



**New Jersey Center for Teaching and Learning**

**Progressive Science Initiative**

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**AP BIOLOGY**



**Investigation #2  
Mathematical Modeling:  
Hardy-Weinberg**

Summer 2014

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**Investigation #2: Mathematical Modeling**

[Click on the topic to go to that section](#)

- **Pacing/Teacher's Notes**
- **Pre-Lab**
- **Guided Investigation**
- **Independent Inquiry**

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## Pacing/Teacher's Notes

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## Teacher's Notes

Lab procedure adapted from College Board AP Biology Investigative Labs: An Inquiry Approach Teacher's Manual

[Click here for CB AP Biology Teacher Manual](#)

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## Pacing

Day (time)	Activity	General Description	Reference to Unit Plan	Notes
Day 1 (HW)	Pre-lab	Pre-Lab Questions	EC Day 6 HW	
Day 2 (40)	Steps 1-3	Qualitatively describe the system	EC Day 7	If time permits, begin spreadsheet
Day 3 (80)	Steps 4-7	Setting up spreadsheet	EC Day 8	Students will experience in spreadsheet software may not need entire lab period. If necessary, the example spreadsheet can be shared with students
Day 4 (40)	Independent Investigation	Set up spreadsheet to test independent question	EC Day 10	
Day 5 (40)	Independent Investigation	Analysis of question and reporting	EC Day 11	
Day 6 (20)	Assessment	Lab Quiz	EC Day 12	

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## Pre-Lab

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## Question/Objectives

***How can mathematical models be used to investigate the relationship between allele frequencies in populations of organisms and evolutionary change?***

In this lab we will:

- Use a data set that reflects a change in the genetic makeup of a population over time and apply mathematical methods and conceptual understandings to investigate the cause(s) and effect(s) of this change.
- Apply mathematical methods to data from a real or simulated population to predict what will happen to the population in the future.
- Evaluate data-based evidence that describes evolutionary changes in the genetic makeup of a population over time.
- Use and justify data from mathematical models based on Hardy-Weinberg equilibrium to analyze genetic drift and the effect of selection in the evolution of specific populations.
- Describe a model that represent evolution within a population.
- Evaluate data sets that illustrate evolution as an ongoing process.

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## Pre-Lab Questions

Read the background information and answer the following questions in your lab notebook.

1. Describe the life cycle of a diploid organism.
2. Do all organisms complete their life cycle? Why or why not
3. According to the Hardy-Weinberg equilibrium, if the frequencies of alleles in the population ( $p$  and  $q$ ) change, a population is evolving. Under what conditions would a population evolve?
4. Give a brief outline of this investigation.

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## Safety

To avoid frustration, periodically save your work.

When developing and working out models, save each new version of the model with a different file name. That way, if a particular strategy doesn't work, you will not necessarily have to start over completely.

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## Guided Investigation

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## Materials

- Computer with spreadsheet software
- Laboratory notebook

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## Building a Simple Mathematical Model

- Step 1** Formulate the question.
- Step 2** Determine the basic ingredients.
- Step 3** Qualitatively describe the biological system.
- Step 4** Quantitatively describe the biological system.
- Step 5** Analyze the equations.
- Step 6** Perform checks and balances.
- Step 7** Relate the results back to the question.

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## Mathematical Model: Example

### Step 1 Formulate the question.

For guided practice, we will use the following question:

*How do inheritance patterns or allele frequencies change in a population?*

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## Mathematical Model: Example

### Step 2 Determine the basic ingredients.

For this model, assume that all the organism in our hypothetical population are diploid.

This organism has a gene locus with two alleles - A and B.

*We could use A and a, but A and B are easier to work with in the spreadsheet software.*

This imaginary population is sexually reproducing.

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## Mathematical Model: Example

### Step 3 Qualitatively describe the biological system.

For our example: the population consists of diploid, sexually reproducing organisms. All gametes go into one infinite gene pool, and all have an equal chance of taking part in fertilization or formation of a zygote.

All zygotes live to be juveniles, all juveniles live to be adults, and no individuals enter or leave the population; there are also no mutations.

