AP Physics 1 **Universal Gravitation**

Introduction: Astronomy is the oldest science. Practical needs and imagination acted together to give astronomy an early importance. For thousands of years, the motions of the heavenly bodies were carefully observed and recorded. In all sciences, no other field has had such a long accumulation of data as astronomy has had.

In the fourth century B.C., Greek philosophers asked a new question: How can we explain the cyclical changes observed in the sky? That is, what model can consistently and accurately account for all celestial motions? Plato defined the problem this way: The stars – representing eternal, divine, unchanging beings – move at a uniform speed around the earth, as we observe, in that most regular and perfect of all paths, the endless circle. But a few celestial objects, namely the sun, moon and planets, wander across the sky and trace out complex paths, including even retrograde status. Their motions, if not in a perfect circle, must therefore be in some combination of perfect circles.

Prior to the 17th century, people did not understand how the earth moved in relation to the sun. Though Copernicus had asserted in 1543 that the earth was not stationary, his theory was not generally accepted until scientists like Galileo and Tycho Brahe gathered more evidence. Further more, the shape of the earth's orbit and the orbits of the other planets around the sun was still unknown until Jahannes Kepler enunciated his three laws of planetary motion in publications dated 1609 and 1619. Kepler's laws were empirical and it remained for scientists to explain in terms of some theory why these laws were true.

Kepler's Laws of Planetary Motion:

1.) *Law of Ellipses – The planets move in elliptical paths with the sun at one focus.*
2.) *Law of Areas – An imaginary line joining any planet to the sun sweeps out equal areas in equal time intervals.*
3.) *Law of Periods – For any planet in the solar system, the cube of its mean radius from the sun divided by the square of its period of revolution is a constant.* $r^3 / T^2 = K$.

Until the 17th century, gravitational force was thought to be a unique property of the earth. However, Sir Isaac Newton suspected that the earth was not unique among the heavenly bodies. He had already found that motion follows three universal laws. Perhaps the gravitational force of the earth was only one example of a universal force which acts between any two bodies. He found evidence to support this idea in the motion of the heavenly bodies. Using the methods of calculus, a new branch of mathematics he developed, Newton was able to deduce Kepler's three laws of planetary motion from his own law of universal gravitation.

Newton's Law of Universal Gravitation:

Every body in the universe attracts every other body with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Performance Objectives: Upon completion of the readings and activities in this unit, and when asked to respond either orally or on a written test, you will:

- ✓ State Tycho Brahe's contributions to science. Demonstrate an appreciation for the persistence required to gain scientific knowledge.
- ✓ List Kepler's laws of planetary motion. Solve simple problems involving the third law.
- ✓ Display an understanding of how Newton combined basic knowledge to arrive at a completely new law. Recognize the rationale used to determine the relationship between gravitational force and mass. Tell what was new about Newton's law of gravitation in the 17th century and apply the law to a discussion of gravitational fields and weight.
- ✓ Solve problems using the law of gravitation.
- ✓ Clearly indicate an understanding of Newton's test of his law.
- ✓ State the method used by Cavendish to verify Newton's law of universal gravitation. Know the universal gravitation constant.

- ✓ Be able to illustrate how the law of gravitation demands that the acceleration of gravity by a constant in a given location.
- ✓ Describe gravitational fields. Recognize that gravitational fields constitute a method rather than a reality.
- ✓ Understand the difference between true weightlessness and apparent weightlessness.

Textbook Reference: Physics: Chapter 7

"Gravitation can not be responsible for people falling in love."

– Albert Einstein

Problems and Questions: The last page of this packet contains useful information about celestial bodies in our solar system. This data is used in problems throughout this homework packet – so please use it wisely!

