

Differentiate implicitly.

3. $4x^2 + 3y^2 - 3x = 20$

4. $\cos x + \sin 3y = 3$

5. Determine y'' for the equation: $x^2 + 2y^2 - x = 5$

6. Find equations of the tangent and normal lines to $f(x) = \sqrt[3]{1-2x} + x^3$ at $x = 1$. Give your answer in standard form ($Ax + By + C = 0$).

Solutions:

AP Calculus

Related Rates Worksheet

1. 2.5 ft/sec
2. $(2/\pi)$ ft/min
3. 4 ft/sec

AP Calculus

Related Rates/Implicit Differentiation Worksheet

1. 0.0095 ft/min
2. -1.09 ft/sec, 15.39 ft²/sec, 0.09 rad/sec
3. $\frac{dy}{dx} = \frac{3-8x}{6y}$
4. $\frac{dy}{dx} = \frac{\sin x}{3 \cos 3y}$
5. $\frac{d^2y}{dx^2} = \frac{-8y^2 - (1-2x)^2}{16y^3}$
6. tangent: $7x - 3y - 7 = 0$
normal: $3x + 7y - 3 = 0$

Review Sheet 2.5-2.6
Implicit Differentiation
Related Rates

Name _____

You should be able to:

1. Differentiate implicitly with respect to one of the variables in the equation.
2. Differentiate implicitly with respect to a variable not in the equation.
3. Find the equation of the tangent line to a graph at a given point.
4. Apply the use of implicit differentiation to solve related rates problems.
5. Determine the second derivative of an implicit function.

1. Find dy/dx for the given equation.

a. $5x^2 - 2xy + 7y^2 = 0$

b. $y = \frac{x}{x+y}$

c. $y = \sin(x+y)$

2. Differentiate with respect to t .

a. Find dx/dt $5x^2 + 2xy + 3y^2 = 0$

b. Find $d\theta/dt$ $y = \sin^2 2\theta$

c. Find dx/dt $x^2 + y^2 = 50$

3. Find an equation of the tangent line for the graph of the given function at the given point.

a. $x^2 + 2y^2 = 3$ (1,1)

b. $x^4 + 4x^2y^3 + y^2 = 2y$ (1,-1)

c. $y = \sin \frac{x\pi}{2}$ (3,-1)

4.
 - a. As a balloon in the shape of a sphere is being blown up, the volume is increasing at the rate of 4 cubic inches per second. At what rate is the radius increasing when the radius is 1 inch?
 - b. Sand is falling off a conveyor onto a conical pile at the rate of 15 cubic feet per minute. The diameter of the base of the cone is approximately twice the altitude. At what rate is the height of the pile changing when it is 10 feet high?
 - c. Two boats leave the same port at the same time with one boat traveling north at 35 knots per hour and the other boat traveling east at 40 knots per hour. How fast is the distance between the two boats changing after 2 hours?
 - d. The area of a circle is decreasing at a rate of 2 square centimeters per minute. Find the rate of change of the radius with respect to time when the radius is 4 centimeters.
 - e. A 5-meter ladder is leaning against the side of the house. The foot of the ladder is pulled away from the house at a rate of 0.4 m/sec. Determine how fast the top of the ladder is descending when the foot of the ladder is 3 meters from the house.

5. Find the second derivative with respect to x .

a. $-2y + x^3 = 5$

b. $x \sin y = 3$

Solutions:

1.a. $\frac{dy}{dx} = \frac{y-5x}{7y-x}$, **1.b.** $\frac{dy}{dx} = \frac{y}{(x+y)^2 + x}$, **1.c.** $\frac{dy}{dx} = \frac{\cos(x+y)}{1-\cos(x+y)}$, **2.a.** $\frac{dx}{dt} = \frac{dy}{dt} \left(\frac{-x-3y}{5x+y} \right)$,

2.b. $\frac{d\theta}{dt} = \frac{1}{2 \sin 4\theta} \left(\frac{dy}{dt} \right)$, **2.c.** $\frac{dx}{dt} = \frac{-y}{x} \left(\frac{dy}{dt} \right)$, **3.a.** $x + 2y - 3 = 0$, **3.b.** $x - 2y + 3 = 0$,

3.c. $y = -1$, **4.a.** $\frac{1}{\pi}$ inches/second, **4.b.** 0.048 feet/minute, **4.c.** 53.2 knots/hour, **4.d.** $\frac{-1}{4\pi}$ cm/minute,

4.e. -0.3 meters/second, **5.a.** $3x$, **5.b.** $\frac{\tan y(\sec^2 y + 1)}{x^2}$