AP Calculus AB Summer Packet

Name:____________________________

In preparation for a successful year in AP Calculus AB, complete this packet, showing all work. This packet will be collected the second day of class. Mastery of these topics is important in this class. If you are having difficulty with a topic, use your notes, the internet and other resources to learn more.
****Round answers to the nearest .001 except where exact answers are required.****
**Answers are on the back.

1. The graphs of these functions will be used routinely in the course. Practice these graphs until you can demonstrate them from memory. Grid paper is attached at the end of the document.

$a.\ y = x$	$b.\ y = \sqrt[3]{x}$	$c.\ y = e^x$	$d.\ y = \cos x$	$e.\ y = x^2$	$f.\ y = \frac{1}{x}$
$g.\ y = \frac{1}{x^2}$	$h.\ y = \tan x$	$i.\ y = x^3$	$j.\ y = \lvert x \rvert$	$k.\ y = [x]$	$l.\ y = \cot x$
$m.\ y = \sqrt{x}$	$n.\ y = \ln x$	$o.\ y = \sin x$	$p.\ y = \sec x$	$q.\ y = \csc x$	$r.\ y = \sqrt{9 - x^2}$

*** Now go back and classify each function above as even, odd or neither.**

2. There exists a function $y = f(x)$, such that $f(-.4) = -.864$ *and* $f(-.6) = -1.416$.

   a. If *f(x)* is an even function, find *f*(.4) = _______

   b. If *f(x)* is an odd function, find *f*(.6) = _______

3. a. Given $g(x) = -x^2 + 3$, write the equation of the line through $g(-2)$ and $g(3)$.

   b. Write the equation of the line that passes through the *x*-intercept of $2(3x - y) = -3$ and is normal $(\perp)$ to this line.

4. Solve for *x*. **Give exact answers, not decimal.**

   a. $e^{2x} = 5$

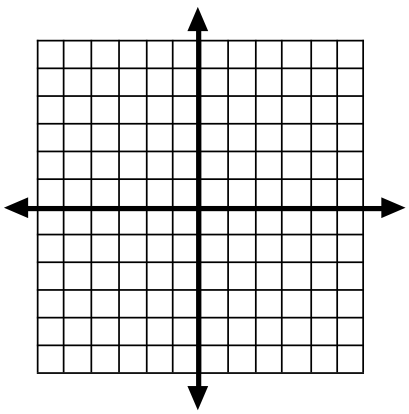
   b. $2 + \cos^2 x = 3\sin^2 x \quad 0 \le x < 2\pi$

   c. $ln(x + 3) = 4$

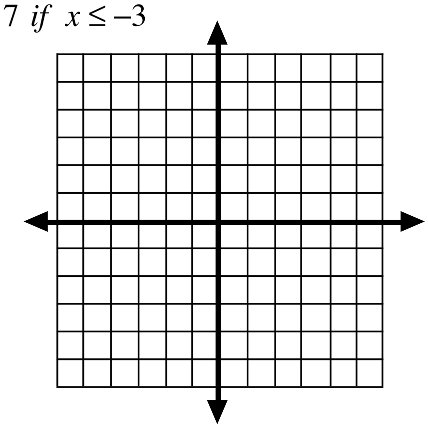
5. Given $f(x) = x^2 + 1$, find $\dfrac{f(x+h) - f(x)}{h}$.

6. Graph:

a. $f(x) = 2|x+3| - 1$



b. $g(x) = \begin{cases} 2x+5 & \text{if } x > -3 \\ -2x-7 & \text{if } x \le -3 \end{cases}$



7. Find the domain and range for the following functions.

a. $f(x) = \begin{cases} \sqrt{9-x^2} & \text{if } 0 \le x < 3 \\ e^x & \text{if } x < 0 \end{cases}$

b. $f(x) = \dfrac{\sin x}{x}$ (Use your calculator)