1.) Compute Kepler's constant for our solar system. The mean radius of the earth's orbit around the sun is 1.49×10^{11} meters. *3.35×10^{18} m^3/s^2.*

2.) The period of revolution for Saturn is 10,759.2 days. Use Kepler's constant for our solar system to compute the mean radius of the orbit of Saturn. *1.43×10^{12} m*

3.) The mean radius of the orbit of Venus is 1.08×10^{11} meters. Calculate the time for one revolution around the sun for the planet Venus. *224.4 days*

4.) An "Astronomical Unit" is another unit for measuring distance. Astronomical Unit is abbreviated AU. One AU is the length of the mean radius of the earth's orbit about the sun. Calculate Kepler's constant for our solar system in AU^3/yr^2.

5.) The mean radius of Pluto's orbit is 39.6 AU. What is the period of revolution in years for Pluto? *249 yr*

6.) Jupiter is 5.2 times farther than Earth is from the sun. Find Jupiter's orbital period in earth years. *12 years*

7.) Which planets – those closer to the sun than the earth or those farther from the sun than the earth – have a period greater than one earth year?

8.) A Satellite is placed in an orbit with a radius that is half the radius of the moon's orbit. Find its orbital period in units of the period of the moon. *$0.35\ T_m$*

9.) How high does a rocket have to go above Earth's surface until its weight is half what it would be on Earth? *about 2600 km*

"I deduced that the forces which keep the planets in their orbs must be reciprocally as the squares of their distances from the centers about which they revolve; and thereby compared the force requisite to keep the Moon in her orb with the force of gravity at the surface of the Earth; and found them answer pretty nearly."

- Isaac Newton

The Universal Law of Gravitation: An *"Inverse-Square"* Law

As legend has it, Newton was struck on the head by an apple while he was napping under a tree. This 'event' supposedly prompted him to imagine that perhaps all of the bodies in the universe are attracted to each other in the same way that the apple was attracted to the earth. With all of the data on the motion of the moon and the planets that had been colleted by this time, Newton (using his own first law of motion) deduced that there had to be a net force acting on our moon – or any of the planets for that matter – that would keep these bodies moving in their elliptical paths! Otherwise, these bodies would be moving in straight lines right? He further deduced that this force (called gravity) was not restricted to the earth and the moon – or to the sun and the earth – but to every mass in the universe! Newton's Universal Law of Gravitation (published in 1687) states that every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. One of many ***"inverse-square"*** laws observed in the physical world.

$$F_G = G\,\frac{m_1 m_2}{d^2} \quad \text{where } (G) \text{ is } 6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2$$

where m_1 and m_2 are the masses of the two particles that are separated by a distance (d). The constant of proportionality (G) – which would not be measured until 1798 by Sir Henry Cavendish – has been measured to be 6.67×10^{-11} N m^2/kg^2. From Cavendish's important experiment, the scientific world gained knowledge as to the mass of the earth!

10.) Two people are standing 2.0 m apart. One has a mass of 80.0 kg. The other has a mass of 60.0 kg. What is the gravitational force between them? *8×10^{-8} N*

11.) a.) What is the gravitational force between two 800.0 kg cars that are parked 5.0 m apart?
b.) How many times smaller is this gravitational attractive force if they were parked 50.0 m apart?
1.7×10^{-6} N 100 times smaller

12.) Two ships are docked next to each other. Their centers of gravity are 40.0 meters apart. One ship weighs 9.8×10^7 N and the other weighs 1.96×10^8 N. What gravitational force exists between them? *8.3 N*

13.) Two space capsules, of equal mass, are put into orbit 30.0 m apart. The gravitational force between them is 2.0×10^{-7} N. a.) What is the mass of each capsule? b.) What is the initial acceleration given to each capsule by this force? c.) Why does this acceleration change?
1643 kg 1.25×10^{-10} m/s^2

14.) The mass of the earth is 6.0×10^{24} kg. If the centers of the earth and the moon are 3.9×10^8 m apart, the gravitational force between them is about 1.9×10^{20} N. What is the approximate mass of the moon?
7.22×10^{22} kg

15.) a.) Calculate the weight of a 100.0 kg person. b.) Calculate the earth's gravitational attraction for this 100.0 kg person. Use 5.98×10^{24} kg for the mass of the earth and 6.38×10^6 m for the radius of the earth.
980 N

