AP CALCULUS BC
Unit 5 Outline – Volume and Arc Length

DATE	**CONCEPT**	**IN-CLASS SAMPLE PROBLEMS**
10/11 10/12	**VOLUME OF SOLIDS WITH KNOWN CROSS SECTIONS**	Notes - Handout
HOMEWORK		Worksheet 25

DATE	**CONCEPT**	**IN-CLASS SAMPLE PROBLEMS**
10/13 DELAYED	**VOLUME OF SOLIDS FORMED BY ROTATION**	Notes - Handout
HOMEWORK		Worksheet 26

DATE	**CONCEPT**	**IN-CLASS SAMPLE PROBLEMS**
10/14	**VOLUME OF SOLIDS FORMED BY ROTATION** **THE SHELL METHOD**	Notes - Handout
HOMEWORK		Worksheet 27

DATE	**CONCEPT**	**IN-CLASS SAMPLE PROBLEMS**
10/15	**ARC LENGTH**	Notes - Handout
HOMEWORK		Worksheet 28

DATE	**CONCEPT**	**IN-CLASS SAMPLE PROBLEMS**
10/16	**REVIEW**	Area Volume Arc Length Average Value
HOMEWORK		Worksheet 29

DATE	**CONCEPT**	**IN-CLASS SAMPLE PROBLEMS**
10/17	**EXAM**	Area Volume Arc Length Average Value
HOMEWORK		None

AP Calculus BC - Worksheet 25 **Volumes with Known Cross Sections**

1. The base of a solid in the *xy*-plane is a right triangle bounded by the axes and $y=-x+2$. Cross sections of the solid perpendicular to the *x*-axis are squares. Find the volume.

2. (Calculator Permitted) The base of a solid *S* is the region enclosed by the graph of $y=\ln x$, the vertical line $x=e$, and the *x*-axis. If the cross sections of *S* perpendicular to the *x*-axis are squares, which of the following gives the best approximation of the volume of *S*?
   (A) 0.718 (B) 1.718 (C) 2.718 (D) 3.171 (E) 7.388

3. (Calculator Permitted) The base of a solid is the region in the first quadrant bounded by the graphs of $y=e^{-x^2}$, $y=1-\cos x$ and the *y*-axis. For this solid, each cross section perpendicular to the *x*-axis is a square. Find the volume of the solid.

4. (Calculator Permitted) The base of a solid is the region in the first quadrant bounded by the graphs of $y=\sqrt{x}$, $y=e^{-3x}$ and the vertical line $x=1$. For this solid each cross section perpendicular to the *x*-axis is a rectangle whose height is 5 times its length of its base. Find the volume of the solid.

5. (Calculator Permitted) The base of a solid is the region in the first quadrant bounded by the *x*-axis, the graph of $y=\sin^{-1}x$, and the vertical line $x=1$. For this solid, each cross section perpendicular to the *y*-axis is a semicircle. What is the volume?
   (A) 0.356 (B) 0.279 (C) 0.139 (D) 1.571 (E) 0.571

6. (Calculator Permitted) The base of a solid is the region bounded by the curve $y=2+\sin x$, the *x*-axis, $x=0$, and $x=\frac{3\pi}{2}$. Find the volume of the solids whose cross sections perpendicular to the *x*-axis are the following:
   a) squares
   b) rectangles whose height is 3 times the base
   c) equilateral triangles
   d) isosceles right triangles with leg on the base
   e) isosceles right triangles with hypotenuse on the base
   f) semi-circles
   g) quarter-circles

Answers:

1. $\frac{8}{3}$	2. A	3. 0.461	4. 1.554
5. C	6. a) 25.20575 b) 75.61725 c) 10.91441 d) 12.60287 e) 6.301437 f) 9.898275 g) 19.79655		

