

Dear Calculus Student:

To be successful in AP Calculus BC, you must be proficient at every skill in this packet. This is a review of Limits, Derivatives, Sequences, and Series. Upon returning to school in the fall, you are expected to have completed this assignment. We will correct and review during the first week of class, and then you will take a test on this content during the second week of class. Please make sure to show all work for each problem to receive full credit.

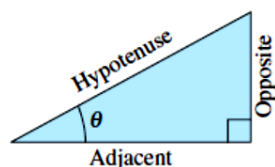
Calculators are NOT ALLOWED on this assignment. Furthermore, you are expected to have memorized the formulas on the sheet attached. While $\frac{2}{3}$ of the AP Calculus test is non-calculator, you are allowed to use a calculator on $\frac{1}{3}$ of the test. Therefore, it is highly recommended that you have a TI-84 when the school-year begins, since we will not have a full classroom set.

AP Calculus is a fast-paced course that is taught at the college-level. Therefore, you must maintain all pre-requisite skills. We will not have time to spend re-teaching the content in this packet. Make sure you are comfortable with this material.

Goodluck!

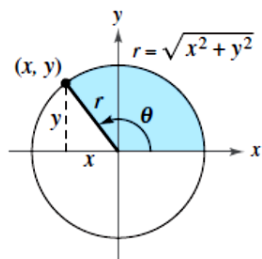
Formulas that must be memorized:

Right triangle definitions, where $0 < \theta < \pi/2$.

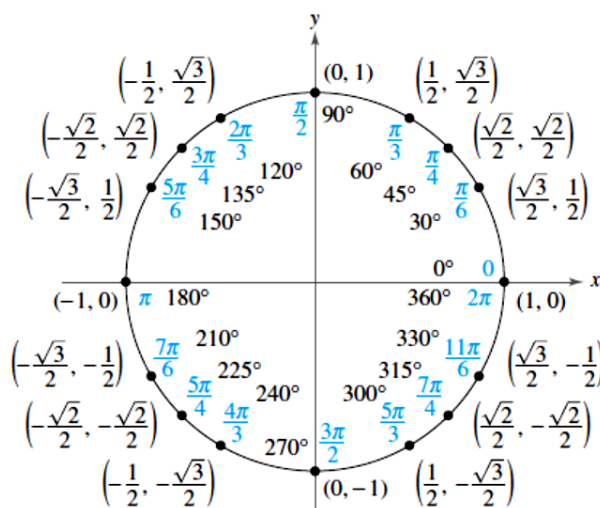


$$\begin{aligned}\sin \theta &= \frac{\text{opp}}{\text{hyp}} & \csc \theta &= \frac{\text{hyp}}{\text{opp}} \\ \cos \theta &= \frac{\text{adj}}{\text{hyp}} & \sec \theta &= \frac{\text{hyp}}{\text{adj}} \\ \tan \theta &= \frac{\text{opp}}{\text{adj}} & \cot \theta &= \frac{\text{adj}}{\text{opp}}\end{aligned}$$

Circular function definitions, where θ is any angle.



$$\begin{aligned}\sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y}\end{aligned}$$



Reciprocal Identities

$$\begin{aligned}\sin x &= \frac{1}{\csc x} & \sec x &= \frac{1}{\cos x} & \tan x &= \frac{1}{\cot x} \\ \csc x &= \frac{1}{\sin x} & \cos x &= \frac{1}{\sec x} & \cot x &= \frac{1}{\tan x}\end{aligned}$$

Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1 \\ 1 + \tan^2 x &= \sec^2 x & 1 + \cot^2 x &= \csc^2 x\end{aligned}$$

Geometric Formulas

Triangle	$A = \frac{1}{2}bh$
Equilateral Triangle	$A = \frac{\sqrt{3}}{4}s^2$
Circle	$A = \pi r^2, C = 2\pi r$
Sphere	$V = \frac{4}{3}\pi r^3, SA = 4\pi r^2$
Cylinder	$V = \pi r^2 h$
Cone	$V = \frac{\pi}{3}r^2 h$

