)$
$= (x - 1)(5x + 9)$

c $= 3x^2 + 21x - 2x - 14$
$= 3x(x + 7) - 2(x + 7)$
$= (3x - 2)(x + 7)$

d $= 7x^2 + 14x - 6x - 12$
$= 7x(x + 2) - 6(x + 2)$
$= (7x - 6)(x + 2)$

e $= 4x^2 + 2x + 10x + 5 = (2x + 5)(2x + 1)$

f $= 8x^2 + 12x - 6x + 5 = (4x - 3)(2x + 3)$

g $= 6x^2 + 4x - 15x - 10 = (2x - 5)(3x + 2)$

3 **a** $(2x + 3)(x - 4)$

b $(x - 5)(3x + 1)$

c $(5x - 2)(x + 3)$

d $(2x + 1)(x + 7)$

e $(3x - 4)(x + 2)$

4 **a** $(2x + 3)(2x + 1)$

b $(3x - 2)(2x + 3)$

c $(3x - 4)(3x + 2)$

d $(4x - 5)(x - 1)$

e $(4x - 3)(x + 3)$

5 **a** $(3x - 2)(3x + 2)$

b $(x - 5y)(x + 5y)$

c $(2a - 5b)(2a + 5b)$

d $(5xy - 3)(5xy + 3)$

e $(1 - 3x)(1 + 3x)$

6 **a** $-4, \frac{1}{2}$

b $-\frac{1}{3}, 3$

c $-2\frac{1}{2}, 2\frac{1}{2}$

d $-1, \frac{3}{4}$

e $-2, \frac{4}{5}$

7 **a** $\frac{2x}{x+2}$

b $\frac{1}{3x-2}$

c $\frac{4}{x+3}$

d $\frac{3}{x+4}$

8 **a** $\frac{2x^2+x-1}{x(x+1)}$

b $\frac{1}{2}, -1$

9 **a** $\frac{5x^2+11x+2}{x(11x+2)}$

b $-2, -\frac{1}{5}$

Developing fluency (page 285)

1 **a** $(7x+8y)(7x-8y)$

b $(x-5)(3x+4)$

c $(5x-2)(x+4)$

d $(2-3x)(x+5)$

e $(6x-5)(1-x)$

f $3x(x-2)$

2 **a** **i** $10(8x-10)(8x+10)$

ii $x = \frac{5}{4}$ or $-\frac{5}{4}$

b **i** $3(x-1)^2$

ii $x = 1$

c **i** $2(3x-2)(x+4)$

ii $x = \frac{2}{3}$ or -4

3 **a** $-\frac{2}{3}, 5$

b $-\frac{1}{2}, \frac{4}{3}$

c $-\frac{3}{4}, -\frac{1}{2}$

d $-5, \frac{2}{5}$

e $-\frac{1}{2}, \frac{1}{2}$

4 **a** $-3, -\frac{1}{4}$

b $-\frac{2}{3}, 2$

c $2, 2\frac{4}{5}$

d $-\frac{4}{3}, \frac{2}{3}$

e $-2, \frac{3}{2}$

5 **a** $(a+b)(c+d)$

b $(x-2)(x+y)$

c $(3x-y)(2x+y)$

d $x(2x-1)(2x+1)$

e $(2a+c)(b-2d)$

f This expression does not factorise.

6 **a** $-5\frac{2}{3}, \frac{1}{2}$

b $\frac{3}{13}, 2$

c $-\frac{3}{22}, 1$

d $-\frac{18}{23}, 6$

e $\frac{3}{37}, 2$

7 **a** This expression does not simplify.

b $\frac{1}{x+3}$

c $\frac{2}{2x+1}$

d $\frac{4}{2x+1}$

e $\frac{1}{3x+2}$

f $\frac{2x-3}{2x+3}$

Problem solving (page 286)

1 $(4x+1)$ metres and $(x+2)$ metres

2 90 m × 120 m and 60 m × 180 m

3 0.4 s and 2 s

4 **a** $\frac{4}{x-2}$

b $x = \frac{2}{3}$ or 4

5 **a** $\frac{(3x-1)+(x+9)}{2} \times (x+3) = \frac{4x+8}{2} \times (x+3) = (2x+4)(x+3) = 2x^2+10x+12$

b 6 cm

Reviewing skills (page 286)

1 **a** $(2x + 3)(x + 5)$

b $2(4x - 5)(x + 5)$

2 **a** $(3x - 4)(3x - 2)$

b $x = \frac{4}{3}, \frac{2}{3}$

3 **a** $\frac{3x+1}{x(x+1)}$

b $\frac{1}{2}, 1$

4 $\frac{1}{2x+3}$

5 The answer is **b**

Practising skills (page 290)

1 a $a = 1, b = -5, c = 4$
b $a = 1, b = 3, c = -2$
c $a = 3, b = 2, c = -5$
d $a = -1, b = 1, c = -2$
e $a = p, b = q, c = r$

2 a 57, two roots
b 0, one (repeated) root
c 144, two roots
d 9, two roots
e −16, no roots

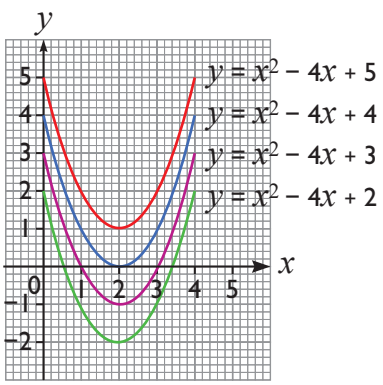
3 a −1 and −4
b 2 and 1
c 3.83 and −1.83 to 3 significant figures
d −0.219 and −2.28 to 3 significant figures
e 1.54 and −0.869 to 3 significant figures

4 a 0.303 and −3.30 to 3 significant figures
b 0.414 and −2.41 to 3 significant figures
c 2 and 1
d 2.37 and −3.37 to 3 significant figures
e 2.56 and −1.56 to 3 significant figures

5 $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ gives $\frac{2 \pm 2\sqrt{2}}{2}$ which is $1 \pm \sqrt{2}$.

Developing fluency (pages 290–292)

1 a i–iv



b **i** No solution
ii $x = 2$
iii $x = 1$ or $x = 3$
iv $x = 3.4$ and 0.6

c **i** No solution
ii $x = 2$
iii $x = 1$ or $x = 3$
iv $x = 2 + \sqrt{2}$ and $2 - \sqrt{2}$

d When $b^2 - 4ac > 0$ (i.e. positive), there are 2 roots.
When $b^2 - 4ac = 0$, there is one root.
When $b^2 - 4ac < 0$ (i.e. negative), there are 0 (real) roots.

2 **a** 1.85 and −4.85 to 3 significant figures
b 3.24 and −1.24 to 3 significant figures
c −0.293 and −1.71 to 3 significant figures
d 0.839 and −0.239 to 3 significant figures
e 1.18 and −0.847 to 3 significant figures

3 **a** $2 \pm \sqrt{7}$
b $3 \pm \sqrt{2}$
c $\frac{1}{2} \pm \sqrt{5}$
d $1 \pm \frac{1}{2}\sqrt{5}$
e $\frac{1}{3} \pm \frac{1}{3}\sqrt{5}$

4 **a** $\sqrt{28}$, 2 roots
b 4, 2 roots
c $24\sqrt{2}$; there are two distinct roots which will be surds.
d error, roots do not exist

5 **a** 2.79 and −1.79 to 3 significant figures
b 0.915 and −2.91 to 3 significant figures
c 2.16 and −4.16 to 3 significant figures
d 3.64 and −0.641 to 3 significant figures
e 0.618 and 1.62 to 3 significant figures

6 **a** 0.694 and −3.36 to 3 significant figures
b 3.19 and −4.69 to 3 significant figures
c 5.06 and −0.395 to 3 significant figures
d 1.65 and −5.15 to 3 significant figures
e 10.5 and −0.349 to 3 significant figures

7 **a** 0.381 seconds
b 1.10 seconds

8 $x = 3.54$

9 **a** Step 1: $x^2 + 2x - 3 = 0$
Step 2: $(x + 1)^2 - 4 = 0$
Step 3: $(x + 1)^2 = 4$
Step 4: $x + 1 = \pm 2$
Step 5: $x = -1 \pm 2$
So $x = 1$ and -3

b Step 1: $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Step 2: $\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$

Step 3: $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - \frac{c}{a}$

Step 4: $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

Step 5: $x + \frac{b}{2a} = \frac{\mp\sqrt{b^2 - 4ac}}{2a}$

Step 6: $x = -\frac{b}{2a} \frac{\mp\sqrt{b^2 - 4ac}}{2a} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$

Problem solving (pages 292–293)

1 **a** 2 roots, $x = 4$ or $x = -4$

b 2 integer roots, $x = 1$ or $x = 0$

c No roots

d 2 equal roots both fractional, $x = \frac{1}{2}$

e No roots

f 2 non-integer roots, $x = 0.3 \pm 1.22$

2 $3x^2 + 5x - 4 = 0$

3 **a** $b^2 - 4ac = -1$

b $b^2 - 4ac = 1$

c $b^2 - 4ac = 0$

4 $x = 3$

5 $x = 20\,\text{cm}$

Reviewing skills (page 294)

1 **a** −0.651 and 1.15

b 0.618 and 1.62

2 **a** $3 \pm \sqrt{5}$

b $1 \pm 2\sqrt{2}$

3 −3.30 and 0.30

4 $c = 3$

Practising skills (pages 299–300)

1 **a** 4
b −3
c 6.5
d 0.5

2 **a** 1
b −4
c 3
d 2

3 **a** (0, 0) and (1, 2)
b 2
c The average velocity upwards of the stone in the first second.
d No, most of the time the stone is moving more quickly.

4 **a** 0.8 seconds
b 0 seconds and 1.62 seconds
c 0.8 seconds
d $-1.8\,ms^{-1}$, downwards

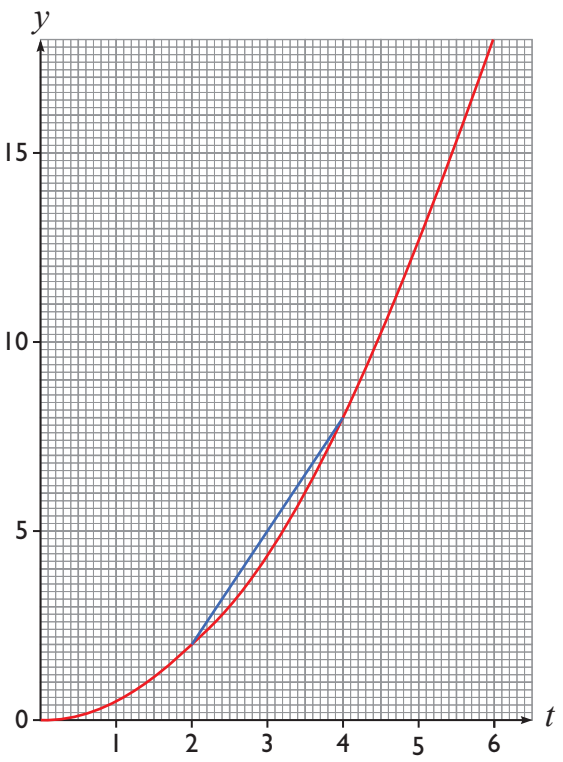
5 **a** 76°C
b at the start
c The tea reaches room temperature.
d −22°C per hour

Developing fluency (pages 300–301)

1 a

t (seconds)	0	1	2	3	4	5	6
y (metres)	0	$\frac{1}{2}$	2	4.5	8	12.5	18

b



c 3

d The average speed of the particle between $t = 2$ and $t = 4$ seconds.

2 b 1.5 m/s^2

c At the start, $t = 0$

d It is where the curve is steepest, the gradient is greatest.

3 a At the end of the period.

b £0.80 per year.

4 a 17°C to 19°C

b At the start.

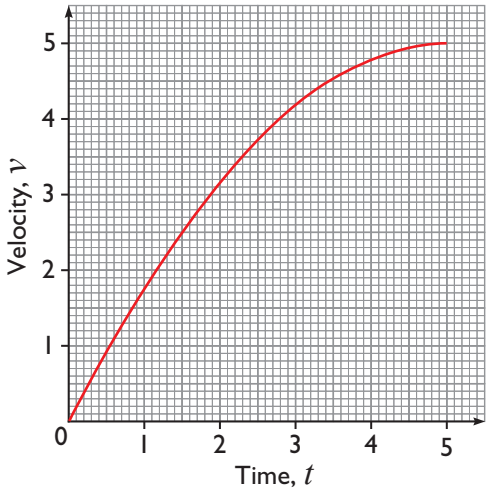
c The gradient of the graph is steepest then and there is a greater temperature difference between the cake and the room at that point. That is because the cake is hottest then.

d −22°C per hour

5 a

t (seconds)	0	1	2	3	4	5
v(m/s)	0	1.8	3.2	4.2	4.8	5

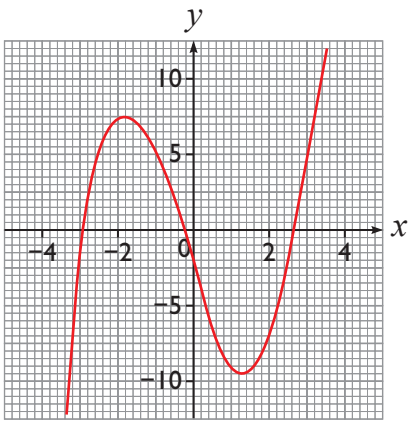
b



c Initially

d $1.2\,\text{ms}^{-2}$

6 a



b 6

c 5

d −11 and −4. Both negative but different in size.

e The gradient of the chord can be a good estimate for the gradient of the tangent but not always.

Problem solving (pages 302–303)

1 a i 9.5 m/s

ii 10 m/s

b $t = 0$ s and $t = 2.4$ s

c 6.25 m/s

2 a The blue curve

b China mug: 0.2 degrees per minute; insulated mug 0.25 degrees/minute

c Insulated mug 1.65 degrees per minute; china mug 2.5 degrees per minute. China mug loses about 1 degree more per minute

3 a $54 \div 10 = 5.4\,\text{m/s}^2$

b $10 - 2.8 = 7.2\,\text{m/s}^2$

c When $t = 6.2$ seconds

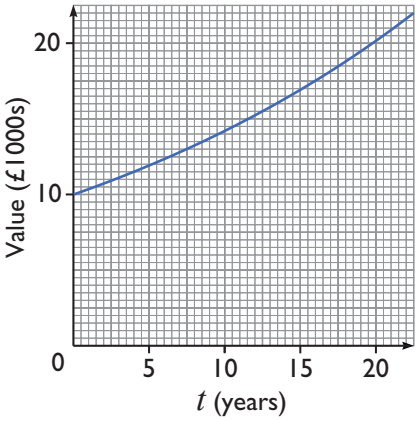
4 a The y axis shows the population in hundreds. Population doubles from $t = 2$ to $t = 4$ and it doubles from $t = 8$ to $t = 10$. Rate of growth at $t = 2, 4, 8, 10$ is 15, 25,100, 180 rabbits per month respectively.

b 48 rabbits/month

c Yes; population is increasing each month by the same factor.

Reviewing skills (page 304)

1 a



b £10 000

c 20 years

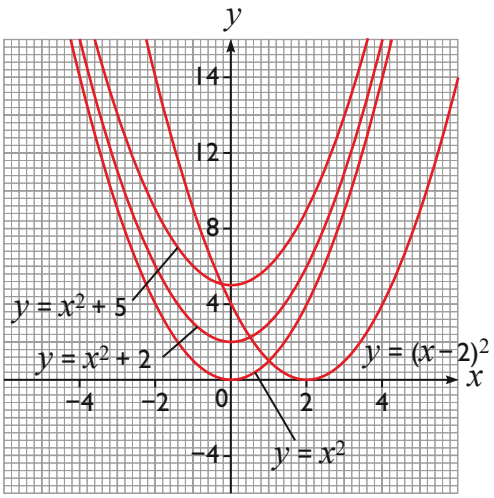
d £600 per year

e £600 per year

f They are the same (or very close). The gradient of the chord straddling the point is approximately the same as that of the tangent.

Practising skills (pages 309–310)

1 a, c, f



b Add 2 to all values, add 5 to all values.

x	−3	−2	−1	0	1	2	3
$f(x) = x^2$	9	4	1	0	1	4	9
$x^2 + 2$	11	6	3	2	3	6	11
$x^2 + 5$	14	9	6	5	6	9	14

d Translated up the y-axis. The + term in the algebra and the table denotes how far up the y-axis the function is translated.

e

x	−1	0	1	2	3	4	5
$f(x) = (x - 2)^2$	9	4	1	0	1	4	9

g Translated to the right by two units. The $f(x)$ numbers of 9,4,1,0,1,4,9 occur at x values of 2 to the positive (right) direction.

2 C

3 a B

b D

c A

d C

4 a D

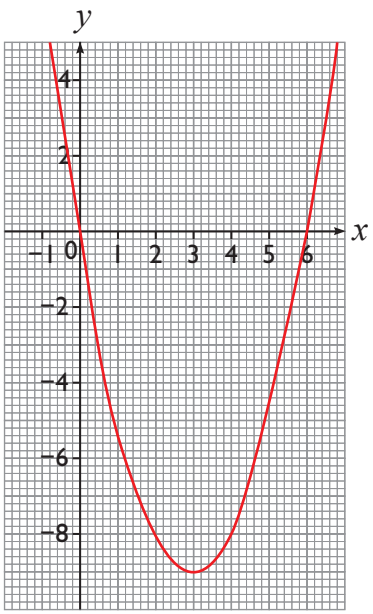
b A

c C

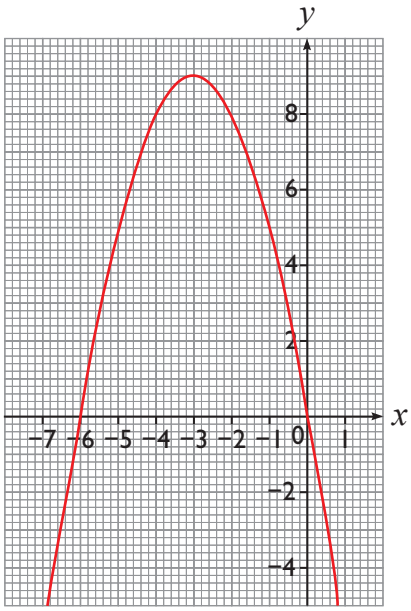
d B

5 a reflection in the x-axis

b



c



d $x = -6x - x^2$

Developing fluency (pages 311–314)

1 a D

b C

c A

d B

e D could be a reflection in the x-axis followed by a translation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$; A could be a reflection in y-axis.

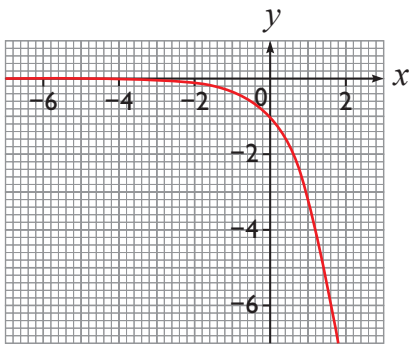
2 a C, minimum not quite at $x = 1$

b A

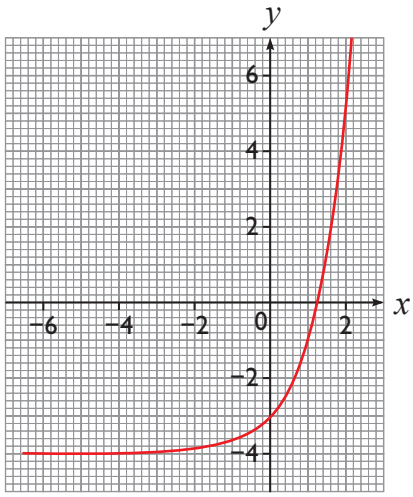
c D, minimum not quite at $x = -1$

d B

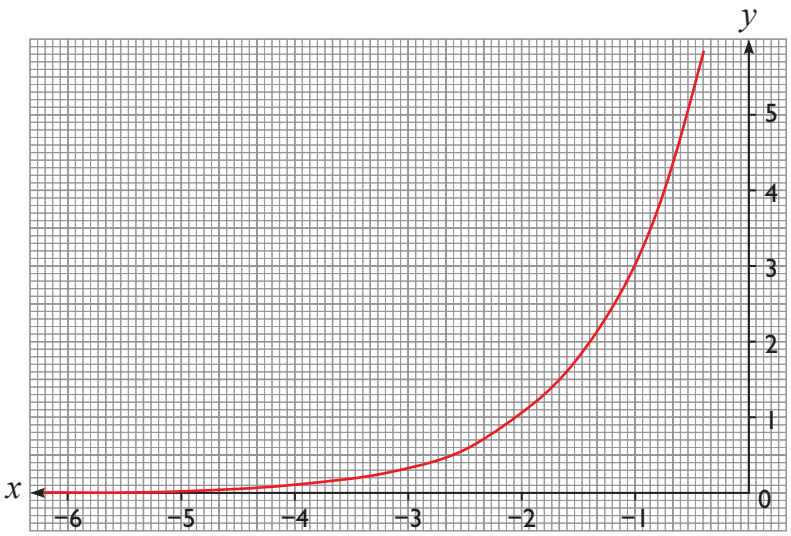
3 a



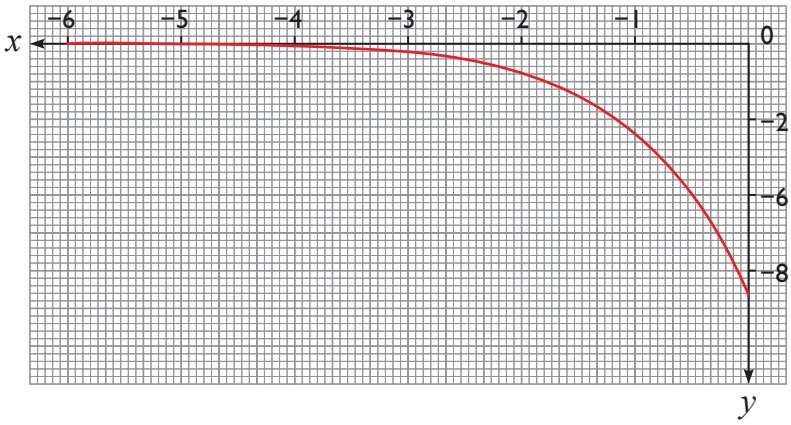
b



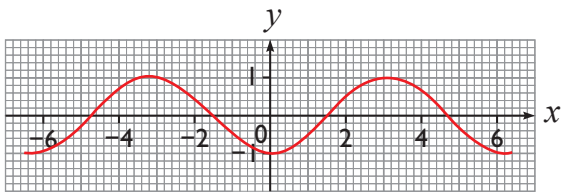
c



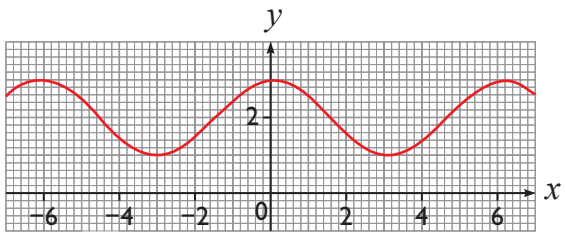
d



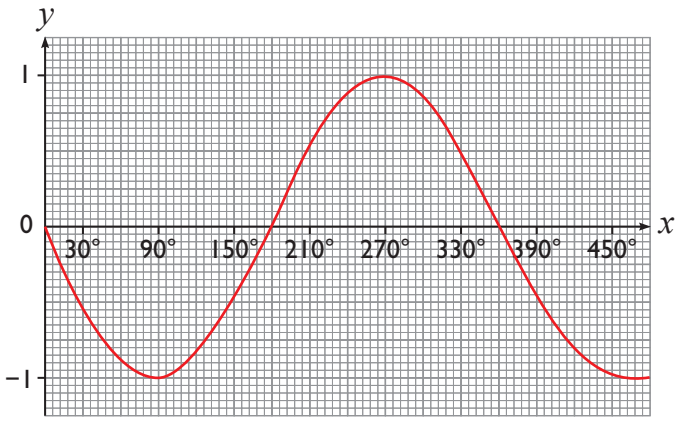
4 a



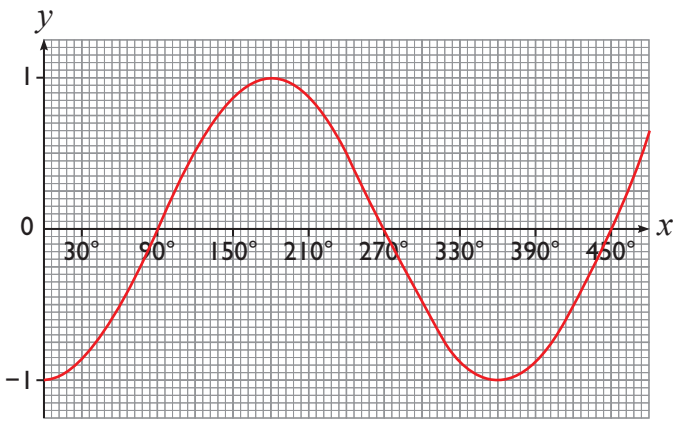
b



c



d



5 **a** $y = 2^{-x} - 3$

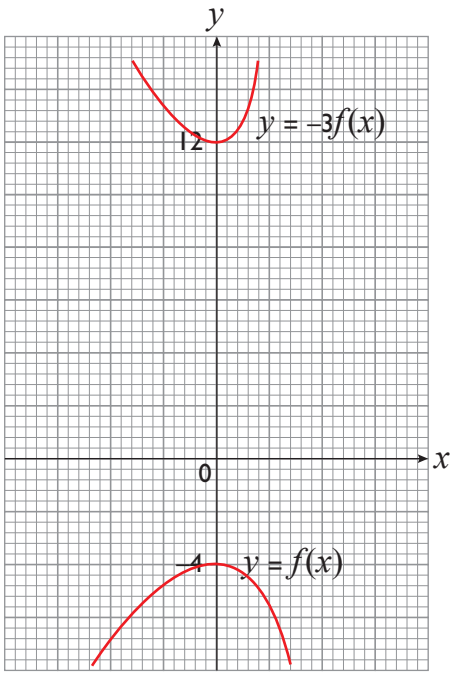
b $y = (x-3)^2 - 1$

c $y = \tan x$

d $y = \frac{1}{-x}$

6 Sometimes true, e.g. for $y = \sin x$. But it is not true for e.g. $y = x^2$. The function needs to have rotational symmetry about the origin.

7



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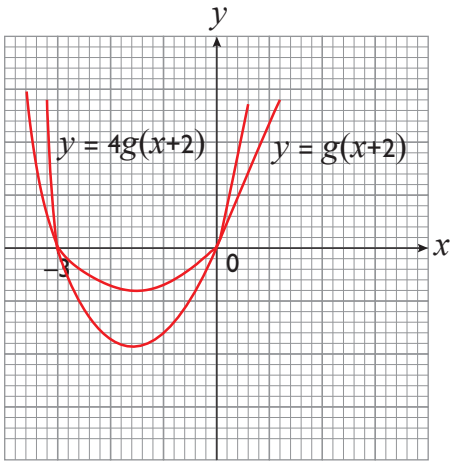

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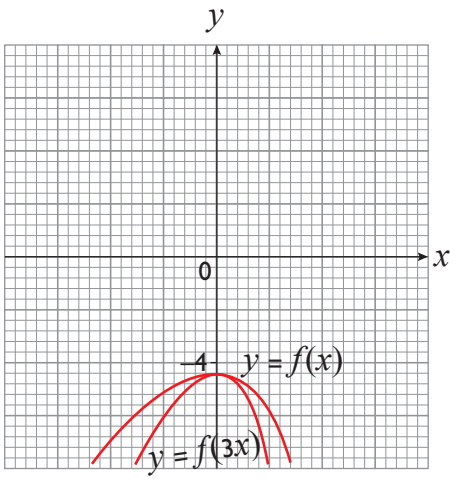
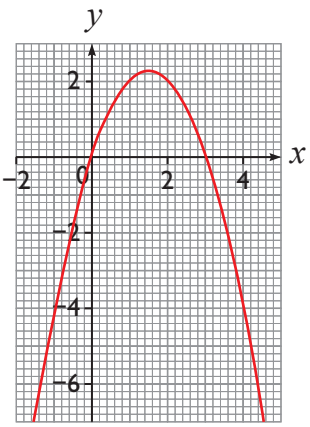
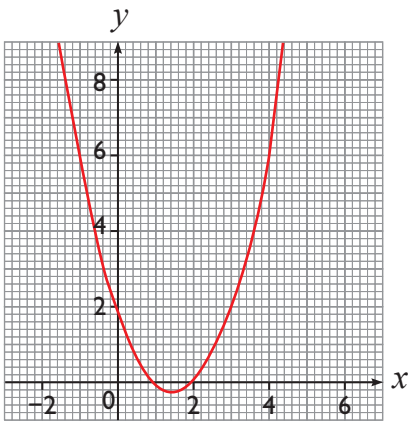


Problem solving (pages 315–316)

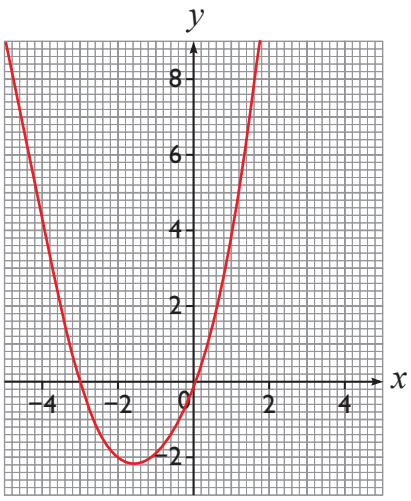
1 a reflection in the x-axis



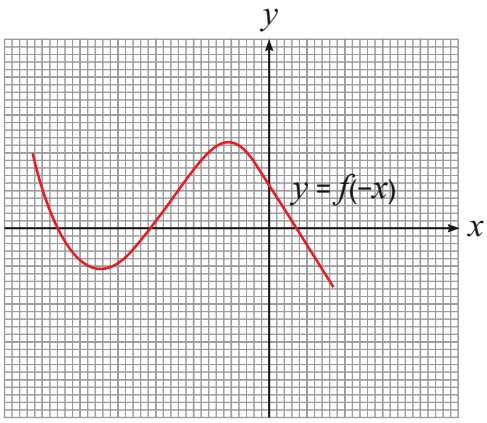
b +2 in the positive y direction



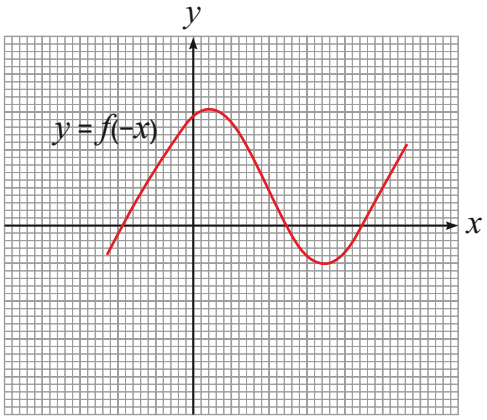
c reflection in the y-axis



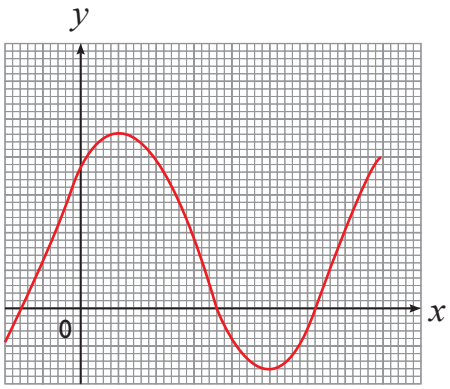
2 a reflection in the x-axis



b reflection in the y-axis



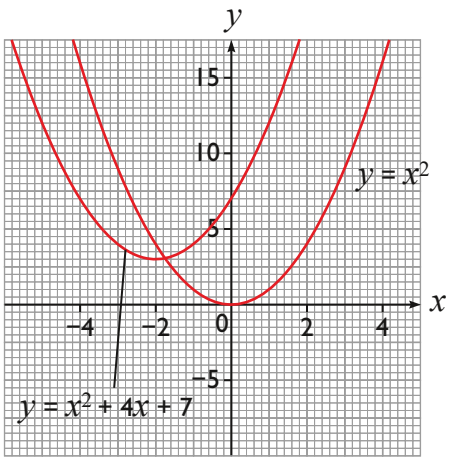
c no change



3 **a** $a = 2; b = 3$

b Translation of +3 in the y direction and −2 in the x direction.

c



d (−2, 3)

4 $y = -2^x + 3$

5 **a** $a = 1, b = 180$

b $x = -90$, 180 and 270

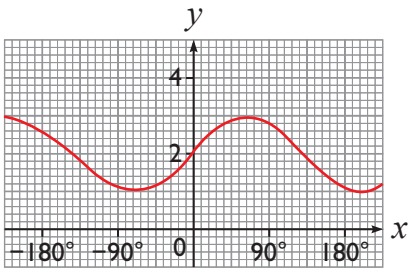
c 2

Reviewing skills (page 316)

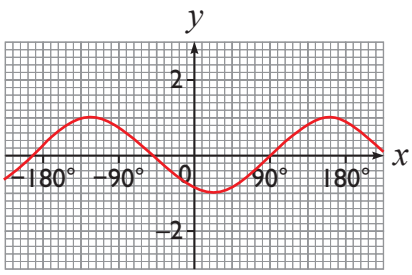
1 a $y=(x-2)^2+1$

b $y=-(x-2)^2-1$

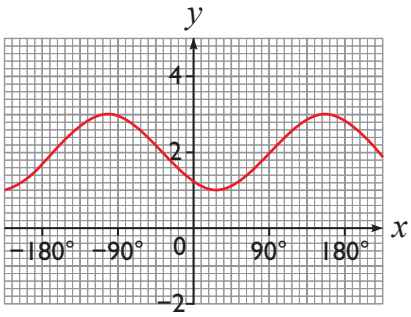
2 a



b



c



Practising skills (pages 323–324)

1 **a** m, metres
b m/s, metres per second
c m/s^2, metres per second per second
d m^2, square metres

2 **a** speed or velocity
b acceleration
c area
d £ per annum, yearly amount

3 **a** metres, which is the distance travelled
b kWh, which is the total amount of electricity used
c £$, which is meaningless, only the gradient has a meaning for this graph.
d litres, which is the total volume of water

4 197.5 miles

5 112

6 84

7 **a** **i** 695 m
ii 500 m
b Both methods have a similar accuracy for this curve.