1	(Calculator Permitted) Let R be the region in the first quadrant bounded by the graph of $y = 8 - x^{3/2}$, the x-axis, and the y-axis. Which of the following gives the best approximation of the volume of the solid generated when R is revolved about the x-axis? (A) 60.3 (B) 115.2 (C) 225.4 (D) 319.7 (E) 361.9
2	Let R be the region enclosed by the graph of $y = x^2$, the line $x = 4$, and the x-axis. Which of the following gives the best approximation of the volume of the solid generated when R is revolved about the y-axis. (A) 64π (B) 128π (C) 256π (D) 360 (E) 512
3	Let R be the region enclosed by the graphs of $y = e^{-x}$, $y = e^{x}$, and $x = 1$. Which of the following gives the volume of the solid generated when R is revolved about the x-axis? (A) $\int_0^1 \left(e^x - e^{-x}\right) dx$ (B) $\int_0^1 \left(e^{2x} - e^{-2x}\right) dx$ (C) $\int_0^1 \left(e^x - e^{-x}\right)^2 dx$ (D) $\pi\int_0^1 \left(e^{2x} - e^{-2x}\right) dx$ (E) $\pi\int_0^1 \left(e^x - e^{-x}\right)^2 dx$
4	(Calculator Permitted) Let R be the region bounded by the curves $y = x^2 + 1$ and $y = x$ for $0 \le x \le 1$. Showing all integral set-ups, find the volume of the solid obtained by rotating the region R about the a) x-axis b) line $y = -1$ c) line $y = 3$ d) y-axis

Answers

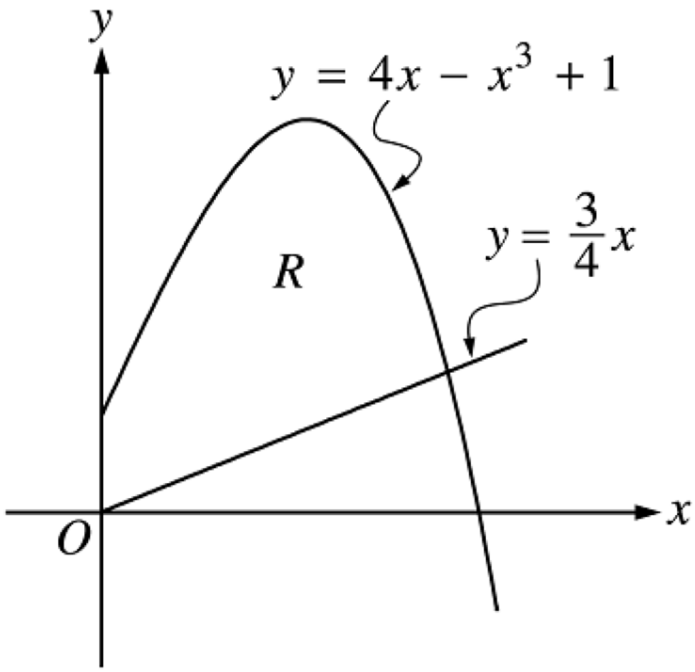
1) E	2) B
3) D	4) a) 4.817 b) 10.053 c) 10.890 d) 2.617

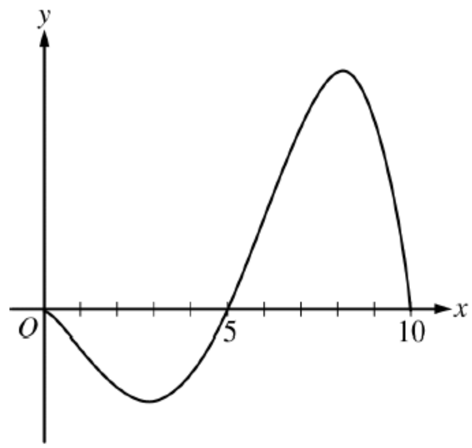
1	Let R be the region in the first quadrant bounded by the graph of $y = 3x - x^2$ and the x-axis. A solid is generated when R is revolved about the vertical line $x = -1$. Set up, but do not evaluate, the definite integral that gives the volume of this solid. (A) $\int_0^3 2\pi(x+1)(3x-x^2)\,dx$ (B) $\int_{-1}^3 2\pi(x+1)(3x-x^2)\,dx$ (C) $\int_0^3 2\pi(x)(3x-x^2)\,dx$ (D) $\int_0^3 2\pi(3x-x^2)^2\,dx$ (E) $\int_0^3 (3x-x^2)\,dx$
2	(Calculator Permitted) Let R be the region bounded by the graphs of $y = \sqrt{x}$, $y = e^{-x}$, and the y-axis. (a) Find the area of R. (b) Find the volume of the solid generated when R is revolved about the line $y = -1$. (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a semicircle whose diameter runs from the graph of $y = \sqrt{x}$ to the graph of $y = e^{-x}$. Find the volume of this solid.
3	(Calculator Permitted) Find the volume of the solid formed when the R enclosed by the curves $y = x$ and $y = x^2$ is rotated about the following axes: a) the x-axis b) the line $y = 2$ c) the line $y = -5$ d) the y-axis e) the line $x = -1$ f) the line $x = 17$.