Equations of lines

Slope-Intercept form $y = mx + b$

Point-Slope form $y - y_1 = m(x - x_1)$

Normal line is perpendicular to tangent line

Double-Angle Formulas

$$\begin{aligned}\sin 2u &= 2 \sin u \cos u \\ \cos 2u &= \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u \\ \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u}\end{aligned}$$

Exponents

$$\begin{aligned}a^0 &= 1, a \neq 0 \\ a^1 &= a \\ a^m \cdot a^n &= a^{m+n} \\ \frac{a^m}{a^n} &= a^{m-n} \\ (a^m)^n &= a^{mn} \\ a^{-m} &= \frac{1}{a^m}, a \neq 0 \\ a^{\frac{m}{n}} &= \sqrt[n]{a^m} = (\sqrt[n]{a})^m\end{aligned}$$

Logarithms

$$\begin{aligned}\ln 1 &= 0 \\ \ln e &= 1 \\ \ln mn &= \ln m + \ln n \\ \ln \frac{m}{n} &= \ln m - \ln n \\ \ln m^n &= n \ln m \\ e^{\ln x} &= x = \ln e^x \\ \log_b x &= \frac{\ln x}{\ln a}\end{aligned}$$

Conversion formula:

$$\begin{aligned}\log_b x &= y \\ &\Leftrightarrow \\ b^y &= x\end{aligned}$$

Radicals

If $x^2 = a$, then $x = \pm\sqrt{a}$

Section I: Limits

Strategies for finding limits.

- Try direct substitution.
- Try to factor & cancel, then use direct substitution.
- Use a limit formula.
- Graph or make a table.

Find each limit algebraically. Do NOT use a calculator.

1. $\lim_{x \rightarrow \infty} \frac{2x^2+5}{3x^2-4}$

2. $\lim_{x \rightarrow 0} \frac{\sqrt{3x+2}-\sqrt{2}}{x}$

3. $\lim_{x \rightarrow \frac{1}{3}} \frac{3x^2-7x+2}{-6x^2+5x-1}$

4. $\lim_{x \rightarrow -1} f(x) = \begin{cases} x^2 - 4x - 12, & x < -1 \\ x - 6, & x \geq -1 \end{cases}$

5. $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$ for $f(x) = -x^2 + 3x$

6. $\lim_{x \rightarrow 2.3} \llbracket x \rrbracket$

7. $\lim_{x \rightarrow \pi} \sin x$

8. Let f be the function defined by the following:

$$f(x) = \begin{cases} \cos x, & x < 0 \\ x^3, & 0 \leq x < 1 \\ 2 - x, & 1 \leq x < 2 \\ x - 4, & x \geq 2 \end{cases}$$

For what values of x is f NOT continuous?

A) 0 only B) 1 only C) 2 only D) 0 and 2 only E) 0, 1, and 2

9. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f . Which of the following statements must be true?

- A) $f(0) = 2$ B) $f(x) \neq 2$ for all $x \geq 0$
C) $f(2)$ is undefined D) $\lim_{x \rightarrow 2} f(x) = \infty$
E) $\lim_{x \rightarrow \infty} f(x) = 2$

True or False. If false, explain why or give an example that shows it false.

10. If $f(c) = L$, then $\lim_{x \rightarrow c} f(x) = L$.

11. If f is undefined at $x = c$, then the limit as x approaches c does not exist.

12. If f is continuous on $[2,3]$, $f(2) > 0$, and $f(3) < 0$, then there must be a point c in $(2,3)$ such that $f(c) = 0$.

Evaluate.