Developing fluency (pages 325–327)

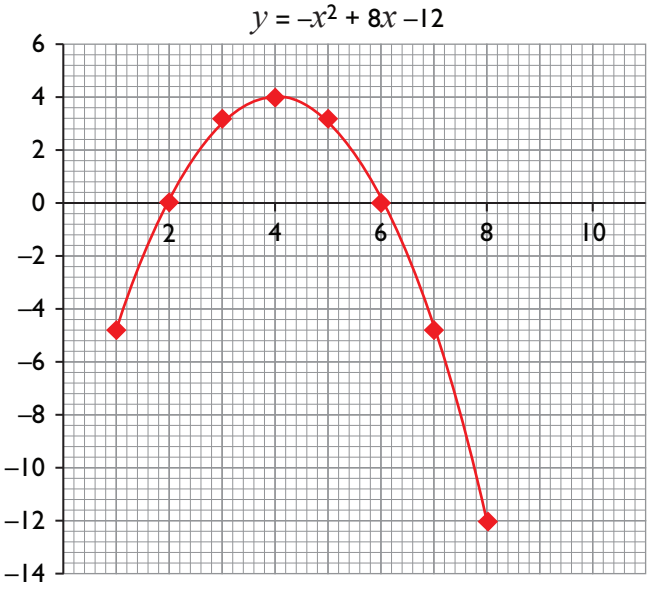
1 **a** 8 mph
b 32 mph/h
c Distance travelled
d 11 miles
e The cyclist takes too long a time to get up to speed. It is unlikely that she will maintain a constant speed for $1\frac{1}{4}$ hours.

2 **a** 100 W
b 0.075 kWh
c 0.415 kWh
d 7p
e The power usage would vary much more than the graph shows.

3 **a**

x	1	2	3	4	5	6	7	8
$y = -x^2 + 8x - 12$	−5	0	3	4	3	0	−5	−12

b



c Using the trapezium rule we have the area as approximately:

$$\left(1\times\frac{(0+3)}{2}\right)+\left(1\times\frac{(3+4)}{2}\right)+\left(1\times\frac{(4+3)}{2}\right)+\left(1\times\frac{(3+0)}{2}\right)=10$$

4

Time:	0	0.5	1	1.5	2	2.5	3	3.5	4
Velocity:	0	0.5	1	2	3	5	4	3.5	3

So the area is approximately:

$$\left(0.5\times\frac{(0+0.5)}{2}\right)+\left(0.5\times\frac{(0.5+1)}{2}\right)+\left(0.5\times\frac{(1+2)}{2}\right)+\left(0.5\times\frac{(2+3)}{2}\right)+\left(0.5\times\frac{(3+5)}{2}\right)+$$

$$\left(0.5\times\frac{(5+4)}{2}\right)+\left(0.5\times\frac{(4+3.5)}{2}\right)+\left(0.5\times\frac{(3.5+3)}{2}\right)=10.25$$

This means that the particle travelled approximately 10.25m

5 **a** 40 mph

b Distance travelled

c 310 miles

d It is unlikely that the car will maintain a constant speed for all three stages of the journey.

6 **a** 60 square units

b Under-estimate as trapezia/triangles all lie under the curve.

7 **a** $a=\frac{v-u}{t}$

b $v = u + at$

c $area=\frac{1}{2}(u+v)t$

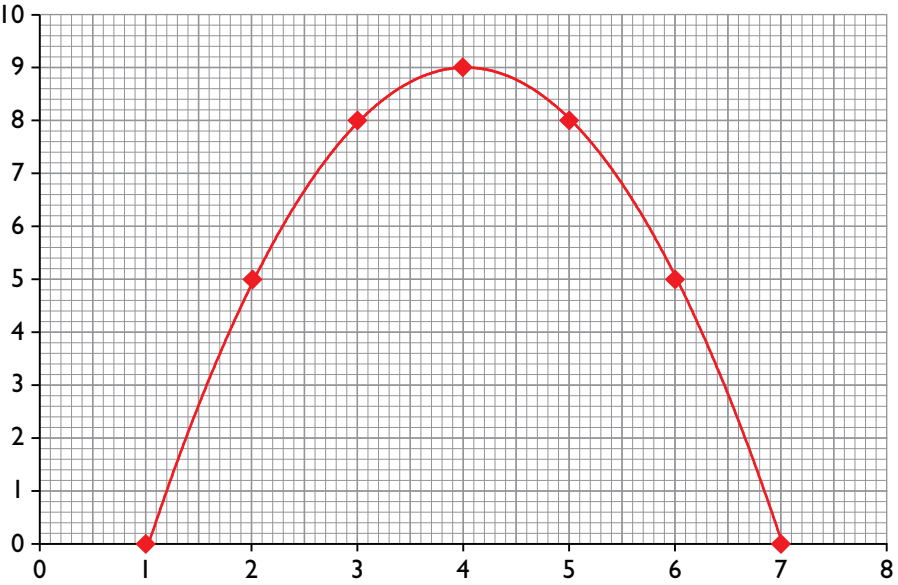
d Distance travelled

Problem solving (pages 327–329)

1 1188 m using 20 second strip width

2 0.5 cm^3

3 **a**



b Using the trapezium rule we have an approximation of the area as:

$$\left(1\times\frac{(0+5)}{2}\right)+\left(1\times\frac{(5+8)}{2}\right)+\left(1\times\frac{(8+9)}{2}\right)+\left(1\times\frac{(9+8)}{2}\right)+\left(1\times\frac{(8+5)}{2}\right)+\left(1\times\frac{(5+0)}{2}\right)=35.$$

So the particle travelled approximately 35m.

4 Using the trapezium rule we have:

$$\left(2\times\frac{(0+18)}{2}\right)+\left(2\times\frac{(18+19)}{2}\right)+\left(2\times\frac{(19+16)}{2}\right)+\left(2\times\frac{(16+7)}{2}\right)+\left(2\times\frac{(7+0)}{2}\right)=120.$$

So the particle went approximately 120m.

5 Using the trapezium rule with trapeziums of width 2.5 we have:

$$\left(2.5\times\frac{(0+5)}{2}\right)+\left(2.5\times\frac{(5+10)}{2}\right)+\left(2.5\times\frac{(10+16)}{2}\right)+\left(2.5\times\frac{(16+15)}{2}\right)+\left(2.5\times\frac{(15+14)}{2}\right)+$$

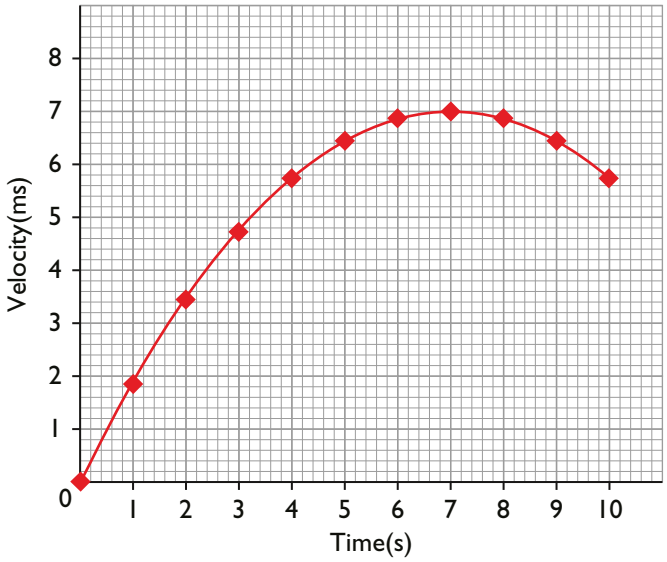
$$\left(2.5\times\frac{(14+13)}{2}\right)+\left(2.5\times\frac{(13+10)}{2}\right)+\left(2.5\times\frac{(10+0)}{2}\right)=207.5$$

So the distance between the two sets of traffic lights is approximately 208m.

6 **a** Car A, as the area under the graph represents the distance travelled.

b 225 m

7 a



b 51 m

Reviewing skills (page 329)

1 33 ±5 square units (33.3)

1 Geometry and Measures Strand 1 • Unit 11 • Answers

Practising skills (page 332)

1 **a** Length
b Volume
c Area
e Area
f Volume
g Volume

2 **b**, **c** and **d**

3 **b** and **d**

4 **a**, **b** and **c**

5 **b** and **c**

6 **d**

7 **a** and **d**

Developing fluency (pages 333–334)

1 **a** $r^2h(4 + \pi)$
b $2r(4h + 4r + \pi h)$

2 $\frac{p}{3}$ is a length divided by 3 so remains a length. $\sqrt{qr}$ is the square root of an area and so is a length. The expression in the brackets is the sum of 2 lengths which makes another length. This is multiplied by a number, 2π, and so remains a length.

3 The answer is **a**

4 **a** 2
b 1
c 1
d 1

5 **a** He needs to use an expression that represents volume, i.e. dimension 3. $4r^2 + h^2$ is dimension 2. Only part of the expressions $\frac{1}{3}r^2h + \pi r^2$ and $r^2(h + \pi)$ have the correct dimension, so these don't represent anything. **i** $r^2h\left(\frac{1}{10} + \pi\right)$ has dimension 3 so is the right answer.
b **i** Only the first part of the expression has the correct dimension for volume.
ii 10 wdh

Problem solving (pages 334–335)

1 Expanding the brackets gives $\pi r^2 + 2\pi r^2h + \pi rh$. The middle term is a volume, not an area.

2 **a** **iii**
b **i**

3 **a** $4\pi R - 4\pi r + 2\pi Rh + 2\pi rh$

b $2\pi(R + r)(R + h - r) = 2\pi(R^2 + Rh - Rr + Rr + rh - r^2)$
$= 2\pi(R^2 + Rh + rh - r^2)$
$= 2\pi R^2 + 2\pi Rh + 2\pi rh - 2\pi r^2$

4 **c**

Reviewing skills (page 335)

1 **a** 3

b 1

c 2

d Incorrect formula

e 1

f Incorrect formula

2 **a**, **b** and **c**

1 Geometry and Measures Strand 1 • Unit 12 • Answers

Practising skills (pages 338–339)

1 **a** 1.5 litres of lemonade for 65p
b 12 for 75p
c 3 kg of grass seed for £4.20
d 1.5 kg for £4.65

2 **a** 3.57 m/s
b 12.86 km/h

3 **a** 2.11 g/cm^3
b 2111.1 kg/m^3

4 0.27 g/cm^3

5 **a** $\frac{n}{t}$ ms^{-1}
b $\frac{3.6n}{t}$ km/h

6 **a** 10.5 g cm^{-3}
b 84 kg

7 **a** It's travelling at a constant speed.
b 6 m/s^2
c acceleration

Developing fluency (pages 339–340)

1 No, as it will take her 4 hr 41 min.

2 In the UK, as the cost is 1.17p per g whereas in the USA it is 1.21p per g.

3 £1.17

4 **a** A
b C
c B

5 No, change in speed is acceleration. So second car is accelerating, whilst first is at constant speed.

6 7.59 g/cm^3

7 **a** Metal A: 2.7 g/cm^3 = 2700 kg/m^3. Metal B: 8900 kg/m^3 = 8.9 g/cm^3.
b Metal B is heavier.

8 1000f N/cm^2

9 $\frac{1000m}{v}$ kg/m^3

10 V × 3600 metres in 1 hour; V × 3600 ÷ 1000 = 3.6V/kilometres in 1 hour = (approximately) 2.25V miles per hour.

11 The population density of (a) is 2 300 000 ÷ 4 500 ≈ 511 people/km^2
The population density of (b) is 48 000 000 ÷ 92 000 ≈ 522 people/km^2.
Therefore (b) has the higher population density.

Problem solving (pages 340–342)

1 **a** 1.728 seconds

b You may have to take the length of the car into account – that would make it a longer time.

2 **a** 10

b 4.67

c 3120

3 The 1.8*l* because the unit price is 0.45 litre per pound as against the large bottle at 0.43 litre per pound.

4 **a** 900 kg/m^3

b Answer $\frac{y}{x}$

5 £171.31

6 Yes, because his BMI is 21.

7 **a** 60 g copper and 40 g zinc

b 7.99 g/cm^3

8 No, as the mass is 989 g which is less than 1000 g.

9 **a** Difference in daily rate is £5 on first day.

b Equation of blue is $5x + 10$

Equation of red is $4x + 5$

Reviewing skills (page 343)

1 Around 13.30

2 54.5 kg/m^3

3 **a** £75

b There is an error on the Student Book. This answer cannot be calculated from the information given.

2 Geometry and Measures Strand 2 • Unit 9 • Answers

Practising skills (pages 348–350)

1 **a** vertically opposite angles
b angles in isosceles triangle
c corresponding angles
d alternate angles
e opposite angles of parallelogram
f angles in isosceles triangle
g opposite angles of parallelogram
h alternate angles
i corresponding angles
j vertically opposite angles

2 **a** equal sides of isosceles triangle
b opposites sides of rectangle
c radii of a circle
d opposite sides of a parallelogram
e sides of an equilateral triangle

3 **a** p and r; s and q
b s and p; p and q; q and r; r and s
c $p + q + r + s = 360°$ (angles round a point)

4 **a** u and r – alternate angles; s and p – alternate angles; q and t must be equal as the triangles are similar (but not congruent) as two angles of the triangle are the same, therefore similar by AAA.
b uts and pqr are angles of a triangle; puq and srt are two angles of a rectangle.

5 **a** SSS
b Not congruent.
c ASA
d SAS
e Not congruent.
f SAS
g Not congruent, but similar triangles AAA.
h Not congruent as 5 cm side isn't the same for both triangles.

6 **a** $a = 50°$ – vertically opp angles; $b = c = g = h = 65°$ – equal angles of equal isosceles triangles as the lines OW, OX, OZ and OY are equal radii of a circle and therefore isosceles triangles OWX and OZY are congruent by SAS; $e = k = 130°$ – angles on a line; $d = f = i = j = 25°$ – equal angles of equal isosceles triangles as the lines OW, OX, OZ and OY are equal radii of a circle and therefore isosceles triangles OXY and OWZ are congruent by SAS.
b Angle bj = angle $gf = 65 + 25 = 90°$ and angle cd = angle $hi = 25 + 65 = 90°$. WX does not equal WZ as they are sides of different isosceles triangles. WXYZ has four 90° angles and two different sides and is therefore a rectangle.
c Three pairs. OWX and OZY; OZW and OXY; WYZ, WXY, XYZ and WXZ.

7 In triangles ABP, ACP: AB = AC (Isosceles triangle); BAP = CAP (given); AP is common. Triangles ABP, ACP are congruent (SAS); PB = PC (corresponding sides of congruent triangles).

Developing fluency (pages 351–353)

1 a AC = CE = 8 cm

ACB = ECD (vertically opposite angles)

BAC = CED (alternate angles on parallel lines)

So, ABC and CDE are congruent (ASA).

b AB and DE are the same, as they are parallel and the triangles are congruent. BC = DC as they are matching sides of congruent triangles.

2 a In triangles DAB and DBC

AB = CB (A rhombus has equal sides)

AD = CD (rhombus has equal sides)

BD is common to both triangles.

So, triangles DAB and DCB are congruent (SSS).

b They are the same.

3 a OA = OB (radii)

ON is common to both triangles

∠ONA = ∠ONB = 90°

So, triangles OAN and OBN are congruent (RHS).

b Point N is in the middle of the line AB as ON bisects AB at 90 degrees, and AN and AB are equal.

4 a The angle must be the one between the equal sides.

b The sides are not in corresponding positions.

c The hypotenuse on the left is equal to one of the shorter sides on the right.

d AAA does not prove congruence. One triangle could be bigger than the other.

5 AD = BC (given)

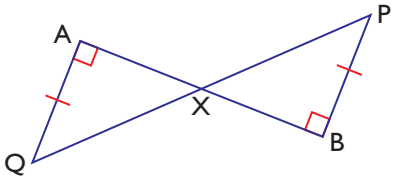
DAB = ABC (given)

AB is common.

Triangles DAB, CBA are congruent (SAS).

BD = AC (corresponding sides of congruent triangles).

6 Let PQ cut AB at X.



In triangles AQX, BPX:

AQ = BP (given)

QAX = PBX = 90° (given)

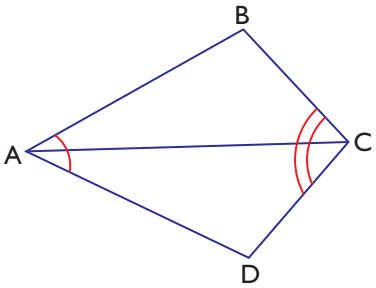
AXQ = BXP (vertically opposite angles)

Triangles AQX, BPX are congruent (ASA)

AX = XB (corresponding sides of congruent triangles)

7 In triangles AMD, BND:
AD = BD (given)
MD = ND (given)
AMD = BND = 90° (given)
Triangles AMD, BND are congruent (RHS)
CAB = CBA (corresponding sides of congruent triangles)
So, ABC is isosceles.

8 BAC = DAC (BAD bisected)



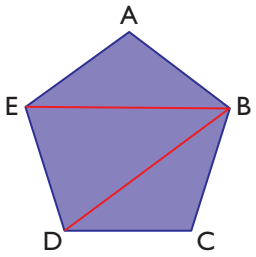
BCA = DCA (BCD bisected)
AC is common.
Triangles ABC, ADC are congruent (ASA)
So, AB = AD, BC = CD (corresponding sides of congruent triangles)
So, ABCD is a kite.

Problem solving (pages 353–354)

1 a They are the same length.

b They are the same size.

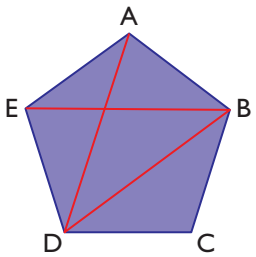
c



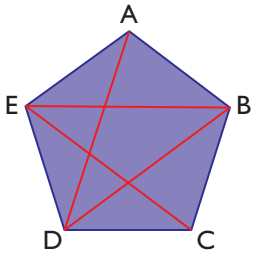
Statements	Reason
AB ≈ BC AE ≈ CD ∠A ≈ ∠C	All sides of a regular pentagon are congruent. All interior angles of a regular pentagon are congruent.
BAE ≈ BCD	SAS (side angle side)
BE ≈ BD	CPCT (corresponding parts of congruent triangles)

d AC, AD, CE

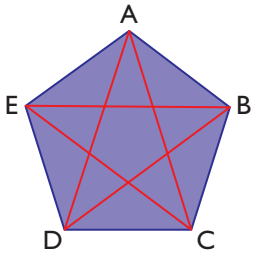
Draw in line AD: AED ≈ BCD (SAS), therefore AD ≈ BD (CPCT)



Draw in line EC: EDC ≈ EAB (SAS), therefore EC ≈ EB (CPCT)



Draw in line AC: ABC ≈ AED (SAS), therefore AC ≈ AD (CPCT)



2 a AB = AC (given), ∠BAD = ∠CAD (given), AD is common (SAS).

b i Congruent triangles mean BD = CD, so D is in the midpoint of BC.

ii Midpoint of the line when ∠BAD = ∠CAD means that the line AD is perpendicular to BC so ADC angle is 90°.

3 a Consider triangles PQS and PRS, angle QPS = angle RPS (given), angle PQS=angle PRS (given), PS is common so triangles PQS and PRS are congruent (SAA). Hence PQ = PR. Triangle PQR is isosceles.

b From a SQ = SR. So triangle SQR is isosceles, so angle SQR = angle SRQ, so angle SQR + angle SPQ = angle SRP + angle SRQ, so angle SQR = angle SRQ

4 Triangles ABC and BDE are congruent (RHS). So DB = BC, hence AD (= AB − DB) = CE (=BE − BC).

5 **a** ∠PQM = ∠RSN (alternate angles), RS = QP (opposite sides of a parallelogram are equal), ∠RNS = ∠PMS (AAS).

b Triangles RNQ and PMS are congruent. Length SP = length RQ. Hence SM = QN due to similar triangles.

6 **a** Draw a rhombus with a line from one vertex to the opposite vertex. This creates two triangles that are congruent by side-side-side congruence. We know that all the acute angles formed from drawing the line are equal in measure. By alternate interior angles, each set of opposite lines is parallel.

b Draw the second diagonal from a different vertex to the opposite vertex. By vertical angles, we know the angle in each pair of opposite triangles at the intersection between all four lines is equal. Then, by angle-angle-side congruence, all four triangles are congruent. Then, since the measures of all four angles at the four-way intersection must sum to 360 degrees and they all have equal measures, each angle measures $\frac{360}{4}$ degrees = 90 degrees.

Reviewing skills (page 355)

1 **a** none of the above

b alternate/corresponding angles

c angles in an isosceles triangle

d angles in an isosceles triangle

e none of the above

f angles in an isosceles triangle

2 **a** DAE = EBC = 90° (rectangle properties)

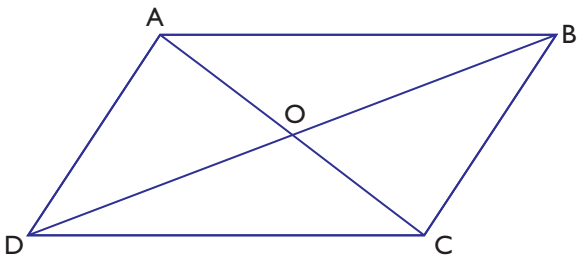
AD = BC (rectangle properties)

ED = EC (given, isosceles triangle)

ADE and EBC are congruent (RHS)

b Point E is in the centre of line AB as AE = EB due to the triangles DAE and EBC being congruent.

3 Draw and label the parallelogram. Draw the diagonals. Prove triangles AOB and DOC are congruent using AAS (corresponding angles and opposite sides of a parallelogram are equal). Therefore lines AO and OC are equal and DO and OB are equal meaning O is at the centre of the lines and the lines bisect each other.



2 Geometry and Measures Strand 2 • Unit 10 • Answers

Practising skills (page 358)

1 M is the odd one out.

A and N: congruent; isosceles triangles with same sides and angles.

B and K: similar; SAS, sides in same ratio.

C and U: similar; all equilateral triangles are similar.

D and V: similar SAS: two sides in same ratio and included angle is equal.

E and S: congruent; all angles the same and same side length.

F and Q: similar; all angles the same.

G and X: congruent; by Pythagoras' theorem, sides of 5 cm, 12 cm and 13 cm.

H and P: congruent SAS

I and O: similar; three equal angles as isosceles triangles with angles 30°, 75° and 75° (AAA), no information about side length so can't state that they are congruent.

J and T: similar both right angled isosceles triangles.

L and W: congruent; by Pythagoras' theorem L is right-angled and both have sides of 7 cm, 24 cm and 25 cm.

R and Y: similar; by Pythagoras' theorem all sides are in the same ratio.

Developing fluency (page 359)

1 In triangles FAB, BCD, DEH:

FA = BC = DE (sides of regular hexagon)

AB = CD = EF (sides of regular hexagon)

FAB = BCD = DEF (angles of regular hexagon)

FAB, BCD, DEH are congruent (SAS)

So, FB = BD = DF (corresponding sides of congruent triangles)

So, BDF is equilateral.

2 **a** It is a rectangle.

b In triangles ABD, BCA:

AD = BC (opposite sides of rectangle)

ABD = BCA = 90° (angles of a rectangle)

AB is common

Triangles ADB and BCA are congruent (SAS).

c Triangles ADB, BCA are therefore also congruent by RHS.

AC = BD (corresponding sides of congruent triangles).

d A rectangle is a parallelogram with four right angles.

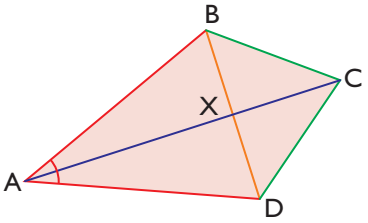
3 a In triangles ABC and ADC:

AB = AD (equal sides of kite)

BC = DC (equal sides of kite)

AC is a common side to both triangles

So, triangles ABC and ADC are congruent (SSS).



b In triangles ABX and ADX:

AB = AD (equal sides of kite)

Angle BAX = Angle DAX (corresponding angles of congruent triangles ABC and ADC)

AX is a common side to both triangles.

So, triangles ABX and ADX are congruent (SAS).

c Triangles ABX and ADX are congruent (SAS), so BX = DX, meaning X is midpoint of BD.

d Isoceles triangles means BX = BD. Hence, midpoint.

Problem solving (pages 359–361)

1 a DB = BC (given)

BA = BE (sides of equilateral triangle)

∠DAB = ∠BEC = 90 (given)

Triangles DAB and CEB congruent by RHS.

b Since, triangles DAB and BEC are congruent (RHS), ∠ABD = ∠CBE, so ∠ABD + ∠ABE = ∠DBE = ∠CBE + ∠ABE = ∠CBA, hence result.

c Yes, providing AB = BE.

2 a ∠BAC = ∠CFG (alternate angles), ∠ACB = ∠GCF (opposite angles), so the third angles are the same and equiangular triangles are similar.

b 5 : 3

3 a ∠OBP = ∠DBA; ∠OPB = ∠DAB (corresponding angles); ∠BOP = ∠BDA (corresponding angles). Triangles are similar (AAA).

b Any of: AOB and DOC; AOD and BOC; AOP and OQC; OPB and OQB; ABC and ADC; ABD and BDC; ABD and OPB and OQB; ABC and AOP and OQC.

c Triangles OPB and DBA are similar; ∠OBP = ∠DBA and ∠OPB = ∠DAB (corresponding angles) (as proved above). The sides are in ratio 1 : 2 as line OB = $\frac{1}{2}$DB is given. Therefore line PB = $\frac{1}{2}$ line AB, thus P is the midpoint of AB.

d Triangles BOQ and BDC are similar, so OQ = $\frac{1}{2}$DC = PB. Triangles ABC and AOP are similar, so OP = $\frac{1}{2}$BC = BQ. Hence POQB is a parallelogram (opposite sides equal), and line PQ bisects OB. Triangles BPQ and ABC are similar as angle PBQ = angle ABC and ∠BPQ = ∠BAC (corresponding angles on parallel lines). The side PB is in 1 : 2 ratio with AB, so the side PQ = $\frac{1}{2}$AC.

4 a SQ = SU. Angle SUQ is 90 degrees due to alternate angles. SU is a shared side, so SQU is congruent to SRU by RHS.

b U is the point where ST cuts QR, therefore angle TUR = 90° (corresponding angles) which means ST is bisecting QR, U is the midpoint of QR. QR : UR = 2 : 1.

c Angle TUR = 90° (corresponding angles). Angle TRU = PRQ as it's the shared angle and so UTR = QPR. TUR and PQR are similar (AAA).

d TUR and PRQ are similar. QR = 2UR, so the triangles are in ratio 1 : 2. PR and TU are corresponding sides of similar triangles and are parallel. Therefore PR = 2TU.

5 a In triangles DEA and DFC:

∠DEA = ∠DFC (corresponding angles on parallel lines)

∠EDA = ∠FDC (common)

So, triangles DEA and DFC are similar (similar triangles have the same angles).

b In triangles BME and CMF:

∠BME = ∠CMF (vertically opp angles)

∠BEM = ∠DEA (vertically opp angles) = ∠DFC (corresponding angles)

Side BM = MC (given)

Therefore triangles are congruent (ASA).

c Triangles DEA and DFC are similar. So ratio of EA to CF is the same as DE to DF. Or, $\frac{EA}{CF} = \frac{DE}{DF}$.
Since, triangles BME and CMF are congruent, CF = EB so, $\frac{EA}{EB} = \frac{DE}{DF}$.

6 a ∠APB = ∠QPR (shared). Side QR = 2AP and side PR = 2PB. Triangles are therefore similar by same angle and same ratio of sides. Therefore AB is parallel to QR.

b Shared angle, ∠PQB = ∠PQX, side QX = 2QB (given), and side PQ = 2QA (given). Triangles QAB and QPX are similar. So AB is parallel to PX. So PX is parallel to QR.
Similarly AB = $\frac{1}{2}$QR and AB = $\frac{1}{2}$PX, so PX = QR.

c Proven that one pair of sides of PXRQ is opposite and parallel. In order to prove it is a parallelogram, need both pairs of opposite sides to be parallel. Need to show that PQ is parallel to XR. We know PQ is parallel to XR, since ∠QXR = ∠PQB (alternate angles) and also ∠QXP = ∠XQR (alternate angles), so ∠PXR = ∠PQR and XR is parallel to PQ.

So yes, with this information on the diagram we can prove that it is a parallelogram. But without the extra geometry, we don't know.