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## Mathematical Model: Example

### Step 4 Quantitatively describe the biological system (setting up the spreadsheet).

A. Bring up a blank spreadsheet on your computer.

Click here an example  
of the spreadsheet in  
Excel

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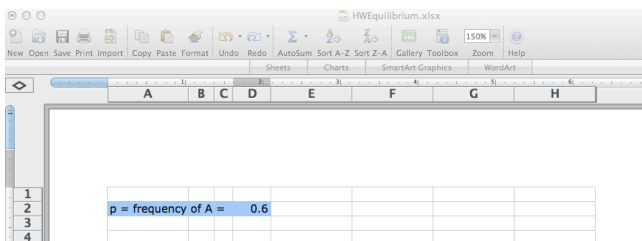
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## Mathematical Model: Example

4B. In cell D2, enter the value for the frequency of the A allele. This value should be between 0 and 1. *Unless otherwise instructed by your teacher, enter 0.6 for now.*

Label this value "p = frequency of A =" as shown. You may also wish to highlight these cells and adjust the column width as shown.




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### Mathematical Model: Example

4C. In cell D3, enter the formula to calculate the value of  $q$ .

*Do not simply enter the value 0.4. You want the spreadsheet to automatically adjust when changes are made to the value of  $p$ .*

Label this value " $q$  = frequency of B =" as shown. You may also wish to highlight these cells.

=1-D2

	A	B	C	D	E	F
1						
2	p = frequency of A =			0.6		
3	q = frequency of B =			0.4		
4						
5						

### Mathematical Model: Example

4D. In any cell enter the following function:

=RAND()

*Note that the parentheses have nothing between them.*

The RAND function returns random numbers between 0 and 1 in decimal format. This is a powerful feature of spreadsheets. It allows us to enter a sense of randomness to our calculation if it is appropriate - and here it is when we are "randomly" choosing gametes from the gene pool.

If you are using a PC, try hitting the F9 key several times and notice that the value in the cell changes. For Macs, enter cmd + or cmd = to force recalculation.

You may delete the RAND function from the cell, or leave it to check the accuracy of your future work.

### Mathematical Model: Example

4E. In cell E5 enter the following function:

=IF(RAND()<=D\$2,"A","B")

In spreadsheet terminology, this says "if the random number is less than or equal to D2, then put A in the cell, if not put B".

Now create the same formula in cell F5, and label these columns "gametes" as shown.

Try recalculating several times, using the F9 or cmd + / cmd = keys.

	A	B	C	D	E	F
1						
2	p = frequency of A =			0.6		
3	q = frequency of B =			0.4		
4						
5					A	A
6						
7						
8						
9						
10						

### Mathematical Model: Example

4F. Copy these two formulas in E5 and F5 down for a total of 16 rows to represent gametes that will form 16 offspring for the next generation, as shown below.

To copy the formulas, click on the bottom right-hand corner of the cell and, with your finger pressed down on the mouse, drag the cell downward.

	A	B	C	D	E	F	G
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
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16							
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### Mathematical Model: Example

4G. In cell G5 enter the following function:

```
=CONCATENATE(E5,F5)
```

This formula combines the values present in E5 and F5.

Copy this formula down as far as you have gametes, and label the column zygotes as shown.

	D	E	F	G	F
	0.6				
	0.4				
			gametes		zygotes
	A	B		AB	
	A	A		AA	
	B	B		BB	
	B	B		BB	
	B	A		BA	
	A	A		AA	
	A	A		AA	
	B	A		BA	
	A	B		AB	
	A	B		AB	
	B	A		BA	
	B	A		BA	
	A	B		AB	
	A	B		AB	
	B	A		BA	
	B	A		BA	

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### Mathematical Model: Example

4H. In cell H5 enter the following function:

```
=IF(G5="AA",1,0)
```

Can you interpret this formula? What does it say in English?

Enter the similar function: =IF(G5="BB",1,0) in cell J5, and label the columns: AA, AB, and BB as shown.

	D	E	F	G	H	I	J
	0.6						
	0.4						
			gametes	zygotes	AA	AB	BB
	B	A	BA		0		0
	A	A	AA				
	A	A	AA				

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### Mathematical Model: Example

**Step 6 Perform checks and balances.**

Try recalculating several times. Check each generation to insure that the data sets are changing as expected.

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### Analyzing & Evaluating Results

**Step 7 Relate the results back to the question.**

In the absence of random event, are the allele frequencies of the original population expected to change from generation to generation?

What happens to allele frequencies in such a population? Is it predictable?

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### Independent Inquiry

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### Designing & Conducting Your Investigation

As you worked through the guided investigation, you were able to use your model to explore how random chance affects the inheritance patterns of alleles.

What other factors can cause allele frequencies to change in a population? How would you model them?

Select a variable to test and generate a testable hypothesis. Alter your model to fit your investigation and collect sufficient data by running your model repeatedly.

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