13. $\lim_{x \rightarrow 0} \frac{x + \tan x}{\sin x}$

14. $\lim_{x \rightarrow 0} \frac{\sin(5x)}{2x}$

15. $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{2 \sin^2 \theta}$

16. $\lim_{\theta \rightarrow 0} \theta \sec \theta$

17. $\lim_{\theta \rightarrow 0} \frac{\cos \theta \tan \theta}{\theta}$

18. $\lim_{x \rightarrow 0} \frac{3(1 - \cos x)}{x}$

19. $\lim_{x \rightarrow 0} \frac{\tan^2 x}{x}$

20. $\lim_{t \rightarrow 0} \frac{\sin 3t}{t}$

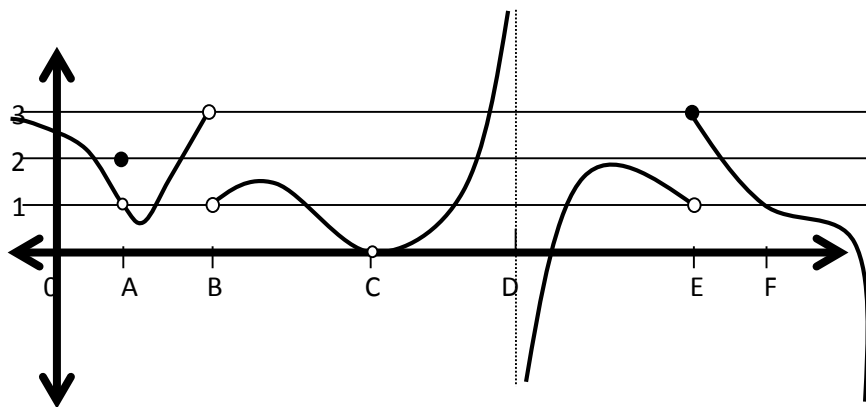
21. $\lim_{x \rightarrow 2} \frac{x-2}{|x-2|}$

22. $\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$

23. $\lim_{x \rightarrow 16} \frac{4 - \sqrt{x}}{x - 16}$

24. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{\sin x - \cos x}$

Use the graph to answer # 25-32.



25. $\lim_{x \rightarrow A^+} f(x)$

26. $\lim_{x \rightarrow B} f(x)$

27. $\lim_{x \rightarrow C} f(x)$

28. $\lim_{x \rightarrow D^-} f(x)$

29. $\lim_{x \rightarrow B^+} f(x)$

30. $\lim_{x \rightarrow E^-} f(x)$

31. $f(E)$

32. $f(A)$

33. $\lim_{x \rightarrow \infty} \frac{2x+5}{3x^2+1}$

34. $\lim_{x \rightarrow \infty} \frac{2x^3+5}{3x^2+1}$

35. $\lim_{x \rightarrow \infty} \sin x$

36. $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$ (Squeeze Thm)

37. $\lim_{x \rightarrow \infty} \frac{a-bx^4}{cx^4+x^2}$

38. $\lim_{x \rightarrow \infty} \left(5 - \frac{2}{x^2} \right)$

39. $\lim_{x \rightarrow \infty} \frac{3x-2}{\sqrt{2x^2+1}}$

40. $\lim_{x \rightarrow 1} \frac{\ln x}{x}$

Section II. Derivatives.

41. Use the limit definition of the derivative to find the derivative of $f(x) = -3x^2 + 5x + 4$.

42. Find $\lim_{h \rightarrow 0} \frac{\sin\left(\frac{3\pi}{4} + h\right) - \sin\left(\frac{3\pi}{4}\right)}{h}$

43. Given $f(x) = \begin{cases} x^3 - 8, & x < 2 \\ 3x^2 - 12, & x \geq 2 \end{cases}$ is f continuous at $x = 2$? Is it differentiable at $x = 2$?

44. Write an equation of the tangent line to the graph of $f(x) = -3x^2 + 5x - 2$ at $x = 3$.

Find each derivative.

45. $y = 2x - x^2 \sin x$

46. $y = 2x\sqrt{3x - 4}$

47. $y = \frac{2x^3 + 3x}{x^2 - 2}$

48. $y = 3(4 - 5x^2)^3$

49. $y = 5\cos^3(2x^4)$

50. $y = (2x - 4)^3(x^2 + 1)$

51. $y = \frac{2x^3 - 2\sqrt{x} + 1}{\sqrt[3]{x}}$

52. $y = \sin(\cos x^2)$

53. $y = \frac{1}{\sqrt{2x+7}}$

54. Find the points at which the function has horizontal and vertical tangent lines. $f(x) = \frac{2x-4}{x^2-3x}$

55. Find the average rate of change of $f(x) = 3x^2 + 4x - 2$ on the interval $[0, 3]$.

56. Find the instantaneous rate of change of $f(x) = 3x^2 + 4x - 2$ at $x = \frac{3}{2}$.

57. Find $\frac{dy}{dx}$. $x^3 - 2xy^2 + 3y^3 = 12$.

58. Write the equation of the tangent line to the curve $2y^2 - 3x^3 = 8$ at $(2, -4)$.

59. An object is dropped from a height of 6400 feet. $[s(t) = -16t^2 + v_0t + s_0]$

a) When does it hit the ground? _____

b) What is the velocity of the object when it hits? _____

60. The position equation for the movement of a particle is given by $s = (2t - 3)^2$ where s is measured in feet and t is measured in seconds. Find the following:

a) velocity at four seconds _____

b) acceleration at four seconds _____

c) Is the particle speeding up or slowing down when $t = 4$? _____

61. The radius of a circle is increasing at rate of 6 cm/min.
Find the rate of change of its area when the radius is 30 cm.

62. The radius of a cone is decreasing at a rate of 5 in/min.
The height is half the radius. Find the rate of change of the
volume when the radius is 40 in. $\left[V = \frac{1}{3} \pi r^2 h \right]$

63. Let f be the function defined by $f(x) = x^4 - 3x^2 + 2$.

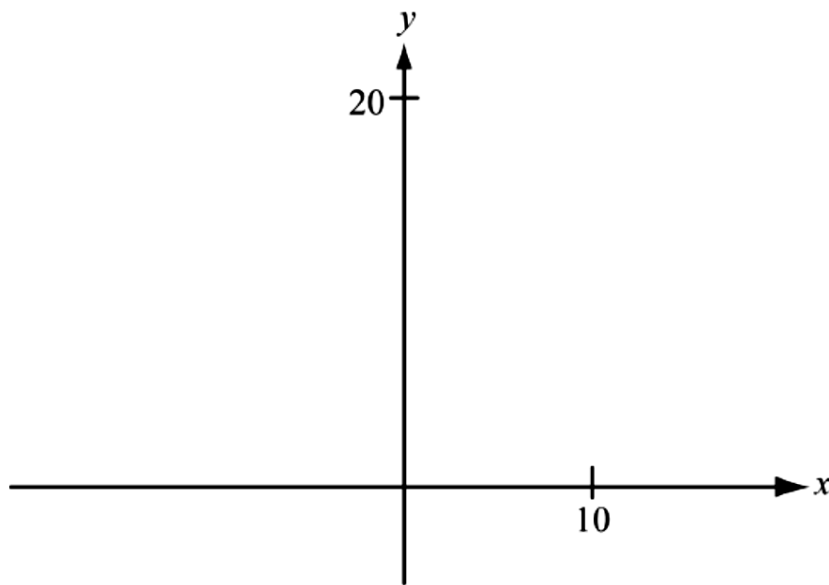
(a) Find the zeros of f .

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$.

(c) Find the x -coordinate of each point at which the line tangent to the graph of f is parallel to the line $y = -2x + 4$.

64. Let f be the function defined by $f(x) = 12x^{\frac{2}{3}} - 4x$.

- (a) Find the intervals on which f is increasing.
- (b) Find the x - and y -coordinates of all relative maximum points.
- (c) Find the x - and y -coordinates of all relative minimum points.
- (d) Find the intervals on which f is concave downward.
- (e) Using the information found in parts (a), (b), (c), and (d), sketch the graph of f on the axes provided.

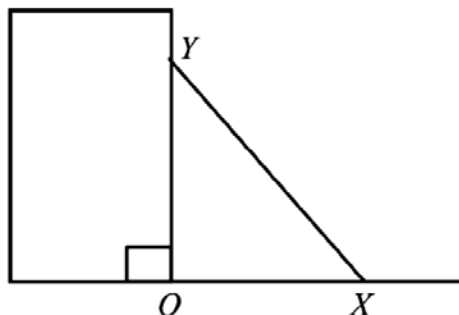


65.

Let f be the function defined by
$$f(x) = \begin{cases} 2x+1, & \text{for } x \leq 2 \\ \frac{1}{2}x^2 + k, & \text{for } x > 2 \end{cases}$$

- (a) For what value of k will f be continuous at $x = 2$? Justify your answer.
- (b) Using the value of k found in part (a), determine whether f is differentiable at $x = 2$. Use the definition of the derivative to justify your answer.
- (c) Let $k = 4$. Determine whether f is differentiable at $x = 2$. Justify your answer.