Reviewing skills (page 361)

1 a In triangles ABX, CDX:

AX = XC (AC is bisected at X)

BX = XD (BD is bisected at X)

Angle AXB = angle CXD (vertically opposite angles)

ABX, CDX are congruent (SAS)

AB is parallel to CD (alternate angles equal)

AB = CD (corresponding sides of congruent triangles)

Angle BAX = DCX (corresponding angles of congruent triangles)

b ABCD is a parallelogram: AB is parallel to CD (alternate angles equal), ∠DAX = ∠XCB (corresponding angles of congruent triangles) therefore AD is parallel to BC (alternate angles equal). Two pairs of parallel sides means parallelogram.

2 Geometry and Measures Strand 2 • Unit 11 • Answers

Practising skills (pages 366–368)

1 **a** 104°, angles at centre of circle
b 90°, angles in a semicircle
c 84°, angles of same segment
d 158°, angles at centre of circle
e 97°, opposite angles of a cyclic quadrilateral
f 63°, angles in a semicircle, angles in an isosceles
g 14°, angles in a semicircle, angles in an isosceles
h 85°, opposite angles of a cyclic quadrilateral
i 170°, angles at centre of circle

2 Angle CDO is 62° because base angles in an isosceles triangle are equal.
Angle COD is 56° because angles in a triangle sum to 180°.
Angle AOB is 56° because vertically opposite angles are equal.

3 Angle ADB is 53° because the angle in a semicircle is a right-angle.
Angle ACB is 53° because angles in the same segment are equal.

4 **a** 43°, angles of same segment
b 103°, opposite angles of a cyclic quadrilateral

5 **a** 52°, angles at centre of circle
b 102°, opposite angles of a cyclic quadrilateral
c 38°, angles in isosceles triangle with angle BCD

6 73°

7 112°

8 35°

Developing fluency (pages 368–370)

1 Angle ABC = 54°, alternate segment to angle ACF.
Angle ACB = 180 – angle CAB – angle ABC = 54°. Therefore triangle ABC is isosceles, with AC = AB.

2 Angle DAC = angle DBC = 26° (same segment). Angle ACD = 180° – angle AED = 64° (opposite angles cyclic quadrilateral). Angle ADC = 180° – angle ACD – Angle DAC = 180° – 26° – 64° = 90°. Therefore triangle ADC is a triangle of the semicircle, and AC is a diameter.

3 Angle DBC = $\frac{1}{2}$ angle DOC = 50°, so angle ABC = 95°.
Opposite angles of cyclic quadrilateral: angle ADC = 85°.
Angle ODC = 40° (isosceles), so angle ADB = 40°, and angle DAB = 95°.
Angle DCB = 85° (either opposite angles of a cyclic quadrilateral, or 4 angles of a quadrilateral).
Since angle ADC = angle DCB and angle DAB = angle ABC, quadrilateral ABCD is a trapezium with parallel sides AB and DC.

4 Angle DAC = Angle DBC = angle DEC = $2x$.

Angle ACB = Angle ADB = x.

Angle ACP = angle DEC (alternate angles parallel lines) = $2x$, so angle CPB = $180 - (2x + x) - 2x = 180 - 5x$.

Therefore angle DPC = 180 – angle CPB = $180 - (180 - 5x) = 5x$.

5 Angle CAB = 60° (alternate segments). Angle ACB = 60° (isosceles triangle, which is actually an equilateral).

Angle COB = 120° (angle at centre of circle). Angle OCB = 30° (isosceles triangle).

Angle OCB = 30° = 0.5 × 60°, so OC bisects angle ACB

6 True. Opposite angles of cyclic quadrilateral add to 180°. In a parallelogram, opposite angles do not add to 180° (but adjacent angles do add to 180°). The only case of a cyclic quadrilateral where adjacent angles add to 180°, and opposite angles add to 180°, is a rectangle, with all angles 90°.

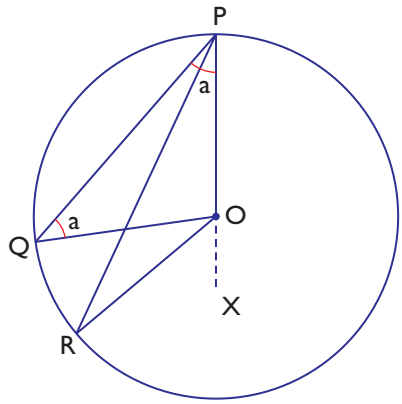
7 a i Triangles OAM and OBM are congruent because OA = OB are both radii and AM = BM as M is the midpoint and OM is common to both triangles.

ii Angle OMA is equal to angle OMB as the triangles are congruent.

Angle OMA is 90° because angles on a straight line add up to 180°.

8 a i Angle OPQ is the same as angle OQP because triangle POQ is isosceles as OP and OQ are both radii.

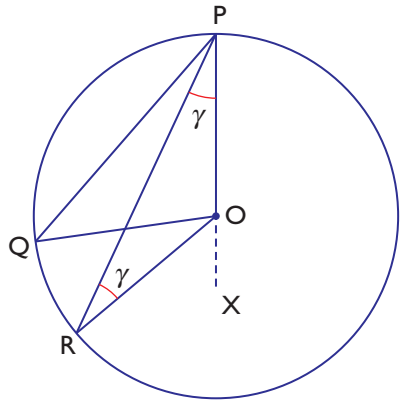
ii



iii angle POQ = 180° – 2a

b i Angle ORP = angle OPR as triangle OPR is isosceles with OP and OR radii.

ii



iii Angle POR = 180° − 2(angle ORP) = 180° − 2γ. (assuming that in **bi**, the angle ORP is one of the γ angles)

c Angle QOR = angle POR – angle POQ = 180° – 2γ- (180° – 2a) = 2a – 2γ.

d Angle QPR = angle OPQ – angle OPR = a – γ, therefore angle QOR = 2 × angle QPR.

Problem solving (pages 371–372)

1 **a** $x = 45°$

b RP and QP are the same length.

2 Angle OCA = 90° tangent and radius
angle OAC = 34° angles of triangle OAC
angle EBA = 108° angles on straight line
angle AEB = 38°, angles of triangle AEB
angle FEG = 38°, opposite angles
angle EFG = 90°, triangle in a semicircle
angle FGE = 52°, angles of triangle FGE
angle COG = 124°, angles on straight line
angle OCG = angle OGC = 28°, angles of isosceles
angle CGF = angle OGC + angle FGE = 80°

3 **a** angle ACB = 90° (triangle in a semi circle) = angle AXC
angle CAX = angle CAB (same angle)
angle ACX = 90° − angle CAX = 90° − angle CAB = angle ABC
triangle AXC is similar to ACB (AAA)

b $\frac{AC}{AX} = \frac{AB}{AC}$, similar triangles ratio of sides
therefore, $AC^2 = AB \times AX$

4 Angle QPX = angle SRX (alternate angles)
angle QPX = angle QPR
angle QPR = angle QSR (same segment)
angle QSR = angle XSR
angle QPR = angle XSR
angle QPX = angle XSR
angle SRX = angle XSR
So, triangle SXR is isosceles and SX = RX.

5 35°

6 angle BAD + angle ADC = 180
angle BAD + angle BCD = 180
So, angle ADC = angle BCD.

7 **a** When $x = 90°$, angle AOC = 0°, which is impossible.

b **i** kite
ii arrow
iii triangle

8 **a** 6.8 cm

b 90°

c 112°

d 62°

e 6.8 tan(34°) = 4.6 cm

Reviewing skills (page 373)

1 84°

2 **a** angle DCB = 90° triangle of a semi circle
angle CBD = 32°
OB = OA isosceles
Angle AOB = 32°, so OA must be parallel with BC to have alternate angles rule.

b Angle CBA = 74° + 32° = 106° ≠ 90°. Therefore AB is not parallel to CD.

3 Geometry and Measures Strand 3 • Unit 5 • Answers

Practising skills (pages 376–378)

1 a z

$z^2 = x^2 + y^2$

b m

$m^2 = n^2 + l^2$

c f

$f^2 = e^2 + g^2$

d r

$r^2 = p^2 + q^2$

2 a $x^2 = 5^2 + 12^2$

$x^2 = 25 + 144$

$x^2 = 169$

$x = \sqrt{169}$

$x = 13\,\text{cm}$

b $y^2 = 6^2 + 8^2$

$y^2 = 36 + 64$

$y^2 = 100$

$y = 10\,\text{cm}$

c $z^2 = 24^2 + 7^2$

$z^2 = 576 + 49$

$z^2 = 625$

$z = 25\,\text{cm}$

3 a 7.2 cm

b 8.6 cm

c 1.4 cm

4 a 3 cm

b 3 cm

c 3 cm

The hypotenuse is a square root of a square number and the sides are expressed as square roots of whole numbers.

5 a $x^2 + 4^2 = 5^2$
$x^2 + 16 = 25$
$x^2 = 25 - 16$
$x^2 = 9$
$x = \sqrt{9}$
$x = 3\,\text{cm}$

b $y^2 + 15^2 = 17^2$
$y^2 + 225 = 289$
$y^2 = 289 - 225$
$y^2 = 64$
$y = 8\,\text{m}$

c $z^2 + 12^2 = 15^2$
$z^2 + 144 = 225$
$z^2 = 225 - 144$
$z^2 = 81$
$z = 9\,\text{cm}$

6 a 9.2 cm
b 6.7 cm
c 6.2 cm

7 a 5 cm
b 6 m
c 1 mile

Developing fluency (pages 378–380)

1 Perimeter = 60 + 62.5 + 17.5 = 140 m
Area = 0.5 × 60 × 17.5 = 525 m^2

2 B is a right-angled triangle because it obeys Pythagoras' theorem:
$7.2^2 + 9.6^2 = 12^2$

3 4 m

4 Diagonal = 3.9 m. Total length of timber = 19.2 m.

5 6.4 km

6 a CD = 5 cm
b 54 cm
c 130 cm^2

7 Perimeter = 20 m
Area = 18 m^2

8 120 cm^2

9 a 7.1 cm
b Each segment = 7.13 cm^2

10 x = 6.5 cm and y = 10.4 cm

Problem solving (pages 380–382)

1 £1510

2 £468

3 26 km

4 Norman is correct because it is 20 460 newtons

5 Beth runs 190 m. Ali goes 57 + 76 + 57 = 190 m

6 **a** Because AC will be more than 15 m, meaning that AC + AB > 25 m

b Difference in height between AD and BC is 7 m

Therefore DC is 7.14 m which means AC is 16.6 m. The total cable she needs is 26.6 m so she does not have enough cable.

7 384 mm^2

8 x = 5.77 cm or 5.8 cm

Reviewing skills (page 382)

1 **a** 8 cm

b 25 cm

c 12 cm

2 **a** 10.2 cm

b 14.3 m

c 10.0 cm

3 perimeter = 669.2 m

area = 19 200 m^2

4 21 + 15.6 + 31.9 = 68.5

3 Geometry and Measures Strand 3 • Unit 6 • Answers

Practising skills (pages 386–387)

1 **a** 7.8 cm
b 18.8 cm
c 19.2 cm
d 33.0 cm

2 **a** **i** perimeter = 17.9 cm
ii area = 19.6 cm^2
b **i** perimeter = 25.7 cm
ii area = 39.3 cm^2
c **i** perimeter = 33.6 cm
ii area = 58.9 cm^2
d **i** perimeter = 35.7 cm
ii area = 39.3 cm^2
e **i** perimeter = 53.6 cm
ii area = 58.9 cm^2

3 **a** 942.5 cm^2
b 42.4 cm^2
c 91.4 cm^2
d 149.0 cm^2

4 **i** arc length = 2.6 cm
area = 9.8 cm^2
ii arc length = 11.0 cm
area = 23.1 cm^2
iii arc length = 1.1 m
area = 0.6 m^2
iv arc length = 2.4 mm
area = 3.5 mm^2

5 **a** 251°
b 60°
c 70°

Developing fluency (pages 387–388)

1 **a** 5 cm
b 55°

2 **a** 9.6 m
b 5.8 m^2

3 Small arcs are $\frac{60}{360} \times 12\pi = 2\pi$ each.

Large arc $= \frac{60}{360} \times 18\pi = 3\pi$

Total perimeter $= 2 \times 2\pi + 3\pi + 18 = 7\pi + 18$ cm

4 7.5 cm

5 $9.234\pi\,\text{cm}^2$

6 112.5°

7 $234.4\,\text{cm}^2$

8 114.6°

Problem solving (pages 389–390)

1 **a** 41.8 m

b $56.0\,\text{m}^2$

2 **a** 52.6 m

b Yes, the volume enclosed is $100.5\,\text{m}^3$.

3 **a** 22.85 m

b $18.85\,\text{m}^2$

4 **a** Height = 3.46 m

area = 6.93m^2

b **i** $8.38\,\text{m}^2$ or $\frac{16}{6}\pi\text{m}^2$

ii $1.45\,\text{m}^2$

c $43.6\,\text{m}^3$

5 $1250\pi\text{cm}^2$

6 115°

7 68 cm

Reviewing skills (page 390)

1 **a** 6.6 cm

b 13.2 cm

c 19.8 cm

d 4.4 cm

2 **i** perimeter = 12.2 cm

area = $8.4\,\text{cm}^2$

ii perimeter = 26.8 cm

area = $37.7\,\text{cm}^2$

iii perimeter = 14.3 cm

area = 3.43 cm

iv perimeter = 28.6 cm

area = $6.9\,\text{cm}^2$

3 Perimeter = $160 + \frac{40}{3}\pi\text{m}$

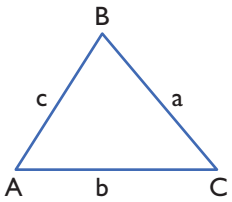
Area = $\frac{1600}{3}\pi\text{m}^2$

4 The answer is **b**

3 Geometry and Measures Strand 3 • Unit 7 • Answers

Practising skills (pages 393–395)

1 a



b 8.9 cm

2 a 14.8 cm

b 1.8 m

c 36.0 cm

3 8.0 cm

4 a −0.8387

b −0.5446

c 0.1736

5 9.6 m

6 a 61.7°

b 57.8°

c 62.2°

7 a A = 52.8°, B = 83.3° C = 43.9°

b A = 40.1°, B = 82.0° C = 57.9°

c A = 27.5°, B = 43.8° C = 108.7°

Developing fluency (pages 395–397)

1 2.2 km

2 a 13.9 cm

b 115.1°

3 127.5°

4 No, the actual angle is 59.0°.

5 a 55.7°

b 77.9°

c 46.5°

6 There is no answer as cos $B = 1.0612$, and cosine cannot be greater than 1. The triangle cannot exist because the longer side is longer than the sum of the two shorter sides.

7 i a AB = c

BD = $c \sin A$

AD = $c \cos A$

b DC = AC – AD = $b - \cos A$

c $(c\sin A)^2 + (b - c\cos A)^2 = a^2$

$(c\sin A)^2 + b^2 - 2bc\cos A + c^2\cos^2 A = a^2$

$c^2\sin^2 A + b^2 - 2bc\cos A + c^2\cos^2 A = a^2$

$a^2 = b^2 - 2bc\cos A + c^2(\sin^2 A + \cos^2 A)$

d $a^2 = b^2 + c^2 - 2bc\cos A$

ii $QR^2 = QP^2 + PR^2$

$1 = \frac{QP^2}{QR^2} + \frac{PR^2}{QR^2}$

$\sin\theta = \frac{QP}{QR}$; $so\ \frac{QP^2}{QR^2} = \sin^2\theta$

likewise $\cos\theta = \frac{PR}{QR}$; so $\frac{PR^2}{QR^2} = \cos^2\theta$

substitute back $1 = \sin^2\theta + \cos^2\theta$

Problem solving (pages 397–399)

1 270 m

2 41.6 km

3 $2\sqrt{7}$

4 1470 m^2

5 Yes as it is 2.96 m above

6 4.9 cm

7 a 59.8°

b 60°

8 13.4 m

9 $\frac{\frac{1}{2}\sqrt{2}}{8}$

10 24.7 km

Reviewing skills (page 400)

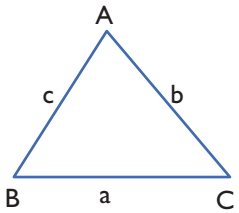
1 97.8°

2 29.5 cm

3 Geometry and Measures Strand 3 • Unit 8 • Answers

Practising skills (pages 403–404)

1 a



b 7.8 cm

2 a 7.2 cm

b 1.0 m

c 11.7 cm

3 5.5 cm

4 a 144°

b 94°

c 73°

d 81°

e 177°

f 89°

5 8.2 m

6 a 57.0°

b 46.4°

7 110.5°

8 a 41.5 cm^2

b 51.9 cm^2

c 83.6 cm^2

Developing fluency (pages 405–406)

1 12.8 km from A, 11.8 km from B

2 102.7 cm^2

3 125°

4 54.5° or 125.5°

5 a 6.6 cm

b 9.0 cm

c 78.6°

6 a 6.7 cm

b 64.4° or 115.6°

c 46.2 cm^2

Problem solving (pages 406–408)

1 14.1 m

2 No, CD = 63.9 m and DB = 97.8 m.

3 26.3 km

4 **a** 10 sin 25°

b Using sine rule in triangle B gives $XY = 10 \times \frac{\sin 50°}{\sin 65°}$. Since XY = 2 × MY

we have $2 \times 10 \sin 25° = 10 \times \frac{\sin 50°}{\sin 65°}$ so $\sin 25° \times \sin 65° = \frac{10 \sin 50°}{2 \times 10}$. Hence result.

5 No as the perimeter is 45.6 m.

6 No, as the cost will be at least £6000.

7 9.03 cm

8 45.6 m

9 8.9 km

10 304.2° or 355.8°

Reviewing skills (page 409)

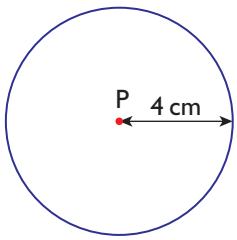
1 63.3°

2 93.4 cm

Geometry and Measures Strand 4 • Unit 4 • Answers

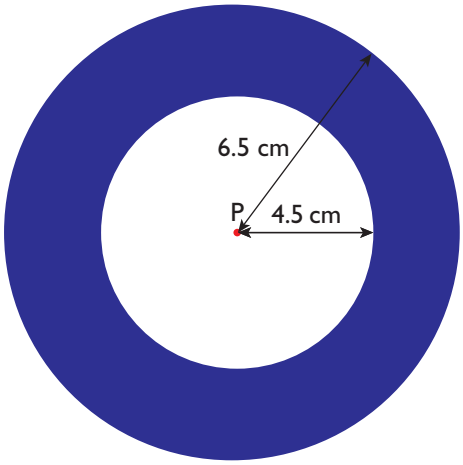
Practising skills (pages 412–414)

1 a

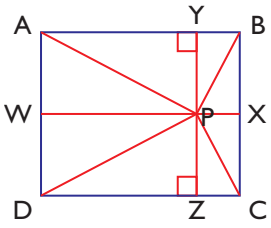


b circle

2



3 To calculate the distances, build right triangles by drawing a line through P perpendicular to two sides of the rectangle, as shown.



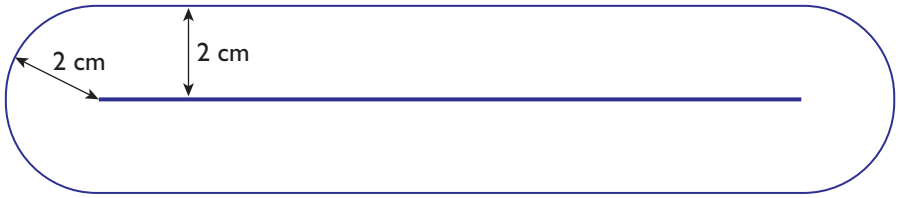
Distance WP = Distance AY = $(AP)^2 - (YP)^2$

Distance XP = Distance BY = $(BP)^2 - (YP)^2$

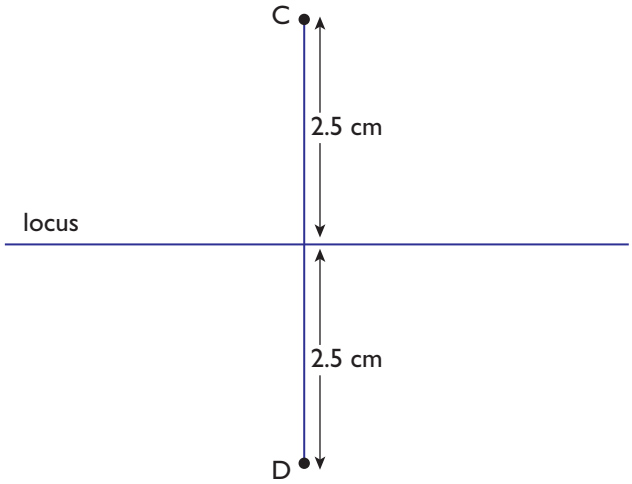
Distance YP = Distance BX = $(BP)^2 - (XP)^2$

Distance ZP = Distance CX = $(CP)^2 - (XP)^2$

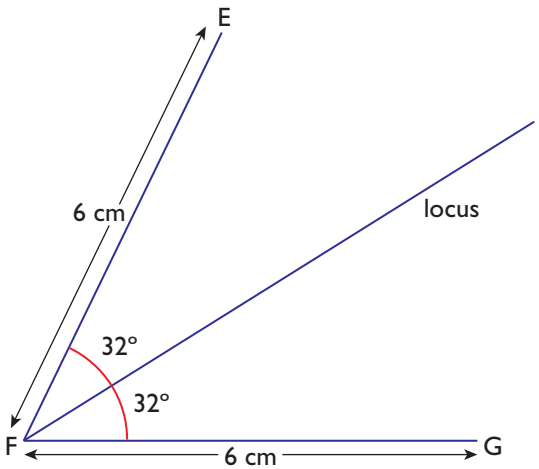
4 a and **b**



5 **a** and **b**

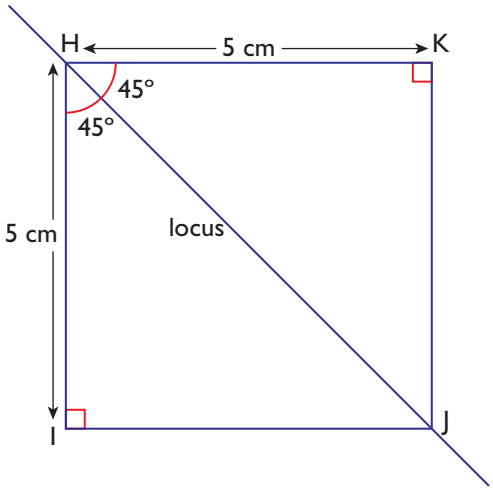


6 **a** and **b**

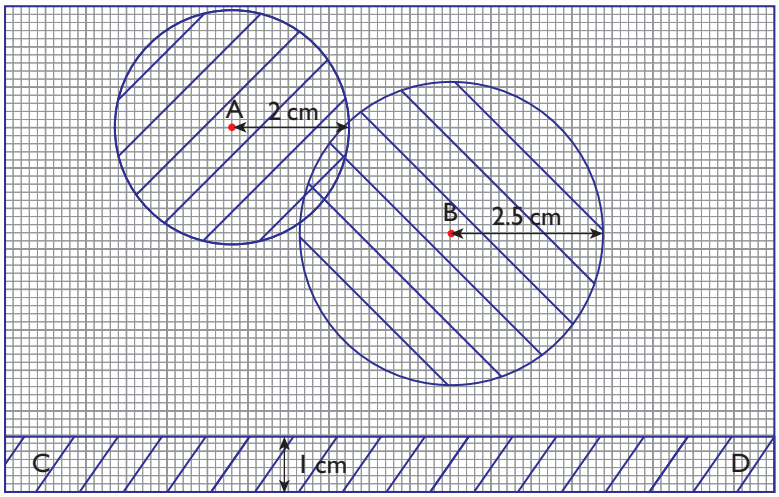


c Bisector of the angle EFG.

7 **a** and **b**



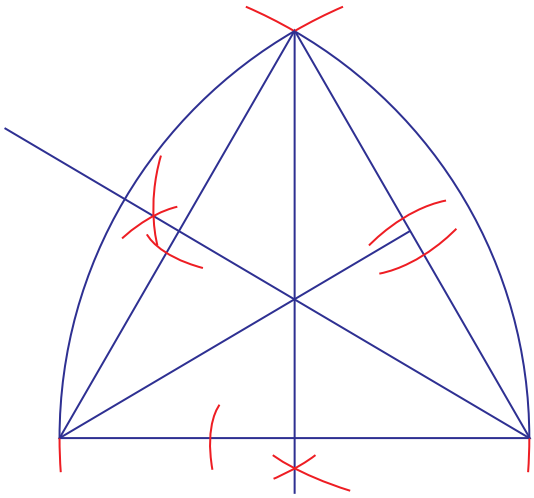
8 **a** and **b**



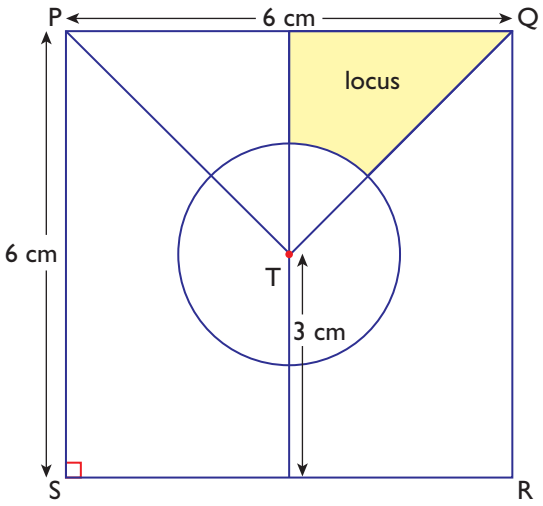
c No

Developing fluency (pages 414–417)

1



2 **a** and **b**



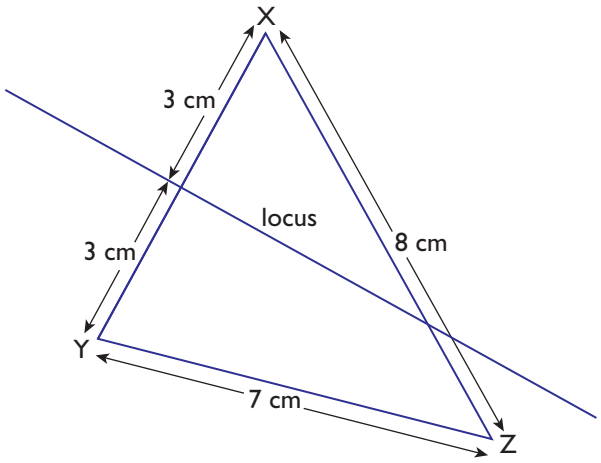
3 **a** XY is the locus of points 1 cm from AB, between 1.5 cm and 7.5 cm from AD and between 1.5 cm and 7.5 cm from BC and 5 cm from CD.

b s

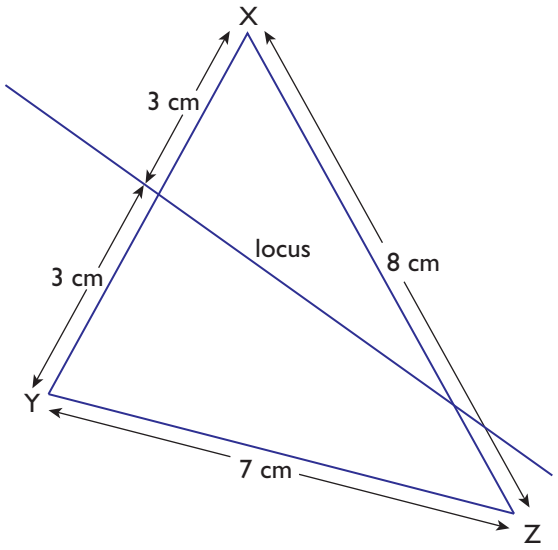
c **i** More than 4.2 cm from AB, more than 4.2 cm from CD, more than 6.3 cm from AD, and more than 6.3 cm from BC.

ii Between 3.3 cm and 4.2 cm from AB, between 3.3 cm and 4.2 cm from CD, between 5 cm and 6.3 cm from AD, and between 5 cm and 6.3 cm from BC.

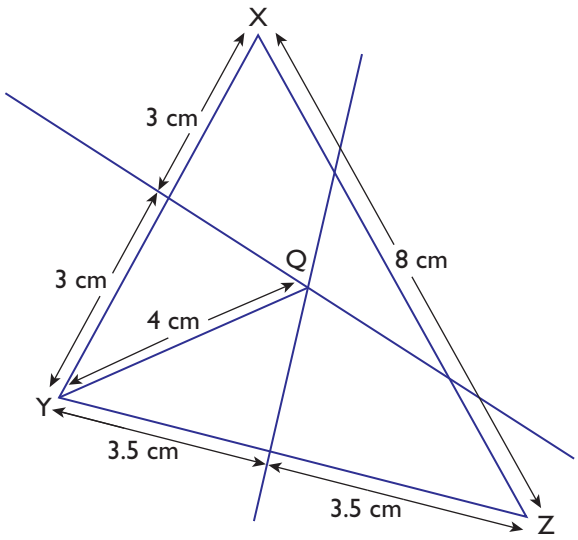
4 **a** and **b**



c

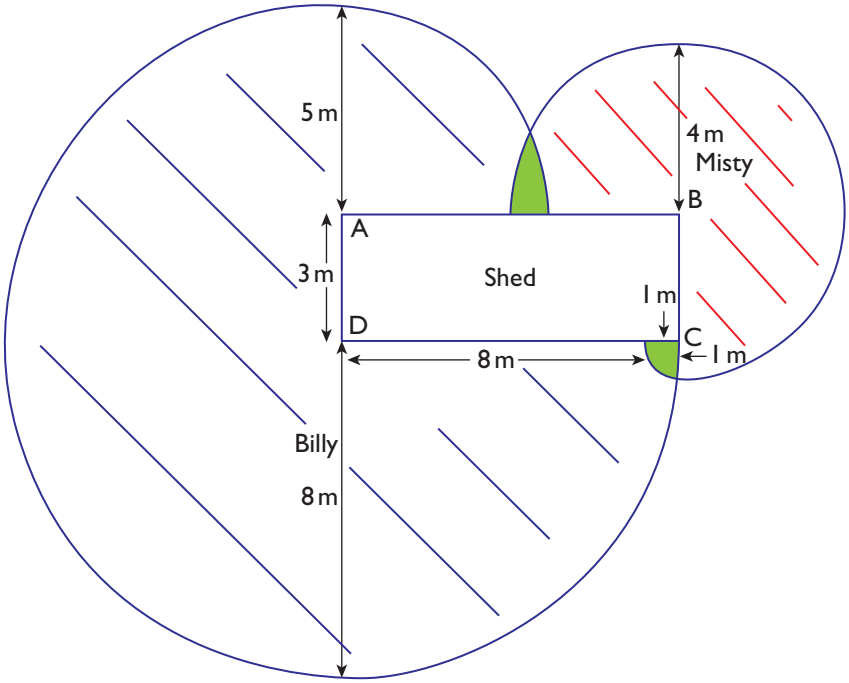


d



5 **a** Careful to show that when Billy moves to point A, his rope is restricted at A, making a new circle locus of radius 5 m. Equally, when Misty reaches point C, her rope is restricted, making a new circle locus of radius 1 m. Where they can graze is not a simple circle about points B and D.

b

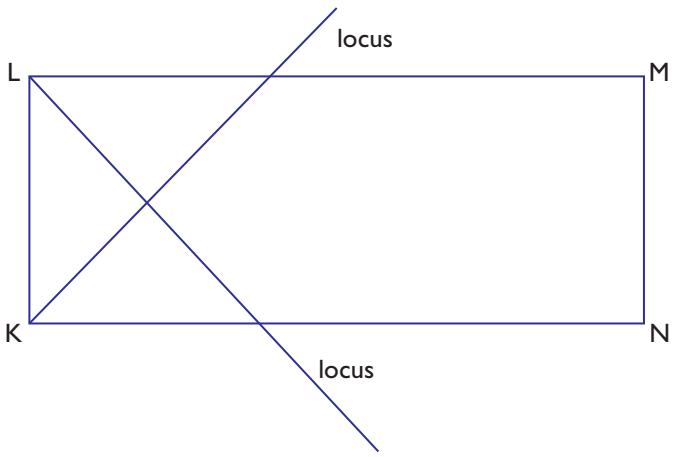


i red shading

ii blue shading

iii green shading

6 **a** and **b**



c one point

d To find the point that is equal distance from three sides, you bisect two angles and the bisectors cross at that point. You cannot bisect three angles and find a point that three bisectors cross in a rectangle.

e square

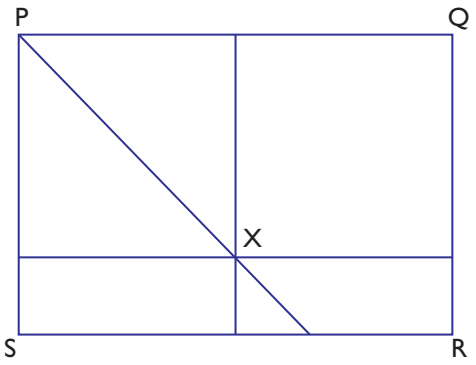
7 a First, points that are more than x km from Hen Rock and more than y km from Rooster Rock. Second, more than z km from The Chicks and more than y km from Rooster Rock. Third, points an equal distance from L and M.

b Hen Rock red region: points 0.75 km from the point Hen Rock.
Rooster Rock: points 1.25 km from the point Rooster Rock.
The Chicks: points 0.5 km from the point The Chicks.

