

AP Calculus BC

Summer Packet



Complete the following exercises throughout the summer. Do not wait until the week before school to start! The skills and concepts represented in this packet have been part of your Algebra 2 and Pre-Calculus courses. It is expected you know how to do every problem in this packet. We will spend one day going through questions in class in the fall. Expect a quiz on this material.

Mr. Corbishley

Mr. Friedman

Section 1 - Trigonometry

Prove the following.

1. $\tan x \sin x + \cos x = \sec x$

2. $\sin x - \sin x \cos^2 x = \sin^3 x$

Convert to Degrees

3. $\frac{17\pi}{6}$

4. 1.4

Convert to Radians

5. 200°

6. 120°

Find the exact value of the following. You will need to be able to know these cold.

7. $\sin 30^\circ$

8. $\cos 2\pi$

9. $\tan 45^\circ$

10. $\cot \frac{5\pi}{4}$

11. $\sin 120^\circ$

12. $\sin \frac{\pi}{3}$

13. $\cos \frac{7\pi}{6}$

14. $\tan \frac{7\pi}{6}$

15. $\sec \frac{11\pi}{6}$

16. $\cot 0$

17. $\csc \frac{21\pi}{4}$

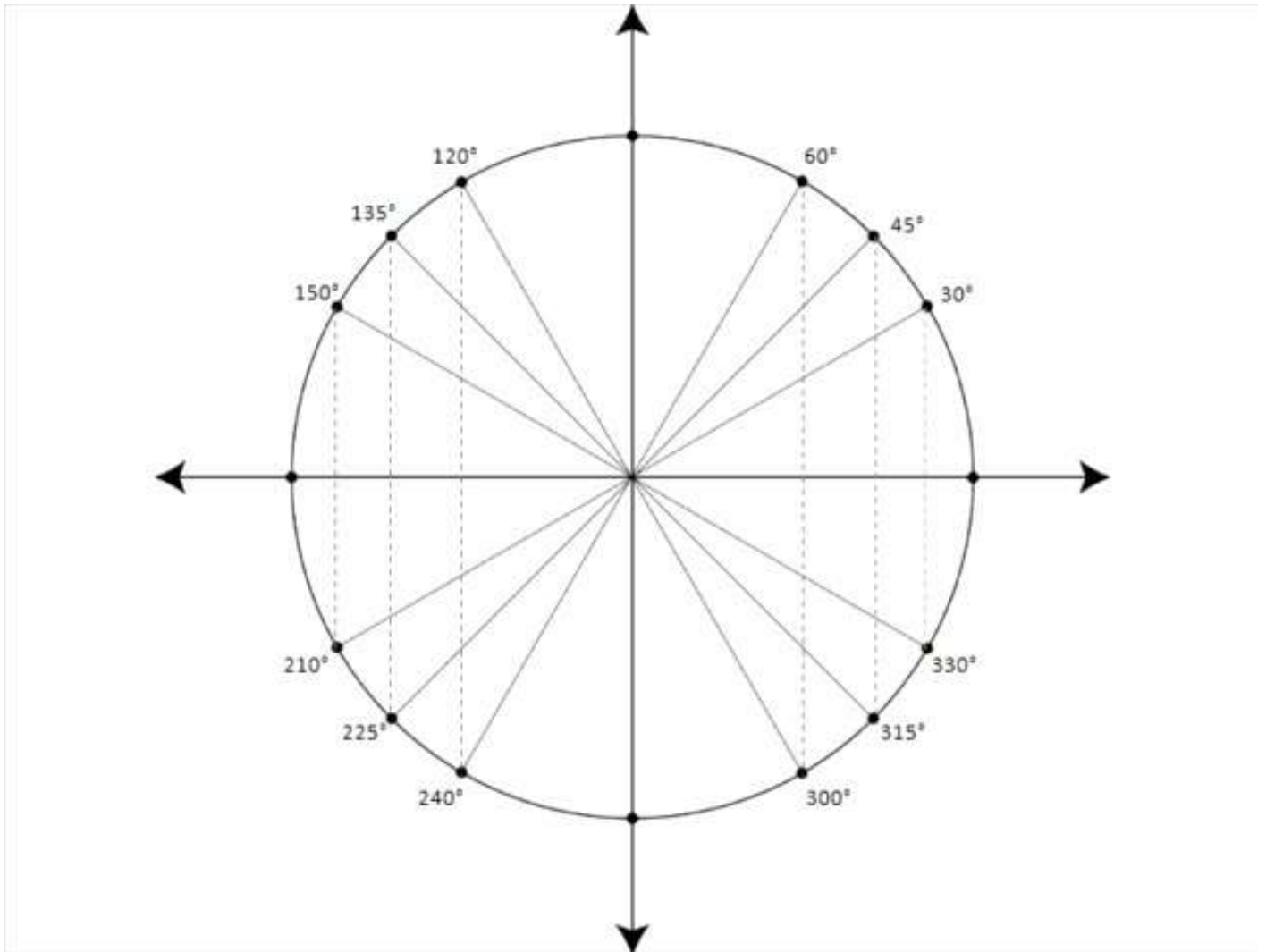
18. $\tan \frac{\pi}{4} + \sin \pi$

19. $\tan^{-1} \sqrt{3}$

20. $\sin^2(4x - 5) + \cos^2(4x - 5)$

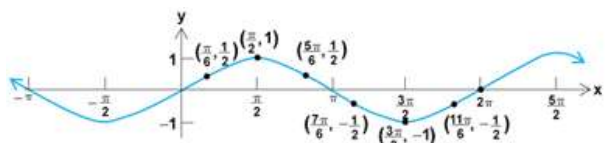
Label on the following unit circle:

- 21.** the angle measurements in radians
the sine of each angle and the cosine of each angle as an ordered pair

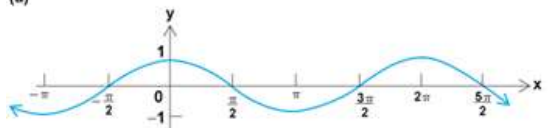


22. Match each function to the corresponding graph.

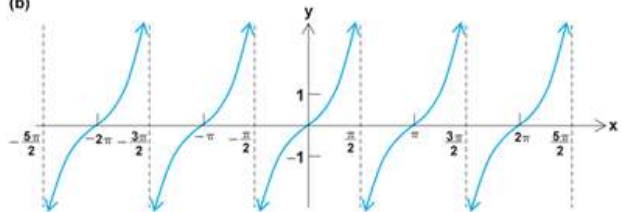
- i. $y = \tan(x)$
- ii. $y = \sec(x)$
- iii. $y = \cos(x)$
- iv. $y = \sin(x)$
- v. $y = \csc(x)$
- vi. $y = \cot(x)$



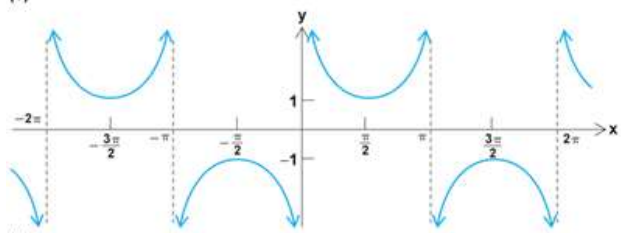
(a)



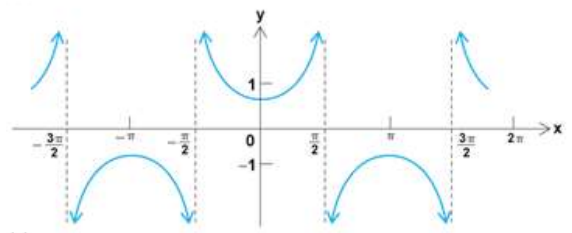
(b)



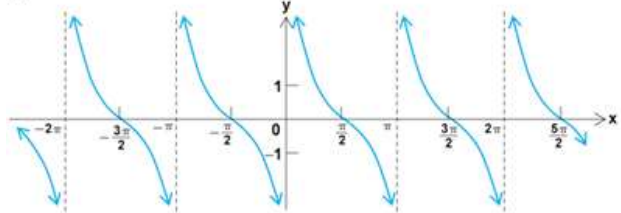
(c)



(d)



(e)



(f)

23. If the amplitude of a sinusoidal function is doubled, does the period change? Justify.

Section 2 - Types of Graphs

Identify the transformations in the following functions:

24. $f(x) = \sqrt{2x - 3}$

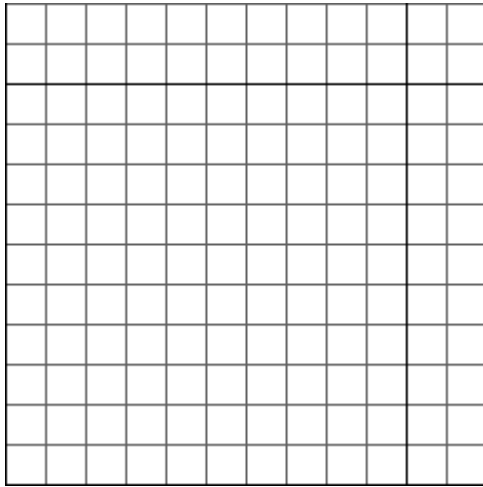
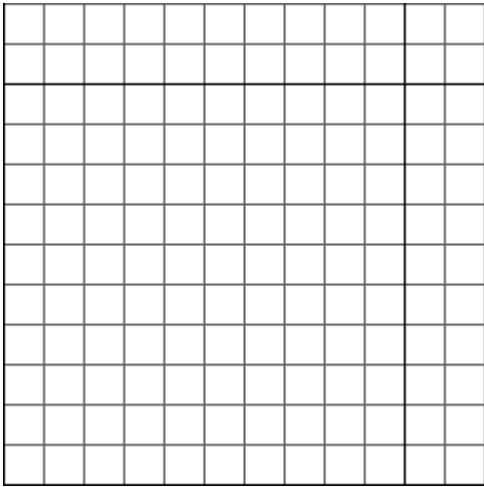
25. $g(x) = (x - 2)^2 + 1$

26. $h(x) = -(x - 12)^4 - 3$

Graph the following functions on the graphs provided below **WITHOUT** the use of a Calculator:

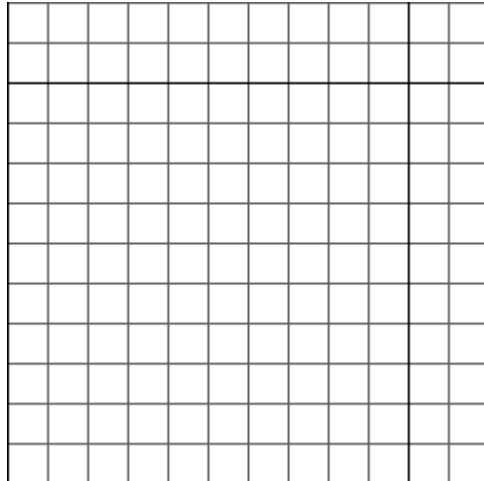
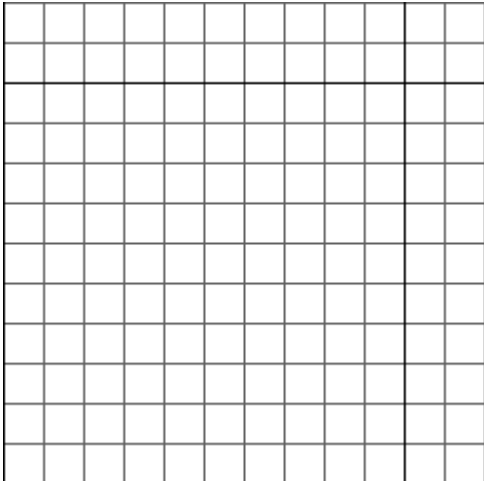
Basic Functions