66.



A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall as shown in the figure. X is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second.

- Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.
- Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building.

The table below gives both the function values and derivative values for three functions.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
0	2	$\frac{1}{2}$	1	$\frac{1}{3}$	5	0
1	3	4	2	-0.75	-1	-1
2	5	$-\frac{1}{2}$	3	4	1	1
3	1	3	5	-3.2	0	-2
4	7	9	-4	7	4	$\frac{1}{2}$

Use the chart to complete each of the following. Show all work.

67. $F'(1)$ when $F(x) = 4f(x) + 3g(x)$

68. $F'(2)$ when $F(x) = f(x)g(x)$

69. $F'(0)$ when $F(x) = \frac{f(x)}{g(x)}$

70. $F'(2)$ when $F(x) = f(g(x))$

71. $F'(1)$ when $F(x) = f(x^2)[g(x)]^2$

72. $F'(2)$ when $F(x) = f(g(h(x)))$

AP-Style Multiple Choice Practice

Part 1: No Calculator

1. Suppose $f(x) = x^4 + ax^2$. What is the value of a if f has a local minimum at $x = 2$?

- A) -24 B) -8 C) -4 D) $-\frac{1}{2}$ E) $-\frac{1}{6}$

2. Suppose $g(x) = \begin{cases} x^3 + 3x^2 - 2x & \text{if } x \leq 0 \\ -x^2 & \text{if } x > 0 \end{cases}$. Then

- A) g is continuous and differentiable at $x = 0$
B) g is continuous but not differentiable at $x = 0$
C) g is not continuous but is differentiable at $x = 0$
D) g is not continuous and not differentiable at $x = 0$
E) Nothing can be said about the differentiability of g at $x = 0$

3. Which of the following is an equation for the tangent line to the graph of $f(x) = \sin(\cos x)$ at $x = \frac{\pi}{2}$?

- A) $y = x - \frac{\pi}{2}$ B) $y = -x + \frac{\pi}{2}$ C) $y = \frac{\pi}{2}$
D) $y = \sin 1 \left(x - \frac{\pi}{2} \right)$ E) $y = \cos 1 \left(x - \frac{\pi}{2} \right)$

4. If $f(x) = \sec(4x)$, then $f'\left(\frac{\pi}{16}\right) =$

- A) $4\sqrt{2}$ B) $\sqrt{2}$ C) 0 D) $\frac{1}{\sqrt{2}}$ E) $\frac{4}{\sqrt{2}}$

5. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

- A) $(-\infty, -1]$ B) $(-\infty, 0)$ C) $[-1, 0)$ D) $(0, \sqrt[3]{2}]$ E) $[\sqrt[3]{2}, \infty)$

6. The derivative of f is $x^4(x - 2)(x + 3)$. At how many points will the graph of f have a relative maximum?

- A) None B) One C) Two D) Three E) Four

7. What is the derivative of $f(t) = \sec \sqrt{t}$?

- A) $\tan^2 \sqrt{t}$ B) $\sec \frac{1}{2\sqrt{t}} \tan \frac{1}{2\sqrt{t}}$ C) $\frac{\sec \sqrt{t} \tan \sqrt{t}}{2\sqrt{t}}$ D) $\sec \sqrt{t} \tan \sqrt{t}$ E) None of these

8. Given $f(x) = 9 - \frac{14}{x}$, find all c in the interval $(2, 7)$ such that $f'(c)$ is parallel to the secant line as guaranteed by the Mean Value Theorem.

- A) $\frac{9}{2}$ B) $\frac{\sqrt{6}}{2}$ C) $\sqrt{14}$ D) $\frac{14}{9}$ E) None of these

9. Evaluate the $\lim_{x \rightarrow 0} \frac{x}{\sin 3x}$.

- A) 0 B) $\frac{1}{3}$ C) 1 D) 3 E) Nonexistent

10. Let $(x) = \sqrt{x}$. What is the equation of the tangent line to f at the point $(4, 2)$?

- A) $y = \frac{1}{4}x + 1$ B) $y = -\frac{1}{2}x + 3$ C) $y = \frac{1}{2}x$ D) $y = 2x - 6$ E) None of these

11. If $\lim_{x \rightarrow c} f(x) = -6$ and $\lim_{x \rightarrow c} g(x) = 3$, find $\lim_{x \rightarrow c} ([f(x)]^2 - 2f(x)g(x) + [g(x)]^2)$.