c. $f(x) = \dfrac{x^2 + x - 12}{x - 3}$

d. $f(x) = \dfrac{x^2 + x - 12}{(x-3)(x+5)}$

e. $f(x) = ln(x-3)$

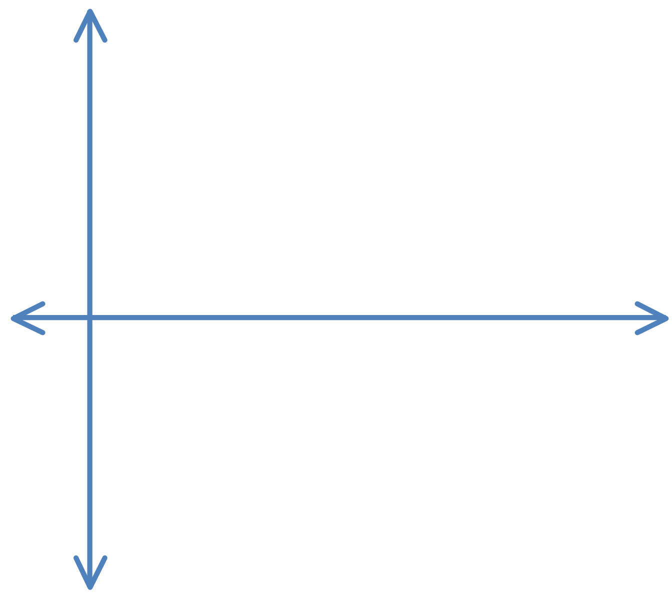
f. How does the graph of $g(x) = x + 4$ compare to the graph of the function in part c. How are their domain/ranges the same or different? What does $g(3)$ have to do with the graph of $f(x)$?

8. Expand using Log Properties:

a. $ln\left(x^2\sqrt{y}\right)$

b. $log_3\left(\frac{x+3}{x^2}\right)$

9. Given: $f\left(x\right)=4\sin\left(\frac{\pi}{2}x\right)-2$



a. Amplitude _______________

b. Period ___________________

c. Graph one period starting at $x=0$..

d. Write the equation of the line through the maximum point and minimum point of the period graphed.

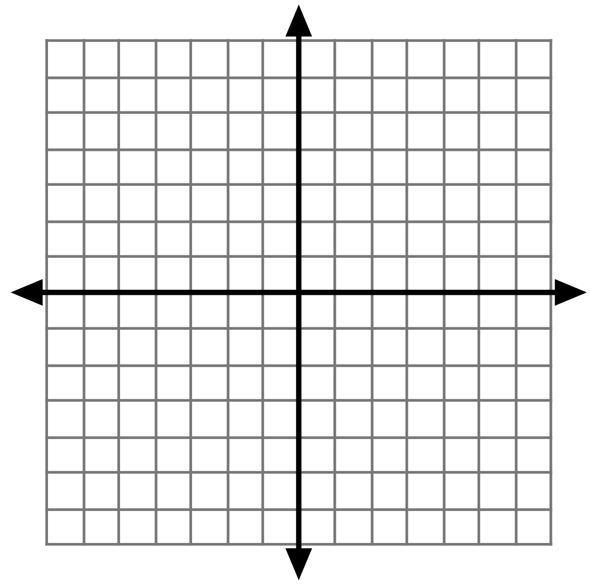
10. Use a division technique (synthetic, long division, or the box method) for dividing the following problems.

a. $\frac{x^3-6x-20}{x+5}$

b. $\frac{x^3-3x^2+4x-9}{x^2+3}$

c. $\frac{x^4+x\ \ -4}{x^2+2}$

11. Given: $y=\frac{2(x+3)(x-2)}{(x-3)(x+3)}$



Identify:

a. any holes ___________________________

b. any vertical asymptotes ___________________

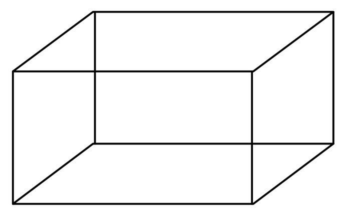
c. any horizontal asymptotes_________________

d. x - intercepts______________________________

e. y - intercepts ______________________________

f. Graph.

