AP CALCULUS BC **CHAPTER 2 REVIEW**

## CHAPTER 2 TEST TOPICS

**In order to master the TEST on Chapter 2, you should be familiar with . . .**

- The formal definition of the derivative
- Applying the above definition
- Tangent vs. secant lines
- Relationships between the graph of a function and its derivative
- Average vs. instantaneous velocity
- Differentiability and continuity
- Basic rules of differentiation
- Basic derivative notation
- The product, quotient and chain rules – **oh my!**
- Any combination of the above three including multiple applications of the chain rule
- Implicit differentiation
- Solving related rate problems
- **All previous mathematical knowledge**

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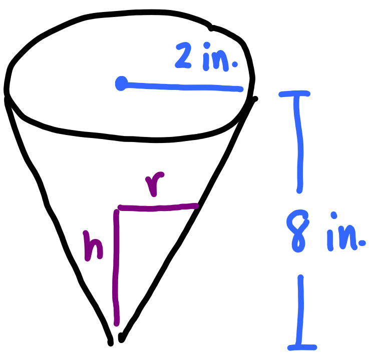
**PART I – GRAPHING CALCULATOR <u>MAY BE USED</u>.**
**IF ROUNDING, THREE DECIMAL PLACES!**

1. Nicole just loves drinking chocolate milk out of her special cone cup ($V_{cone} = \frac{\pi r^2 h}{3}$) – whose radius is 2 inches and whose height is 8 inches. Nicole pours milk into her cone cup at the constant rate of 2.5 in$^3$/sec.

   a. The total amount of milk that this cone cup can hold is 33.510 in$^3$. Find the radius and height of the *milk* when the cone cup is *half full* – i.e. the volume is 16.755 in$^3$.
   b. How fast is the height AND radius of the milk changing at the instant the cone cup is half full of milk?
   c. The milk leaves a ring of chocolate goodness inside the cup as it is poured. How fast is this milk ring moving up the sides of the cone cup at the instant the cone is half full?

a)




$$\frac{dV}{dt} = 2.5 \text{ in}^3/\text{sec}$$

$$\frac{r}{2} = \frac{h}{8}$$

$$h = 4r$$

$$V = \frac{\pi}{3} r^2 h = \frac{\pi}{3} r^2 (4r) = \frac{4}{3}\pi r^3$$

$$16.755 = \frac{4}{3}\pi r^3 \Rightarrow \boxed{\begin{array}{l} r \approx 1.587 \text{ in.} \\ h \approx 0.397 \text{ in.} \end{array}}$$

b) $$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$2.5 = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{2.5}{4\pi r^2}$$

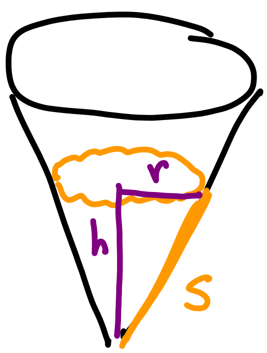
$$\boxed{\frac{dr}{dt} \approx 0.079 \text{ in/sec}}$$

When cup is half full.

$$\frac{dh}{dt} = 4\frac{dr}{dt}$$

$$\boxed{\frac{dh}{dt} \approx 0.316 \text{ in/sec}}$$

c)




$$s^2 = r^2 + h^2$$

$$2s\frac{ds}{dt} = 2r\frac{dr}{dt} + 2h\frac{dh}{dt}$$

$$\frac{ds}{dt} = \frac{r\frac{dr}{dt} + h\frac{dh}{dt}}{s}$$

$$\boxed{\frac{ds}{dt} \approx 0.153 \text{ in/sec}}$$

When the cup is half full.

$s = \sqrt{r^2 + h^2}$

$s \approx 1.636$

2. The position equation of a particle moving along the x-axis is given by $x(t) = t^2 - 3t + 2$ for $t \geq 0$.

   a. Find the velocity equation of the particle.
   b. When is the particle moving to the left? To the right?
   c. Find the position of the particle when the velocity is 0.
   d. Find the speed of the particle when the position is 0.

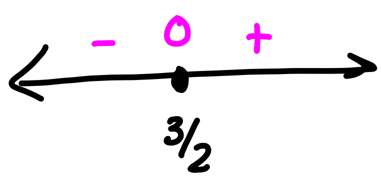
a) $V(t) = S'(t) = 2t - 3$ $\boxed{V(t) = 2t-3}$

b) Left $(V(t) < 0)$

$0 \leq t < 3/2$

$2t - 3 = 0$

$t = 3/2$




Right $(V(t) > 0)$

$t > 3/2$

c) $V(t) = 0 \Rightarrow t = 3/2$

$S(3/2) = (3/2)^2 - 3(3/2) + 2 = \frac{9}{4} - \frac{9}{2} + 2 = \frac{9 - 18 + 8}{4}$