27. Graph: $f(x) = e^x$ and $g(x) = e^{-x}$



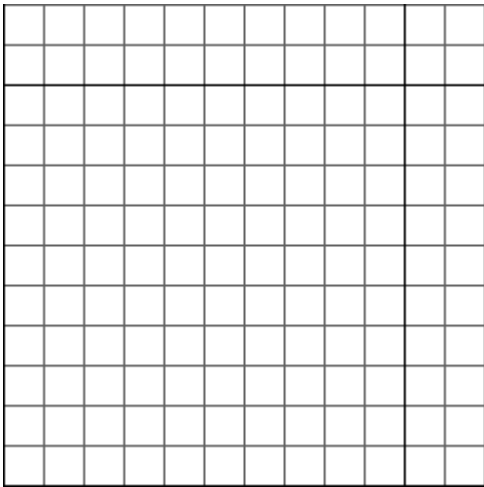
28. Graph: $f(x) = \ln(x)$

29. Graph: $f(x) = |x|$



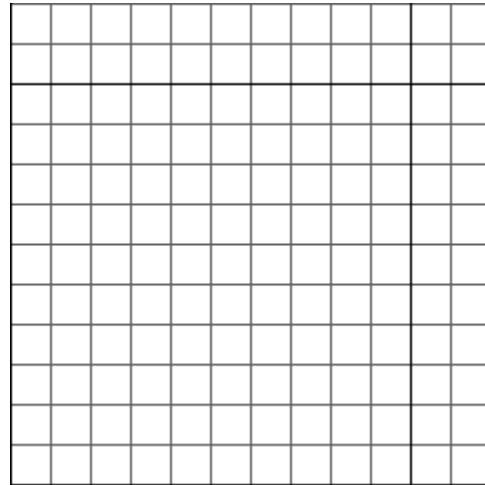
30. Square Root Functions:

$$y = \sqrt{x + 2}$$



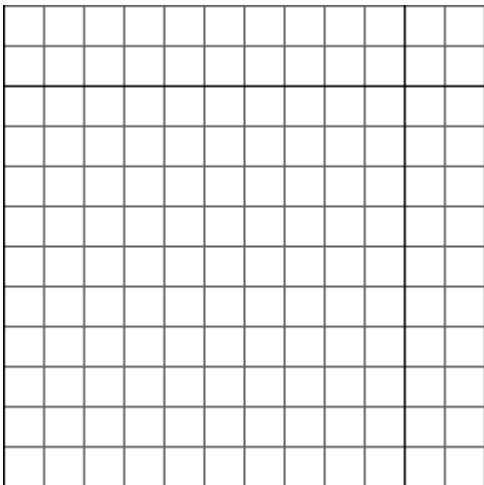
31. Cubic Functions:

$$y = x^3 + 3x^2 + x$$



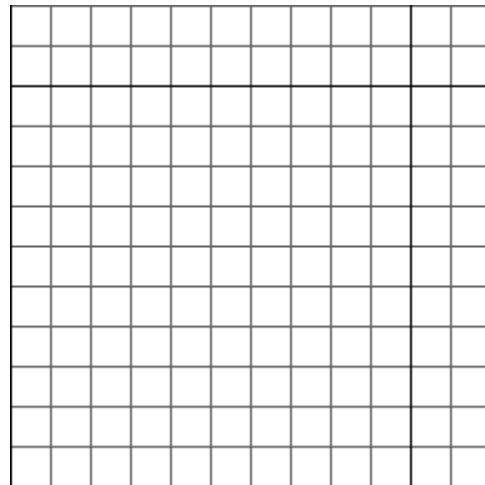
32. Absolute Value Functions:

$$y = |x^2 - 3x - 4|$$



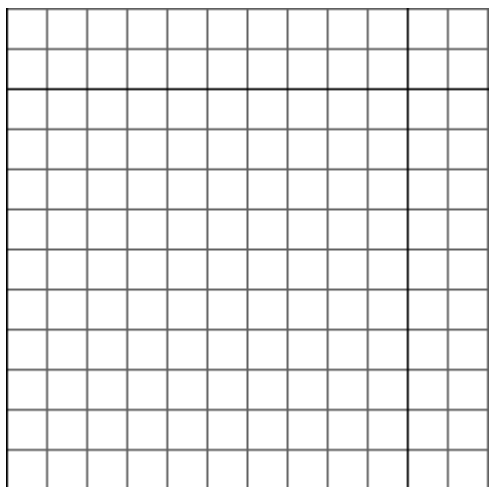
33. Exponential Functions:

$$y = 3^x + 4$$



34. Piecewise Functions:

$$f(x) = \begin{cases} 2x^2 - 1, & x < 1 \\ x + 4, & x \geq 1 \end{cases}$$



Write the equation of the function according to each condition if $f(x) = e^x$

35. Translate left 4 units and down 2

36. Translate right 3 units and stretch vertically by 3

Section 3 - Functions

Use the table to find the following composite functions.