12. A **closed** box with a square base of side (x) and height (y) has a surface area of 100 sq. ft. **Round to three decimal places.**



a. Express the volume of the box as a function of x. V(x) =

b. Domain of V(x) = ___________________ Range of V(x) = ____________________

13. Simplify and include restrictions:

a. $\frac{x^3-125}{x-5}$

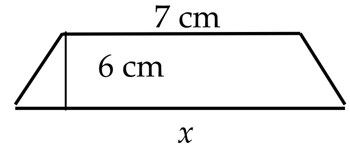
b. $\frac{|x|}{\sqrt{x^2}}$ (Hint: graph)

c. $\frac{\frac{1}{x+3}-\frac{1}{3}}{x}$

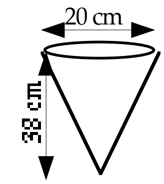
d. Rationalize the denominator and simplify:

$$\frac{x}{2-\sqrt{4-x}}$$

14. The area of the trapezoid at right is is $30\sqrt{2}\ \ cm^2$. Find the exact value of x.



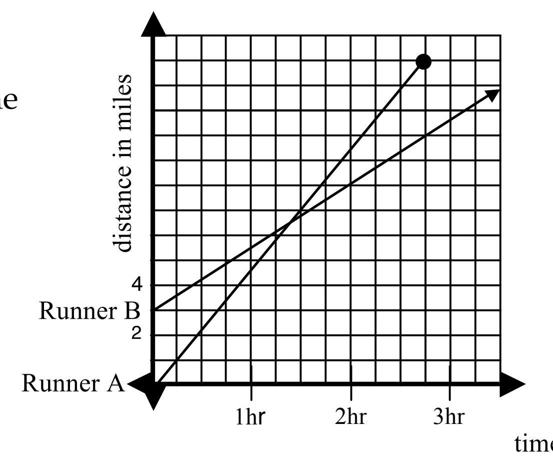
15. Water is poured into a conical container with a diameter of 20 cm and a height of 38 cm. If the volume of a cone is one-third the volume of a cylinder, write an equation for the volume of water as a function of the height of the water in the container.



16. Given $f(x) = 3x - 5$ and $g(x) = 2(x-4)^2 + 2x$, calculate the following.

a.) $g(f(x))$ b.) $f(g(3))$ c.) $f^{-1}(x)$

17. The graph below shows the time versus distance plot for two runners (Runner A and Runner B).



a. Which runner has a head start? How far did Runner A go by the end of the race?

b. Which runner ran fastest? How much faster?

c. How much longer did it take for the loser to cross the finish line?

d. How far did the winner win by?

Important Trig Identities

$\sin^2 x + \cos^2 x = 1$	$\tan^2 x + 1 = \sec^2 x$	$1 + \cot^2 x = \csc^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\cos 2x = 2\cos^2 x - 1$	$\cos 2x = 1 - 2\sin^2 x$
$\sin 2x = 2\sin x \cos x$	$\tan x = \dfrac{\sin x}{\cos x}$	$\cot x = \dfrac{\cos x}{\sin x}$

18. Evaluate:

a. $sin\left(\dfrac{2\pi}{3}\right)$ b. $cos\left(-\dfrac{5\pi}{4}\right)$ c. $tan\left(\dfrac{11\pi}{6}\right)$ d. $csc\left(\dfrac{7\pi}{6}\right)$

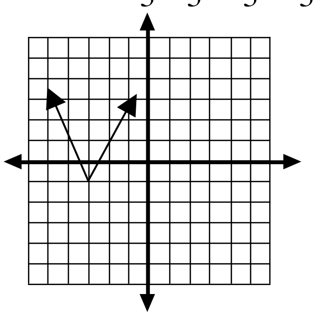
19. Simplify: a. $\frac{1-cos^2 x}{sec^2 x - 1}$ b. $sec\,x \cdot sin\,2x$

Answers: 1. Even: d, e, g, j, p, r Odd: a, b, f, h, i, l, o, q Neither: c, k, m, n

2a. $-.864$ 2b. 1.416 3a. $y=-x-3$ 3b. $y=-\frac{1}{3}x-\frac{1}{6}$

4a. $x=\frac{\ln 5}{2}$ 4b. $\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}$ 4c. $x=e^4-3$ 5. $2x\ +h$

6. a. and b.



7a. $D:(-\infty,3)\ \ R:(0,3]$

7b. $D:\mathbb{R}, x\neq 0\ \ R:[-.21723,1)$

7c. $D:\mathbb{R}, x\neq 3\ \ R:\mathbb{R}, y\neq 7$

7d. $D:\mathbb{R}, x\neq 3,-5\ \ R:\mathbb{R}, y\neq 7/8$

7e. $D:(3,\infty)\ \ R:(-\infty,\infty)$

7f. Graph are the same except *g(x)* does not have a hole at (3,7) and *f(x)* does.

