AP Calculus – Final Review Sheet

When you see the words ….	**This is what you think of doing**
1. Find the zeros	Set function = 0, factor or use quadratic equation if quadratic, graph to find zeros on calculator
2. Find equation of the line tangent to $f(x)$ on $[a,b]$	Take derivative - $f'(a)=m$ and use $y-y_1=m(x-x_1)$
3. Find equation of the line normal to $f(x)$ on $[a,b]$	Same as above but $m=\frac{-1}{f'(a)}$
4. Show that $f(x)$ is even	Show that $f(-x)=f(x)$ - symmetric to y-axis
5. Show that $f(x)$ is odd	Show that $f(-x)=-f(x)$ - symmetric to origin
6. Find the interval where $f(x)$ is increasing	Find $f'(x)$, set both numerator and denominator to zero to find critical points, make sign chart of $f'(x)$ and determine where it is positive.
7. Find interval where the slope of $f(x)$ is increasing	Find the derivative of $f'(x)=f''(x)$, set both numerator and denominator to zero to find critical points, make sign chart of $f''(x)$ and determine where it is positive.
8. Find the minimum value of a function	Make a sign chart of $f'(x)$, find all relative minimums and plug those values back into $f(x)$ and choose the smallest.
9. Find the minimum slope of a function	Make a sign chart of the derivative of $f'(x)=f''(x)$, find all relative minimums and plug those values back into $f'(x)$ and choose the smallest.
10. Find critical values	Express $f'(x)$ as a fraction and set both numerator and denominator equal to zero.
11. Find inflection points	Express $f''(x)$ as a fraction and set both numerator and denominator equal to zero. Make sign chart of $f''(x)$ to find where $f''(x)$ changes sign. (+ to – or – to +)
12. Show that $\lim\limits_{x\to a} f(x)$ exists	Show that $\lim\limits_{x\to a^-} f(x)=\lim\limits_{x\to a^+} f(x)$
13. Show that $f(x)$ is continuous	Show that 1) $\lim\limits_{x\to a} f(x)$ exists ($\lim\limits_{x\to a^-} f(x)=\lim\limits_{x\to a^+} f(x)$) 2) $f(a)$ exists 3) $\lim\limits_{x\to a} f(x)=f(a)$
14. Find vertical asymptotes of $f(x)$	Do all factor/cancel of $f(x)$ and set denominator = 0
15. Find horizontal asymptotes of $f(x)$	Find $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$
16. Find the average rate of change of $f(x)$ on $[a,b]$	Find $\frac{f(b)-f(a)}{b-a}$

17. Find instantaneous rate of change of $f(x)$ at a	Find $f'(a)$
18. Find the average value of $f(x)$ on $[a,b]$	Find $\dfrac{\int_a^b f(x)dx}{b\text{-}a}$
19. Find the absolute maximum of $f(x)$ on $[a,b]$	Make a sign chart of $f'(x)$, find all relative maximums and plug those values back into $f(x)$ as well as finding $f(a)$and $f(b)$ and choose the largest.
20. Show that a piecewise function is differentiable at the point *a* where the function rule splits	First, be sure that the function is continuous at $x = a$. Take the derivative of each piece and show that $\lim\limits_{x \to a^-} f'(x) = \lim\limits_{x \to a+} f'(x)$
21. Given $s(t)$ (position function), find $v(t)$	Find $v(t) = s'(t)$
22. Given $v(t)$, find how far a particle travels on $[a,b]$	Find $\int_a^b \lvert v(t) \rvert dt$
23. Find the average velocity of a particle on $[a,b]$	Find $\dfrac{\int_a^b v(t)dt}{b-a} = \dfrac{s(b)-s(a)}{b-a}$
24. Given $v(t)$, determine if a particle is speeding up at $t = k$	Find $v(k)$and $a(k)$. Multiply their signs. If both positive, the particle is speeding up, if different signs, then the particle is slowing down.
25. Given $v(t)$ and $s(0)$, find $s(t)$	$s(t) = \int v(t)dt + C$ Plug in $t = 0$ to find C
26. Show that Rolle's Theorem holds on $[a,b]$	Show that *f* is continuous and differentiable on the interval. If $f(a) = f(b)$, then find some *c* in $[a,b]$ such that $f'(c) = 0$.
27. Show that Mean Value Theorem holds on $[a,b]$	Show that *f* is continuous and differentiable on the interval. Then find some *c* such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.
28. Find domain of $f(x)$	Assume domain is $(-\infty,\infty)$. Restrictable domains: denominators $\neq 0$, square roots of only non negative numbers, log or ln of only positive numbers.
29. Find range of $f(x)$ on $[a,b]$	Use max/min techniques to rind relative max/mins. Then examine $f(a), f(b)$
30. Find range of $f(x)$ on $(-\infty,\infty)$	Use max/min techniques to rind relative max/mins. Then examine $\lim\limits_{x \to \pm\infty} f(x)$.
31. Find $f'(x)$ by definition	$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$ or $f'(x) = \lim\limits_{x \to a} \dfrac{f(x)-f(a)}{x-a}$