16.) What would a 980 N (as measured on the surface of the earth) person weigh on a dense planet that was twice as massive as the planet earth but only had one-third that of the earth's radius? What about if he were now on a planet that was only one-half as massive as the earth but had a radius 4 times that of the earth?
17,640 N 30.63 N

17.) Compute the gravitational force between a proton and an electron in a hydrogen atom. The mass of a proton is 1.67×10^{-27} kg. The mass of an electron is 9.11×10^{-31} kg. Use 5.29×10^{-11} m as the radius of orbit of the electron.
3.63×10^{-47} N

18.) The moon's mass is 7.35×10^{22} kg and it moves around the earth approximately in a circle of radius 3.82×10^{5} km. The time required for one revolution is 27.3 days. a.) Calculate the centripetal force that must act on the moon. b.) Verify numerically that gravitational attraction supplies the necessary force calculated in part (a) using the additional data that the mass of the earth is 5.98×10^{24} kg. *1.99×10^{20} N* *2.01×10^{20} N*

19.) When a satellite revolves in an orbit around the earth, the centripetal force equals the gravitational attraction. a.) Derive a formula for the period of a satellite in a circular orbit. (Your formula should include the radius "r" of the orbit, the mass M of the earth, and the gravitational constant G. Note that the mass of the satellite cancels out.) b.) Is your formula consistent with Kepler's third law? c.) Use your formula to calculate the period of an earth satellite in a circular orbit of radius 7000 km. *5826 sec*

20.) With what speed does the earth orbit the sun? The mass of the sun is 2.0×10^{30} kg and the radius of the earth's orbit is 1.5×10^{11} m. *29,800 m/s*

21.) a.) Calculate the gravitational force of the sun on the moon. b.) Calculate the gravitational force of the earth on the moon. c.) How is it that the moon goes around the earth and not around the sun? …or does it? d.) How many times larger is your answer to "a" than "b"? Your answer might indicate to you that the sun should be the major factor in causing tides on earth.
4.35×10^{20} N *2.01×10^{20} N*

22.) The weight of an apple near the surface of the earth is 1.0 N. What is the weight of the earth in the gravitational field of the apple?

23.) The earth and the moon are attracted to each other by gravitational force. Does the more massive earth attract the less massive moon with a force that is greater, smaller, or the same as the force with which the moon attracts the earth? Explain.

24.) Strictly speaking, you weigh a tiny bit less when you are in the lobby of a massive skyscraper. Why?

APPARENT WEIGHTLESSNESS

It is clear from our definition of weight, W = mg, that because the mass remains constant, the weight depends upon the particular value of g at the place where the weighing is being made. As the body moves further and further from the center of the earth, the gravitational field strength, and hence the weight, decreases. From $g \propto 1/R^2$ we can see that g is zero when R is infinitely far from the earth (and all other bodies in space). From this we can conclude that a body is never truly weightless. Yet we often speak of astronauts in orbiting spaceships as being "weightless". The confusion is this use of the term arises not from our definition of weight, but from our method of measuring weight. That is, the force of the supporting structure must be considered if the term "weightless" is to make any sense. When you stand on a platform scale, the scale pushes up with a force equal to your weight. You feel this force and you can read its value on the scale. An orbiting astronaut is freely falling around the earth – as is everything with him/her in the orbiting spaceship. Because everything has the same inward acceleration, he senses no upward supporting forces, and he therefore feels "weightless". A better term to describe this situation is to say that his *apparent weight* is zero. His true weight is still mg; it simply depends upon his distance from the center of the earth – not his motion.

As the capsule spirals in to touch down, it is no longer in free fall. The resistance of the earth's atmosphere and the retrorockets combine to exert a tremendous force on the capsule slowing it down. The capsule pushes on the astronaut, and he feels several times his true weight. Later, when the capsule's parachute opens, a tremendous upward force is exerted on the capsule and the astronaut. In this situation, his apparent weight is said to be many g's, that is - many times the weight he normally experiences at the earth's surface.