8a. $2\,ln\,x+\frac{1}{2}ln\,y$ 8b. $log_3(x+3)-2\,log_3 x$ 9a. Amp=4 9b. Period=4 9d. $y=-4x+6$

10a. $x^2-5x+19-\frac{115}{x+5}$ 10b. $x-3+\frac{x}{x^2+3}$ 10c. $x^2-2+\frac{x}{x^2+2}$

11a. $\left(-3,\frac{5}{3}\right)$ 11b. $x=3$ 11c. $y=2$ 11d. $(2,0)$ 11e. $\left(0,\frac{4}{3}\right)$

12a. $V(x)=\frac{1}{4}(100x-2x^3)$ 12b. $(0,7.071)$ 12c. $(0,68.041]$

13a. $x^2+5x+25,\ x\neq 5$ 13b. $1,\ x\neq 0$ 13c. $\frac{-1}{3(x+3)},\ x\neq 0,-3$ 13d. $2+\sqrt{4-x},\ x\leq 4$

14. $x=10\sqrt{2}-7$ 15. $V(h)=\frac{25\pi h^3}{1083}$ 16a. $18x^2-102x+152$ 16b. 19 16c. $f^{-1}(x)=\frac{x+5}{3}$

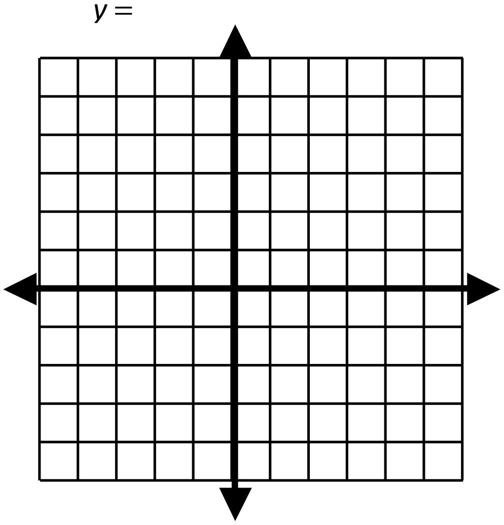
17a. Runner B / the race was 13 miles for Runner A 17b. Runner A runs $\frac{24}{11}$ *mph* faster than Runner B

17c. 70.714 minutes 17d. 3 miles

18a. $\frac{\sqrt{3}}{2}$ 18b. $-\frac{\sqrt{2}}{2}$ 18c. $-\frac{\sqrt{3}}{3}$ 18d. -2 19a. $cos^2 x$ 19b. $2\,sin\,x$

Grids for Problem #1.

$y =$



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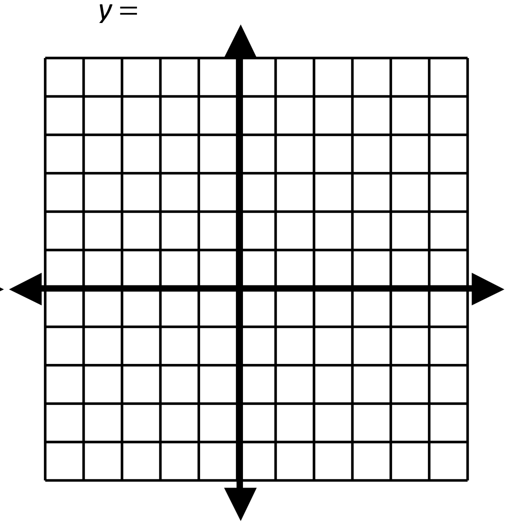

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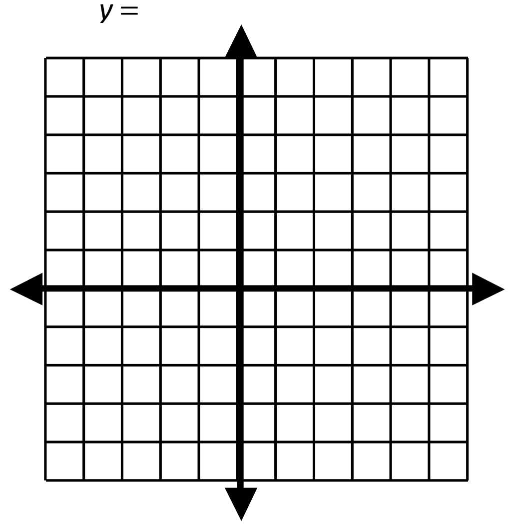