$\boxed{S(3/2) = -1/4}$

d) $S(t) = 0$

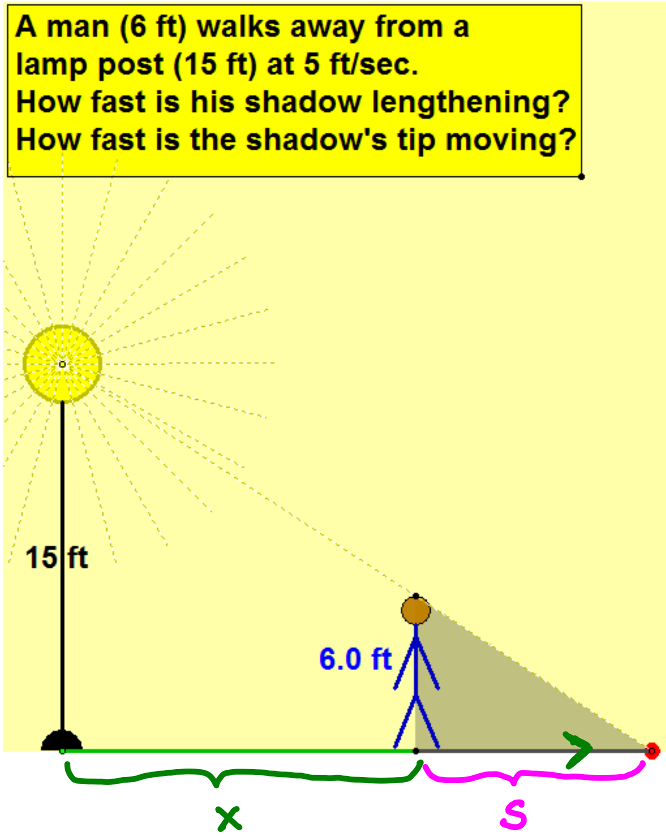
$t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

$t = 2 \quad t = 1$

$\left.\begin{array}{l} V(2) = 2(2) - 3 = 1 \\ V(1) = 2(1) - 3 = -1 \end{array}\right\}$ $\boxed{\text{At both instances the speed equals 1.}}$

3.




$$\frac{dx}{dt} = 5\,ft/sec$$

a) $\frac{ds}{dt}$ – ?

$$\frac{s}{6} = \frac{x+s}{15}$$

$$15s = 6x + 6s$$

$$9s = 6x$$

$$s = \frac{2}{3}x \qquad \frac{ds}{dt} = \frac{2}{3}\frac{dx}{dt}$$

$$\boxed{\frac{ds}{dt} = \frac{10}{3}\ ft/sec}$$

b) $\frac{d}{dt}(x+s)$ – ?

$$\frac{d}{dt}(x+s) = \frac{dx}{dt} + \frac{ds}{dt}$$

$$= 5 + \frac{10}{3} = \frac{25}{3}$$

$$\boxed{\frac{d}{dt}(x+s) = \frac{25}{3}\ ft/sec}$$

**PART I – GRAPHING CALCULATOR MAY NOT BE USED.**

4. Find the derivative of the following four functions:

a. $f(x) = (\sin(x) + 3x^2)(\tan(x) + 2x)$

b. $f(x) = \dfrac{3x - 2}{\cos x}$

cos (x)

c. $f(x) = x\tan(x^2)$

d. $f(x) = \dfrac{1}{\sqrt{x+3}}$

a) $f'(x) = (\cos x + 6x)(\tan x + 2x) + (\sin x + 3x^2)(\sec^2 x + 2)$

$f'(x) = \sin x + 2x\cos x + 6x\tan x + 12x^2 + \sin x\sec^2 x + 2\sin x + 3x^2\sec^2 x + 6x^2$

$\boxed{f'(x) = 3\sin x + 2x\cos x + 6x\tan x + 18x^2 + \sin x\sec^2 x + 3x^2\sec^2 x}$

b) $f'(x) = \dfrac{3\cos x + \sin x(3x-2)}{\cos^2 x} = \boxed{\dfrac{3\cos x - 2\sin x + 3x\sin x}{\cos^2 x}}$

c) $f'(x) = \tan(x^2) + x\sec^2(x^2)\cdot 2x = \boxed{\tan(x^2) + 2x^2\sec^2(x^2)}$

d) $f(x) = (x+3)^{-1/2}$

$f'(x) = -\frac{1}{2}(x+3)^{-3/2} = \boxed{-\dfrac{1}{2\sqrt[3]{(x+3)^2}}}$

5. Consider $x^2 - 3y^2 = 10$

a. Find the derivative implicitly

b. Find the second derivative

a) $2x - 6y \cdot y' = 0$

$y' = \frac{2x}{6y}$

$\boxed{\frac{dy}{dx} = \frac{x}{3y}}$

b) $y'' = \frac{3y - x \cdot 3y'}{9y^2}$

$= \frac{y - xy'}{3y^2} = \frac{y - x \cdot \frac{x}{3y}}{3y^2}$

$=$

6. Let *f* be the function given by $f(x) = \tan(x)$ and let *g* be the function given by $g(x) = x^2$. At what value of *x* in the interval $0 \le x \le \pi$ do the graphs of *f* and *g* have parallel tangent lines?