Answers:

1) A	2) a) 0.161 b) 1.630 c) 0.034	3) a) 0.418 b) 1.675 c) 5.654 d) 0.523 e) 1.570 f) 17.278

1	('88 BC) The length of the curve $y = x^3$ from $x = 0$ to $x = 2$ is given by (A) $\int_0^2 \sqrt{1+x^6}\,dx$ (B) $\int_0^2 \sqrt{1+3x^2}\,dx$ (C) $\pi\int_0^2 \sqrt{1+9x^4}\,dx$ (D) $2\pi\int_0^2 \sqrt{1+9x^4}\,dx$ (E) $\int_0^2 \sqrt{1+9x^4}\,dx$
2	('03 BC) The length of a curve from $x = 1$ to $x = 4$ is given by $\int_1^4 \sqrt{1+9x^4}\,dx$. If the curve contains the point $(1,6)$, which of the following could be an equation for this curve? (A) $y = 3+3x^2$ (B) $y = 5+x^3$ (C) $y = 6+x^3$ (D) $y = 6-x^3$ (E) $y = \frac{16}{5}+x+\frac{9}{5}x^5$
3	(Calculator Permitted) Which of the following gives the best approximation of the length of the arc of $y = \cos(2x)$ from $x = 0$ to $x = \frac{\pi}{4}$? (A) 0.785 (B) 0.955 (C) 1.0 (D) 1.318 (E) 1.977
4	Which of the following gives the length of the graph of $x = y^3$ from $y = -2$ to $y = 2$? (A) $\int_{-2}^{2} \left(1+y^6\right)dy$ (B) $\int_{-2}^{2} \sqrt{1+y^6}\,dy$ (C) $\int_{-2}^{2} \sqrt{1+9y^4}\,dy$ (D) $\int_{-2}^{2} \sqrt{1+x^2}\,dx$ (E) $\int_{-2}^{2} \sqrt{1+x^4}\,dx$
5	Find the length of the curve described by $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 8$. (A) $\frac{26}{3}$ (B) $\frac{52}{3}$ (C) $\frac{512\sqrt{2}}{15}$ (D) $\frac{512\sqrt{2}}{15}+8$ (E) 96
6	Which of the following expressions should be used to find the length of the curve $y = x^{2/3}$ from $x = -1$ to $x = 1$? (A) $2\int_0^1 \sqrt{1+\frac{9}{4}y}\,dy$ (B) $\int_{-1}^{1} \sqrt{1+\frac{9}{4}y}\,dy$ (C) $\int_0^1 \sqrt{1+y^3}\,dy$ (D) $\int_0^1 \sqrt{1+y^6}\,dy$ (E) $\int_0^1 \sqrt{1+y^{9/4}}\,dy$

7	(AP BC 2002B-3) (Calculator Permitted) Let R be the region in the first quadrant bounded by the y-axis and the graphs of $y = 4x - x^3 + 1$ and $y = \frac{3}{4}x$.  (a) Find the area of R. (b) Find the volume of the solid generated when R is revolved about the x-axis. (c) Write an expression involving one or more integrals that gives the perimeter of R. Do not evaluate.