$f(5)$	$g(6)$	$h(6)$
6	5	4

37. $f(g(6))$

38. $h(f(g(6)))$

$f(x)$	$g(x)$	$h(x)$	$j(x)$
$x^2 + 2x$	\sqrt{x}	$x^2 + 5x + 8$	$\frac{x+1}{x^2}$

39. $f(j(1))$

40. $f(g(x))$

41. $f(g(h(j(1))))$

Find the following inverse functions.

42. Find the inverse of $f(x) = 4x + 12$

43. Find $f^{-1}(x)$ if $f(x) = \ln|x| + 7$

Identify if the functions are even/odd/neither

44. $f(x) = 2x^4 - 7x^2 + 5$

45. $g(x) = \sin(2x)$

Section 4 - Characteristics of Rational Functions

Without graphing the function, state the asymptotes and the domain/range of the function.

$$46. y = \frac{-2x^2 + 1}{2x^3 + 4x^2}$$

$$47. y = \frac{x}{(x-1)(x+2)}$$

$$48. y = \frac{5}{(x+2)^2}$$

$$49. y = \frac{2x+4}{x-1}$$

Let $r(x) = f(x)/g(x)$ be a rational function where $f(x) = 8x + 3$ and $g(x)$ is either linear or quadratic.

50. Choose $g(x)$ so that $r(x)$ has one horizontal at $y = 2$ and one vertical asymptote at $x = 0$.

51. Choose $g(x)$ so that $r(x)$ has one horizontal at $y = 0$ and two vertical asymptotes at $x = -3$ and $x = 3$, respectively.

52. Choose $g(x)$ so that $r(x)$ has one horizontal at $y = 0$ and one vertical asymptote at $x = 0$.

53. Choose $g(x)$ so that $r(x)$ has one horizontal at $y = 1$ and one vertical asymptote at $x = 1$.

Find the x-intercepts

$$54. f(x) = (x + 4)(x + 2)(x - 1)$$

Find the y-intercept

$$55. f(x) = \frac{x^2 - x - 6}{x^2 - 1}$$

Identify the hole in the rational function, written as an ordered pair.

$$56. f(x) = \frac{x^2 + 2x - 8}{x^2 - x - 2}$$

Determine the left and right-hand behavior of the graph

$$57. f(x) = -x^3 + 4x$$

Rewrite the following rational functions using partial fraction decomposition.

58. $\frac{3x+5}{x^2+3x+2}$

59. $\frac{1}{x^2+2x-3}$

Section 5 - Properties of Exponents / Logarithmic and Exponential Functions

Simplify the following expressions involving exponents.

60. $(2a^{12}b^3)(3a^2b^4)$

61. $\left(\frac{3x^4y^{-3}z^2}{4x^{-3}y^{10}z}\right)^2$

62. $\left(\frac{b^3\sqrt{5b+2}}{a-b}\right)^2$

Rewrite as an equivalent expression

63. $2\ln(e^2)$

64. $\log_5 125$

65. $\log_4 \frac{1}{2}$

66. $\log 1000000$

67. $\log_b 1$

68. $\ln e^x$

69. $\log_{10}\sqrt{10}$

70. $\frac{1}{2}\log x + \log y - 3\log z$

71. $\log_3 81 + \log 0.001$

72. $\log_4 16 + \log_4 64$

73. $\log_x x^2 + \log_x x^3$

Suppose $x = \log(A)$ and $y = \log(B)$, write the following expressions in terms of x and y .

74. $\log(AB) =$

75. $\log(A) \log(B) =$

76. $\log\left(\frac{A}{B^2}\right) =$

Determine which functions (if any) are equivalent (without a calculator):

77. $f(x) = 3^{x-2}$ $g(x) = 3^x - 9$ $h(x) = 3^x / 9$

78. $f(x) = 5^{-x} + 3$ $g(x) = 5^{3-x}$ $h(x) = -5^{x-3}$

Using either the model given in the problem or a model you create, answer the following problems.

79. A recent study revealed that the amount of time a person spent working on math over the summer directly affected the grade they received on their first test. This relationship can be modeled by the equation $G = 5e^{kt}$ where G is the grade, t is the number of hours the person spent working on math over the summer, and k is a constant.

- a) Suppose a student spent 8 hours on math over the summer and a grade of 75 is earned on the first test. What is the value of k ?
- b) Using the same value of k , determine the amount of hours one should spend studying in order to earn a 90 on the first test.

80. A researcher collects data on rabbit population over a 22-month period. The population (in thousands) is given in the table below:

Month	0	2	4	6	8	10	12	14	16	18	20	22
Number	10	12	14	16	22	30	35	39	44	48	50	51

- a) Draw a scatter-plot of the data and find a logistic regression model.
- b) What can you conclude about the limit of the rabbit population growth in this area? Justify.

Section 6 - Limits

Evaluate the following limits.