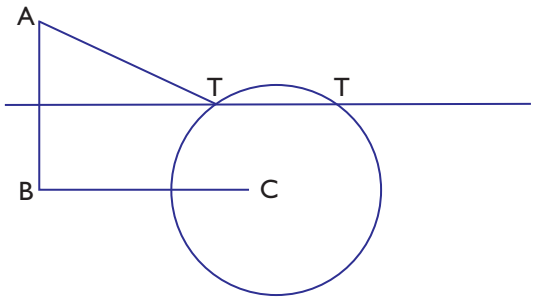
c Any answers such as: Sail due East until you reach an even distance between Hen Rock and Rooster Rock. Sail due south, maintaining equal distance from Hen Rock and Rooster Rock. At point that is an equal distance from The Chicks and Rooster Rock, sail South East. At a point 1.5 km due East of the Chicks, sail due South, maintaining equal distance from L and M.

Problem solving (pages 417–419)

1 58.3 m

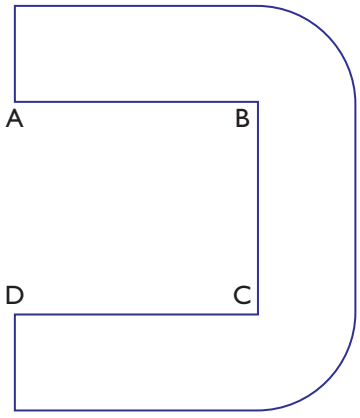


2 a



b 16 miles

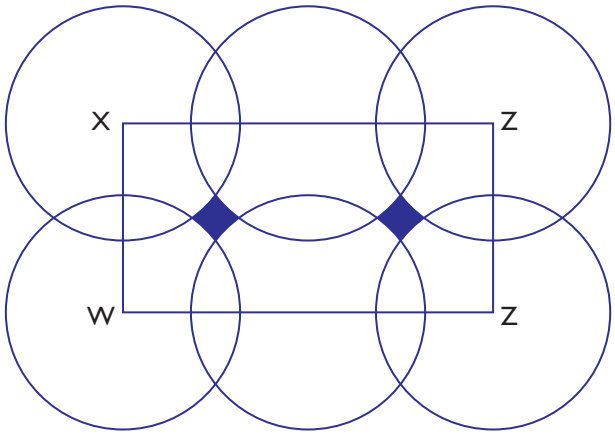
3 a



b 50.28 m^2

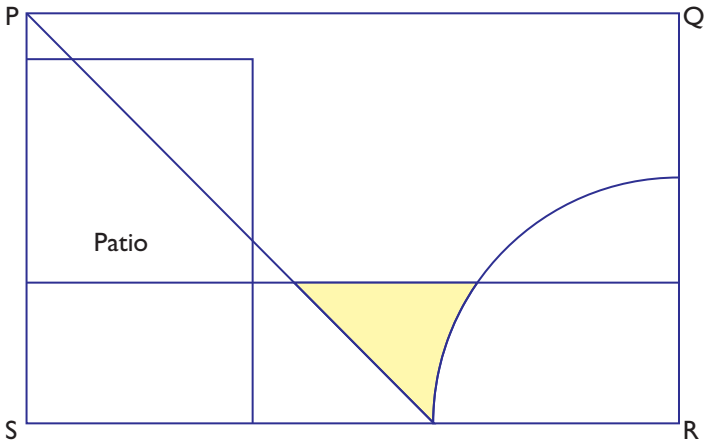
c The locus of the house is the inside of the path.
It misses the fourth side, AD.

4 **a** and **b**

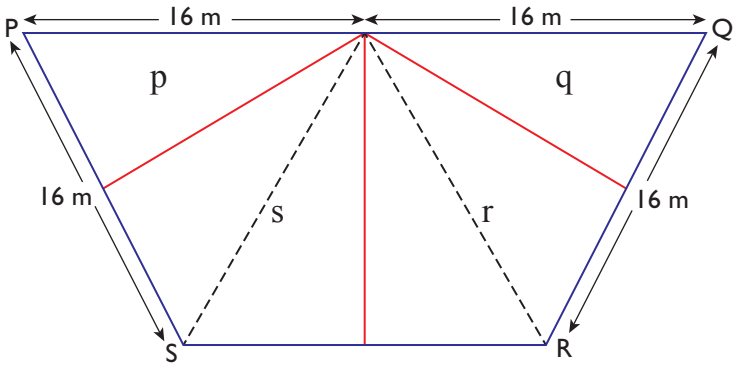


c e.g. move the lights at W and at Z 10 m towards X and Z.

5 (This is an approximate location for the tree.)



6 **a** and **b**



c 42 m

d $p = 56\,m^2$, $q = 56\,m^2$, $r = 105\,m^2$, $s = 105\,m^2$

e See diagram above.

Reviewing skills (page 419)

1 **a** points equal distance from AB and AC

b points equal distance from P and Q

c points 2 squares away from the line XY

d points 5 squares away from point Q

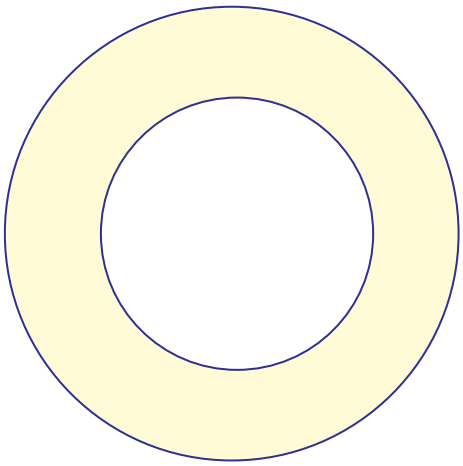
2 **a** points within the square and closer to BC than AD

b points closer to the line OL than the line OM

c points within 2 squares of point O

d points more than 3 squares but less than 5 squares away from line UV

3 **a**



b 5027 m^2

5 Geometry and Measures Strand 5 • Unit 7 • Answers

Practising skills (pages 423–426)

1 A and P; B and Q; C neither; D and Q; E and P; F and P; G and P; H neither; I and Q; J and P.

2 **a** $y = 7.5$
b $z = 9.6$
c $x = 3$
d similar

3 **a** $y = 12$
b $z = 16$
c $x = 5$
d similar
e Both are Pythagoras triangles.

4 **a** 2
b $x = 10$
c $w = 8$
d The same.
e They are similar triangles.

5 **a** Similar; size ratio of both sides is the same, factor 3.
b Not similar; the size ratio of the sides is different $5 \times 2 = 10$ but 2×2 does not = 5.
c Similar; regular pentagons are similar, factor 1.5.
d Not similar; side 4×3 = side 12 but the scale factor is not 3 for other sides.
e Not similar; scale factor different for each side.
f Not similar; angles different.
g Similar as angles are the same.

6 **a** Not similar as the given angle is not the same between the two shapes.
b Similar – the given angle is the same, and the sides scale with factor 2.
c Not similar – the given angle is the same, but the sides scale with different factors.

Developing fluency (pages 426–428)

1 B and C

2 2.5 m

3 **a** 121.5 cm^2
b 337.5 cm^2

4 **a** 12
b AEC, FBD; AVZ, FYU, BTX, YUC, XET, DVZ
c $27\sqrt{3}$ cm^2 or 46.8 cm^2

5 **a** 24 cm
b 48 cm
c A = 24 cm^2 and B = 96 cm^2

6 **a** P and A and B and C are congruent.
b P and D are similar.

Problem solving (pages 428–430)

1 **a** Using the sum of the angles in a triangle is 180°, the triangles have equal angles and so are similar.
b 3

2 48 cm

3 **a** Angle C is common, Angle CDE = angle CAB (= 90°) and angle CED = angle CBA (from angle sum of a triangle = 180°)
b 1.4 m

4 63 cm

5 3.2 m

6 394 cm

7 **a** Angle ACB = angle DCE (vertically opposite angles)
Angle ABC = angle CDE
Therefore the triangles are similar (AAA)
b 2
c $\frac{2}{3}$

8 Angle BAC = angle XAY (shared)
Angle AXY = angle ABC (corresponding angles)
Therefore AXY is similar to ABC.
As AX = $\frac{1}{2}$AB, it means that AB = 2AX, so the scale factor is 2. Therefore 2AY = AC meaning AY = $\frac{1}{2}$AC, and in the same way XY= $\frac{1}{2}$BC.

9 **a** Since M, N are midpoints, MN is parallel to BC, so angle MNA = angle CBA (corresponding angles)
Angle BAC = angle NAM (shared)
Therefore they are similar by AAA.
b All similar triangles
c 4:1

Reviewing skills (page 431)

1 **a** 10
b 12.5
c 7.2

2 **a** Unknown angle in T is 40; unknown angle in V is 30. Three equal angles therefore V and T are similar.
b 4.5 cm
c 5 cm

3 3.36 m

Practising skills (pages 434–436)

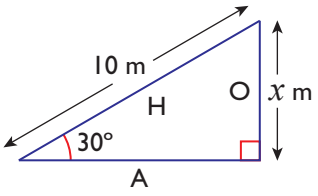
1 a i hypotenuse = a
ii opposite = b
iii adjacent = c
b i $h = f$
ii $o = e$
iii $a = d$
c i $h = h$
ii $o = i$
iii $a = g$
d i $h = j$
ii $o = l$
iii $a = k$
e i $h = o$
ii $o = n$
iii $a = m$
f i $h = p$
ii $o = r$
iii $a = q$

2 a $\sin\theta = \frac{3}{5}$ is true
b $\cos\theta = \frac{4}{5}$ is true
c $\tan\theta = \frac{3}{4}$ is true

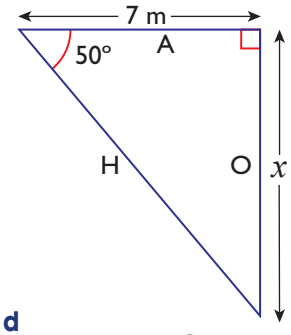
3 a $\tan\alpha = \frac{5}{12}$ is true
b none are true
c none are true

4 **i** and **ii**

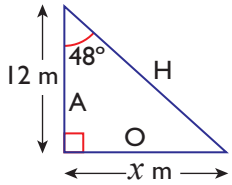
a



b



c



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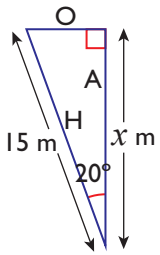
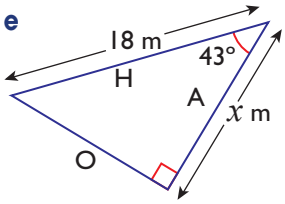


e



iii and **iv**

a $\sin 30 = \frac{x}{10}$; $10 \sin 30 = x$; $x = 5$ m

b $\tan 50 = \frac{x}{7}$; $7 \tan 50 = x$; $x = 8.3$ m

c $\tan 48 = \frac{x}{12}$; $12 \tan 48 = x$; $x = 13.3$ m

d $\cos 20 = \frac{x}{15}$; $15 \cos 20 = x$; $x = 14.1$ m

e $\cos 43 = \frac{x}{18}$; $18 \cos 43 = x$; $x = 13.2$ m

5 **a** 30°

b 45°

c 60°

d 40°

e 72°

f 82°

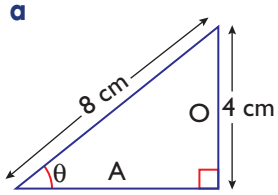
g 53°

h 48°

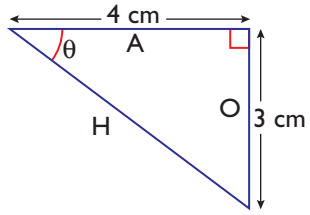
i 61°

6 **i** and **ii**

a



b



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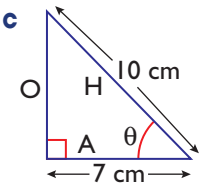

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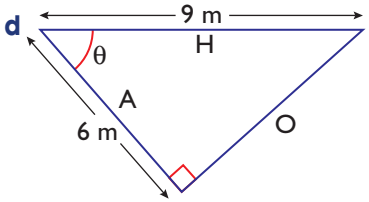

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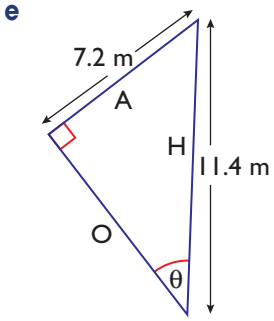
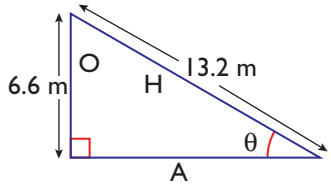


f



iii and **iv**

a $\sin\theta = \frac{4}{8}$; $\theta = 30°$

b $\tan\theta = \frac{3}{4}$; $\theta = 36.9°$

c $\cos\theta = \frac{7}{10}$; $\theta = 45.6°$

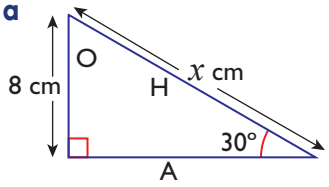
d $\cos\theta = \frac{6}{9}$; $\theta = 48.2°$

e $\sin\theta = \frac{7.2}{11.4}$; $\theta = 39.2°$

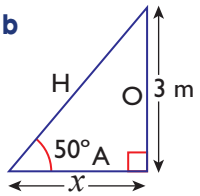
f $\sin\theta = \frac{6.6}{13.2}$; $\theta = 30°$

7 **i** and **ii**:

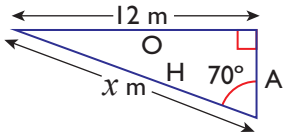
a



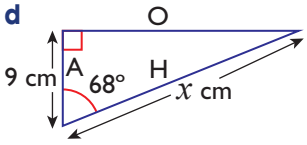
b



c



d



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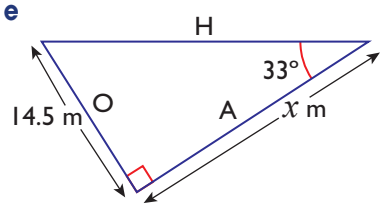
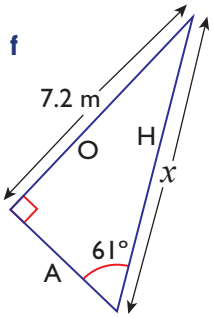


f



iii and **iv**

a $\sin 30 = \frac{8}{x}; \frac{8}{\sin 30} = x; x = 16.0\,\text{m}$

b $\tan 50 = \frac{3}{x}; \frac{3}{\tan 50} = x; x = 2.5\,\text{m}$

c $\sin 70 = \frac{12}{x}; \frac{12}{\sin 70} = x; x = 12.8\,\text{m}$

d $\cos 68 = \frac{9}{x}; \frac{9}{\cos 68} = x; x = 24.0\,\text{m}$

e $\tan 33 = \frac{14.5}{x}; \frac{14.5}{\tan 33} = x; x = 22.3\,\text{m}$

f $\sin 61 = \frac{7.2}{x}; \frac{7.2}{\sin 61} = x; x = 8.2\,\text{m}$

Developing fluency (pages 436–438)

1 **a** 11.0 m (1dp)

b 44.0 m^2 (1dp)

c 31.4 m (1dp)

2 perimeter = 53.5 m (1dp)

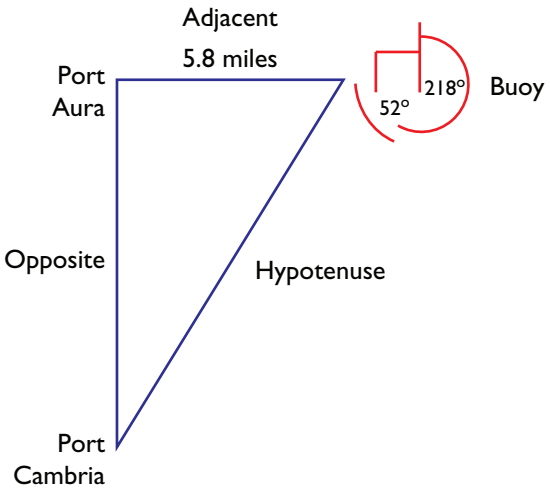
area = 109.8 m^2 (1dp)

3 (assuming the diagonal is included) 27.3 m

4 **a** 15.5 m (1dp)

b 32.3°

5 5.8 tan(52) = 7.4 miles



6 **a** 158.5 m (1dp)

b 133.6 m further

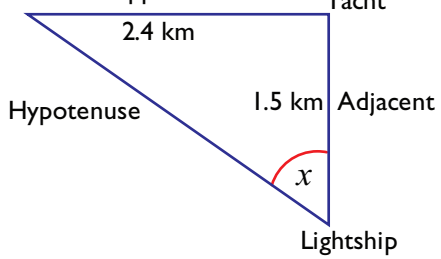
7 **a** **i** 71.1°

ii 12 cm

b 12 cm by $x^2 + 35^2 = 37^2$

c yes

8 Lighthouse Opposite Yacht



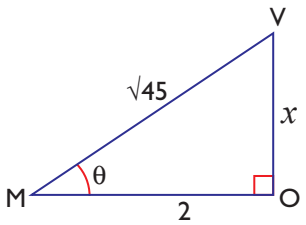
a $\sqrt{2.4^2 + 1.5^2} = 2.8$ km

d $\tan(x) = \frac{2.4}{1.5}$ gives x as 58°. Therefore the heading from the lightship to the lighthouse is 360 − 58 = 302°.

9 2314 m

10 **a** Pythagoras – $x^2 + 2^2 = 7^2$

b



c 72.7°

d 6.4 cm

Problem solving (pages 439–440)

1 0.350 m

2 3.14 km

3 **a** 68.8 cm

b 79.7 cm

4 **a** 19.8 m

b 20.2 m

5 **a** 33.7°

b e.g. the number of steps cancels and you are left with $\frac{20}{30}$

6 16.16 m

Reviewing skills (pages 440–441)

1 **a** $x = 4$ cm

i the 8 cm side is the hypotenuse

ii the side x is the opposite

iii the third side is the adjacent

b $x = 19.3$ cm

i the unlabelled side is the hypotenuse

ii the side x is the opposite

iii the 9 cm side is the adjacent

c $x = 12.3$ m

i the 20 m side is the hypotenuse

ii the unlabelled side is the opposite

iii the side x is the adjacent

2 **a** **ii** H: 8 cm side; O: unmarked side; A: 5 cm side

iii and iv $\cos\theta = \frac{5}{8}$, so $\theta = 51.3°$

b **ii** H: 11 cm side; O: 8.5 cm side; A: unmarked side

iii and iv $\sin\theta = \frac{8.5}{11}$, so $\theta = 50.6°$

c **ii** H: unmarked side; O: 5.6 m side; A: 7.4 m side

iii and iv $\tan\theta = \frac{5.6}{7.4}$, so $\theta = 37.1°$

d **ii** H: 9 m side; O: unmarked side; A: 6.4 m side

iii and iv $\cos\theta = \frac{6.4}{9}$, so $\theta = 44.7°$

3 **a** 5.5 cm

b 10.4 m

c 7.8 m

4 **a** 10.3 cm

b 10.6 m

c 8.3 m

5 8.77 m

5 Geometry and Measures Strand 5 • Unit 9 • Answers

Practising skills (page 444)

1 **a** When triangle A is rotated 90° clockwise about (2, 7), it moves to position B.
b When triangle B is rotated 90° anticlockwise about (4, 8), it moves to position C.
c When triangle C is rotated 90° anticlockwise about (6, 7), it moves to position E.
d When triangle E is rotated 90° clockwise about (9, 5), it moves to position D.
e When triangle B is rotated 180° clockwise about (5, 6), it moves to position E.
f A and D

2 **a** (1.4, 4) 90° clockwise
b (0, 5) 180°
c (−3, 2.5) 180°
d (1.5, 2) 180°
e (4.5, −0.5) 180°
f (0, −3) 180°
g (1.5, 0.5) 90° clockwise

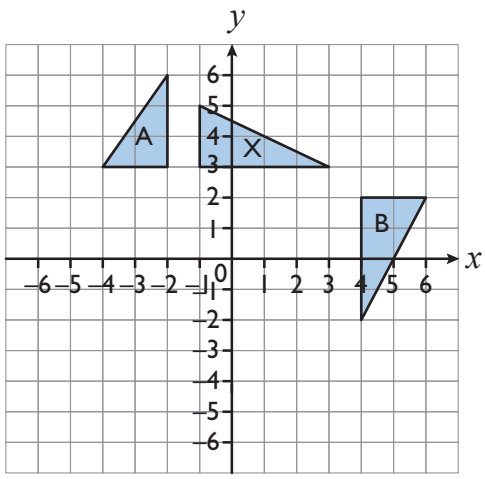
Developing fluency (pages 445–448)

1 **a** **i** (0, 2)
ii (−1, 1)
iii (−2, 0)
b lie on a straight line

2 **a** Q
b T
c P
d S
e R

3 **a** 90° clockwise rotation about (−3, 4)
b 90° anticlockwise about (−3, 4)

4 **a** and **b**



c 180° (either direction) rotation about (1, 2)

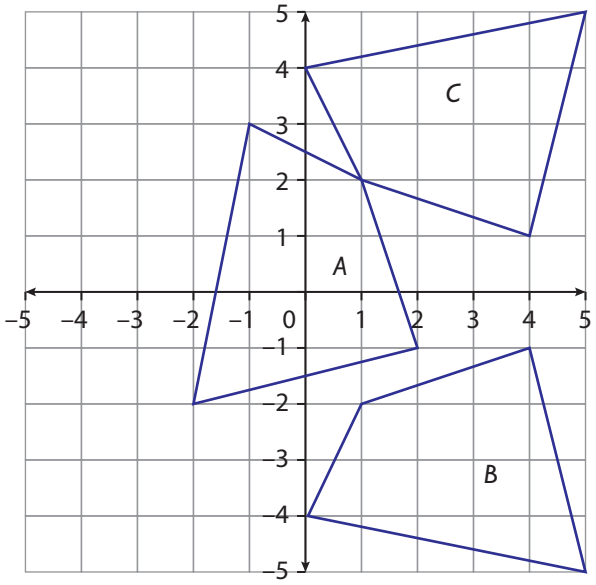
5 **a** 90° clockwise about (4, 6)

b 180° about (5,7)

c 90° clockwise about (7, 5)

d 180° about (4, 5)

6 **a**



b You cannot find a rotation for a shape that is reflected.

c 90° anticlockwise about (3, 0)

7 **a** **i** Translate (7, −8)

ii 180° about (3.5, −4)

iii Reflection in the y axis.

iv Reflect in x axis.

v Rotate 90° clockwise.

vi 90° clockwise about (−3, 0)

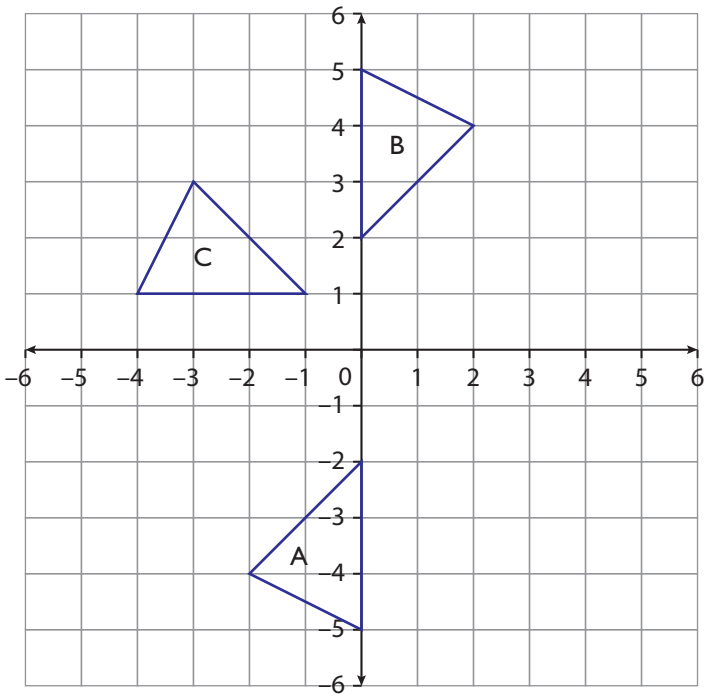
vii 180° about (3.5, −4)

viii Translate (−7, 8)

b B and D

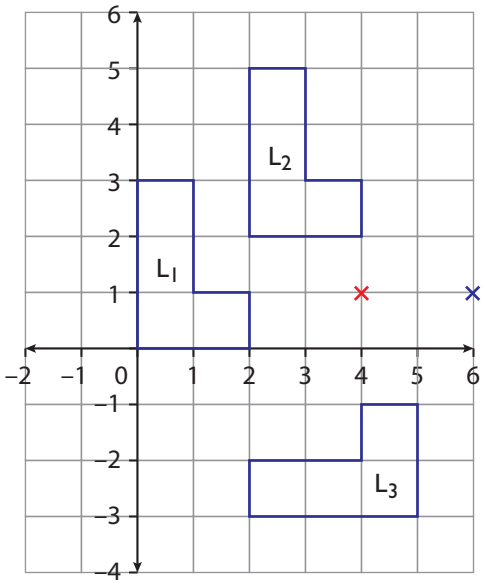
Problem solving (pages 448–449)

1 **a** and **b**



c a rotation of 90° anticlockwise about (1, 0)

2 **a** and **b**



c a rotation of 90°clockwise about (4, 1)

Reviewing skills (page 450)

1 **a** 90° clockwise about (−1, −1)

b 90° anticlockwise about (−1, −1)

c 90° clockwise about (4, 2)

d 180° about (3, −2)

2 **a** a rotation of 90° anticlockwise about the origin

b a rotation of 90° anticlockwise about (−2, 4)

c a rotation of 180° about (1, 3)

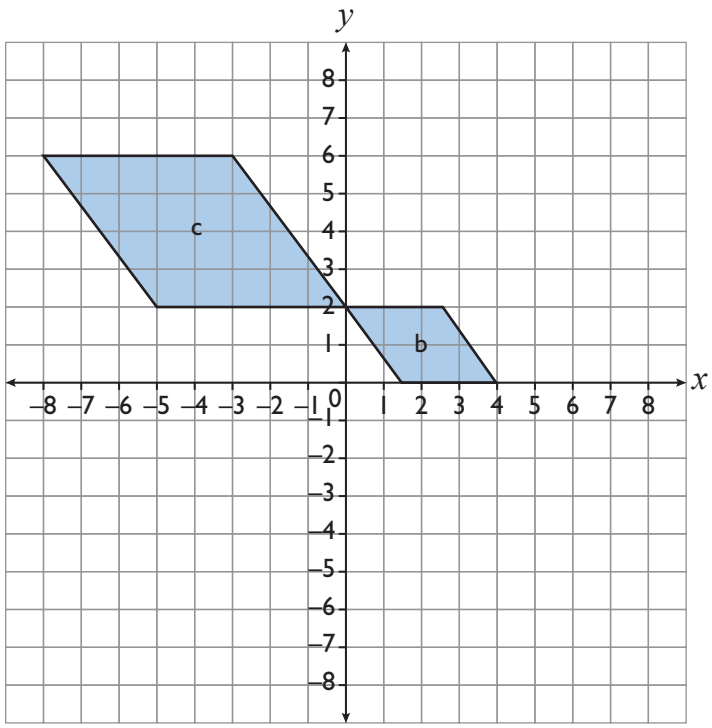
Practising skills (pages 454–455)

1 Scale factor −2, centre (−2, 3)

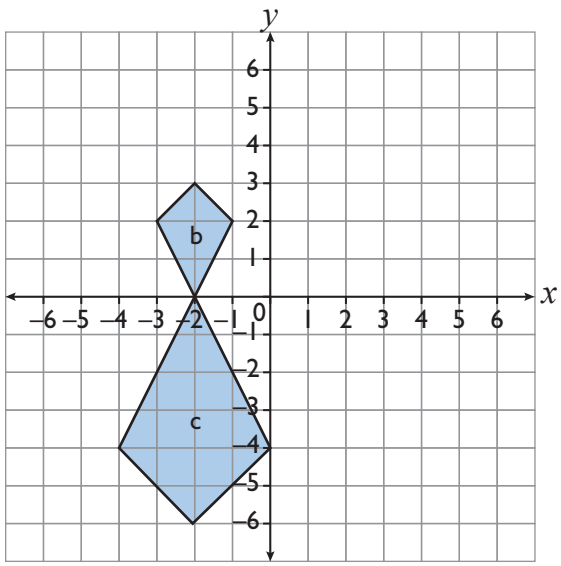
2 Scale factor −3, centre (1, 0)

3 Scale factor −3, centre (2, −1)

4 a, b, c



5 a, b, c



6 **a** Scale factor −3, centre (−2, 3)

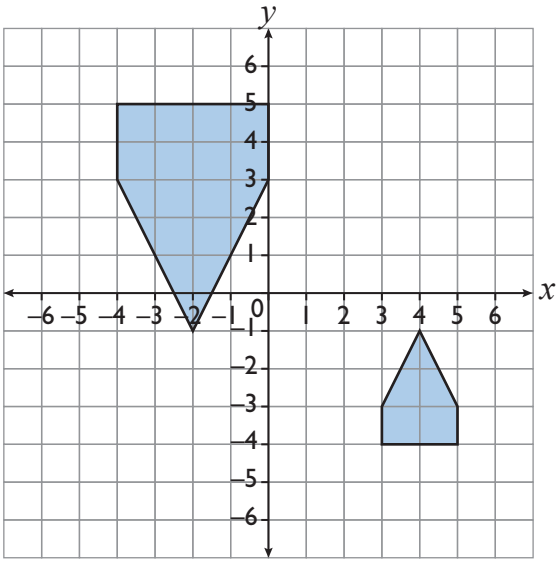
b Scale factor −2, centre (−2.5, 2)

c Scale factor $\frac{2}{3}$, centre (−6, −5)