Graph of f

4. The graph of the differentiable function $y = f(x)$ with domain $0 \le x \le 10$ is shown in the figure above. The area of the region enclosed between the graph of f and the x-axis for $0 \le x \le 5$ is 10, and the area of the region enclosed between the graph of f and the x-axis for $5 \le x \le 10$ is 27. The arc length for the portion of the graph of f between $x = 0$ and $x = 5$ is 11, and the arc length for the portion of the graph of f between $x = 5$ and $x = 10$ is 18. The function f has exactly two critical points that are located at $x = 3$ and $x = 8$.

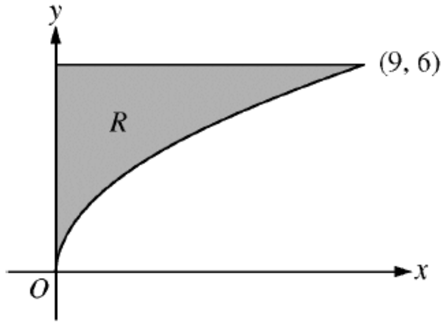
   (a) Find the average value of f on the interval $0 \le x \le 5$.

   (b) Evaluate $\int_0^{10} (3f(x) + 2)\, dx$. Show the computations that lead to your answer.

   (c) Let $g(x) = \int_5^x f(t)\, dt$. On what intervals, if any, is the graph of g both concave up and decreasing? Explain your reasoning.

   (d) The function h is defined by $h(x) = 2f\left(\frac{x}{2}\right)$. The derivative of h is $h'(x) = f'\left(\frac{x}{2}\right)$. Find the arc length of the graph of $y = h(x)$ from $x = 0$ to $x = 20$.

No Calculator on this problem



1. Let *R* be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the $y-\text{axis}$, as shown in the figure above.

   a) Find the area of *R*.

   b) Write, but do not solve, an integral expression that can be used to find the perimeter of *R*.

   c) Region *R* forms the base of solid. Write, but do not evaluate, an integral expression that can be used to find the volume of this solid if cross-sections taken perpendicular to the *x*-axis are semicircles.

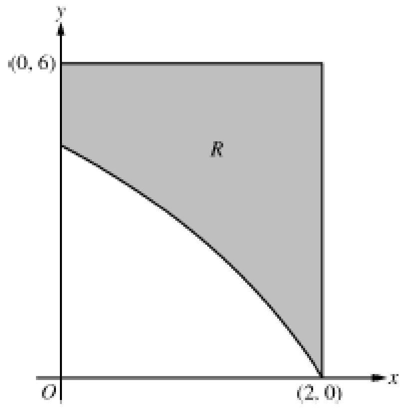
   d) Region *R* is rotated around the line $y = 6$ to form a solid. Write, but do not solve, an integral expression that can be used to find the volume of the solid.

   e) Region *R* is rotated around the *x*-axis to form a solid. Write, but do not solve, an integral expression that can be used to find the volume of the solid.

   f) Region *R* is rotated around the *y*-axis to form a solid. Write, but do not solve, an integral expression that can be used to find the volume of the solid.

   g) Region *R* is rotated around the line $x = 9$ to form a solid. Write, but do not solve, an integral expression that can be used to find the volume of the solid.

A calculator may be used for these problems.



2. In the figure above, *R* is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3-x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

   a) Find the area of *R*.

   b) Find the perimeter of *R*.

   c) The region *R* is the base of a solid. For this solid, each cross section perpendicular to the *x*-axis is a square. Find the volume of the solid.

   d) The region *R* is the base of a solid. For this solid, each cross section perpendicular to the *y*-axis is a quarter-circle. Find the volume of the solid.

   e) Find the volume of the solid generated when *R* is revolved about the horizontal line $y = 6$.

   f) Find the volume of the solid generated when *R* is revolved about the horizontal line $y = 8$.

   g) Find the volume of the solid generated when *R* is revolved about the vertical line $x = 2$.

   h) Find the volume of the solid generated when *R* is revolved about the vertical line $x = -11$.