81. $\lim_{x \rightarrow 0} \frac{2-x}{x^2+4}$

82. $\lim_{x \rightarrow 4} f(x)$ such that $f(x) = \begin{cases} 3x - 2, & x \neq 4 \\ 15, & x = 4 \end{cases}$

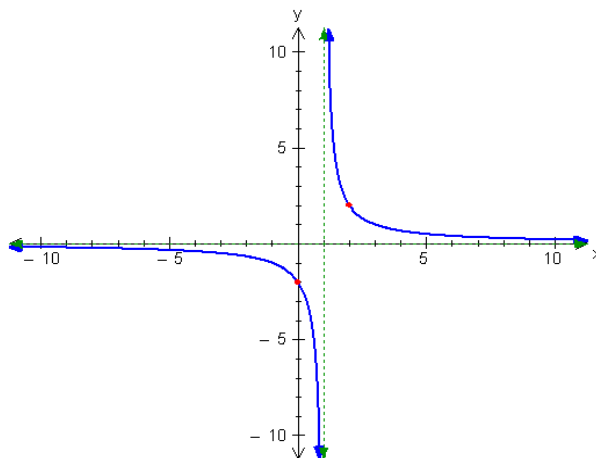
83. The following is a graph of $f(x)$

a) $\lim_{x \rightarrow 1} f(x) =$

b) $\lim_{x \rightarrow 1+} f(x) =$

c) $\lim_{x \rightarrow 1-} f(x) =$

d) $\lim_{x \rightarrow \infty} f(x) =$

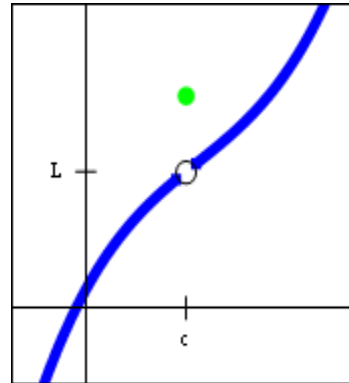


84. $\lim_{x \rightarrow 0} \frac{\sqrt{3-x} - \sqrt{3}}{x}$

85. $\lim_{x \rightarrow \infty} \frac{x^2 - 4x + 15}{3x + 4x^2}$

86. $\lim_{x \rightarrow -\infty} \frac{6x^3 - 10}{4x + 1}$

87. The following is a graph of $f(x)$:



a) $\lim_{x \rightarrow c} f(x)$

b) Does $\lim_{x \rightarrow c} f(x) = f(c)$?

88. $\lim_{x \rightarrow 9} \sin\left(\frac{\pi}{18}x\right)$

89. $\lim_{x \rightarrow 3} f(x)$ such that $f(x) = \begin{cases} x^2 + 1, & x \leq 3 \\ 7x - 3, & x > 3 \end{cases}$

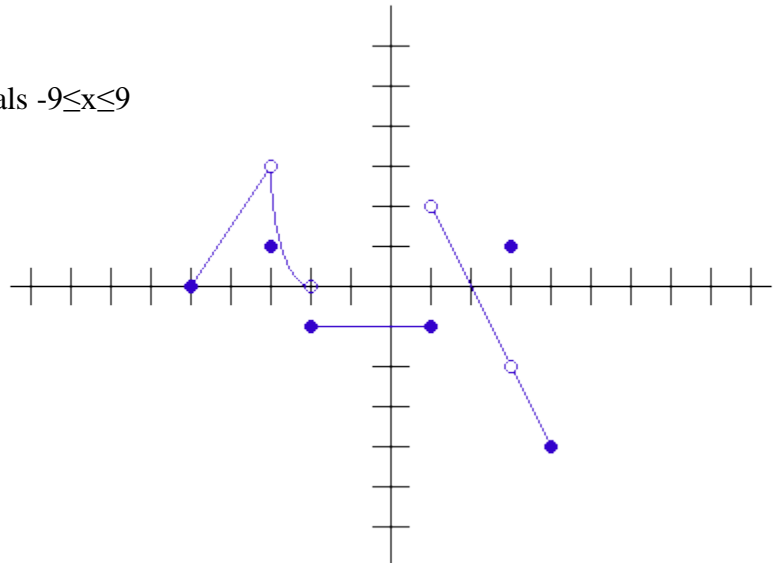
90. Pictured is a graph of $f(x)$ on the intervals $-9 \leq x \leq 9$

a) $\lim_{x \rightarrow 0} f(x) =$

b) $\lim_{x \rightarrow 3} f(x) =$

c) $\lim_{x \rightarrow -3} f(x) =$

d) $\lim_{x \rightarrow -2} f(x) =$



91. For the following problems, $g(x) = \sqrt{x^2 - 36}$

a) $\lim_{x \rightarrow 6} g(x) =$

b) $\lim_{x \rightarrow 6^+} g(x) =$

c) $\lim_{x \rightarrow 6^-} g(x) =$

$$92. \lim_{x \rightarrow \infty} \frac{x^2 - 5}{x^5 + x^3 + x^2 + 3} - 5$$

$$93. \lim_{x \rightarrow 3} x$$

$$94. \lim_{x \rightarrow \infty} c^x$$

$$95. \lim_{x \rightarrow 3} \frac{5x^2 - 8x - 13}{x^2 - 5}$$

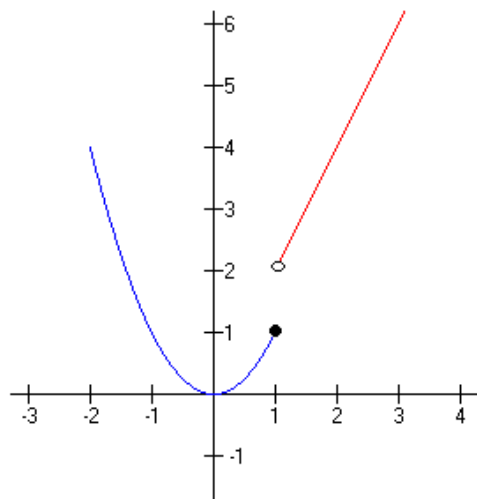
$$96. \lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x^3 + 8}$$

$$97. \lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - x - 2}$$

$$98. \lim_{x \rightarrow \infty} \frac{14x - 3}{3x^5 - 2x^4 - 3x - 1}$$

$$99. \lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$$

- 100.
- a) $\lim_{x \rightarrow 1^-} f(x) =$
 - b) $\lim_{x \rightarrow 1^+} f(x) =$
 - c) $\lim_{x \rightarrow 1} f(x) =$
 - d) $f(1) =$



Section 7 - Sequences and Series

Read through the following information and answers the questions embedded within the notes.