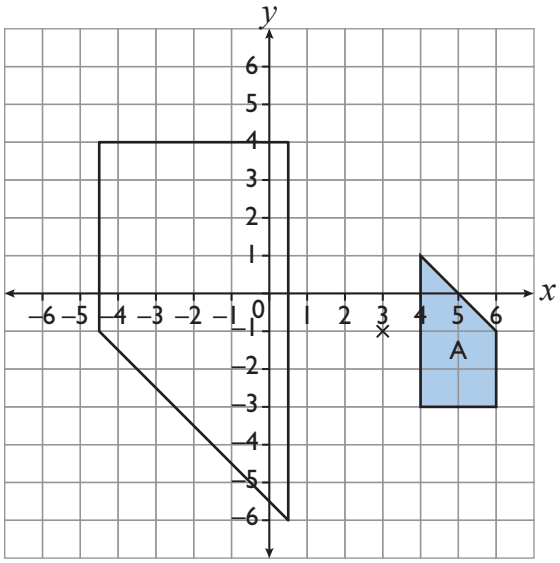
Developing fluency (pages 456–457)

1 Scale factor −3, centre (−3, 3.5)

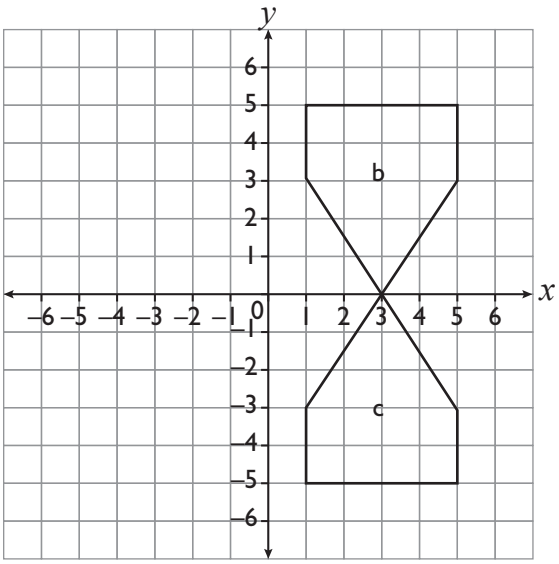
2



3



4 a, b, c

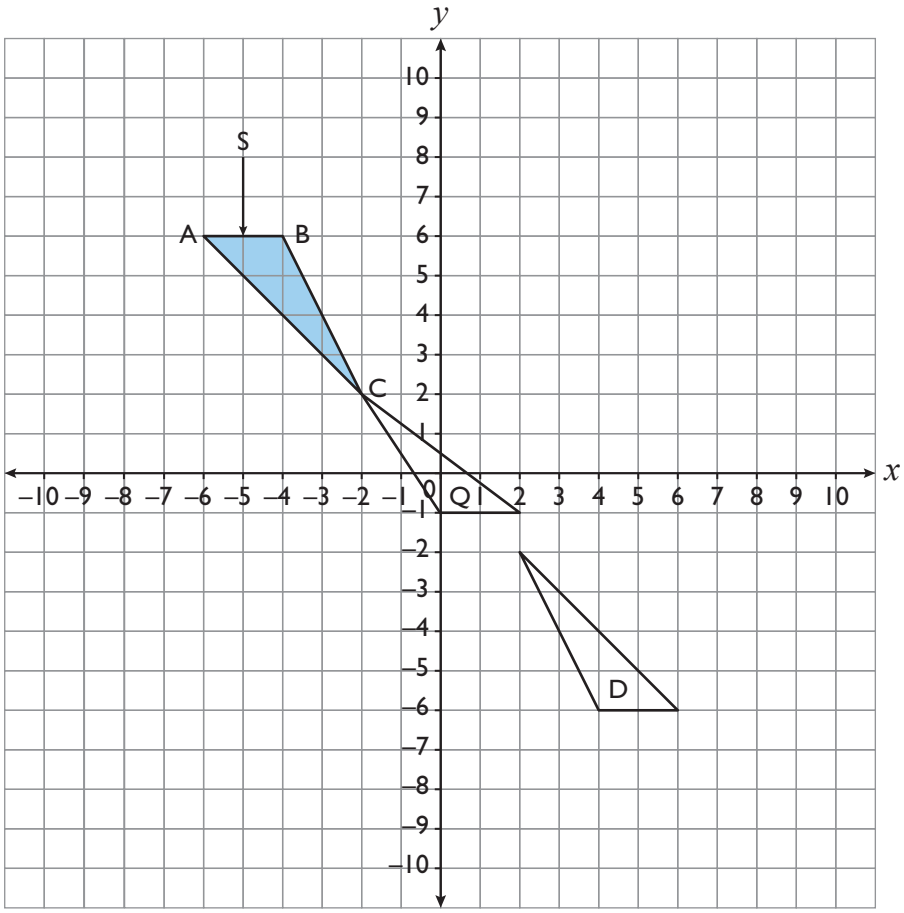


5 a Scale factor $-1\frac{1}{3}$, centre (0, −2)

b Scale factor $-\frac{2}{3}$, centre (3, 1)

Problem solving (pages 458–459)

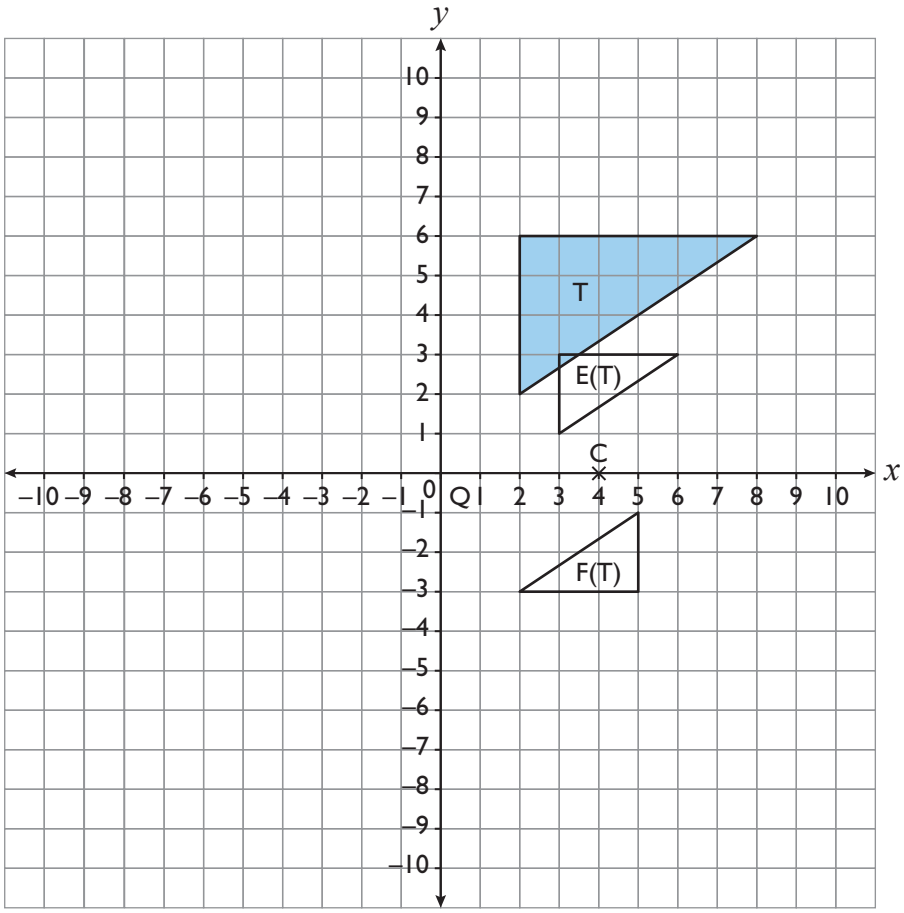
1 a and c



b a rotation of 180° about the origin

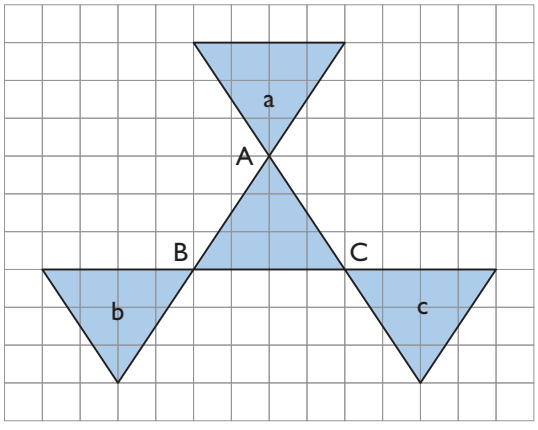
d a rotation of 180° about (2, 2)

2 a and b



c a rotation of 180° about C (4, 0),
an enlargement of scale factor –1, centre C

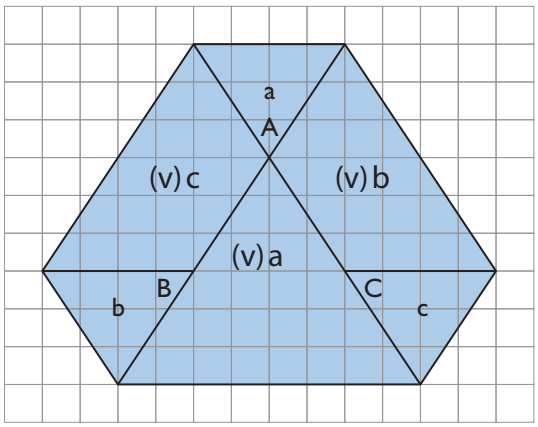
3 i and ii, a, b and c



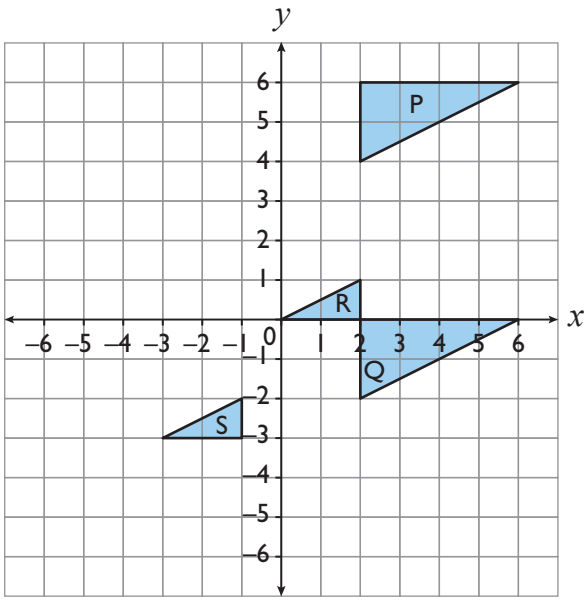
iii rotational symmetry order 3

iv yes, it would be the same

v, a, b and c



4 a, b, c, and d

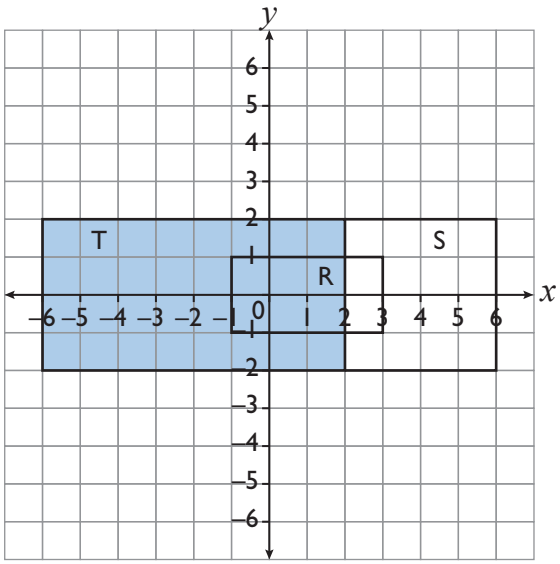


d transformation $\begin{pmatrix} 0 \\ -6 \end{pmatrix}$, (−6 in the y axis direction)

f transformation $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$, (3 in the positive x axis direction, 2 in the positive y axis direction)

5 There is an error in the Student Book. This is not a rectangle, as the second coordinate for R should be (3,1)

i, ii and iii



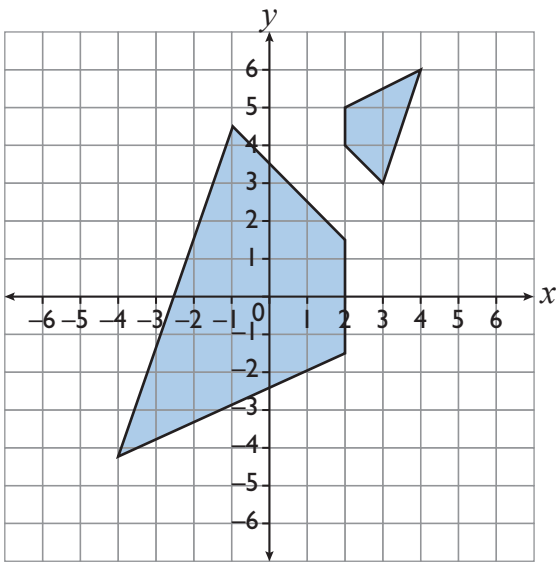
iv a Reflection in y-axis

b Enlargement scale factor $\frac{1}{2}$ centre (0,0)

c Enlargement scale factor $-\frac{1}{2}$ (0,0)

Reviewing skills (page 459)

1 a, b, c



Geometry and Measures Strand 5 • Unit 11 • Answers

Practising skills (pages 462–463)

1 26.1 cm

2 **a** 21.2 cm
b 22.6 cm
c 62.1°

3 18.1 cm (to 1 d.p.)

4 **a** 20.7 cm
b 68.1°

5 **a** 20.1 cm
b 25.5 cm
c 32.4 cm
d 51.7°

Developing fluency (pages 463–465)

1 **a** 17.6 cm
b 19.9°

2 **a** 72.5°
b 17.0 cm
c 64.9°

3 **a** 26.3 cm
b 171.3°
c 160.0°

4 31.1°

5 19.95 cm

6 **a** 20.0 cm
b 16.9 cm

Problem solving (pages 465–468)

1 28 105 m^2

2 **a** $\tan BCE = \frac{80\,cm}{120\,cm}$, angle BCE = 33.7°
b 199 cm

3 532 km/h

4 26.2 m

5 25.7 m

6 Angle BCE = 30.2°

7 39.6°

8 AXB = 2 $tan^{-1}\left(\frac{8}{6}\right)$ = 106°

9 a The congruent faces are VAB, ABC and VBC (SAS)

b Let AB = a. Area of triangle VAB is $\frac{a^2}{2}$, sides of triangle VAC are $\sqrt{2}$, area of triangle VAC is

$\frac{1}{2}\times\sqrt{2}\times\sqrt{2}\times\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{2}a^2$; TSA $=\frac{3a^2}{2}+\frac{3a^2}{2}=\frac{3+\sqrt{3}}{2}a^2$;

ratio is $\frac{\sqrt{3}}{2}:\frac{3+\sqrt{3}}{2}=1:\frac{3+\sqrt{3}}{2}\times\frac{2}{\sqrt{3}}=1:\frac{3}{\sqrt{3}}+1=1:\sqrt{3}+1$.

10 Angles between the edges on the square base are 90° because it is a square. Length of the diagonal of the square is $\times\sqrt{2}$. Angle at vertex V at the top is 90° from the cosine rule.

Reviewing skills (page 468)

1 a 7.4 cm

b 11.8°

6 Geometry and Measures Strand 6 • Unit 5 • Answers

Practising skills (pages 472–473)

1 **a** **i** $18\,cm^2$
ii $144\,cm^3$
b **i** $20\,cm^2$
ii $240\,cm^3$
c **i** $16\,cm^2$
ii $176\,cm^3$

2 **a** **i** $7.07\,cm^2$
ii $70.7\,cm^3$
b **i** $50.3\,cm^2$
ii $351.9\,cm^3$
c **i** $78.5\,cm^2$
ii $628.3\,cm^3$
d **i** $153.9\,cm^2$
ii $923.6\,cm^3$

3 $54\,cm^3$

4 8 cm

5 **a** $14\,cm^2$
b 20 cm

6 **a** **i** $50\,m^2$; $40\,m^2$; $30\,m^2$
ii $120\,m^2$
b **i** 12 m
ii $120\,m^2$
c They are equal
d **i** $6\,m^2$
ii Find the volume = $60\,m^3$

7 **a** $210\,cm^3$
b $168\,cm^3$
c $96\,cm^3$
d $62.8\,cm^3$

Developing fluency (pages 474–475)

1 $3.5\,m^3$

2 **a** 62.8 cm

b $1885\,cm^2$

c $314.2\,cm^2$

d $2513\,cm^2$

3 $370\,cm^3$

4 **a** Q

b $301.6\,cm^3$

5 **a** $2.4\,m^3$

b $8.48\,m^2$

6 **a** Anna's

b $0.033\,m^3$

7 **a** Yes, rectangle

b Yes, triangle

c Yes, circle

d No

e No

f Yes, square

g No

Problem solving (pages 476–477)

1 **a** 47.52 m^2

b 12.96 m^3

2 Yes, as 50 bins have a capacity of 154 m^3

3 **a** 1676 trips

b **i** 25 133 m^2

ii 2482 m^3

4 **a** 2.8 m^2

b 0.3 m^3 or 300 000 cm^3

5 **a** 201 cm^2

b 1407 cm^2; £1688.77

6 **a** **i** 900 cm^2

ii 180 m^3 or 180 000 000 cm^3

b 420 m^3

Reviewing skills (page 478)

1 **a** **i** 21 cm^2

ii 210 cm^3

b **i** 28 cm^2

ii 252 cm^3

2 46 cm^2

3 **a** **i** 113.1 cm^2

ii 1018 cm^3

b **i** 63.6 cm^2

ii 763 cm^3

c **i** 28.3 cm^2

ii 226 cm^3

4 189 cm^3

6 Geometry and Measures Strand 6 • Unit 6 • Answers

Practising skills (pages 481–482)

1 **a** 1 : 2

b **i** A = 32 cm; B = 64 cm;

ii 1 : 2

c **i** A = 48 cm^2; B = 192 cm^2

ii 1 : 4

2 **a** 30 cm

b small = 225 cm^2; large = 900 cm^2

c 1 : 4

3 **a** 1 : 3

b 1 : 9

4 **a** 64 cm^3

b 8000 cm^3

c 1 : 5

d 1 : 125

e 125

5 **a** Area enlarges by the square of the length

b 8

Developing fluency (pages 482–483)

1 **a** 1.5

b $x = 6$, $y = 4.5$, $z = 8$

c 240 cm³, 810 cm³

d 8 : 27

2 **a** 6 cm and 10 cm

b 3 : 5

3 **a** 24 m × 16 m

b 15.36 cm²

c 384 m² or 3 840 000 cm²

d 1 : 250 000

4 **a** **i** 3.75 cm

ii 135 cm³

b **i** 1458 cm²

ii 3645 cm³

5 **a** 12 cm

b 30 cm

c 2

d 1 : 8

Problem solving (pages 484–485)

1 $18\,m^2$

2 **a** $90\,cm \times 36\,cm$

b 18

c $3240\,cm^2$

3 **a** 100 cm

b Block A $9600\,cm^2$; Block B $60\,000\,cm^2$

c Area block A $= 6 \times 40^2 = 9600\,cm^2$, Area block B $= 6 \times 100^2 = 60\,000\,cm^2$

Ratio is $9600 : 60\,000 = 1 : 6.25$

4 **a** 20 cm

b Width = 6 m; height = 2.4 m

c $115.2\,m^3$

d Width = 15 cm; height = 6 cm; volume = $1800\,cm^3$

e Model = $0.0018\,m^3$; real = $115.2\,m^3$; ratio is $0.0018 : 115.2 = 1 : 64\,000 = 1 : 40^3$

5 **a** 2 : 3

b Areas are 96 and 216 cm^2 with a ratio of $96 : 216 = 4 : 9$ which is not the same as 2 : 3 (=4 : 6)

c Masses are $64d$ and $216d$, where d is the density

Ratio is $64d : 216d = 8 : 27$

6 **a** 24 cm and 36 cm

b **i** 1 : 3

ii 1 : 9

c Resulting picture is 32 cm × 44 cm. This is a different ratio of sides to the original (1 : 1.375 vs 1 : 1.5), so it is not an enlargement of B

Reviewing skills (page 486)

1 **a** 5 cm × 5 cm × 15 cm
b 56 cm^2
c 350 cm^2
d 24 cm^3
e 375 cm^3
f 1 : 6.25
g 1 : 15.625

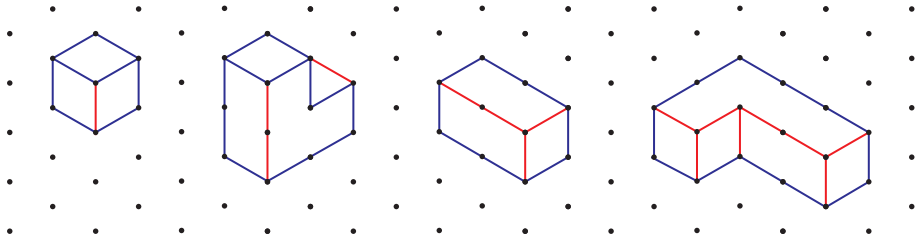
2 112.5 (i.e. 112 cubes)

Geometry and Measures Strand 6 • Unit 7 • Answers

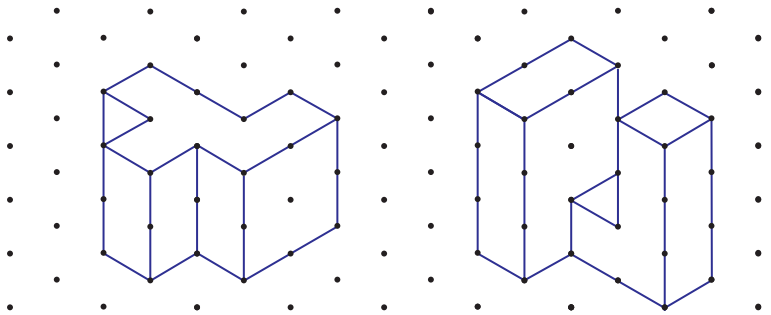
Practising skills (pages 489–490)

1 **a** = iv **b** = vi **c** = v **d** = i **e** = iii **f** = ii

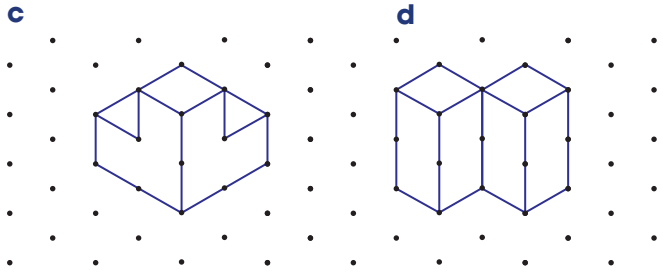
2 **a** **b** **c** **d**



3 **a** **b**



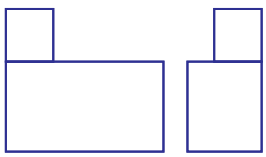
c **d**



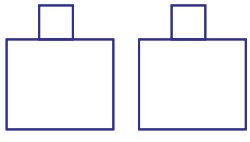
4

Shape	Front elevation	Side elevation	Plan
A	1	2	4
B	9	8	5
C	3	6	7

5 **a**

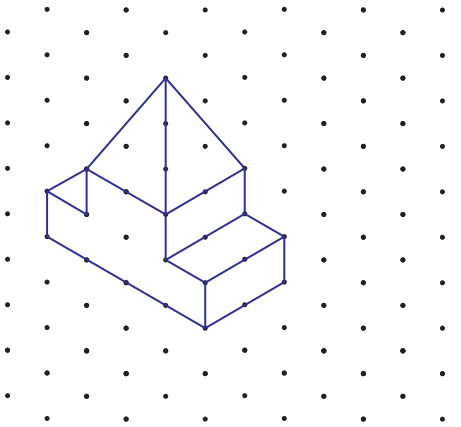


b

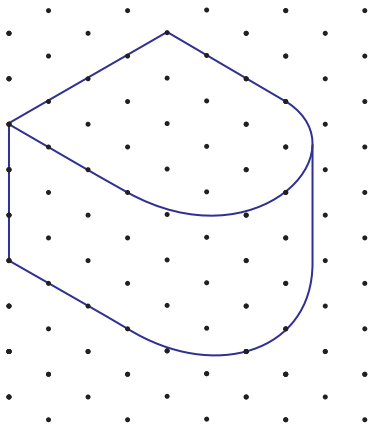


Developing fluency (pages 491–492)

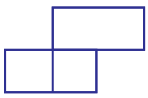
1 a

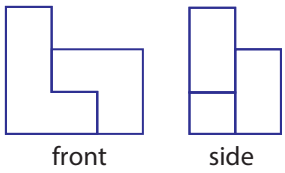


b

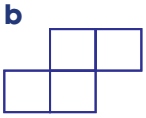


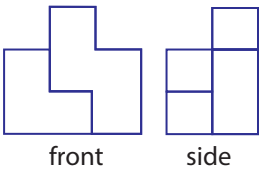
2 a



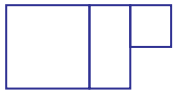


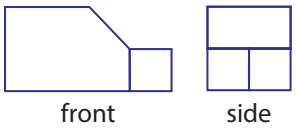
b



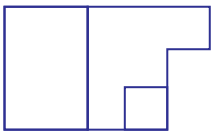


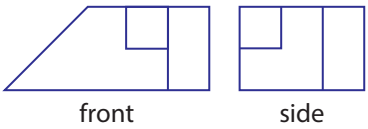
3 a





b





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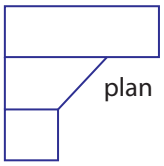

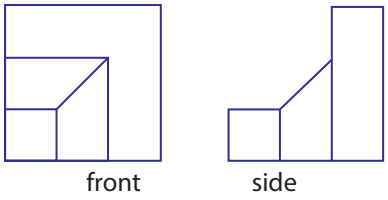

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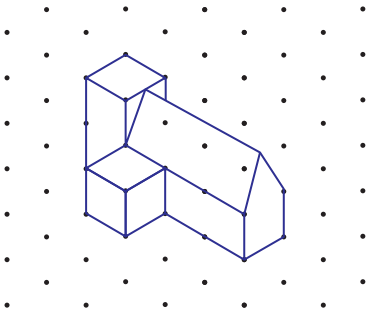
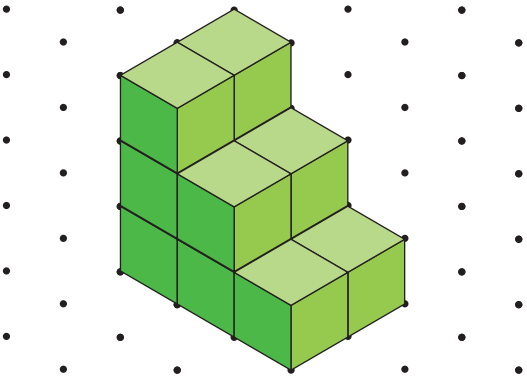


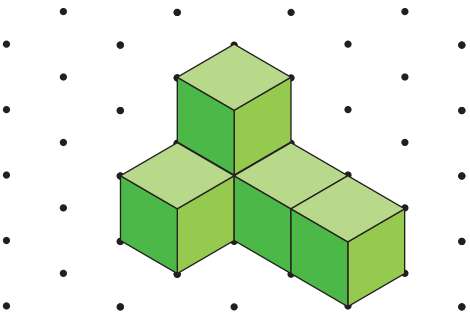
6 a



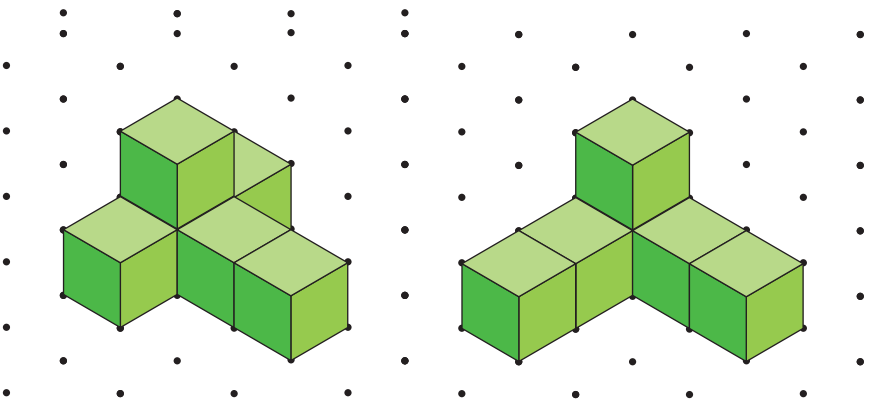
b i volume = 12 cm^3

ii surface area = 36 cm^2

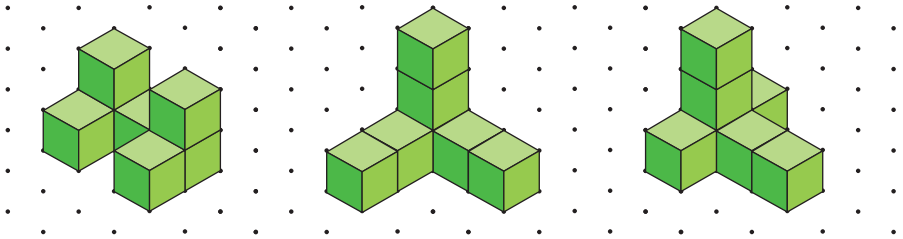
7 a

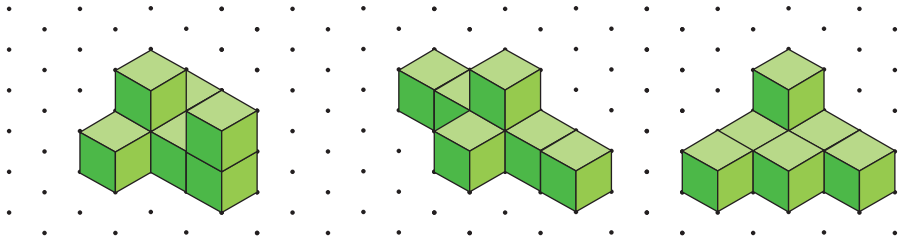


b



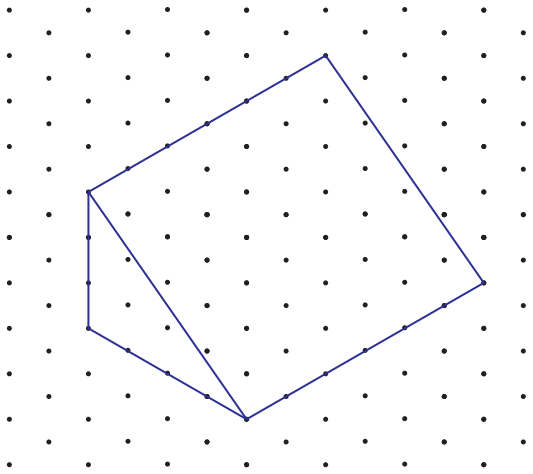
c Several possible answers:





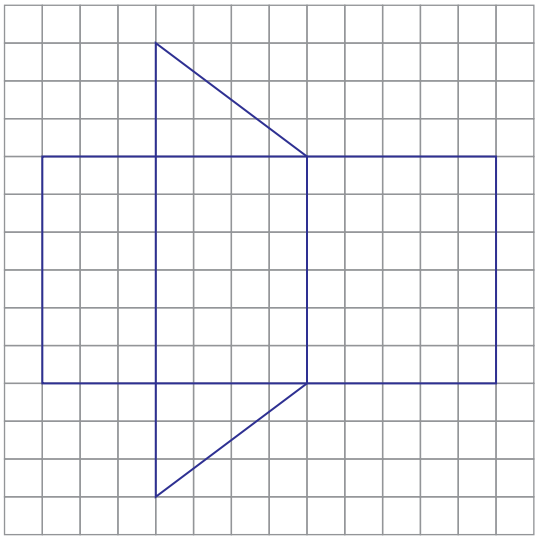
Problem solving (page 493)

1 a triangular prism



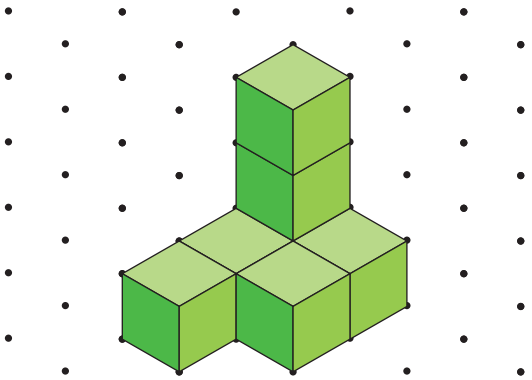
b 36 cm^3

c

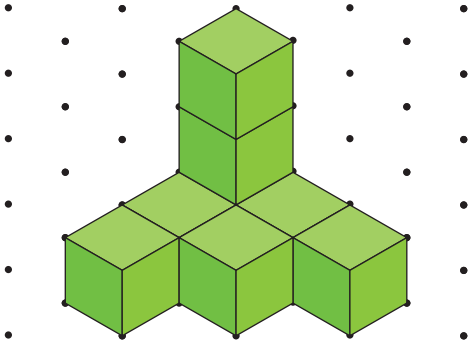


d 84 cm^2

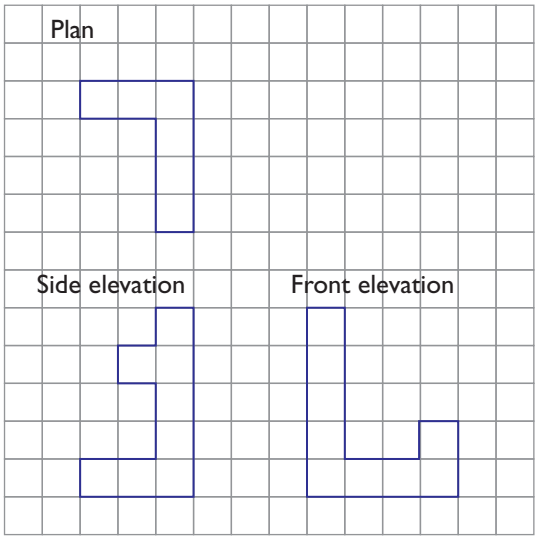
2 a



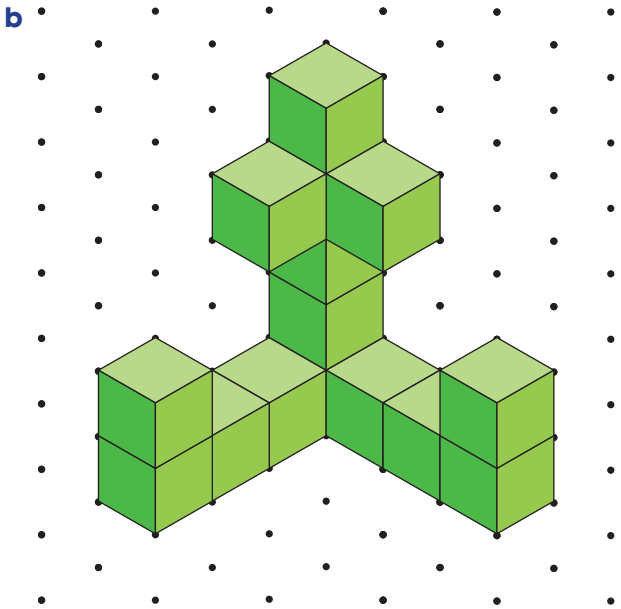
b



3 a

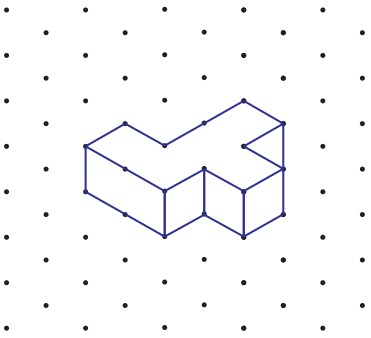


b

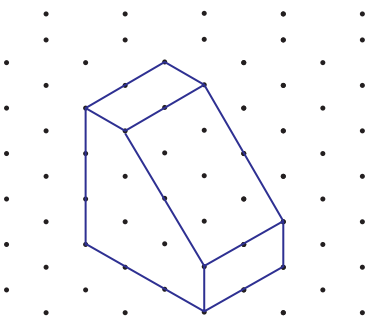


Reviewing skills (page 494)

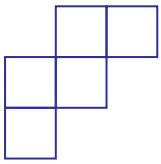
1



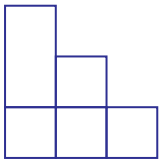
2



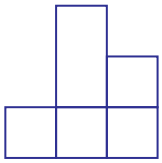
3



plan



front



side

6 Geometry and Measures Strand 6 • Unit 8 • Answers

Practising skills (page 497)

1 605 cm^3

2 **a** 1018 cm^3

b 679 cm^2

3 **a** 81 81 cm^3

b 1964 cm^2

4 7 cm

5 **a** 348π cm^3

b 180π cm^2

Developing fluency (pages 497–498)

1 **a** 1847 cm^2

b 5747 cm^3

2 14 cm

3 **a** 435.6π cm^2

b 1064.8π cm^3

4 2304π cm^3

5 **a** 405π cm^3

b $189\pi + 108$ cm^2

6 1830π cm^3

7 **a** $\frac{4}{3}\pi r^3 = 2\frac{1}{3}\pi r^2 h$; $4r = 2h$; $h = 2r$

b $l^1 = r^2 + h^1$; $h = 2r$ $l^2 = r^2 + (2r)^2 = 5r^2$; $l = \sqrt{5}r$

c surface area of cone $= \pi r^2 + \pi rl = \pi(r^2 + \sqrt{5}r^2) = \pi r^2(1 + \sqrt{5})$

surface area of sphere $= 4\pi r^2$

$4\pi r^2 : \pi r^2(1 + \sqrt{5}) = 4 : (1+\sqrt{5})$

Problem solving (pages 499–500)

1 **a** $3.6 \times 10^8\,\text{km}^2$

b No, because both rounding up and rounding down have taken place.

2 $763\,\text{m}^3$

3 **a** Let r be the radius. Volume of the cone is $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \times r = \frac{1}{3}\pi r^3$

radius of the hemisphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}r^3$

b CSA of the cone $= \pi r l = \pi r\sqrt{(r^2 + r^2)} = \sqrt{2}\pi r^2$

CSA of the hemisphere $= \frac{1}{2} \times 4\pi r^2 = 2\pi r^2$. Ratio is $2 : \sqrt{2} = \sqrt{2} : 1$

4 **a** $47\,909\,\text{cm}^3$

b $8875\,\text{cm}^2$

5 $0.17\,\text{m}^3$

6 172 800

7 $88.3\,\text{m}^2$

Reviewing skills (page 500)

1 $1511\,\text{cm}^2$

2 **a** $4712\,\text{cm}^3$

b $1885\,\text{cm}^2$

6 Geometry and Measures Strand 6 • Unit 9 • Answers

Practising skills (pages 503–504)

1 **a** 9 : 16
b 27 : 64
2 120 cm^2
3 **a** 2 : 3
b 4 : 9
c 8 : 27
4 **a** 4 : 5
b 250 cm^2
c 192 cm^3
5 21.2 cm^2

Developing fluency (pages 504–505)

1 3750
2 It holds 0.046 875 litres less than 2 litres.
3 1 : 216
4 1 : 50 000
5 6 cm
6 972 cm^3

Problem solving (pages 505–508)

1 £405
2 **a** Angle AED = angle ACB and angle ADE = angle ABC (corresponding angles); angle A is common, so the triangles are similar.
b 14 m^2
3 **a** 16 m
b 3.22 m^2
4 £25
5 Hywel is correct. The error in Gwil's answer is that $\pi\left(r+\frac{1}{2}r\right)^2 l \neq \pi r^2 l + \frac{1}{4}\pi r^2 l$
6 20.7 kg
7 540 cm^2
8 245 cm^2
9 **a** 32 m^3
b Yes, as area is 51.2 m^2

10 a $a^3 - b^3$

b Let H cm be the height of the large pyramid and h cm the height of the small pyramid.

$V = \frac{1}{3}a^2 H - \frac{1}{3}b^2 h$ *and* $\frac{H}{h} = \frac{a}{b} =$ *and* $d = H - h; V = \frac{H}{3a}(a^3 - b^3) =$

$\frac{H}{3a}(a-b)(a^2+ab+b^2) = \frac{1}{3}(H-h)(a^2+ab+b^2) = \frac{d}{3}(a^2+ab+b^2)$

Reviewing skills (page 508)

1 a 64 : 27

b 593 g

2 18.9 cm

1 Statistics and Probability Strand 1 • Unit 4 • Answers

Practising skills (pages 511–513)

1

Height (cm), h	Frequency, f	Midpoint, m	$m \times f$
$150 \leqslant h < 156$	3	153	459
$156 \leqslant h < 162$	6	159	954
$162 \leqslant h < 168$	8	165	1320
$168 \leqslant h < 174$	3	171	513
$174 \leqslant h < 180$	2	177	354
Totals	22		3600

Mean height $= \frac{3600}{22} = 163.6\,\text{cm}$

2 a

Length of call (minutes), l	Frequency, f	Midpoint, m	$m \times f$
$0 \leqslant l < 10$	1	5	5
$10 \leqslant l < 20$	5	15	75
$20 \leqslant l < 30$	3	25	75
$30 \leqslant l < 40$	5	35	175
$40 \leqslant l < 50$	5	45	225
$50 \leqslant l < 60$	1	55	55
Totals	20		610

b $30 \leqslant l < 40$

c $\frac{610}{20} = 30.5$

3 a i 19 and 185 seconds

ii 166 seconds

b 83.5 seconds

c

Time (seconds), t	Frequency, f	Midpoint, m	$m \times f$
$0 \leqslant t < 40$	3	20	60
$40 \leqslant t < 80$	8	60	480
$80 \leqslant t < 120$	9	100	900
$120 \leqslant t < 160$	1	140	140
$160 \leqslant t < 200$	3	180	540
Totals	24		2120

d $80 \leqslant t < 120$

e $\frac{2120}{24} = 88.3$

f Their times are above 63 seconds and below 114 seconds.

4 a

Speed (mph), v	Frequency, f	Midpoint, m	$m \times f$
$0 \leqslant v < 20$	13	10	130
$20 \leqslant v < 30$	22	25	550
$30 \leqslant v < 40$	4	35	140
$40 \leqslant v < 60$	1	50	50
Totals	40	120	870

Mean speed $= \frac{870}{40} = 21.75$ mph

b 30 mph

c 40%

5 a $\frac{410}{45} = 9$ hours

b 1.5 hours

Developing fluency (pages 513–515)

1 a $\frac{1791}{20} = 89.55$ and 81

b

Number of people, n	Frequency, f	Midpoint, m	$m \times f$
$0 \leqslant n < 50$	3	25	75
$50 \leqslant n < 100$	10	75	750
$100 \leqslant n < 150$	5	125	625
$150 \leqslant n < 200$	2	175	350
Totals	20		1800

c $50 \leq n < 100$

d 90

e The means are very close, the median in part **a** is more accurate and is close to both means.

f For small samples you would calculate it exactly and for large samples you would estimate it, or any other correct explanation.

2 a

Time (minutes), t	Frequency, f	Midpoint, m	$m \times f$
$0 \leqslant t < 20$	1	10	10
$20 \leqslant t < 30$	8	25	200
$30 \leqslant t < 40$	14	35	490
$40 \leqslant t < 50$	7	45	315
Totals	30	115	1015

Mean time $= \frac{1015}{30} = 33.8$ minutes

b range = less than 50 minutes

c The second club were quicker on average but more spread out / varied in their times.

3 a Mean waiting time $= \frac{502.5}{35} = 14.4$ minutes

b 13 times

c for example, hold-up/accident/power failure and it does not represent a normal journey

4 a

Distance (metres), d	Frequency, f	Midpoint, m	$m \times f$
$0 \leqslant d < 2$	6	1	6
$2 \leqslant d < 3$	12	2.5	30
$3 \leqslant d < 4$	9	3.5	31.5
$4 \leqslant d < 5$	8	4.5	36
Total	35	11.5	103.5

Mean distance jumped $= \frac{103.5}{35} = 2.96$ metres

b $(14 \times 3) - 35 = 7$ foul jumps

c At most, 8 jumpers qualified.

5 B has a larger mean yield of 854.2 g vs 812.5 g for A

6 a

	Current salary	Scheme 1 increase	Scheme 2 increase	Scheme 3 increase
Sandra	£8000	£8400	£8870	£8750
Shameet	£2200	£2310	£3070	£2950
Comfort	£3600	£3780	£4470	£4350

b Scheme 2 benefits Sandra, Shameet and Comfort by the greatest amount.

c The MD should choose Scheme 1 because 5% increase in employees' current salary total is less than 5% of the mean salary or the median salary so the increases will be smaller and therefore cost the company less.

Problem solving (pages 515–518)

1

Score, s	Frequency, f	Midpoint, m	$m \times f$
$0 \leqslant s < 10$	11	5	55
$10 \leqslant s < 20$	6	15	90
$20 \leqslant s < 30$	12	25	300
$30 \leqslant s < 40$	10	35	350
$40 \leqslant s < 50$	9	45	405
Totals	48		1200

a 25

b 19

c The mean is an estimate, therefore the cut-off for interview is also an estimate, which means the number of applicants interviewed is also an estimate.

2 a

Distance travelled, d km	Frequency f	Midpoint m	$m \times f$
$20 < d \leqslant 30$	6	25	150
$30 < d \leqslant 40$	12	35	420
$40 < d \leqslant 50$	20	45	900
$50 < d \leqslant 60$	26	55	1430
$60 < d \leqslant 70$	11	65	715
Total	75	Total	3615

b $40 < d \leqslant 50$

c 48.2 km

d 72.3 minutes

3 a $20 < t \leqslant 25$

b 25 minutes

c 20.1 minutes

d 10% of people are geniuses

4 a 100 people

b $170 < h \leqslant 180$

c Incorrect, the range is 50 cm, as you take the midpoints of the classes to calculate the range

d 175.5 cm

e 7%

5 a $30 < c \leqslant 40$

b $40 < c \leqslant 50$

c 53.7

d 61.1

e 26%

6 Class Y1. Y1 has mean of 56, whereas X1 has a mean of 54. Y1's median is 70, whereas X1's median is 50.

Reviewing skills (page 518)

1 a Mean = 51.7

Score, s	Midpoint m	Frequency f	$m \times f$
$0 \leqslant s < 20$	10	6	60
$20 \leqslant s < 40$	30	8	240
$40 \leqslant s < 60$	50	15	750
$60 \leqslant s < 80$	70	14	980
$80 \leqslant s < 100$	90	5	450
Totals		48	2480

b 5

c Their marks are lower than 50

Statistics and Probability Strand 1 • Unit 5 • Answers

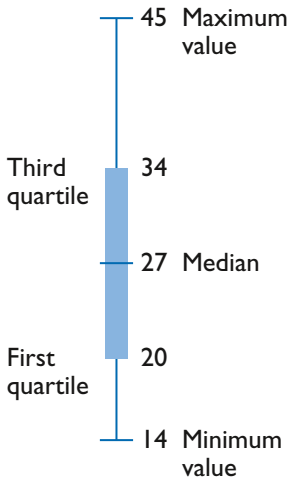
Practising skills (pages 522–523)

1 a i 27

ii 34, 20

iii 14

b



c Any sensible suggestion, e.g. exam scores.

2 a i 14.1

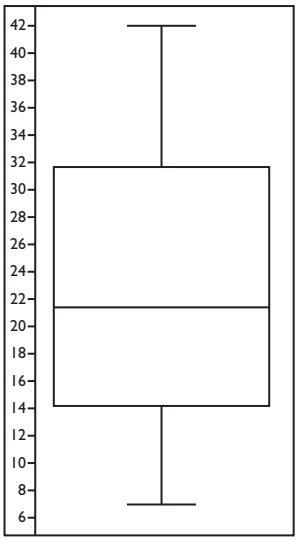
ii 14.8, 13.5

iii 1.3

iv 3

b Any sensible suggestion, e.g. ages of players in an under 15 football team.

3 a



b i 42

ii 21.5

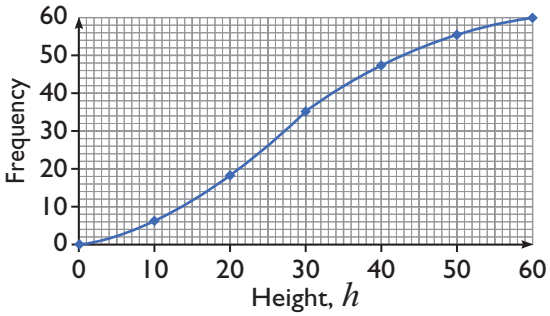
iii 31.5, 14.5

iv 17

v 35

4 a 47, 56, 60

b

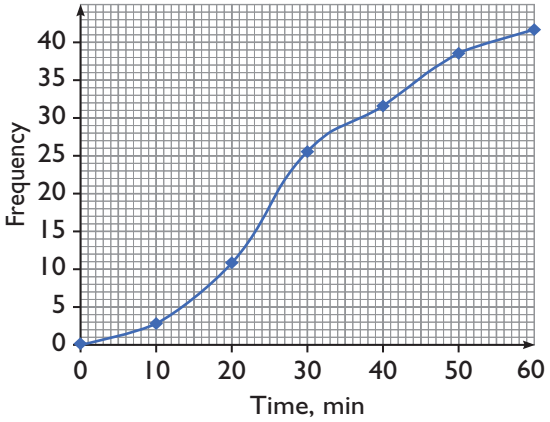


c i 27

ii 23

d 21 m

5 a



b i 27

ii 19–20

c It has increased on average by about 5 minutes but the inter-quartile range has decreased so the times are less spread out. Some people are living further from work/the traffic has got worse.

d Those living closer to work got to work faster than normal, perhaps indicating a reduction in local traffic whilst those living further away took longer than normal, perhaps indicating an accident or traffic jam.

Developing fluency (pages 524–526)

1 a 100

b median = 72 mph; lower quartile = 65 mph; upper quartile = 79 mph

c 79 and 65 mph

d 58%

e Travelling at 55 or less – 8; travelling at 90 or more – 10.

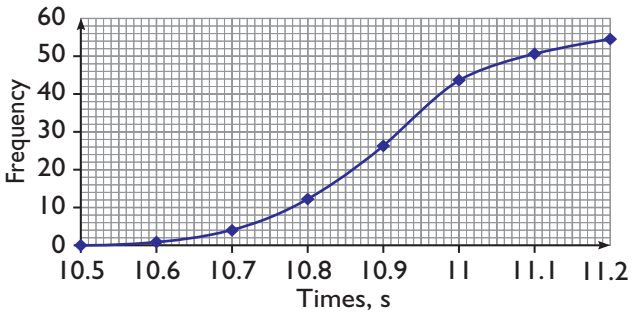
2 a 26 for both, A: 14, B: 7

b They have the same median, so similar average/total rainfall but Town A has more very wet weeks.

c Not exactly the same due to particularly wet/dry years, climate change etc

3 a 1 4 12 26 43 50 54

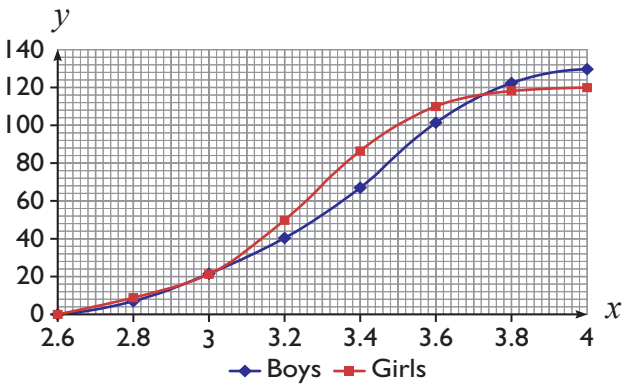
b



c 10.92, 0.15

d 17–18

4 a Boys 6 21 40 67 102 123 130; Girls 9 21 49 86 110 118 120

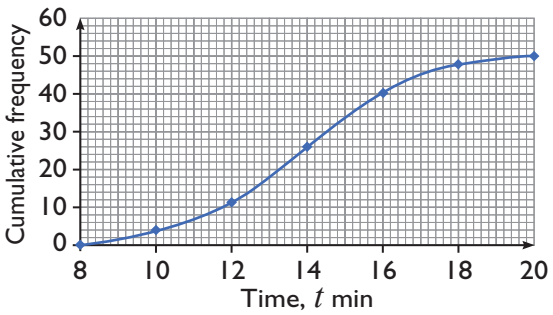


b Boys 3.38–3.40 and 0.45; Girls 3.25–3.27 and 0.37

c Boys are heavier on average by about 0.14 kg and their weights are slightly more spread out.

d No, but you can estimate that it is more likely to be a boy.

5 a



b median = 13.8-14 ; IQR = 3.3

c 15.5 minutes

d The journey back is about 2 minutes quicker on average and has similar spread. They were rowing upstream in the first part.

6 a True. The definition of the quartiles is that half the data lie between them.

b False. The range can be very different to the IQR and is not related to the IQR.

c False. Box and whisker divides data into one box of two parts and two whiskers which represent the range.

d True. Definition of cumulative is that there is always addition of the frequency.

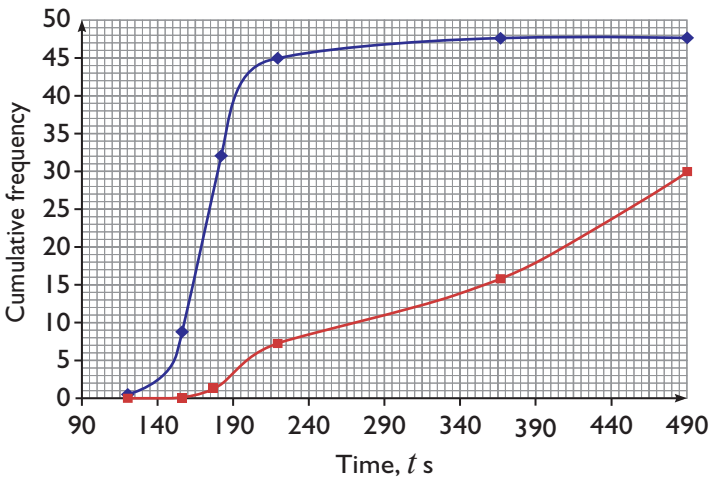
e True, if by midway you mean midway between the frequency and not midway between the variable.

7 a $150 \leq t < 180$

b

time, t	frequency dogs	cumulative frequency dogs	frequency owners	cumulative frequency owners
$90 \leq t < 120$	1	1	0	0
$120 \leq t < 150$	5	6	0	0
$150 \leq t < 180$	27	33	2	2
$180 \leq t < 210$	12	45	5	7
$210 \leq t < 360$	3	48	9	16
$360 \leq t < 480$	0	48	14	30

c



d median = 170 s; IQR = 25 (22 – 27) s

e dogs – 147 s; owners – 197 s

f Dogs – most of the dogs fall around the median of 170, with 45 of the 48 dogs below 210 s. Owners are more concentrated to the high times, 23 of the 30 owners are above 210 s. The IQR is very large for the owners.

Problem solving (pages 527–529)

1 a 100

b 20

c The upper quartile or the 75 percentile is a mark of 15 so James is correct.

2 a 3.35 cm

b i

wing span	$2.0 < w \leq 2.5$	$2.5 < w \leq 3.0$	$3.0 < w \leq 3.5$	$3.5 < w \leq 4.0$	$4.0 < w \leq 4.5$	$4.5 < w \leq 5.0$
frequency	8	18	20	18	14	2

ii 3.36 cm

3 a 80

b i median = 13.5 s; upper Q = 14 s; lower Q = 12.9 s

ii median = 15.45 s; upper Q = 16.1 s; lower Q = 14.7 s

c man – 11.5 s; woman – 13 s

d 29

e 12

4 Male range 6500–4500 = 2000, median = 5300, inter-quartile range = 950

Female range = 5000−3200 = 1800, median = 4000, inter-quartile range = 600

The female distribution is negatively skewed. The male distribution is positively skewed.

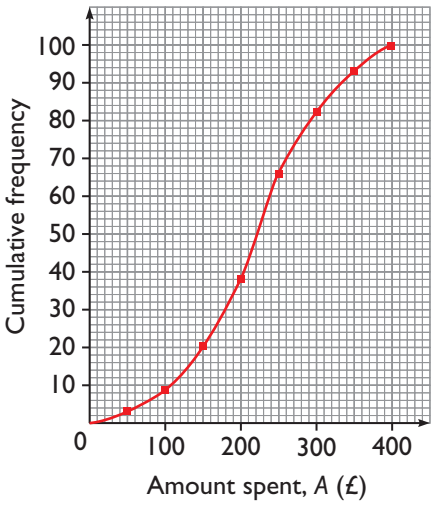
The females' weight is more consistent as both the range and the inter-quartile range are smaller.

The males on average weigh more than the females as shown by the median.

5 Both distributions are positively skewed but more prominent to the right in the group that had drugs.

Recovery was slightly less variable in the group that had drugs than in the other group. In the drug group, recovery took 2 to 14 days (range = 12) versus 5 to 18 days (range = 13) for the other group. The inter-quartile range for the drug group was 7 days and for the other group 9 days. So in general the drug group had a more consistent recovery time. The median recovery time was 6 days for the drug group and 13 days for the other group. It appears that the drug had a positive effect on patient recovery.

6 a

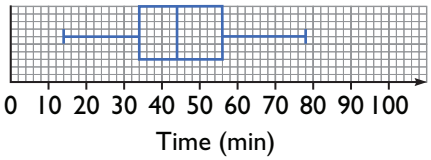


b i £220

ii £115

c The median amount spent was lower at Christmas as families tended to go out less, however the range is greater suggesting a couple of families didn't go out at all or else had one or two expensive outings. The inter-quartile range was lower at Christmas than at Easter.

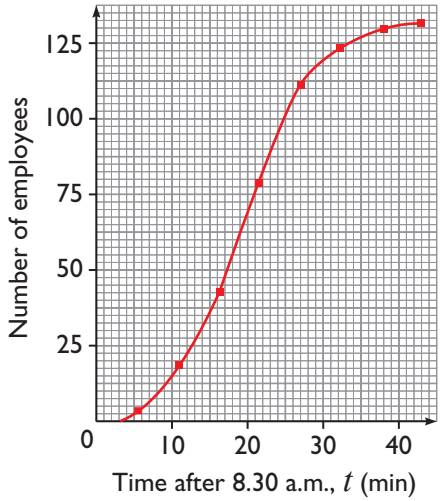
7



Slowest time	15 minutes
Fastest time	78 minutes
Range	63 minutes
Median	42 minutes
Upper quartile	54 minutes
Lower quartile	32 minutes
Inter-quartile range.	22 minutes

Reviewing skills (page 530)

1 a



b 8.47

c **i** 13 min

ii 10 min

d 8.38 a.m.

e 5 employees

Statistics and Probability Strand 2 • Unit 4 • Answers

Practising skills (pages 534–535)

1 a i discrete

ii continuous

iii discrete

iv discrete

b i discrete

ii continuous

iii continuous

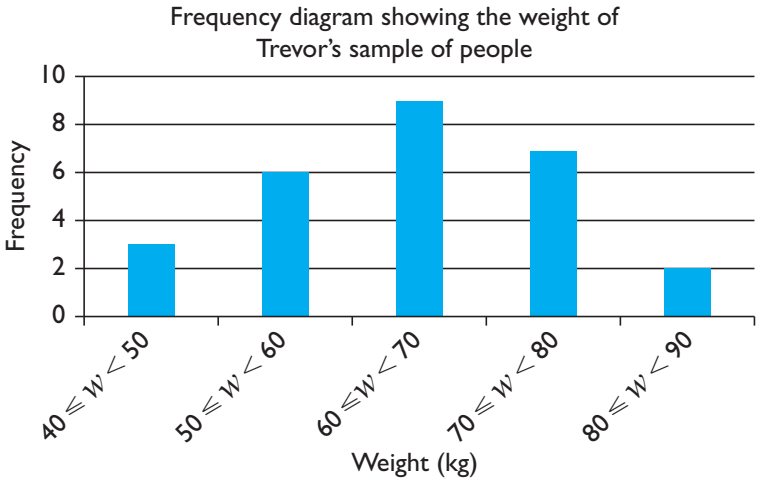
iv continuous

2 A: 150 in two classes. B: 150 not in any class. C: unclear – 150 likely to be in two classes. D: 150 not in any class. $140 \leqslant$ height < 150, $150 \leqslant$ height < 160, etc.

3 a Frequencies are 3, 6, 9, 7, 2.

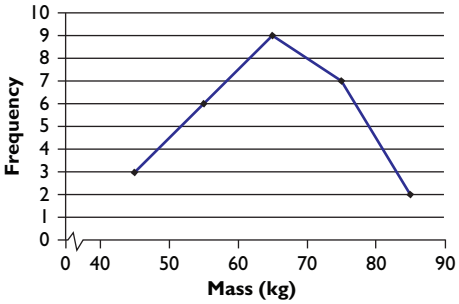
b $60 \leqslant w < 70$

c



d Chart as it is summarised in order.

e



4 a Frequencies are 1, 6, 5, 4, 4, 1.

b $30 \leqslant n < 40$

c 7

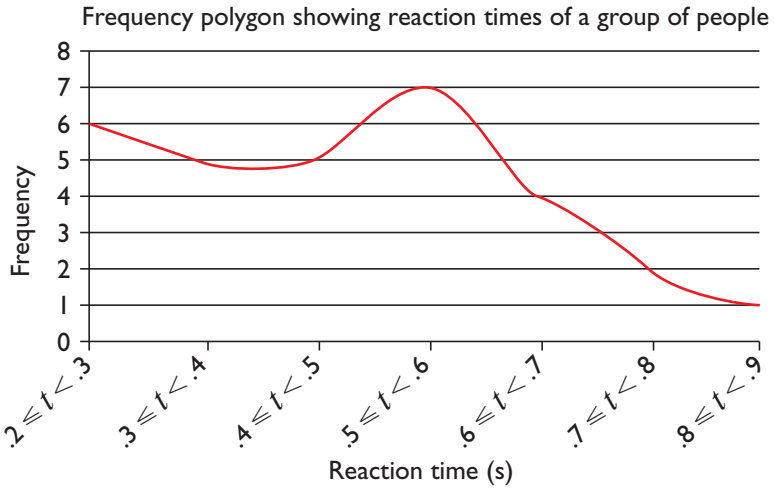
d There are generally between 30 and 70 people in the gym on any day.

