AP Calculus
Related Rates Worksheet

1. A small balloon is released at a point 150 feet from an observer, who is on level ground. If the balloon goes straight up at a rate of 8 feet per second, how fast is the distance from the observer to the balloon increasing when the balloon is 50 feet high?
2. Water is pouring into a conical tank at the rate of 8 cubic feet per minute. If the height of the tank is 12 feet and the radius of its circular opening is 6 feet, how fast is the water level rising when the water is 4 feet deep?
3. A child is flying a kite. If the kite is 90 feet above the child's hand level and the wind is blowing it on a horizontal course at 5 feet per second, how fast is the child letting out the cord when 150 feet of cord is out? (Assume the cord forms a straight line - i.e. there is no slack in the string.)
4. Sand is falling off a conveyor belt at a rate of 12 cubic feet per minute into a conical pile. The diameter of the pile is four times the height. At what rate is the height of the pile changing when the pile is 10 feet high?
5. A ladder 25 feet long is leaning against a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second.
a. How fast is the top of the ladder moving down the wall when the base of the ladder is 12 feet from the wall?
b. Consider the triangle formed by the side of the house, the ladder, and the ground. Find the rate at which the area of the triangle is changing when the base is 12 feet from the wall.
c. Find the rate at which the angle between the ladder and the wall of the house is changing when the base of the ladder is 12 feet from the wall.

Differentiate implicitly.
3. $4 x^{2}+3 y^{2}-3 x=20$
4. $\cos x+\sin 3 y=3$
5. Determine $y^{\prime \prime}$ for the equation: $x^{2}+2 y^{2}-x=5$
6. Find equations of the tangent and normal lines to $f(x)=\sqrt[3]{1-2 x}+x^{3}$ at $\mathrm{x}=1$. Give your answer in standard form $(\mathrm{Ax}+\mathrm{By}+\mathrm{C}=0)$.

Solutions:
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1. $2.5 \mathrm{ft} / \mathrm{sec}$
2. $(2 / \pi) \mathrm{ft} / \mathrm{min}$
3. $4 \mathrm{ft} / \mathrm{sec}$

## AP Calculus

Related Rates/Implicit Differentiation Worksheet

1. $\quad 0.0095 \mathrm{ft} / \mathrm{min}$
2. $\quad-1.09 \mathrm{ft} / \mathrm{sec}, 15.39 \mathrm{ft}^{2} / \mathrm{sec}, 0.09 \mathrm{rad} / \mathrm{sec}$
3. $\frac{d y}{d x}=\frac{3-8 x}{6 y}$
4. $\frac{d y}{d x}=\frac{\sin x}{3 \cos 3 y}$
5. $\frac{d^{2} y}{d x^{2}}=\frac{-8 y^{2}-(1-2 x)^{2}}{16 y^{3}}$
6. tangent: $7 \mathrm{x}-3 \mathrm{y}-7=0$
normal: $3 x+7 y-3=0$

You should be able to:

1. Differentiate implicitly with respect to one of the variables in the equation.
2. Differentiate implicitly with respect to a variable not in the equation.
3. Find the equation of the tangent line to a graph at a given point.
4. Apply the use of implicit differentiation to solve related rates problems.
5. Determine the second derivative of an implicit function.
6. Find $d y / d x$ for the given equation.
a. $5 x^{2}-2 x y+7 y^{2}=0$
b. $y=\frac{x}{x+y}$
c. $y=\sin (x+y)$
7. Differentiate with respect to $t$.
a. Find dx/dt $\quad 5 x^{2}+2 x y+3 y^{2}=0$
b. Find $\mathrm{d} \theta / \mathrm{dt} \quad y=\sin ^{2} 2 \theta$
c. Find dx/dt $\quad x^{2}+y^{2}=50$
8. Find an equation of the tangent line for the graph of the given function at the given point.
a. $x^{2}+2 y^{2}=3$
b. $x^{4}+4 x^{2} y^{3}+y^{2}=2 y$
c. $y=\sin \frac{x \pi}{2}$
9. a. As a balloon in the shape of a sphere is being blown up, the volume is increasing at the rate of 4 cubic inches per second. At what rate is the radius increasing when the radius is 1 inch?
b. Sand is falling off a conveyor onto a conical pile at the rate of 15 cubic feet per minute. The diameter of the base of the cone is approximately twice the altitude. At what rate is the height of the pile changing when it is 10 feet high?
c. Two boats leave the same port at the same time with one boat traveling north at 35 knots per hour and the other boat traveling east at 40 knots per hour. How fast is the distance between the two boats changing after 2 hours?
d. The area of a circle is decreasing at a rate of 2 square centimeters per minute. Find the rate of change of the radius with respect to time when the radius is 4 centimeters.
e. A 5-meter ladder is leaning against the side of the house. The foot of the ladder is pulled away from the house at a rate of $0.4 \mathrm{~m} / \mathrm{sec}$. Determine how fast the top of the ladder is descending when the foot of the ladder is 3 meters from the house.
10. Find the second derivative with respect to $x$.
a. $-2 y+x^{3}=5$
b. $x \sin y=3$

## Solutions:

1.a. $\frac{d y}{d x}=\frac{y-5 x}{7 y-x}$, 1.b. $\frac{d y}{d x}=\frac{y}{(x+y)^{2}+x}$, 1.c. $\frac{d y}{d x}=\frac{\cos (x+y)}{1-\cos (x+y)}$, 2.a. $\frac{d x}{d t}=\frac{d y}{d t}\left(\frac{-x-3 y}{5 x+y}\right)$,
2.b. $\frac{d \theta}{d t}=\frac{1}{2 \sin 4 \theta}\left(\frac{d y}{d t}\right)$, 2.c. $\frac{d x}{d t}=\frac{-y}{x}\left(\frac{d y}{d t}\right)$, 3.a. $\mathrm{x}+2 \mathrm{y}-3=0$, 3.b. $\mathrm{x}-2 \mathrm{y}+3=0$,
3.c. $\mathrm{y}=-1$, 4.a. $\frac{1}{\pi}$ inches/second, 4.b. 0.048 feet/minute, 4.c. 53.2 knots/hour, 4.d. $\frac{-1}{4 \pi} \mathrm{~cm} /$ minute,
4.e. -0.3 meters/second, 5.a. $3 x$, 5.b. $\frac{\tan y\left(\sec ^{2} y+1\right)}{x^{2}}$