A. ~~0~~ B. ~~0.660~~ $\boxed{\text{C. } 2.083}$ D. 2.194 E. 2.207

$$\left.\begin{aligned} f'(x) &= \sec^2 x \\ g'(x) &= 2x \end{aligned}\right\} \quad 2x = \sec^2 x$$

$\sec(0) = 1$
$2(0) = 0$

$(\sec(0.660))^2 \approx 1.602$
$2(0.660) = 1.32$

$(\sec(2.083))^2 \approx 4.163$
$2(2.083) \approx 4.166$

7. What do the following limits find in terms of derivatives?

a. $\lim_{h \to 0} \frac{\cos(3x+3h) - \cos(3x)}{h}$

b. $\lim_{h \to 0} \frac{(3+h)^3 - 3^3}{h}$

a) $\lim_{h \to 0} \frac{\cos(3(x+h)) - \cos(3x)}{h} = \boxed{\frac{d}{dx}\cos(3x)}$

b) $\lim_{h \to 0} \frac{(3+h)^3 - 3^3}{h} = \frac{d}{dx} x^3 \Big|_{x=3}$

8. Give an example of a function that is continuous at x = 3, but not differentiable at x = 3.

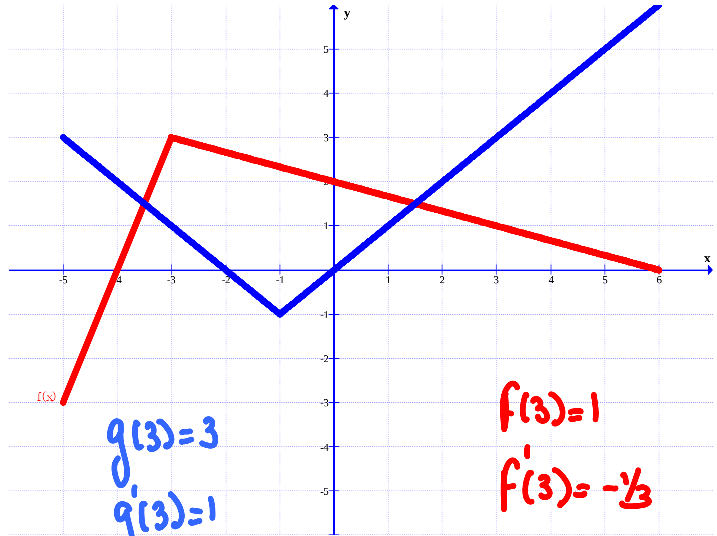
$g(x) = |x-3|$

9. Find the second derivative of y = sin (3x + 2)

$y' = 3\cos(3x+2)$

$\boxed{y'' = -9\sin(3x+2)}$

10. The functions *f(x)* and *g(x)* are piecewise linear functions whose graphs are shown above. If $h(x) = f(x)g(x)$, then $h'(3) =$




$g(3) = 3$

$g'(3) = 1$

$f(3) = 1$

$f'(3) = -\frac{1}{3}$

A. $-\frac{8}{3}$

B. $-\frac{1}{3}$

C. $\frac{2}{3}$

D. $0$

E. $\frac{8}{3}$

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

$$h'(3) = f'(3)g(3) + f(3)g'(3)$$

$$h'(3) = -\frac{1}{3}(3) + 1(1) = -1 + 1 = 0$$

$$\boxed{h'(3) = 0}$$

11. Let $f(t)=\frac{1}{t}$ for $t>0$. For what value of $t$ is $f'(t)$ equal to the average rate of change of $f$ on the closed interval [a, b]?

A. $-\sqrt{ab}$ B. $\boxed{\sqrt{ab}}$ C. $\frac{1}{\sqrt{ab}}$ D. $-\frac{1}{\sqrt{ab}}$ E. $\sqrt{\frac{1}{2}\left(\frac{1}{b}-\frac{1}{a}\right)}$

$$f'(t)=-\frac{1}{t^2}$$

$$\frac{\frac{1}{b}-\frac{1}{a}}{b-a}=\frac{\frac{a-b}{ab}}{b-a}=-\frac{\cancel{(b-a)}}{ab\cancel{(b-a)}}=-\frac{1}{ab}$$

$$-\frac{1}{t^2}=-\frac{1}{ab}$$

$$t^2=ab$$

$$t=\sqrt{ab}$$