AP CALCULUS AB & BC
FORMULA LIST

Definition of e: $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

Absolute value: $|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$

Definition of the derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \qquad \text{(Alternative form)}$$

Definition of continuity: f is continuous at c iff

1) $f(c)$ is defined;
2) $\lim_{x \to c} f(x)$ exists;
3) $\lim_{x \to c} f(x) = f(c)$.

Average rate of change of $f(x)$ on [a, b] $= \frac{f(b) - f(a)}{b - a}$

Rolle's Theorem: If f is continuous on $[a, b]$ and differentiable on (a, b) and if $f(a) = f(b)$, then there is at least one number c on (a, b) such that $f'(c) = 0$.

Mean Value Theorem: If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c on (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Intermediate Value Theorem: If f is continuous on $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c between a and b such that $f(c) = k$.

$\sin 2x = 2 \sin x \cos x$

$$\cos 2x = \begin{cases} \cos^2 x - \sin^2 x \\ 1 - 2\sin^2 x \\ 2\cos^2 x - 1 \end{cases} \qquad \cos^2 x = \frac{1 + \cos 2x}{2} \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$

Definition of a definite integral: $\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x_i) \cdot (\Delta x_i)$

$$\frac{d}{dx}[c]=0$$

$$\frac{d}{dx}\left[x^n\right]=nx^{n-1}$$

$$\frac{d}{dx}[uv]=uv'+vu'$$

$$\frac{d}{dx}\left[\frac{u}{v}\right]=\frac{vu'-uv'}{v^2}$$

$$\frac{d}{dx}\left[f\left(g\left(x\right)\right)\right]=f'\left(g\left(x\right)\right)\cdot g'\left(x\right)$$

$$\frac{d}{dx}[\sin u]=\cos u\,\frac{du}{dx}$$

$$\frac{d}{dx}[\cos u]=-\sin u\,\frac{du}{dx}$$

$$\frac{d}{dx}[\tan u]=\sec^2 u\,\frac{du}{dx}$$

$$\frac{d}{dx}[\cot u]=-\csc^2 u\,\frac{du}{dx}$$

$$\frac{d}{dx}[\sec u]=\sec u\tan u\,\frac{du}{dx}$$

$$\frac{d}{dx}[\csc u]=-\csc u\cot u\,\frac{du}{dx}$$

$$\frac{d}{dx}[\ln u]=\frac{1}{u}\,\frac{du}{dx}$$

$$\frac{d}{dx}[\log_a u]=\frac{1}{u\ln a}\,\frac{du}{dx}$$

$$\frac{d}{dx}[e^u]=e^u\,\frac{du}{dx}$$

$$\frac{d}{dx}[a^u]=a^u\ln a\,\frac{du}{dx}$$

$$\frac{d}{dx}[\arcsin u]=\frac{1}{\sqrt{1-u^2}}\,\frac{du}{dx}$$

$$\frac{d}{dx}[\arccos u]=-\frac{1}{\sqrt{1-u^2}}\,\frac{du}{dx}$$

$$\frac{d}{dx}[\arctan u]=\frac{1}{1+u^2}\,\frac{du}{dx}$$

$$\frac{d}{dx}[\operatorname{arc}\cot u]=-\frac{1}{1+u^2}\,\frac{du}{dx}$$

$$\frac{d}{dx}[\operatorname{arc}\sec u]=\frac{1}{|u|\sqrt{u^2-1}}\,\frac{du}{dx}$$

$$\frac{d}{dx}[\operatorname{arc}\csc u]=-\frac{1}{|u|\sqrt{u^2-1}}\,\frac{du}{dx}$$

$$\left(f^{-1}\right)'\left(a\right)=\frac{1}{f'\left(f^{-1}\left(a\right)\right)}$$

$$\int\cos u\,du=\sin u+C$$

$$\int\sin u\,du=-\cos u+C$$

$$\int\sec^2 u\,du=\tan u+C$$

$$\int\csc^2 u\,du=-\cot u+C$$

$$\int\sec u\tan u\,du=\sec u+C$$

$$\int\csc u\cot u\,du=-\csc u+C$$

$$\int\frac{1}{u}\,du=\ln|u|+C$$

$\int \tan u \, du = -\ln|\cos u| + C$

$\int \cot u \, du = \ln|\sin u| + C$

$\int \sec u \, du = \ln|\sec u + \tan u| + C$

$\int \csc u \, du = -\ln|\csc u + \cot u| + C$

$\int e^u du = e^u + C$

$\int a^u du = \dfrac{a^u}{\ln a} + C$

$\int \dfrac{du}{\sqrt{a^2 - u^2}} = \arcsin \dfrac{u}{a} + C$

$\int \dfrac{du}{u^2 + a^2} = \dfrac{1}{a} \arctan \dfrac{u}{a} + C$

$\int \dfrac{du}{u\sqrt{u^2 - a^2}} = \dfrac{1}{a} arc\sec \dfrac{|u|}{a} + C$

<u>Definition of a Critical Number</u>:
Let f be defined at c. If $f'(c) = 0$ or if f' is undefined at c, then c is a critical number of f.

<u>First Derivative Test</u>:
Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

1) If $f'(x)$ changes from negative to positive at c, then $f(c)$ is a **relative minimum** of f.
2) If $f'(x)$ changes from positive to negative at c, then $f(c)$ is a **relative maximum** of f.

<u>Second Derivative Test</u>:
Let f be a function such that the second derivative of f exists on an open interval containing c.

1) If $f'(c) = 0$ and $f''(c) > 0$, then $f(c)$ is a relative minimum.
2) If $f'(c) = 0$ and $f''(c) < 0$, then $f(c)$ is a relative maximum.