We see, then, that apparent weight is due to changes in supporting force, while true weight depends upon position only. It is not necessary to be in a space ship to experience weightlessness, amusement park rides can also cause a person to feel "weightless"!

SATELLITES OF THE PLANETS

		DISCOVERY	AVERAGE RADIUS OF ORBIT	PERIOD OF REVOLUTION			DIAMETER
EARTH:	Moon		238,857 miles	27d	7h	43m	2160 miles
MARS:	Phobos	1877, Hall	5,800	0	7	39	10?
	Deimos	1877, Hall	14,600	1	6	18	5?
JUPITER:	V	1892, Barnard	113,000	0	11	53	150?
	1 (Io)	1610, Galileo	262,000	1	18	28	2000
	II (Europa)	1610, Galileo	417,000	3	13	14	1800
	III (Ganymede)	1610, Galileo	666,000	7	3	43	3100
	IV (Callisto)	1610, Galileo	1,170,000	16	16	32	2800
	VI	1904, Perrine	7,120,000	250	14		100?
	VII	1905, Perrine	7,290,000	259	14		35?
	X	1938, Nicholson	7,300,000	260	12		15?
	XII	1951, Nicholson	13,000,000	625			14?
	XI	1938, Nicholson	14,000,000	700			19?
	VIII	1908, Melotte	14,600,000	739			35?
	IX	1914, Nicholson	14,700,000	758			17?
SATURN:	Mimas	1789, Herschel	115,000	0	22	37	300?
	Enceladus	1789, Herschel	148,000	1	8	53	350
	Tethys	1684, Cassini	183,000	1	21	18	500
	Dione	1684, Cassini	234,000	2	17	41	500
	Rhea	1672, Cassini	327,000	4	12	25	1000
	Titan	1655, Huygens	759,000	15	22	41	2850
	Hyperion	1848, Bond	920,000	21	6	38	300?
	Phoebe	1898, Pickering	8,034,000	550			200?
	Iapetus	1671, Cassini	2,210,000	79	7	56	800
URANUS:	Miranda	1948, Kuiper	81,000	1	9	56	—
	Ariel	1851, Lassell	119,000	2	12	29	600?
	Umbriel	1851, Lassell	166,000	4	3	28	400?
	Titania	1787, Herschel	272,000	8	16	56	1000?
	Oberon	1787, Herschel	364,000	13	11	7	900?
NEPTUNE:	Triton	1846, Lassell	220,000	5	21	3	2350
	Nereid	1949, Kuiper	3,440,000	359	10		200?

THE SOLAR SYSTEM

	RADIUS	MASS	AVERAGE RADIUS OF ORBIT	PERIOD OF REVOLUTION
Sun	6.95×10^{8} meters	1.98×10^{30} kilograms	—	—
Moon	1.74×10^{6}	7.34×10^{22}	3.8×10^{8} meters	2.36×10^{6} seconds
Mercury	2.57×10^{6}	3.28×10^{23}	5.79×10^{10}	7.60×10^{6}
Venus	6.31×10^{6}	4.83×10^{24}	1.08×10^{11}	1.94×10^{7}
Earth	6.38×10^{6}	5.98×10^{24}	1.49×10^{11}	3.16×10^{7}
Mars	3.43×10^{6}	6.37×10^{23}	2.28×10^{11}	5.94×10^{7}
Jupiter	7.18×10^{7}	1.90×10^{27}	7.78×10^{11}	3.74×10^{8}
Saturn	6.03×10^{7}	5.67×10^{26}	1.43×10^{12}	9.30×10^{8}
Uranus	2.67×10^{7}	8.80×10^{25}	2.87×10^{12}	2.66×10^{9}
Neptune	2.48×10^{7}	1.03×10^{26}	4.50×10^{12}	5.20×10^{9}
Pluto	?	?	5.9×10^{12}	7.28×10^{9}