- A sequence is nothing more than a list of numbers written in a specific order. The list may or may not have an infinite number of terms in them.
- General sequence terms are denoted as follows:

$$\begin{aligned}
 & a_1 - \text{first term} \\
 & a_2 - \text{second term} \\
 & \quad \vdots \\
 & a_n - n^{\text{th}} \text{ term} \\
 & a_{n+1} - (n+1)^{\text{st}} \text{ term} \\
 & \quad \vdots
 \end{aligned}$$

In the notation above we need to be very careful with the subscripts. The subscript of $n+1$ denotes the next term in the sequence and NOT one plus the n^{th} term! In other words:

$$a_{n+1} \neq a_n + 1$$

so be very careful when writing subscripts to make sure that the “+1” doesn’t migrate out of the subscript! This is an easy mistake to make when you first start dealing with this kind of thing.

Each of the following are equivalent ways of denoting a sequence:

$$\{a_1, a_2, \dots, a_n, a_{n+1}, \dots\} \quad \{a_n\} \quad \{a_n\}_{n=1}^{\infty}$$

In the second and third notations above a_n is usually given by a formula.

- First, note the difference between the second and third notations above. If the starting point is not important or is implied in some way by the problem it is often not written down as we did in the third notation.
- Next, we used a starting point of $n = 1$ in the third notation only so we could write one down. There is absolutely no reason to believe that a sequence will start at $n = 1$. A sequence will start where ever it needs to start.

Example 1 - Write down the first five terms of each of the following sequences.

(a) $\left\{ \frac{n+1}{n^2} \right\}_{n=1}^{\infty}$

(b) $\left\{ \frac{(-1)^{n+1}}{2^n} \right\}_{n=0}^{\infty}$

(c) $\{b_n\}_{n=1}^{\infty}$, where $b_n = n^{\text{th}}$ digit of π

Convergence:

Given the sequence $\{a_n\}$ if we have a function $f(x)$ such that $f(n) = a_n$ and

$\lim_{x \rightarrow \infty} f(x) = L$ then $\lim_{n \rightarrow \infty} a_n = L$

1. $\lim_{n \rightarrow \infty} (a_n \pm b_n) = \lim_{n \rightarrow \infty} a_n \pm \lim_{n \rightarrow \infty} b_n$
2. $\lim_{n \rightarrow \infty} c a_n = c \lim_{n \rightarrow \infty} a_n$
3. $\lim_{n \rightarrow \infty} (a_n b_n) = \left(\lim_{n \rightarrow \infty} a_n \right) \left(\lim_{n \rightarrow \infty} b_n \right)$
4. $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \frac{\lim_{n \rightarrow \infty} a_n}{\lim_{n \rightarrow \infty} b_n}$, provided $\lim_{n \rightarrow \infty} b_n \neq 0$
5. $\lim_{n \rightarrow \infty} a_n^p = \left[\lim_{n \rightarrow \infty} a_n \right]^p$ provided $a_n \geq 0$

The sequence $\{r^n\}_{n=0}^{\infty}$ converges if $-1 < r \leq 1$ and diverges for all other value of r . Note that this

sequence is geometric. It will converge according to the following: $\lim_{n \rightarrow \infty} r^n = \begin{cases} 0 & \text{if } -1 < r < 1 \\ 1 & \text{if } r = 1 \end{cases}$

Example 2 - Determine if the following sequences converge or diverge. If the sequence converges determine its limit.

(a) $\left\{ \frac{3n^2 - 1}{10n + 5n^2} \right\}_{n=2}^{\infty}$

(b) $\left\{ (-1)^n \right\}_{n=0}^{\infty}$

Series – The Idea and Notation

A series is built from a sequence, but differs from it in that the terms are *added together*. For example

$$1, 4, 7, 11, \dots$$

Is a *sequence*, but

$$1 + 4 + 7 + 11 + \dots$$

Is a *series*.

A series can be finite (for example, it might only have 25 terms) or infinite, and the notation needs to allow for both.

The easiest way to get used to series *notation* is with an example.

Example: Find the sum of the series

$$\sum_{i=1}^5 3i + 4$$

Convergence and Divergence

If the series goes on forever (and in real world applications many do) adding the terms might seem a bit pointless, since they seem to just add up to infinity. But some series, even though they go on forever, *have a finite sum*. For example, consider the series

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

You can check by adding up this series for a couple more terms that it never climbs above 2, and in fact it approaches 2 the more terms you add up. This is called a *convergent* series, and this series *converges* to 2. You'll meet more convergent series in calculus.²

The sequences we have met generally go on for ever (at least in theory, in practise we only work with the first few terms most of the time), so a sequence can converge or diverge too. For example, the *sequence*

$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$$

Converges to 0, but the sequence

$$2, 4, 6, 8, \dots$$

Diverges, since it's terms don't tend to a constant value as n gets large.

Arithmetic Series

If we have an arithmetic *sequence*, adding up the terms gives us an arithmetic *series*. Once we realize it's arithmetic, and we get the rule for each term in the form

$$a_n = a + (n-1)d$$

Then there's an easy formula for adding up the first n terms of an arithmetic series. It's

$$S_n = n \left(\frac{a + a_n}{2} \right)$$

² Figuring out whether a series is convergent or not is harder than it looks. It's not enough for the terms to be getting smaller as n gets large. For example, the series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{n} + \dots$ looks like it might converge (i.e. have a finite sum), but it does not. A series that doesn't converge is said to *diverge*. Wait till Calc 3 to look at rules for testing series.