Developing fluency (pages 536–537)

1 a

Reaction time, t	Frequency
$0.2 \leqslant t < 0.3$	6
$0.3 \leqslant t < 0.4$	5
$0.4 \leqslant t < 0.5$	5
$0.5 \leqslant t < 0.6$	7
$0.6 \leqslant t < 0.7$	4
$0.7 \leqslant t < 0.8$	2
$0.8 \leqslant t < 0.9$	1

b



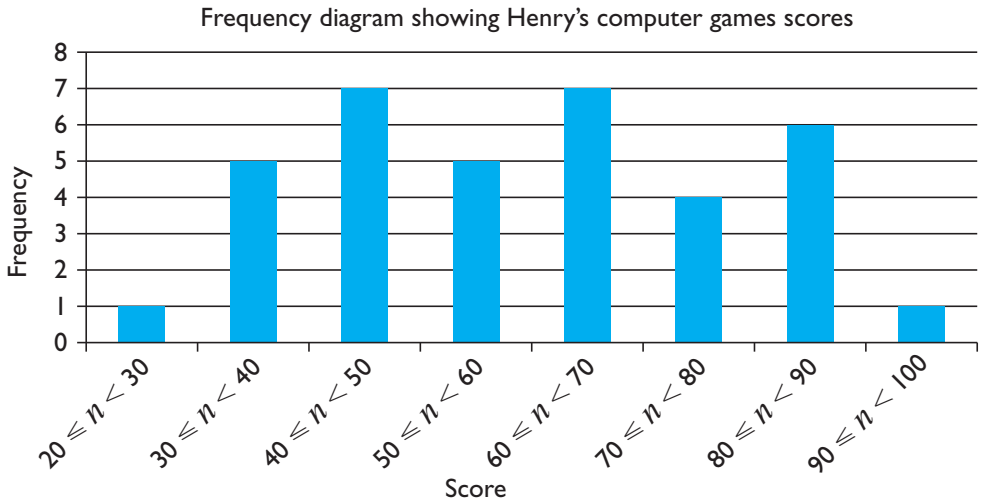
c 20%

d Fairly level until 0.6, with a modal class at $0.5 \leqslant t < 0.6$, then it falls away.

2 a

Score, n	Frequency
$20 \leqslant n < 30$	1
$30 \leqslant n < 40$	5
$40 \leqslant n < 50$	7
$50 \leqslant n < 60$	5
$60 \leqslant n < 70$	7
$70 \leqslant n < 80$	4
$80 \leqslant n < 90$	6
$90 \leqslant n < 100$	1

b



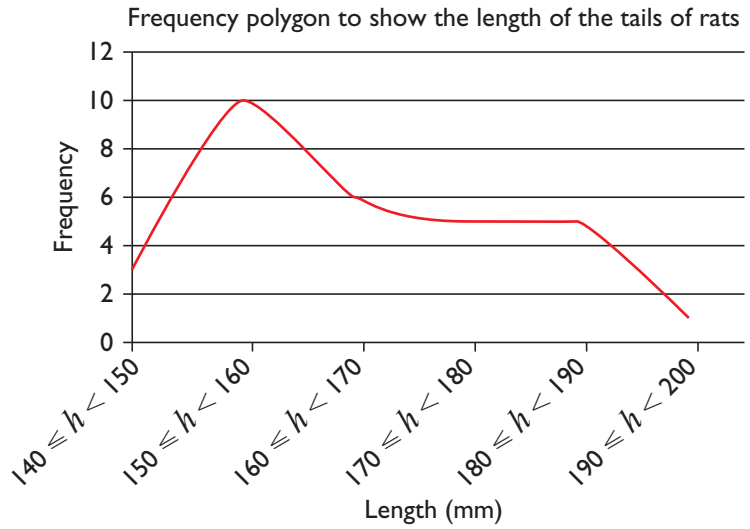
c Table helps to narrow the median down to between 50 and 70 but the original data gives the actual scores so you can work out the median as $\frac{(58 + 60)}{2} = 59$.

d Fairly even between 30 and 90 with only one score above and one below that. Grouped data.

3 a

height, h	Frequency
$140 \leqslant h < 150$	3
$150 \leqslant h < 160$	10
$160 \leqslant h < 170$	6
$170 \leqslant h < 180$	5
$180 \leqslant h < 190$	5
$190 \leqslant h < 200$	1

b



c 11

d $150 \leqslant h < 160$

4 a 21

b

Age, a	Frequency
$0 \leqslant a < 10$	4
$10 \leqslant a < 20$	10
$20 \leqslant a < 30$	15
$30 \leqslant a < 40$	19
$40 \leqslant a < 50$	16
$50 \leqslant a < 60$	5
$60 \leqslant a < 70$	2

c Lots of people of working age.

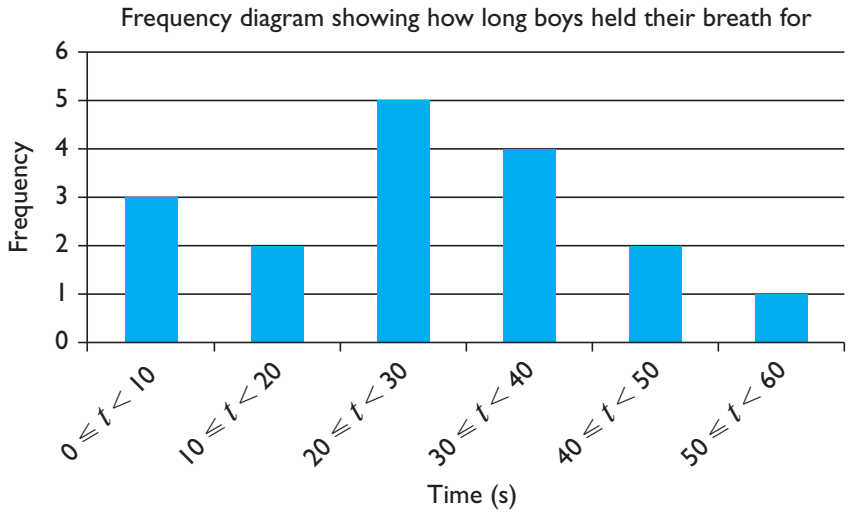
d More retired people and less of working age.

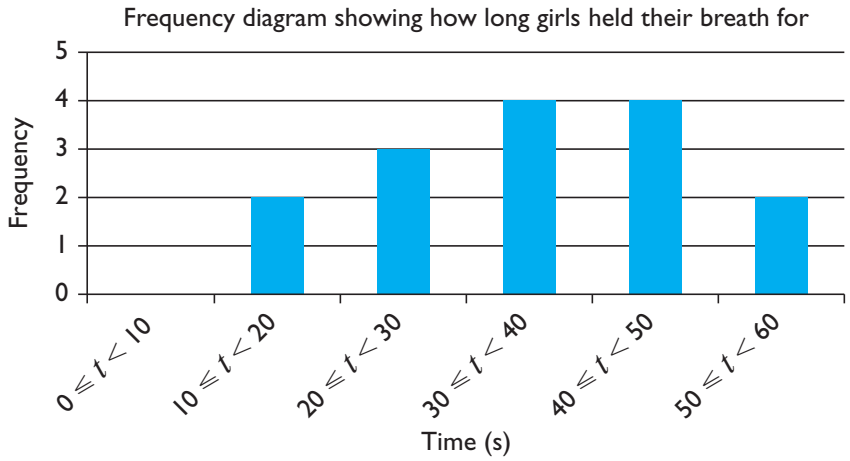
5 a

Time, t (seconds)	Boys	Girls
$0 \leqslant t < 10$	3	0
$10 \leqslant t < 20$	2	2
$20 \leqslant t < 30$	5	3
$30 \leqslant t < 40$	4	4
$40 \leqslant t < 50$	2	4
$50 \leqslant t < 60$	1	2

b Boys $20 \leqslant t < 30$. Using the original data, the median is 25. Girls $30 \leqslant t < 40$. The median is 35.

c





d Girls – they have a higher average.

6 **a** Discrete – a hair is a hair.

b Time is continuous, but discrete values of age are usually easier to work with.

c Discrete – you cannot have half a person.

Problem solving (pages 538–539)

1 **a** 150 (Accept: there might be one area of the field which was particularly densely populated. Reject: 150 is so much higher than all the other values that it is likely to be an error.)

b

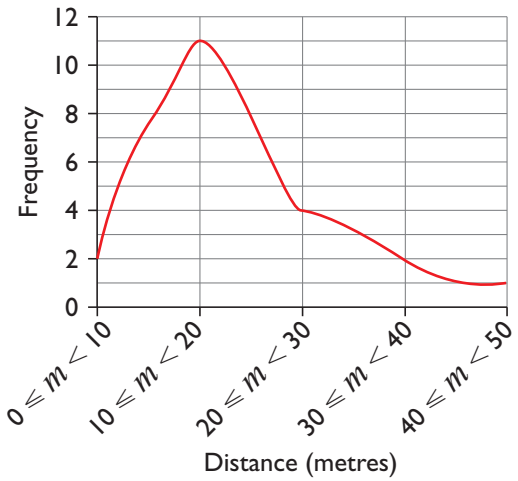
Number of poppies, p	Frequency
$0 \leqslant p < 10$	0
$10 \leqslant p < 20$	3
$20 \leqslant p < 30$	1
$30 \leqslant p < 40$	2
$40 \leqslant p < 50$	4
$50 \leqslant p < 60$	5
$60 \leqslant p < 70$	6
$70 \leqslant p < 80$	5
$80 \leqslant p < 90$	3
$90 \leqslant p < 100$	1

c

d The mode is 15 and median is 61. The mode is more representative. Modal class is 60–70 which is also representative of the data.

e 50%

2 a

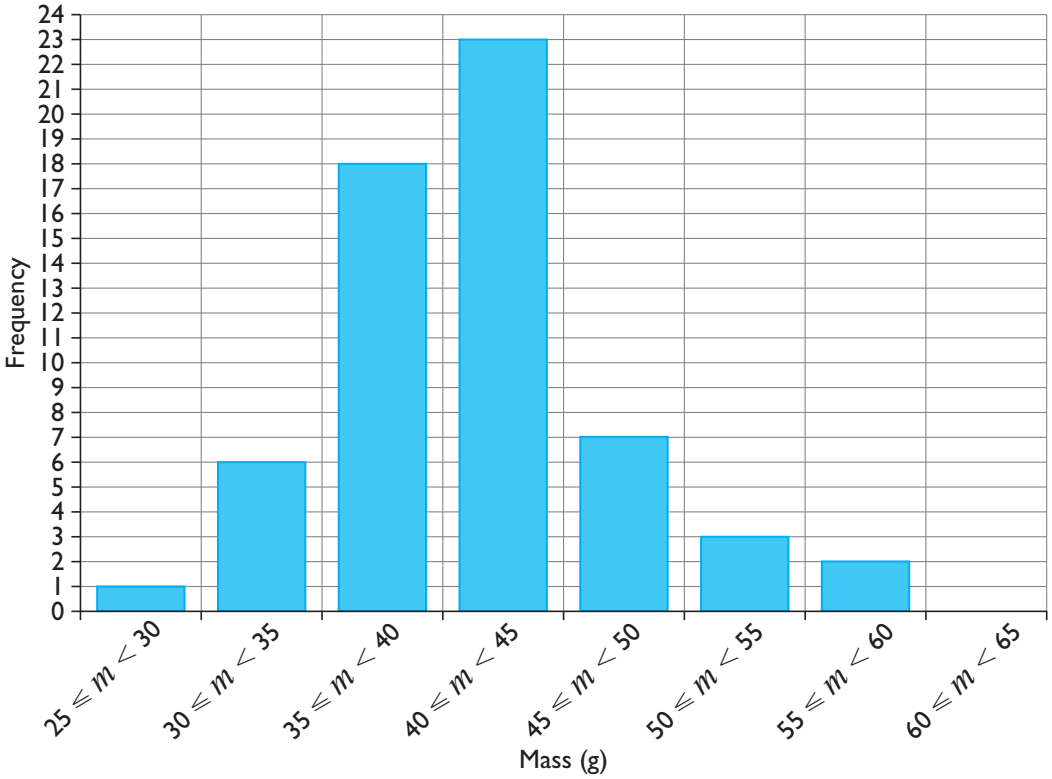


b Most of them can only swim less than 20 m and few of them can swim further. It is skewed towards the lower end (positively skewed).

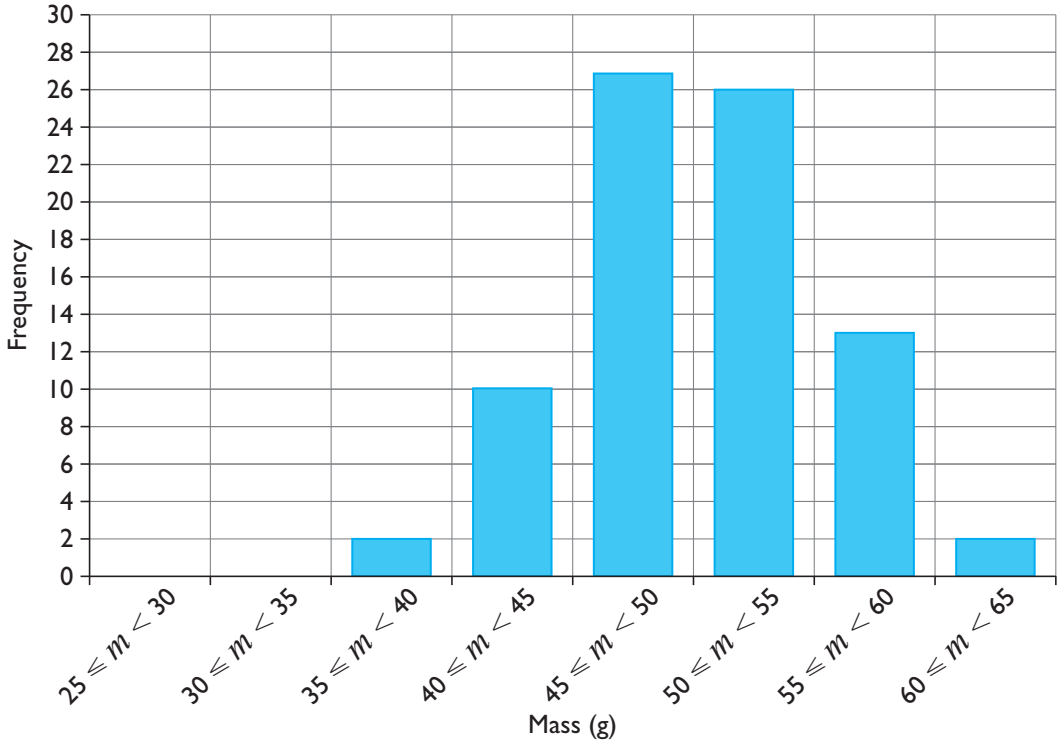
c 55%

d The better swimmers have slightly increased the distance they can swim underwater; the weaker swimmers have performed much the same as before.

3 a Female chicks:



Male chicks:



b Diagrams show that male chicks tend to be heavier than female chicks. Modal class for female chicks is $40 \leqslant m < 45$ and for male chicks is $45 \leqslant m < 50$.

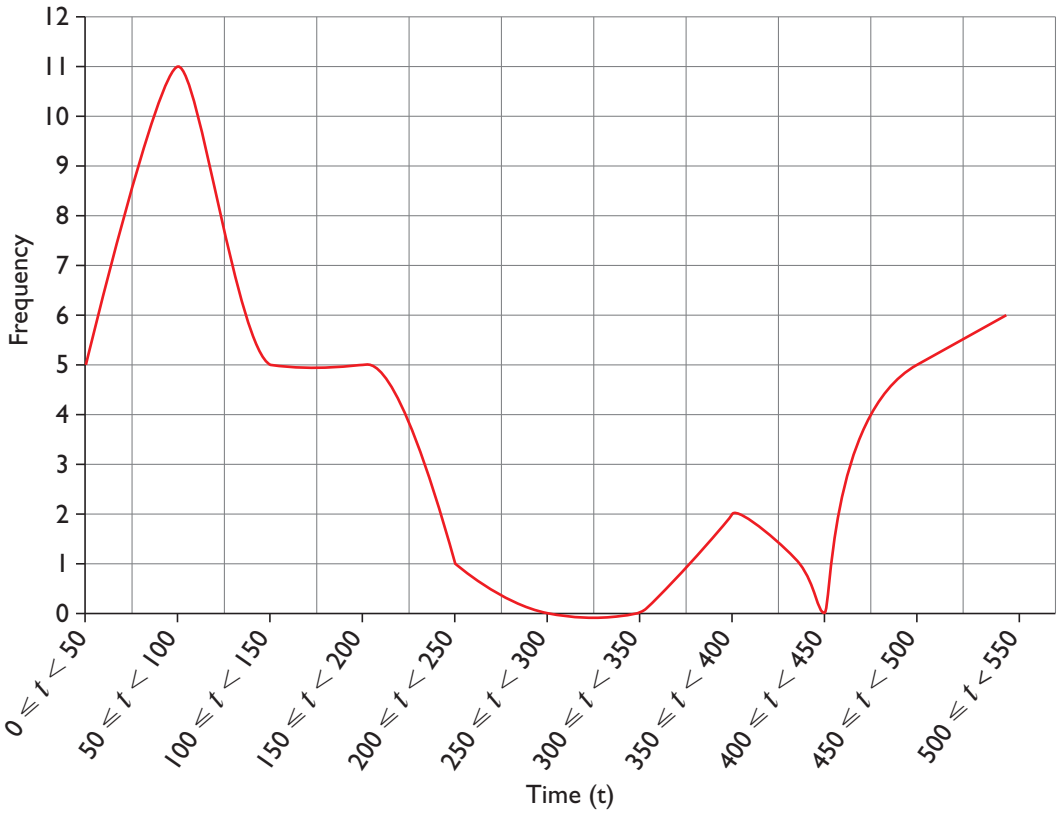
c **i** 20%

ii 82.8%

4 **a**

Time (t minutes)	Frequency
$0 \leqslant t < 50$	5
$50 \leqslant t < 100$	11
$100 \leqslant t < 150$	5
$150 \leqslant t < 200$	5
$200 \leqslant t < 250$	1
$250 \leqslant t < 300$	0
$300 \leqslant t < 350$	0
$350 \leqslant t < 400$	2
$400 \leqslant t < 450$	0
$450 \leqslant t < 500$	5
$500 \leqslant t < 550$	6

b



c The distribution is bimodal with two distinct groups. Most people don't spend a lot of time on computer games but there is a group who spend a lot of time on them.

d In August the students are on holiday, so the people in the group that spend less time playing games might spend more time outdoors, but the people who are very keen on playing games will probably play for even longer. The graph will probably be a similar shape, but with lower bars on the left and higher bars on the right.

5 a Prys has used the computer every day, Anna hasn't. However, when Anna does use the computer she generally spends a long time on it.

b They don't really need a second computer as the mean time they each spend on the computer is 50 minutes, so there should be plenty of time to use the computer in the evening.

Reviewing skills (page 540)

1 a $0.5\,\text{kg} \leqslant n < 1.0\,\text{kg}$

b $1.5\,\text{kg} \leqslant n < 2.0\,\text{kg}$

c

Weight, n (kg)	Frequency
$0 \leqslant n < 0.5$	3
$0.5 \leqslant n < 1.0$	5
$1.0 \leqslant n < 1.5$	7
$1.5 \leqslant n < 2.0$	10
$2.0 \leqslant n < 2.5$	9
$2.5 \leqslant n < 3.0$	6
$3.0 \leqslant n < 3.5$	4
$3.5 \leqslant n < 4.0$	4
$4.0 \leqslant n < 4.5$	1
$4.5 \leqslant n$ 5.0	1

d 50

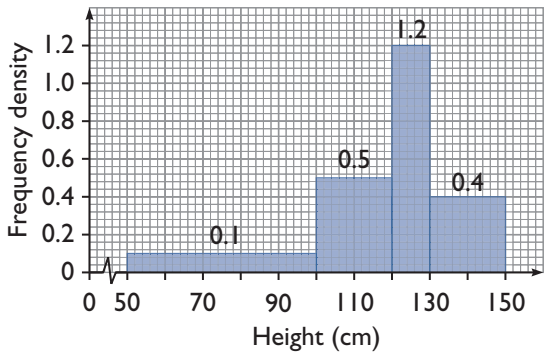
Statistics and Probability Strand 2 • Unit 7 • Answers

Practising skills (pages 544–545)

1 **a** No, bar chart
b Yes
c Yes
d No, vertical line chart
e No, bar chart
f Yes
g Yes

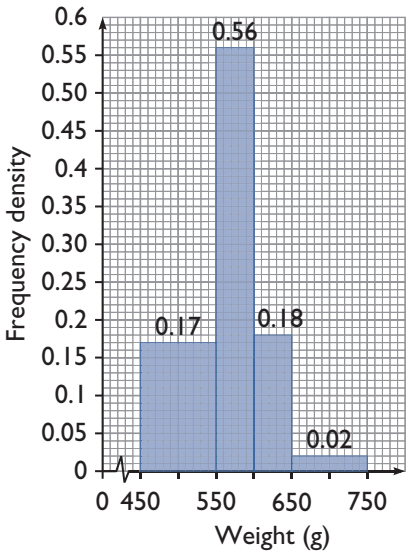
2

Height, h (cm)	Frequency	Class width	Frequency density
$50 < h \leqslant 100$	5	50	0.1
$100 < h \leqslant 120$	10	20	0.5
$120 < h \leqslant 130$	12	10	1.2
$130 < h \leqslant 150$	8	20	0.4



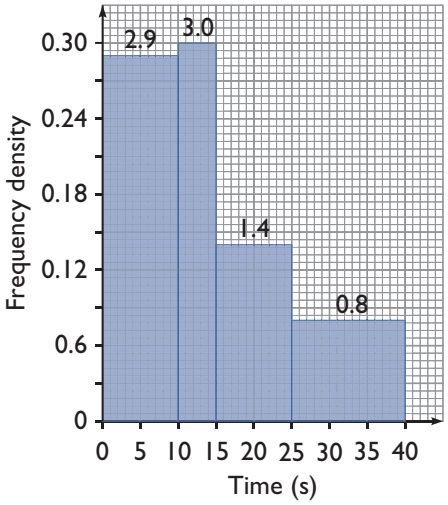
3

Weight, w (g)	Frequency	Class width	Frequency density
$450 < w \leqslant 550$	17	100	0.17
$550 < w \leqslant 600$	28	50	0.56
$600 < w \leqslant 650$	9	50	0.18
$650 < w \leqslant 750$	2	100	0.02



4

Time, t (s)	Frequency	Class width	Frequency density
$0 < t \leqslant 10$	29	10	2.9
$10 < t \leqslant 15$	15	5	3.0
$15 < t \leqslant 25$	14	10	1.4
$25 < t \leqslant 40$	12	15	0.8

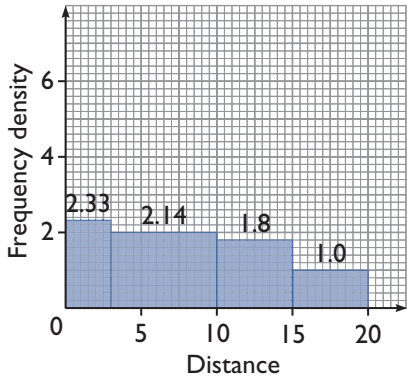


5

Height, h (cm)	Frequency	Class width	Frequency density
$0 < h \leqslant 10$	18	10	1.8
$10 < h \leqslant 25$	27	15	1.8
$25 < h \leqslant 30$	6	5	1.2
$30 < h \leqslant 50$	3	20	0.15

Developing fluency (pages 546–548)

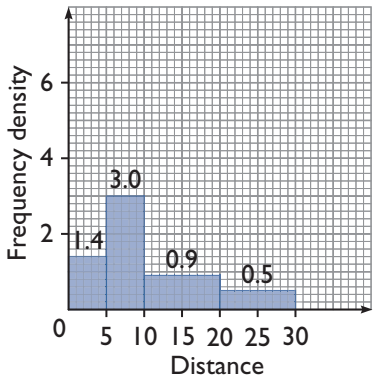
1 a i



ii highest frequency – $3 < d \leqslant 10$; highest frequency density – $0 < d \leqslant 3$

iii $3 < d \leqslant 10$

b i



ii $5 < d \leqslant 10$

c Company 2 has the employees that travel furthest to get to work. Both have the same number of employees that travel less than or equal to 10 km, but on average (mean) company 1's employees are closer.

d Company 1 is in a city as its employees are mean closer. Company 2 has employees up to 30 km away.

2 a 15

b 6

c (depends what one considers hot). 64 days were above 25 °C. This is hot weather for the UK, so summer that year was hot.

3

mass, w (g)	Frequency
$100 < w \leqslant 110$	40
$110 < w \leqslant 120$	78
$120 < w \leqslant 130$	61
$130 < w \leqslant 150$	36

4 a

time	frequency density	frequency
$0 < t \leqslant 10$	0.5	5
$10 < t \leqslant 30$	0	0
$30 < t \leqslant 40$	0.8	8
$40 < t \leqslant 50$	0.2	2

b $30 < t \leqslant 40$

5 a

mass, w (g)	frequency
$0 < w \leqslant 50$	50
$50 < w \leqslant 100$	250
$100 < w \leqslant 150$	200
$150 < w \leqslant 200$	100
$200 < w \leqslant 250$	50

b Mean = 113.5. Yes, the new mean is higher which shows on average the weight has increased by 5.5 grams.

Problem solving (pages 548–550)

1 a

height, cm	frequency density	frequency
$160 < h \leqslant 175$	2	30
$175 < h \leqslant 185$	8	80
$185 < h \leqslant 190$	10	50
$190 < h \leqslant 195$	4	20
$195 < h \leqslant 205$	2	20

b i $175 < h \leqslant 185$

ii $185 < h \leqslant 190$

c $175 < h \leqslant 185$

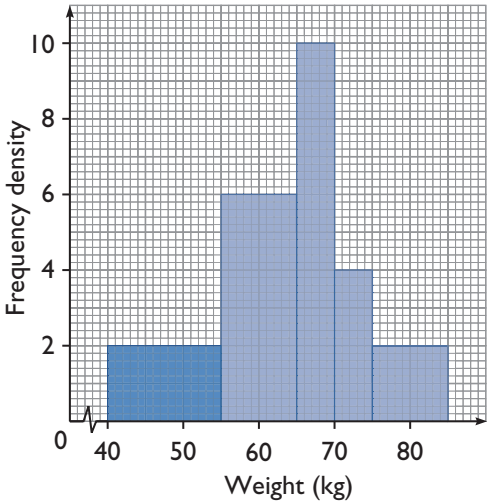
d 183.25 cm

2 a $p = 0.02$; $q = 0.04$; $r = 0.06$; $s = 0.08$

b 90

c 3944 hours

3 a



b 180 people

c 65 kg

4 a 10, 60, 30, 10, 15

b 10 – 25 min

5 a 5900

b Positive skew, median 32 years.

6 a

	Median	Lower quartile	Upper quartile	Inter-quartile range
Women	102.5	93	110.5	16.5
Men	105.3	96.5	110.5	14

b The median for the women is higher than the mean showing that on average the women took less time to complete the puzzle. The inter-quartile range is the same, however the men's lower quartile is lower showing the men are more skewed to the longer times and the women to the shorter times.

Reviewing skills (page 550)

1 1.6

2 a 0.26

b 69

3 Statistics and Probability Strand 3 • Unit 3 • Answers

Practising skills (pages 554–555)

1 It is not a random sample because not everyone has a chance of being chosen.

2 **a** The numbers in the sample from each group must be in the same proportion as the numbers in the population.

b

Age (years)	0-20	21-40	41-60	61-80	81+
Sample	20	27	16	10	2

3 **a** Possible reasons include:

- If they go to work by car they will drive straight there and not be in the town centre.
- The sample will only contain people who live or work in the town centre.
- It may include people who are unemployed or retired and so do not go to work.

b Conduct an online survey or ask employers to do a survey.

4

Under 11 years	11-12 years	13-14 years	15 years and over
3	5	10	7

5

Sport	Committee members
Squash	3
Tennis	5
Bowls	4
Golf	3
Hockey	5

Developing fluency (pages 555–556)

1 No, as not everyone can be selected and so it is not a random sample.
Also, she will ask students from the same family and so she will get duplicate data.

2 **a** This will only use 16 people which is too small a sample.

b Method B, as all customers have equal chance.

3 **a** The views will fairly represent the two different groups.

b 22 one/two nights, 8 week.

c Examples include gender; or diet (omnivore / vegetarian).

4 35

5 **a** Number all the population and use random numbers.

b 196

Problem solving (page 556)

1 Huw is correct. Not everyone is in the telephone book as they might not have a phone or be ex-directory. Not everyone can be chosen, so it is not a random sample.

2 **a** Students who don't eat the meals because they don't like them cannot be asked.

b She could number all the students on the college roll and use a random number generator to select students.

3 Approximately 97, but it could be any integer, n, such that $94 \leq n \leq 100$

4 A stratified sampling method gives these results:

Village	Committee members
Aberwen	3
Bronllew	8
Caerwys	1
Dunwen	1
Edenton	11

This gives a committee of 24 so it might be best to increase Edenton's representation to 12 as they are the largest and also have been rounded down from 11.45.

5 Oliver should interview 10 tennis players

Reviewing skills (page 557)

1 This is a systematic sample and is not random. Most people have no chance of being selected.

2 A stratified sample make sure that groups are represented in proportion to the size of the group, so reduces the risk of bias.

3

Country	Number of employees
Austria	53
Sweden	27
Norway	20
Italy	11
United Kingdom	9

4 3 from School A, 5 from School B and 6 from School C.

5 A stratified sample would yield these results:

Country	Number of employees
Wales	6
Scotland	4
England	12
Northern Ireland	2

This makes a total of 24, so an extra representative should be invited. The extra representative could be form England as the largest country and also has been rounded down the most (from 12.3214).