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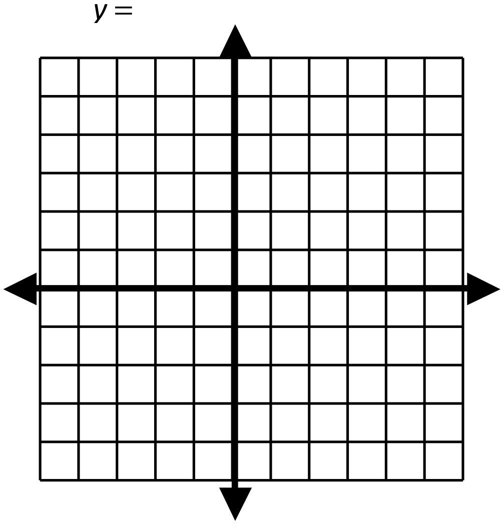

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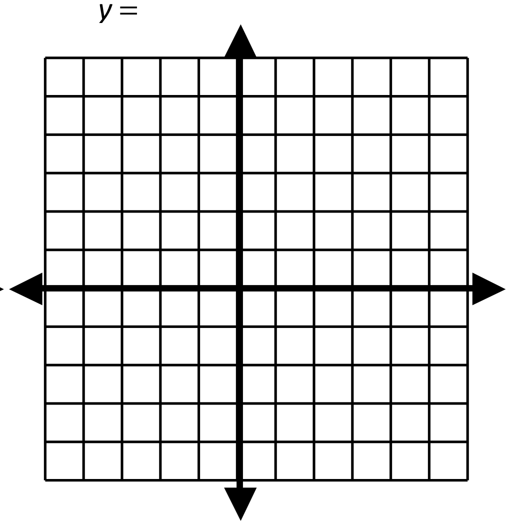

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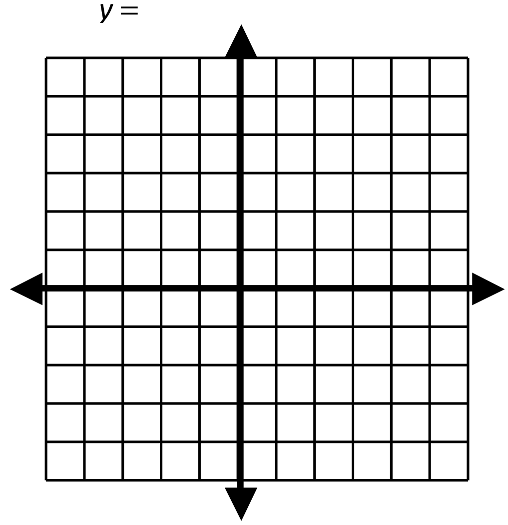

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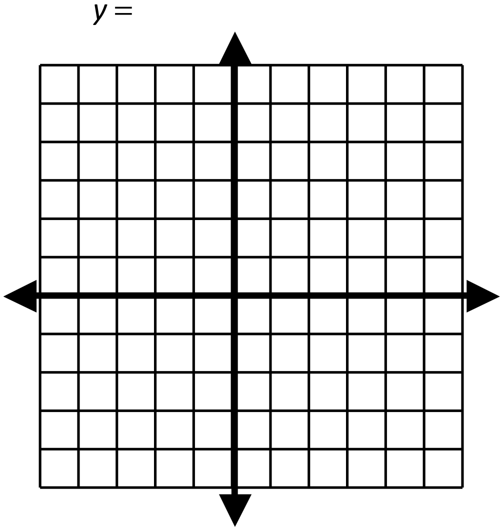

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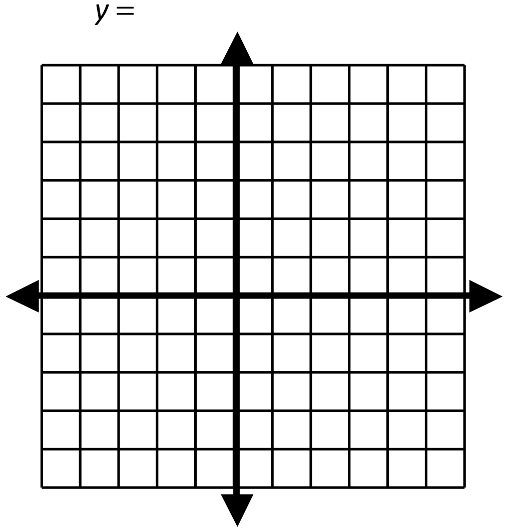