Example: Find the sum of the series

$$\sum_{i=1}^{25} 3i + 4$$

Geometric Series

Similarly If we have an geometric *sequence*, adding up the terms gives us an geometric *series*. Once we realize it's geometric, there's a formula for the total sum.

Recall that a geometric sequence is for example: $3, 6, 12, 24, \dots$

where the *ratio* between successive terms is constant. The ratio was labeled r and as before, the first term is labeled a . Then the i th term of the sequence is from before

$$a_i = ar^{i-1}$$

And the formula for the sum of the first n terms of a geometric series is

$$S_n = \frac{a(1-r^n)}{1-r}$$

Furthermore, if we want to add all the terms of an *infinite* geometric series it's given by

$$S_\infty = \frac{a}{1-r}, \quad |r| < 1 \quad (*)$$

So as long as the ratio has an absolute value less than 1 the geometric series *converges*. (Otherwise it *diverges*).

Example: For the series

$$2 + \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \dots$$

(a) Find the sum of the first 10 terms.

(b) If it converges, find S_{∞} .

(a): The series is geometric since the common ratio is $\frac{1}{4}$ - each term gets multiplied by $\frac{1}{4}$ to get the next term. So $a = 2$ and $r = \frac{1}{4}$. Let's plug $n=10$ into the above formula:

$$S_{10} = \frac{2\left(1 - \left(\frac{1}{4}\right)^{10}\right)}{1 - \frac{1}{4}} = 2.666664124$$

(c) The sum to infinity is $S_{\infty} = \frac{2}{1 - \frac{1}{4}} = \frac{2}{\frac{3}{4}} = \frac{8}{3}$.

Other Important Skills:

Find the n th term (n starts at 1):

1. $9, 11/8, 13/27, 15/64, 17/125, \dots$

2. $1, 1, 9/7, 16/10, 25/13, 36/16, \dots$

Write the following using series notation:

1. $1 + 4/3 + 2 + 16/5 + 32/6 + 64/7$

2. $-1/5 + 4/7 - 1 + 16/11$

Find the n th term (n starts at 1):

3. $9, 11/8, 13/27, 15/64, 17/125, \dots$

4. $1, 1, 9/7, 16/10, 25/13, 36/16, \dots$

Section 8 - Parametric Equations

Read through the following information and answers the questions embedded within the notes

Parametric equation -When x and y are functions of a third variable t , called a parameter, (*note: t usually but does not need to stand for time*) then the set of points $(x, y) = (f(t), g(t))$ is a parametric curve.

Circles:

The equation $x^2 + y^2 = r^2$ is parametrized by $x = a(\cos(t))$ and $y = a(\sin(t))$

Ex:

Describe the graph of the relation determined by $x = 2\cos t$ and $y = 2\sin t$ for $0 \leq t \leq 2\pi$. Find the initial and terminal points, if any, and indicate the direction in which the curve is traced. Find a Cartesian equation for a curve that contains the parametrized curve.

Solution:

Begin by creating a table of $x(t)$ and $y(t)$ for the common trig values from $0 \leq t \leq 2\pi$. This produces a circle with radius 2. The graph is traced exactly once counterclockwise. The initial point at $t = 0$ is $(2, 0)$ and the terminal point at $t = 2\pi$ is also $(2, 0)$.

To find a Cartesian equation you must eliminate the variable t .

$$\begin{aligned}x^2 + y^2 &= 4\cos^2 t + 4\sin^2 t \\ &= 4(\cos^2 t + \sin^2 t) \\ &= 4\end{aligned}$$

AP Need to Know

- Sketching the path of a particle and indicating direction of motion given a parametric representation.
- Finding the Cartesian equation of a parametric curve.

Example 1

A particle is moving in the coordinate plane in such a way that $x(t) = 2t - 5$ and $y(t) = 4\sin\left(\frac{\pi}{t+1}\right)$ for $0 \leq t \leq 5$. Sketch the path of the particle and indicate the direction of the motion.

Solution

t	0	1	2	3	4	5
x(t)	-5	-3	-1	1	3	5
y(t)	0	4	3.464	2.828	2.351	2

Plot the points $(x(t), y(t))$ and sketch the path as a smooth curve. Indicate the direction of motion moving from $t = 0$ to $t = 5$ by using small arrows on the curve.

Example 2

A parametric curve is defined by $x = 2 + e^t$ and $y = e^{3t}$. Find the Cartesian equation of the curve

Solution

You need to get one of the equations so that t is alone. Solve $x = 2 + e^t$ so that $t = \ln(x - 2)$.

Substitute $t = \ln(x - 2)$ into $y = e^{3t}$

$$y = e^{3\ln(x-2)} = e^{\ln(x-2)^3} = (x-2)^3$$

Finally be careful of the domain: $t = \ln(x - 2)$ is only defined for $x > 2$.

The equation of the curve is therefore $y = (x - 2)^3$ with domain $(2, \infty)$

Additional Help

To understand parametric equations, first experiment with a graphing utility and its “param mode”. Press mode and change “Func” to “Par”. Now put $x(t) = \cos t$ and $y(t) = \sin t$. A curve should be generated over an interval of t -values. For each value of t , the coordinates of a point (x, y) were calculated. Because the position of the point (x, y) varies with t , the variable t is called the parameter. Both x and y are dependent variables and are functions of t .

You should have a sense of what a parametric equation is after experimenting with the graphing utility. If not, here is an exact definition:

Let f and g be two functions defined on an interval I . The relation of all ordered pairs $(f(t), g(t))$ for t in I is called a curve C . The equations $x=f(t)$ and $y=g(t)$ (for t in I) are called parametric equations for C , and the variable t is the parameter.