<u>Definition of Concavity</u>:
Let f be differentiable on an open interval I. The graph of f is **concave upward** on I if f' is increasing on the interval and **concave downward** on I if f' is decreasing on the interval.

<u>Test for Concavity</u>:
Let f be a function whose second derivative exists on an open interval I.

1) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward in I.
2) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward in I.

<u>Definition of an Inflection Point</u>:
A function f has an inflection point at $(c, f(c))$

1) if $f''(c) = 0$ or $f''(c)$ does not exist <u>and</u>
2) if f'' changes sign from positive to negative or negative to positive at $x = c$

   <u>OR</u> if $f'(x)$ changes from increasing to decreasing or decreasing to increasing at $x = c$.

First Fundamental Theorem of Calculus: $\int_a^b f'(x)\,dx = f(b) - f(a)$

Second Fundamental Theorem of Calculus: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

Chain Rule Version: $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)$

Average value of $f(x)$ on $[a, b]$: $f_{AVE} = \frac{1}{b-a}\int_a^b f(x)dx$

Volume around a horizontal axis by discs: $V = \pi\int_a^b [r(x)]^2\,dx$

Volume around a horizontal axis by washers: $V = \pi\int_a^b ([R(x)]^2 - [r(x)]^2)dx$

Volume by cross sections taken perpendicular to the x-axis: $V = \int_a^b A(x)dx$

If an object moves along a straight line with position function $s(t)$, then its

Velocity is $v(t) = s'(t)$

Speed = $|v(t)|$

Acceleration is $a(t) = v'(t) = s''(t)$

Displacement (change in position) from $x = a$ to $x = b$ is Displacement = $\int_a^b v(t)\,dt$

Total Distance traveled from $x = a$ to $x = b$ is Total Distance = $\int_a^b |v(t)\,dt|$

or Total Distance = $\left|\int_a^c v(t)\,dt\right| + \left|\int_c^b v(t)\,dt\right|$, where $v(t)$ changes sign at $x = c$.

CALCULUS BC ONLY

Differential equation for logistic growth: $\frac{dP}{dt} = kP(L-P)$, where $L = \lim_{t\to\infty} P(t)$

Integration by parts: $\int u\,dv = uv - \int v\,du$

Length of arc for functions: $s = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$

If an object moves along a curve, its

Position vector = $(x(t),\ y(t))$

Velocity vector = $(x'(t),\ y'(t))$

Acceleration vector = $(x''(t),\ y''(t))$

Speed (or magnitude of velocity vector) = $|v(t)| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

Distance traveled from $t = a$ to $t = b$ (or length of arc) is $s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

In polar curves, $x = r\cos\theta$ and $y = r\sin\theta$

Slope of polar curve: $\dfrac{dy}{dx} = \dfrac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$

Area inside a polar curve: $A = \dfrac{1}{2}\int_a^b r^2 d\theta$

Definition of a Taylor polynomial:

If f has n derivatives at c, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the **nth Taylor polynomial for f at c.**

Lagrange Error Bound for a Taylor Polynomial (or Taylor's Theorem Remainder):

If f is differentiable through order $n+1$ in an interval I containing c, then for each x in I, there exists z between x and c such that

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n + R_n(x),$$

where $R_n(x) = \dfrac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}$. $R_n(x)$ gives a bound for the size of the error found by the nth degree Taylor polynomial.

The remainder represents the difference between the function and the polynomial. That is,

$$|R_n| = |f(x) - P_n(x)| .$$

Alternating Series Remainder:

If a series has terms that alternate, decrease in absolute value, and have a limit of 0 (so that the series converges by the Alternating Series Test), then the absolute value of the remainder R_n involved in approximating the sum S by S_n is less than the first neglected term. That is,

$$|R_n| = |S - S_n| < a_{n+1} .$$

Maclaurin series that you must know:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

Summary of Tests for Series

Test	Series	Condition(s) of Convergence	Condition(s) of Divergence	Comment
nth-Term	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	This test cannot be used to show convergence.
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\lvert r\rvert < 1$	$\lvert r\rvert \geq 1$	Sum: $S = \frac{a}{1-r}$
Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{n\to\infty} b_n = L$		Sum: $S = b_1 - L$
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1}a_n$	$0 < a_{n+1} \leq a_n$ and $\lim_{n\to\infty} a_n = 0$		Remainder: $\lvert R_N\rvert \leq a_{N+1}$
Integral (f is continuous, positive, and decreasing)	$\sum_{n=1}^{\infty} a_n$, $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	Remainder: $0 < R_N < \int_N^{\infty} f(x)\,dx$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} > 1$	Test is inconclusive if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = 1$.
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert < 1$	$\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert > 1$	Test is inconclusive if $\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert = 1$.
Direct Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Limit Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	

Taylor's Theorem: If a function f is differentiable through order $n+1$ in an interval I containing c, then for each x in I, there exists a number z between x and c such that

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \ldots + \frac{f^{(n)}(c)(x-c)^n}{n!} + R_n(x)$$

where $\lvert R_n(x)\rvert = \left\lvert \frac{f^{(n+1)}(z)(x-c)^{n+1}}{(n+1)!} \right\rvert$. $\lvert R_n(x)\rvert = \left\lvert \frac{f^{(n+1)}(z)(x-c)^{n+1}}{(n+1)!} \right\rvert$ is called the Lagrange Form of the Remainder or the Taylor's Theorem Remainder.

Alternating Series Remainder: For a convergent alternating series whose terms are decreasing, the absolute value of the remainder R_n involved in approximating the sum S by S_n is less than or equal to the first neglected (truncated) term. In other words, $\lvert S - S_n\rvert = \lvert R_n\rvert \leq a_{n+1}$.