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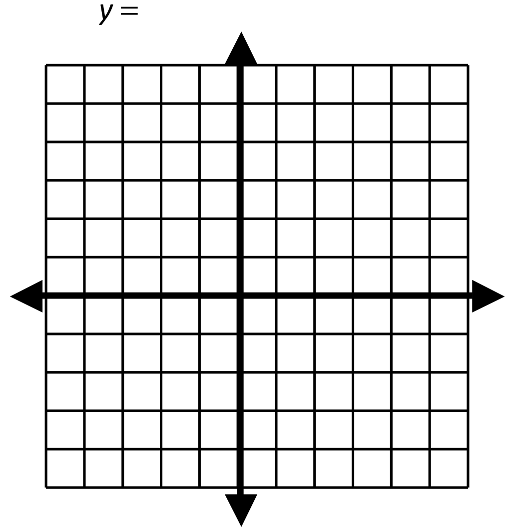

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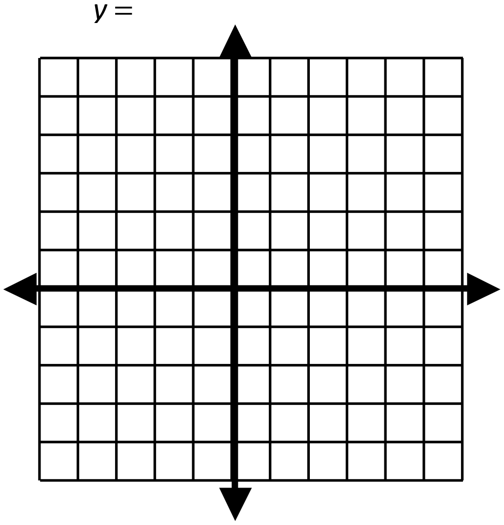

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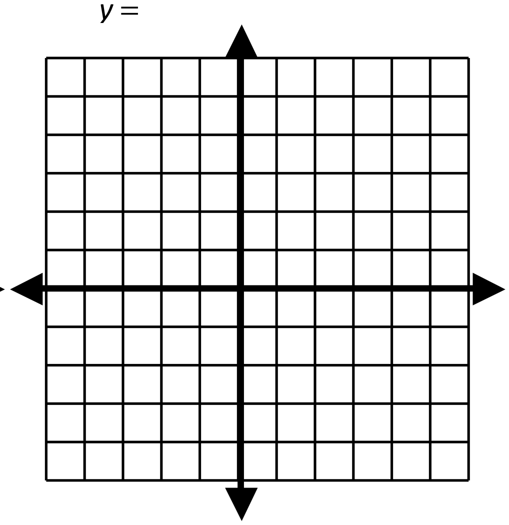

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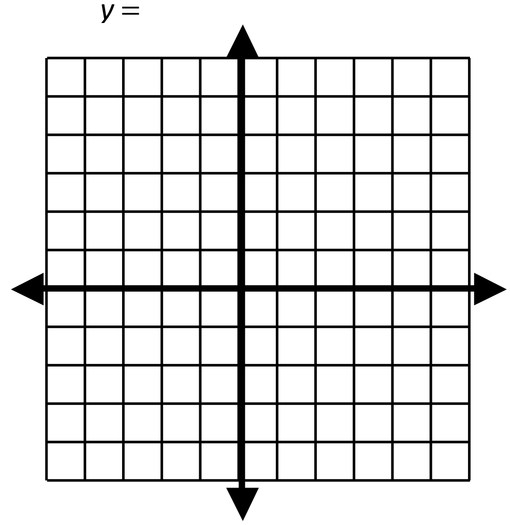