Statistics and Probability Strand 4 • Unit 5 • Answers

Practising skills (pages 562–564)

1 a $\frac{1}{6}$

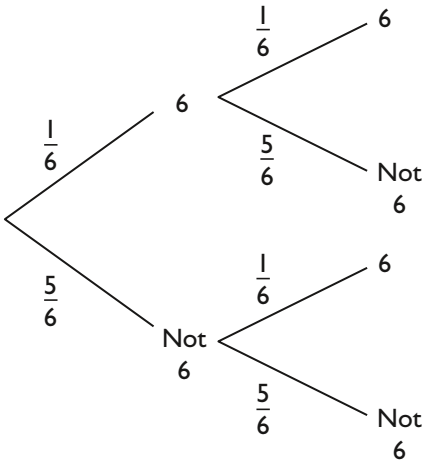
b $\frac{1}{6}$

c $\frac{1}{6}$

d $\frac{1}{6}$

e $\frac{1}{6}$

2 a



b $\frac{1}{36}$

c $\frac{25}{36}$

d $\frac{11}{36}$

e $\frac{10}{36}$

3 a $\frac{4}{52}$

b $\frac{4}{51}$

c $\frac{4}{50}$

d $\frac{3}{49}$

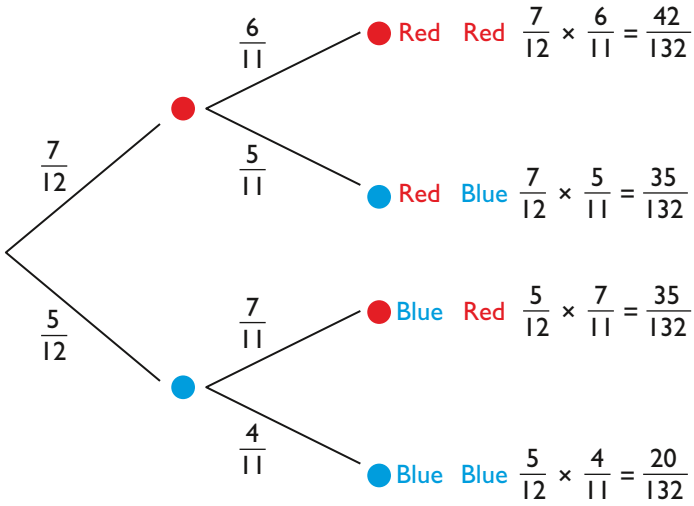
4 a $\frac{1}{12}$

b $\frac{5}{12}$

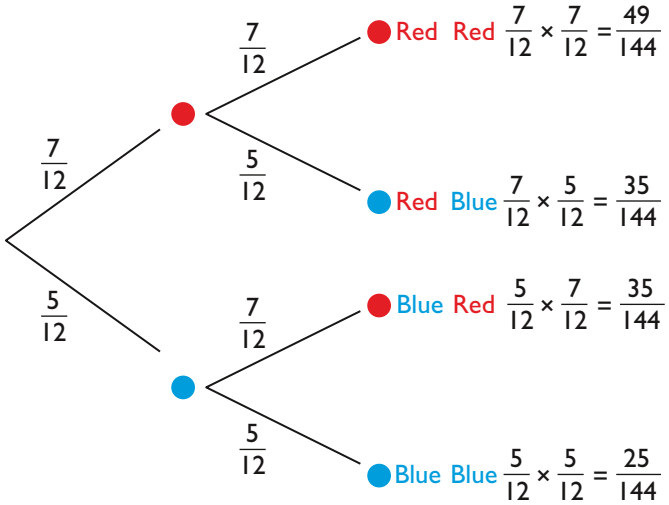
c $\frac{1}{12}$

d $\frac{5}{12}$

5 a



b



6 a $\frac{4}{9}$

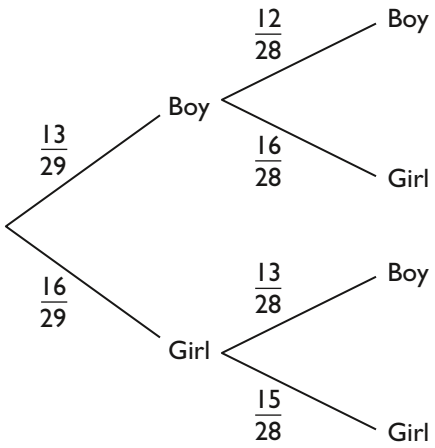
b There are 3 red and 5 yellow counters left in the bag.

c i $\frac{2}{7}$

ii $\frac{5}{7}$

Developing fluency (pages 564–566)

1 a



i $\frac{240}{812}$

ii $\frac{156}{812}$

iii $\frac{572}{812}$

iv $\frac{416}{812}$

2 a independent

b independent

c dependent

d independent

3 a i $\frac{1}{8}$

ii $\frac{1}{6}$

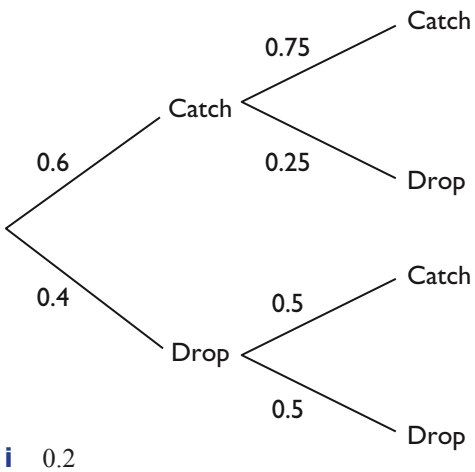
b i $\frac{1}{8}$

ii $\frac{1}{6}$

c $\frac{1}{8}$

d independent

4 a



b i 0.2

ii 0.35

iii 0.45

5 **a** $1 - 0.05 = 0.95$

b $1 - 0.4 = 0.6$

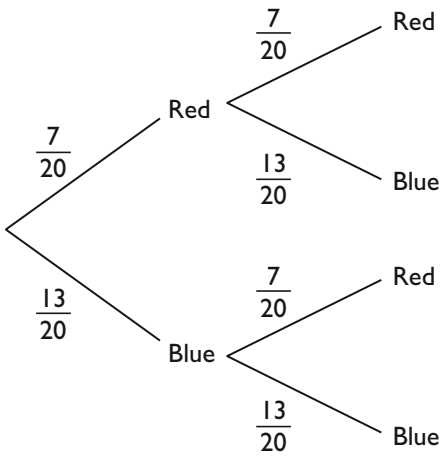
c $0.4 \times 0.05 = 0.02$

d $1 - (0.95 \times 0.6) = 0.43$

6 It is not impossible – each time James tossed the coin it had a $\frac{1}{2}$ chance of being a head, it just never landed on heads. Each coin toss was independent of the previous one.

Problem solving (pages 566–567)

1 **a**



i $\frac{169}{400}$

ii $\frac{49}{400}$

iii $\frac{182}{400}$

iv $\frac{351}{400}$

2 **a** $\frac{4}{15}$

b $\frac{2}{15}$

c $\frac{1}{3}$

3 **a** $\frac{1}{14} \times \frac{1}{13} = \frac{1}{182}$

b $\frac{1}{48}$

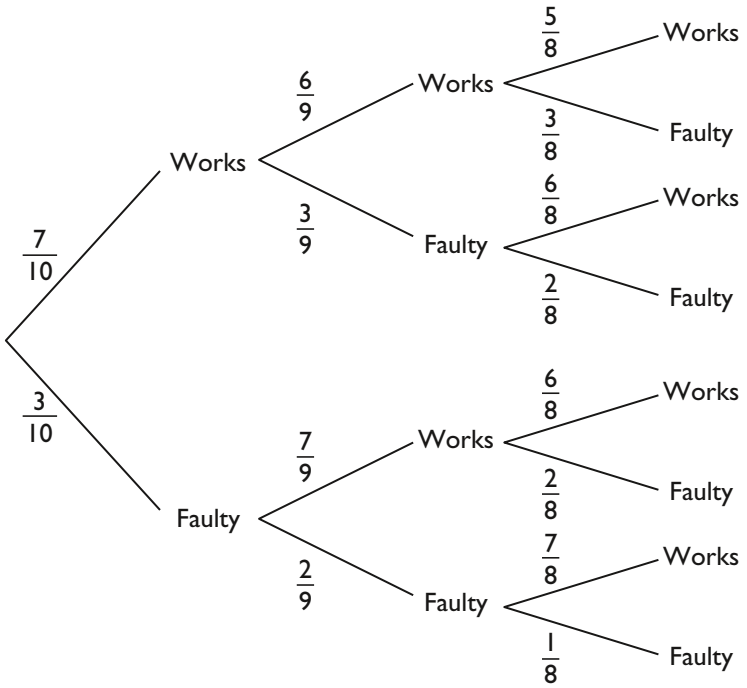
4 **a** Yes, because $0.3 \times 0.8 = 0.24$ and $\frac{6}{25} = \frac{24}{100} = 0.24$.

b 0.62

5 **a** $\frac{31}{220}$ or 0.14

b Yes. It is expected that Naela's train will be late 30% of the time when it's raining, but it might not have been late at all!

6 a



b i $\frac{1}{120}$

ii $\frac{7}{40}$

iii $\frac{21}{40}$

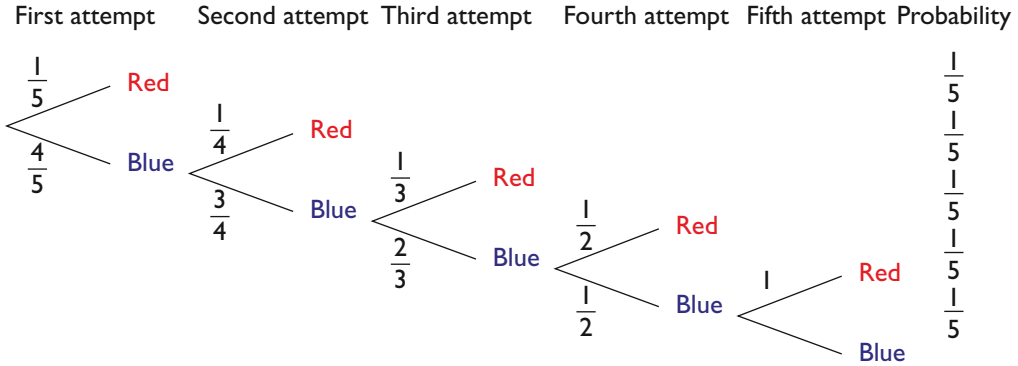
iv $\frac{7}{24}$

c There is a $\frac{119}{120}$ chance that at least one bulb will work and 70% chance that at least two will.

7 a $\frac{7}{15}$

b 8 – assuming he doesn't put the socks back into the drawer, if he pulled out 6 blue socks in a row then all that are left are red socks.

8 a



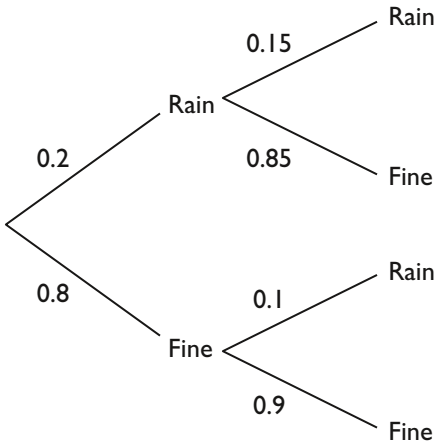
b Each of the outcomes is just as likely as any other. The total of the probabilities tells us that there are no other options in outcomes, since they add to 1.

c Instead of the probabilities changing as counters are removed they will stay the same at $\frac{1}{5}$ for a red and $\frac{4}{5}$ for a blue.

d $\frac{2101}{3125} \approx 0.67$

Reviewing skills (page 568)

1 a



b $1 - (0.2 \times 0.15) = 0.97$

2 a $\frac{2}{21} \approx 0.095$

b $\frac{9}{21} \approx 0.43$

c $\frac{10}{21} \approx 0.475$

d $\frac{19}{21} \approx 0.905$

4 Statistics and Probability Strand 4 • Unit 6 • Answers

Practising skills (pages 571–573)

1 a $\frac{3}{15}$

b $\frac{2}{15}$

c $\frac{1}{3}$

2 a $\frac{1}{60}$

b $\frac{7}{300}$

c $\frac{1}{25}$

3 a $\frac{57}{600} = \frac{19}{200}$

b $\frac{3}{25}$

c $\frac{129}{600} = \frac{43}{200}$

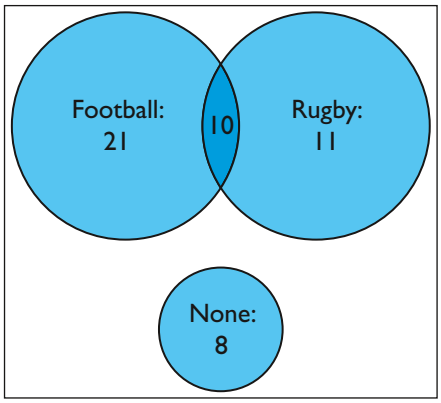
d $\frac{471}{600} = \frac{157}{200}$

4 a $\frac{1}{3}$

b $\frac{1}{3}$

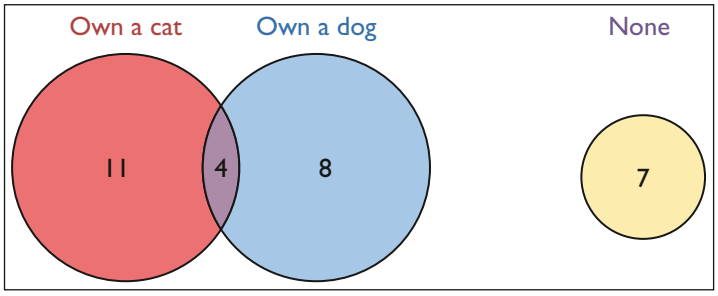
c $\frac{2}{3}$

5 a



b $\frac{11}{50}$

6 a



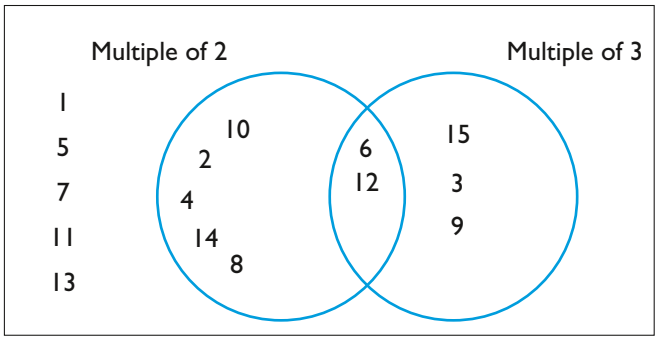
b i $\frac{12}{30} = \frac{2}{5}$

ii $\frac{4}{30} = \frac{2}{15}$

iii $\frac{23}{30}$

iv $\frac{7}{30}$

7 a



b i $\frac{7}{15}$

ii $\frac{1}{3}$

iii $\frac{2}{15}$

iv $\frac{2}{3}$

v $\frac{1}{3}$

Developing fluency (pages 573–574)

1 a $\frac{1}{4}$

b $\frac{3}{4}$

c $\frac{3}{4}$

d $\frac{8}{52} = \frac{2}{13}$

e $\frac{44}{52} = \frac{11}{13}$

2 The events sun and rain are not mutually exclusive.

3 a Wears glasses total: 7. Does not wear glasses total: 22.

b i $\frac{12}{29}$

ii $\frac{15}{29}$

iii $\frac{13}{29}$

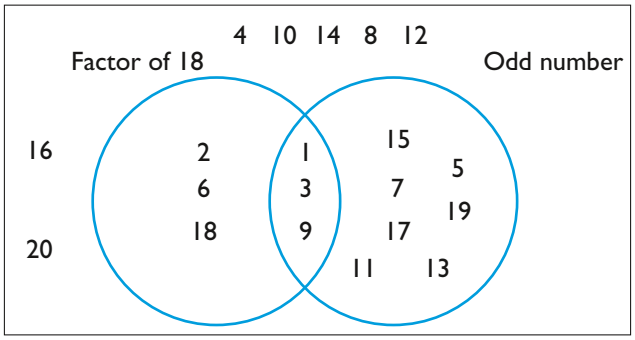
iv $\frac{10}{29}$

v 0

4 a $\frac{2}{3}$

b $\frac{1}{3}$

5 a



b i $\frac{1}{2}$

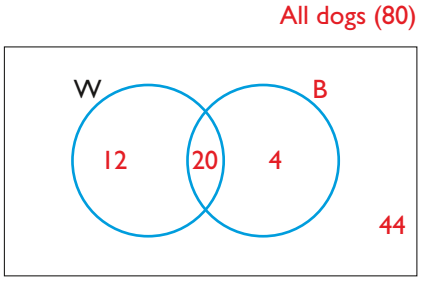
ii $\frac{6}{20}$

iii $\frac{3}{20}$

iv $\frac{13}{20}$

v $\frac{7}{20}$

6 a



W = dogs with weak hips

B = dogs with bad eyesight

b Estimated probability $= \frac{20}{80} = \frac{1}{4}$.

It is an estimate because this is based on a sample not on all the dogs of the breed.

c i In the general formula the sets are A and B. Here they are W and B.

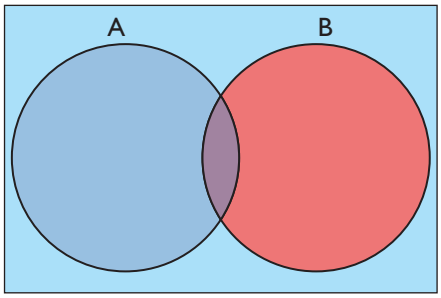
$p(W) = \frac{32}{80}$, $p(B) = \frac{24}{80}$, $p(W \text{ and } B) = \frac{20}{80}$

$p(W \text{ or } B) = p(W) + p(B) - p(W \text{ and } B)$

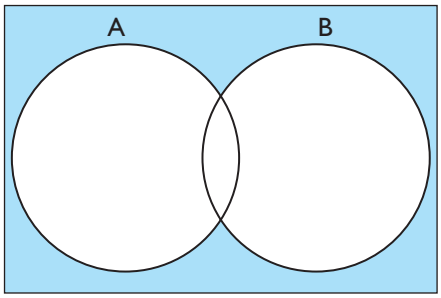
$= \frac{32}{80} + \frac{24}{80} + \frac{20}{80} = \frac{36}{80} = 0.45$

ii Using the Venn diagram: the number of dogs with one or more defect is 12 + 20 + 4 = 36. So the probability is $\frac{36}{80} = 0.45$. You get the same answer both ways of course.

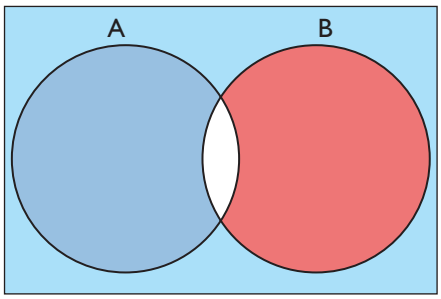
7 a



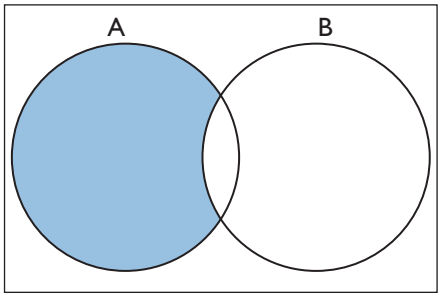
b



c

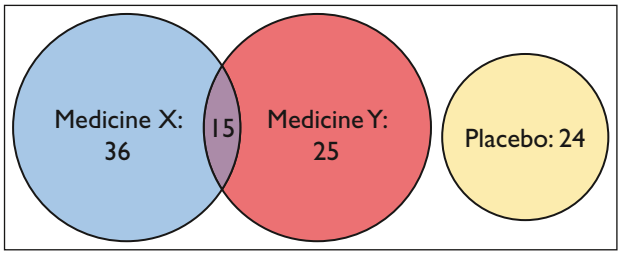


d



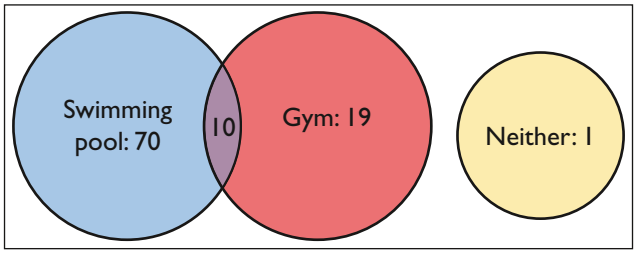
Problem solving (pages 575–576)

1 a



b $P(\text{neither medicine}) = \frac{24}{100}$

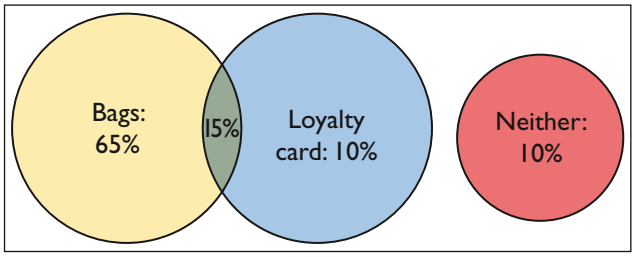
2 a



b $\frac{29}{100}$

c $\frac{89}{100}$

3 a



b i 0.1

ii 0.65

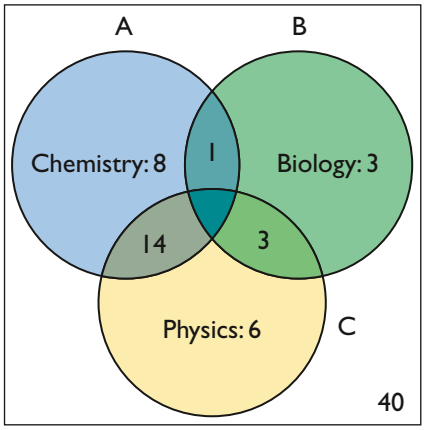
iii 0.9

iv 0.1

4 a $\frac{5}{12}$

b $\frac{3}{4}$

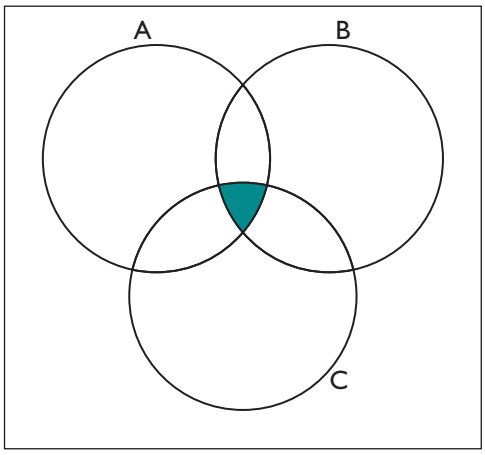
5 a and **b** Correct diagram:



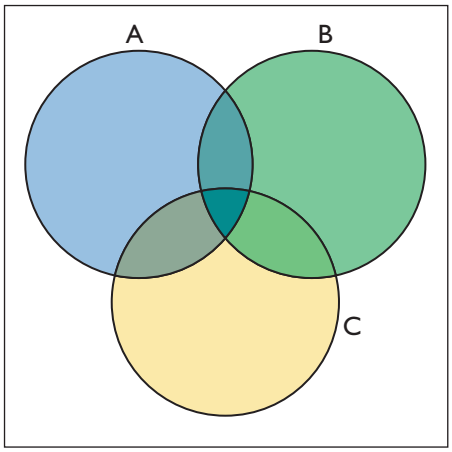
c $\frac{23}{80}$

d Neither; only half of the students do one or more science so the probability of choosing one who takes science is $\frac{1}{2}$.

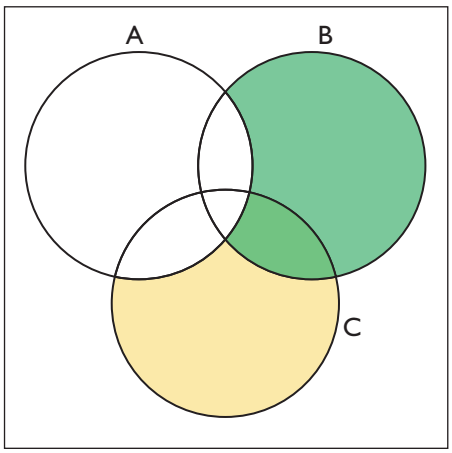
6 a



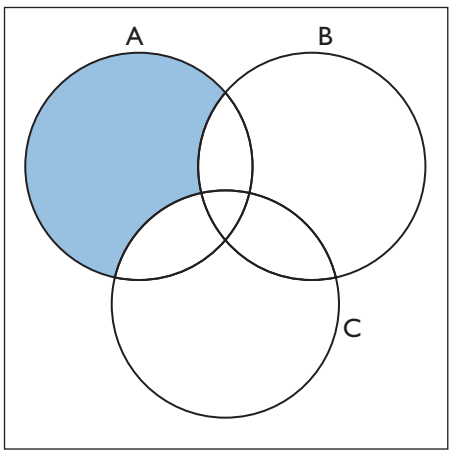
b



c



d



Reviewing skills (page 576)

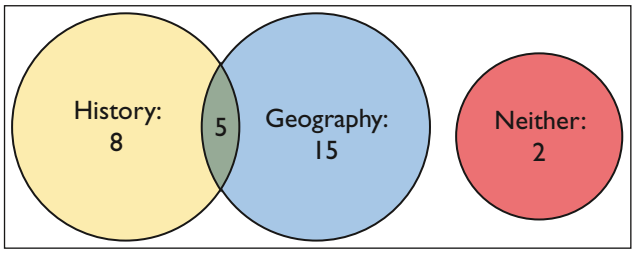
1 a i $\frac{1}{2}$

ii $\frac{1}{2}$

iii $\frac{5}{6}$

b The events 'even' and '3 or less' are not mutually exclusive as '2' is both even and less than 3.

2 a Correct diagram:



b $\frac{23}{30}$

4 Statistics and Probability Strand 4 • Unit 7 • Answers

Practising skills (pages 581–583)

1 120

2 No, there are 676 possible pairs of initials, so it is possible for each student to have unique initials.

3 **a** 1296

b **i** $\frac{1}{1296}$

ii $\frac{1}{1296}$

iii $\frac{1}{1296}$

4 **a** 12

b $\frac{1}{12}$

c $\frac{1}{6}$

d Assumption: that each pair of routes is equally likely.

5 **a** **i** $\frac{1}{6}$

ii $\frac{1}{6}$

iii $\frac{3}{6}$

iv $\frac{2}{6}$

v $\frac{3}{6}$

b **i** $\frac{1}{3}$

ii 0

iii $\frac{1}{3}$

iv $\frac{2}{3}$

v 0

6 **a** $\frac{4}{36}$

b 0

7 **a** $\frac{13}{30}$

b $\frac{4}{13}$

c $\frac{2}{13}$

d $\frac{5}{15}$

e $\frac{5}{11}$

f $\frac{5}{17}$

8 a $\frac{7}{27}$

b $\frac{2}{11}$

c $\frac{2}{7}$

d $\frac{4}{16}$

e $\frac{4}{9}$

9 a 9000

b $\frac{1}{9000}$

c $\frac{1}{100}$

10 a

	Party A	Party B	Party C	Party D	Total
Voters in their teens and 20s	261	192	93	104	650
Voters in their 30s	203	241	198	47	689
Voters in their 40s	154	275	89	12	530
Voters in their 50s and older	73	315	83	56	527
Total	691	1023	463	219	2396

b i $\frac{691}{2396}$

ii $\frac{261}{650}$

iii $\frac{83}{527}$

iv $\frac{241}{1023}$

v $\frac{227}{691}$

Developing fluency (pages 583–585)

1 a i $\frac{46}{50}$

ii $\frac{46}{91}$

iii $\frac{1}{25}$

iv $\frac{1}{50}$

b i A single digit or a number that is not even nor prime is chosen.

ii A prime number is chosen.

2 a

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

b i $\frac{2}{6}$

ii $\frac{2}{6}$

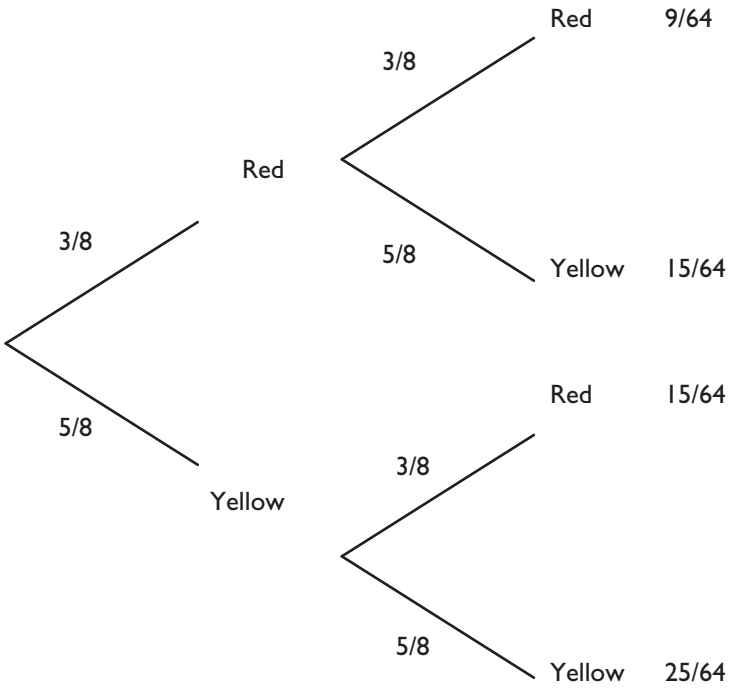
iii $\frac{3}{6}$

iv $\frac{4}{6}$

v $\frac{2}{8}$

vi 0

3 a



b i $\frac{3}{8}$

ii $\frac{5}{8}$

iii $\frac{3}{8}$

4 a

	Katie	Carol	Hannah	Total
Year 7	31	48	45	124
Year 8	43	65	31	139
Year 9	48	68	23	139
Year 10	30	73	38	141
Year 11	55	51	49	155
Total	207	305	186	698

b **i** $\frac{31}{124}$

ii $\frac{31}{207}$

iii $\frac{23}{139}$

iv $\frac{23}{186}$

v $\frac{73}{141}$

vi $\frac{73}{305}$

c A student in year 11 (total = 155) who voted for Carol has a probability of $\frac{51}{155}$. A student who voted for Carol (total = 305) is in year 11 is $\frac{51}{305}$. They are different probabilities.

5 **a** combinations = 1×10^9

b combinations = 2.08×10^{11}

c combinations = 7.84×10^{10}

So b, eight letter characters is the most secure.

6 **a** Not all 1000 people are equally likely to get pregnant because 500 of them are men. So you can't use relative frequency to work out probability. The student should have said 'The probability of a woman born in 1965 getting pregnant during her life was $\frac{400}{500} = 0.8$' and perhaps the obvious 'The probability of a man born in 1965 getting pregnant during his life was 0'.

b The probability of 0.4 applies to no one. The probability is 0 if the person is a man and 0.8 if the person is a woman. So two conditional probabilities are needed – otherwise the statement is meaningless.

7 **a** **i** $\frac{3}{16}$

ii $\frac{1}{61}$

iii $\frac{14}{19}$

b Yes, Ben's train is often cancelled [P(train cancelled) = $\frac{19}{80}$] so he should get an earlier train as it is unreliable.

8 $\frac{2}{3}$

9 $\frac{17}{38}$

Problem solving (pages 586–587)

1 **a** $\frac{4}{9}$

b $\frac{5}{9}$

c $\frac{1}{6}$

2 **a** $\frac{9}{44}$

b $\frac{8}{55}$

3 **a** $\frac{13}{145}$

b $\frac{55}{406}$

c $\frac{5}{28}$

4 0.44

5 Mazzie is right as P(at least one sunny day) = $0.55 \times 0.8 + 0.55 \times 0.2 + 0.45 \times 0.1 = 0.595 > 0.5$.

6 a $\frac{1}{36}$

b $\frac{215}{216}$

7 $P(\text{black}) = \frac{p(p-1)}{90} + \frac{2p(10-p)}{90} = \frac{(p^2 - p - 2p^2 + 20p)}{90} = \frac{19p - p^2}{90}$

8 a $\frac{25}{49}$

b $\frac{37}{49}$

9 0.487

10 $\frac{10}{11}$

Reviewing skills (pages 587–588)

1 a $\frac{3}{10}$

b $\frac{782}{1617}$

2 a i $\frac{5}{24}$

ii $\frac{13}{24}$

b $\frac{2}{5}$

3 a $\frac{304}{677}$

b $\frac{11}{45}$

c $\frac{19}{73}$

4 a 0.28

b 0.72

c 0.973