- A) -9 B) 45 C) 63 D) 81 E) None of these

12. Which of the following is not true given $f(x) = 1 + x^{2/3}$?

- A) f is continuous for all real numbers
B) f has a minimum at $x = 0$
C) f is increasing for $x > 0$
D) $f'(x)$ exists for all x
E) $f''(x)$ is negative for $x > 0$

13. If $f(x) = \cos x \sin 3x$, then $f'\left(\frac{\pi}{6}\right) =$

- A) $\frac{1}{2}$ B) $-\frac{\sqrt{3}}{2}$ C) 0 D) 1 E) $-\frac{1}{2}$

14. Rolle's Theorem does not apply to $f(x) = |x - 3|$ on $[1, 4]$ because of which of the following reasons?

- I. $f(x)$ is not continuous on $[1, 4]$
II. $f(x)$ is not differentiable on $(1, 4)$
III. $f(1) \neq f(4)$

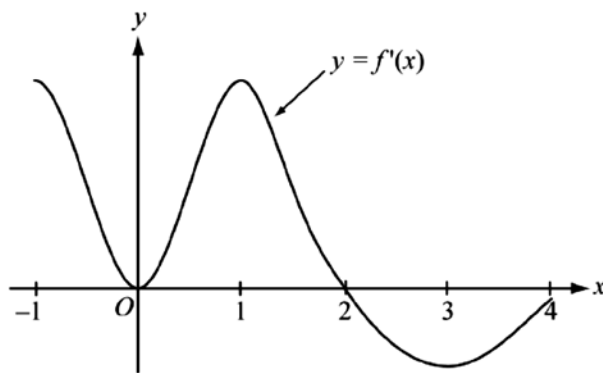
- A) I only B) II only C) III only D) I and II E) II and III

15. If $h(x) = f^2(x) - g^2(x)$ and $f'(x) = -g(x)$, then $h'(x) =$

- A)** 0 **B)** $2g(x)(f'(x) - g'(x))$ **C)** $-4f(x)g(x)$ **D)** $(-g(x))^2 - (f(x))^2$ **E)** $-2g(x)(f(x) + g'(x))$

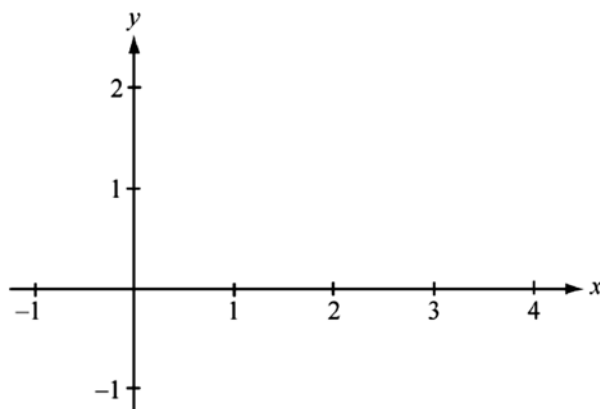
Free Response-No Calculator

1.



Let f be a function that has domain the closed interval $[-1, 4]$ and range the closed interval $[-1, 2]$. Let $f(-1) = -1$, $f(0) = 0$, and $f(4) = 1$. Also let f have the derivative function f' that is continuous and shown in the figure above.

- a) Find all values of x for which f assumes a relative maximum. Justify your answer.
- b) Find all values of x for which f assumes an absolute minimum. Justify your answer.
- c) Find the intervals on which f is concave downward.
- d) Give all the values of x for which f has a point of inflection.
- e) On the axes provided, sketch the graph of f .



2. Consider the curve given by $y^2 = 2 + xy$.

a) Find $\frac{dy}{dx}$.

b) Find all points (x, y) on the curve where the line tangent to the curve has a slope $\frac{1}{2}$.

c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At the time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.