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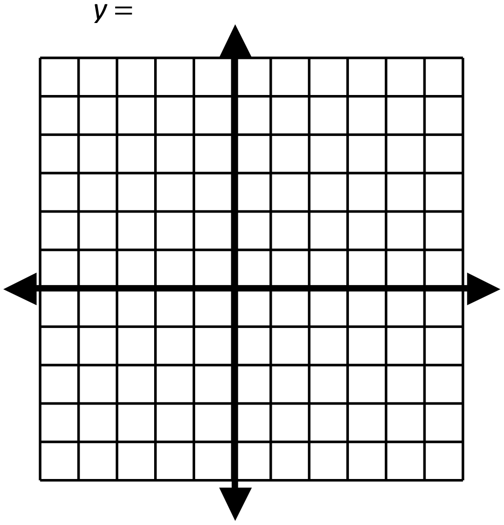

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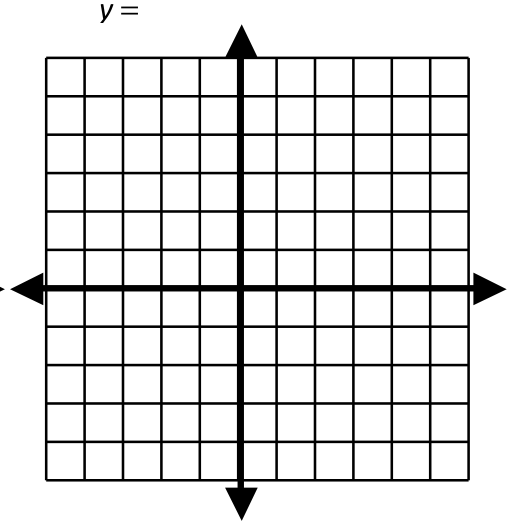

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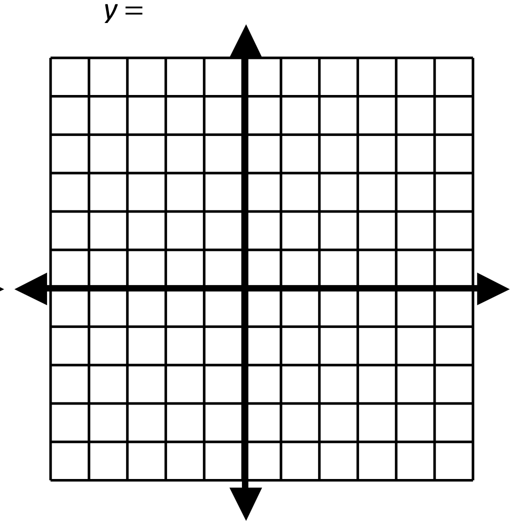

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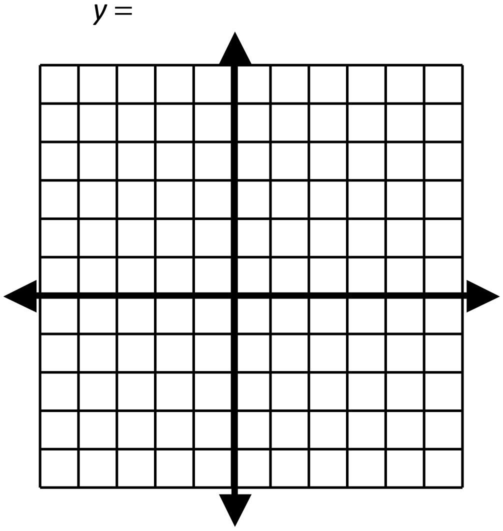

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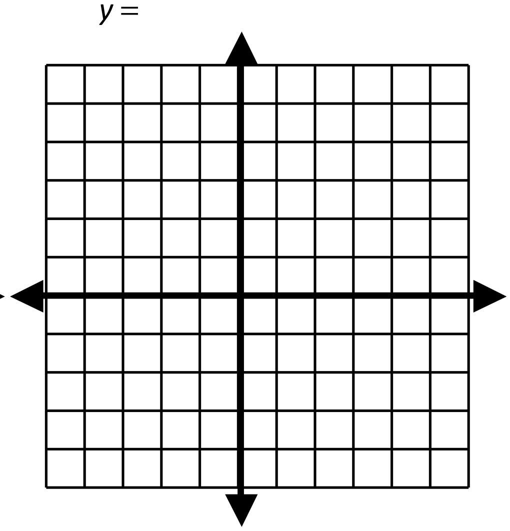

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