32. Find derivative of inverse to $f(x)$ at $x = a$	Interchange *x* with *y*. Solve for $\frac{dy}{dx}$ implicitly (in terms of *y*). Plug your *x* value into the inverse relation and solve for *y*. Finally, plug that *y* into your $\frac{dy}{dx}$.
33. y is increasing proportionally to y	$\frac{dy}{dt} = ky$ translating to $y = Ce^{kt}$
34. Find the line $x = c$ that divides the area under $f(x)$ on $[a,b]$ to two equal areas	$\int_a^c f(x)dx = \int_c^b f(x)dx$
35. $\frac{d}{dx}\int_a^x f(t)dt =$	2nd FTC: Answer is $f(x)$
36. $\frac{d}{dx}\int_a^u f(t)dt$	2nd FTC: Answer is $f(u)\frac{du}{dx}$
37. The rate of change of population is …	$\frac{dP}{dt} = ...$
38. The line $y = mx + b$ is tangent to $f(x)$ at (x_1, y_1)	Two relationships are true. The two functions share the same slope ($m = f'(x)$) and share the same *y* value at x_1.
39. Find area using left Reimann sums	$A = base[x_0 + x_1 + x_2 + ... + x_{n-1}]$
40. Find area using right Reimann sums	$A = base[x_1 + x_2 + x_3 + ... + x_n]$
41. Find area using midpoint rectangles	Typically done with a table of values. Be sure to use only values that are given. If you are given 6 sets of points, you can only do 3 midpoint rectangles.
42. Find area using trapezoids	$A = \frac{base}{2}[x_0 + 2x_1 + 2x_2 + ... + 2x_{n-1} + x_n]$ This formula only works when the base is the same. If not, you have to do individual trapezoids.
43. Solve the differential equation …	Separate the variables – *x* on one side, *y* on the other. The *dx* and *dy* must all be upstairs.
44. Meaning of $\int_a^x f(t)dt$	The accumulation function – accumulated area under the function $f(x)$ starting at some constant *a* and ending at *x*.
45. Given a base, cross sections perpendicular to the *x*-axis are squares	The area between the curves typically is the base of your square. So the volume is $\int_a^b (base^2)dx$
46. Find where the tangent line to $f(x)$ is horizontal	Write $f'(x)$ as a fraction. Set the numerator equal to zero.
47. Find where the tangent line to $f(x)$ is vertical	Write $f'(x)$ as a fraction. Set the denominator equal to zero.

48. Find the minimum acceleration given $v(t)$	First find the acceleration $a(t)=v'(t)$. Then minimize the acceleration by examining $a'(t)$.
49. Approximate the value of $f(0.1)$ by using the tangent line to *f* at $x=0$	Find the equation of the tangent line to *f* using $y-y_1=m(x-x_1)$ where $m=f'(0)$ and the point is $(0, f(0))$. Then plug in 0.1 into this line being sure to use an approximate $(\approx)$sign.
50. Given the value of $F(a)$ and the fact that the anti- derivative of *f* is *F*, find $F(b)$1	Usually, this problem contains an antiderivative you cannot take. Utilize the fact that if $F(x)$is the antiderivative of *f*, then $\int_a^b F(x)dx = F(b)-F(a)$. So solve for $F(b)$ using the calculator to find the definite integral.
51. Find the derivative of $f(g(x))$	$f'(g(x))\cdot g'(x)$
52. Given $\int_a^b f(x)dx$, find $\int_a^b [f(x)+k]dx$	$\int_a^b [f(x)+k]dx = \int_a^b f(x)dx + \int_a^b kdx$
53. Given a picture of $f'(x)$, find where $f(x)$ is increasing	Make a sign chart of $f'(x)$ and determine where $f'(x)$ is positive.
54. Given $v(t)$ and $s(0)$, find the greatest distance from the origin of a particle on $[a,b]$	Generate a sign chart of $v(t)$ to find turning points. Then integrate $v(t)$ using $s(0)$ to find the constant to find $s(t)$. Finally, find *s*(all turning points) which will give you the distance from your starting point. Adjust for the origin.
55. Given a water tank with *g* gallons initially being filled at the rate of $F(t)$ gallons/min and emptied at the rate of $E(t)$ gallons/min on $[t_1,t_2]$, find a) the amount of water in the tank at *m* minutes	$g+\int_t^{t_2}(F(t)-E(t))dt$
56. b) the rate the water amount is changing at *m*	$\frac{d}{dt}\int_t^m (F(t)-E(t))dt = F(m)-E(m)$
57. c) the time when the water is at a minimum	$F(m)-E(m)$=0, testing the endpoints as well.
58. Given a chart of *x* and $f(x)$ on selected values between *a* and *b*, estimate $f'(c)$ where *c* is between *a* and b.	Straddle *c*, using a value *k* greater than *c* and a value *h* less than *c*. so $f'(c)\approx \frac{f(k)-f(h)}{k-h}$
59. Given $\frac{dy}{dx}$, draw a slope field	Use the given points and plug them into $\frac{dy}{dx}$, drawing little lines with the indicated slopes at the points.
60. Find the area between curves $f(x), g(x)$ on $[a,b]$	$A=\int_a^b [f(x)-g(x)]dx$, assuming that the *f* curve is above the *g* curve.

61. Find the volume if the area between $f(x), g(x)$ is rotated about the x-axis	$A = \int_a^b \left[(f(x))^2 - (g(x))^2 \right] dx$ assuming that the f curve is above the g curve.

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