To plot a set of parametric equations by hand you need to follow the steps below:

- **First**, you need to find or determine the range over which t varies. Sometimes this is given in the problem and other times you need to determine it by just picking some values and trying them in the equations.
- **Second**, you need to go through each of the values for t and plug them into the equations $x(t)$ and $y(t)$ so that you find the ordered pairs $(x(t), y(t))$.
- **Third**, the ordered pairs you have found in step two need to be plotted on a standard x - y coordinate plane.

Practice Problems:

1. Eliminate the parameter from the following set of parametric equations.

$$x = t^2 + t \qquad y = 2t - 1$$

2. Use parametric equations to describe a curve that is the graph of $y = 3x^2 - 4x + 5$
3. Eliminate the parameter from the curve C defined by $x(t) = t - 1$ and $y(t) = t^2$ for t .
4. Describe the graph of $y = 3x^4 + 7x^3 - 8x^2 - 3$ with parametric equations
5. Sketch the parametric curve for the following set of parametric equations.

$$x = t^2 + t \qquad y = 2t - 1$$

Section 9 - Polar Functions

Read through the following information and answers the questions embedded within the note

The Basics:

Instead of graphing something using a traditional Cartesian coordinate system, sometimes we may choose to graph something using a completely different coordinate system – polar coordinates.

Cartesian coordinates are of the form (x, y) while polar coordinates are of the form:

$$(r, \theta)$$

r = the directed distance (think radius)

θ = the directed angle from the initial ray

Graphing with Polar Coordinates:

Your graphing calculator can be put into polar mode and you can always graph that way.

Be familiar with these basic polar equations:

- $r = \text{constant}$
This is a graph of a circle around origin (the radius is always the same value)
- $\theta = \text{any radian measure}$
This is a graph of a line through the origin since the angle must always be the same value.

Polar-Rectangular Conversion Formulas:

You need to MEMORIZE these formulas to convert

$$x = r \cos \theta \qquad y = r \sin \theta \qquad r^2 = x^2 + y^2 \qquad \tan \theta = \frac{y}{x}$$

Parametric Equations of Polar Curves: (Note that r = any function in term of θ)

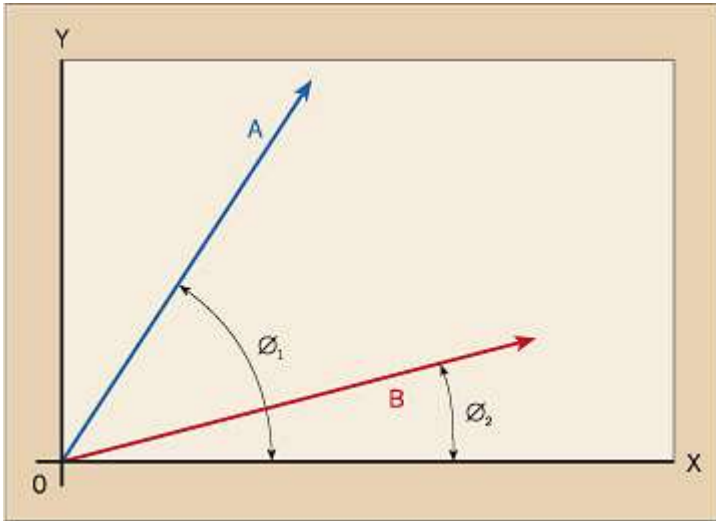
The polar graph of $r = f(\theta)$ is the curve defined parametrically by:

$$\begin{aligned} x &= r \cos \theta = f(\theta) \cos \theta \\ y &= r \sin \theta = f(\theta) \sin \theta \end{aligned}$$

Overview of Vectors and Polars

So what exactly is a vector?

-A vector represents a displacement in regards to BOTH magnitude and direction. The length, r , of the vector is the magnitude. The angle the vector makes with the x -axis (θ) gives its direction. An arrow is drawn to represent the vector. (If you took Physics, this should be fairly familiar).



Picture illustrating vectors

-However, the vector can be written in terms of horizontal and vertical components. In order to do so, two simple relationships must be MEMORIZED. Thus, any vector in the plane can be represented by the ordered pair $\langle x, y \rangle$. Basically you can transform the magnitude and direction to (x, y) . This is a good relationship for converting Cartesian coordinates to polar coordinates.

$$x = r \cos(\theta)$$

$$y = r \sin(\theta)$$

(Practice problems/solutions will elucidate confusions. If you are asking yourself, “What is the point of vectors” or “What does r , θ , x , y mean” then direct yourself to examples and practice problems to see how vector problems are done. It is easier to understand this topic through examples)

-Two more relationships can be used to find the magnitude and direction of a vector. In Calculus AB/BC, these equations aren't really used, but they can be used as a basis for further Calculus things (Example, it is used to derive the formula of arc length [something you will learn in Calc]). It is important to memorize these equations:

$$\text{To find magnitude: } r = \sqrt{x^2 + y^2}$$

$$\text{To find direction: } \tan \theta = \frac{y}{x}$$

Practice Problems of Vectors and Polars:

1. Find the magnitude and direction of the vector represented by $\langle 6, -3 \rangle$
2. Find the magnitude and direction of the vector represented by $\langle -5, 5 \rangle$
3. Find the ordered pair representation of a vector of magnitude 12 and direction $-\frac{\pi}{4}$
4. Find the ordered pair representation of a vector of magnitude 69 and direction 0
5. Change these polar coordinates to Cartesian coordinates
 - a) $(1, \pi)$
 - b) $(2, \frac{-2\pi}{3})$