## AP Physics C

Momentum

## Free Response Problems



1. A bullet of mass $m$ moves at a velocity $v_{0}$ and collides with a stationary block of mass $M$ and length $L$. The bullet emerges from the block with a velocity of $v_{0} / 3$. Assuming the friction between the block and the surface is negligible, answer the following:
a. Find the velocity of the block after the collision.
b. Find the change in kinetic energy of the bullet-block system.

The bullet emerges from the block with a velocity $\mathrm{v}_{0} / 2$ when the block is held fixed.
c. Find the change in kinetic energy of the bullet-block system.
d. Assuming that the retarding force in both trials is the same, find the minimum thickness $L_{\text {min }}$ of the block in order to stop the bullet.
e. Explain the difference in answers (b) and (c).

2. A rifle bullet with a mass $m=9.00 \mathrm{~g}$ strikes and embeds into a wooden block with a mass $\mathrm{M}=1.00 \mathrm{~kg}$ that rests on a frictionless, horizontal surface and is attached to a spring. After the collision the spring compresses by a distance of 18.0 cm . Assuming the spring obeys Hook's Law and has a spring constant $k=300 \mathrm{~N} / \mathrm{m}$ :
a. Find the velocity of the bullet-block system after the collision.
b. Find the velocity of the bullet before the collision.
c. Find the energy loss due to the collision.
d. Find the period of oscillations.

3. A bullet with a mass $m$ is fired into a wooden block that lies on the top of a frictionless horizontal table. The bullet becomes embedded after the collision with the block and the block slides off the table, landing at a distance $x$ from the edge of the table. The height of the table is $h$.
a. Find the velocity of the bullet-block system after the collision.
b. Find the initial velocity of the bullet before the collision.
c. Find the ration between the initial kinetic energy of the bullet and the kinetic energy of the bullet-block system.

In the second experiment the bullet is fired with the same velocity and the block is placed at a distance $L$ from the right end of the table. The friction is present in this trial.
d. Find the minimum coefficient of kinetic friction so that the block doesn't leave the table.

4. A dart with a mass $m$ is fired into a wooden block of mass $M$ suspended at the end of a light string of length L. After the dart strikes the block it becomes embedded in the block. The block swings to the right and the string makes an angle $\theta$ with the vertical.
a. Determine the velocity of the block after the collision.
b. Determine the velocity of the dart before the collision.
c. Determine the ratio of the initial kinetic energy to the final kinetic energy.
d. Determine the tension in the string when the block returns to the equilibrium point.
5. A wooden block of mass 3 m lies on the bottom of a frictionless circular track with a radius $R$. A small bullet of mass $m$ is fired from a gun with an initial velocity $v_{0}$. The bullet strikes the block and remains embedded in the block as it circles the loop.
a. Find the minimum velocity of the bullet that the block can rise to the top of the loop.
b. Find the minimum velocity of the bullet that the block can complete the loop.
c. Find the normal force on the block from the ramp when it returns to the lowest point of the loop.


Figure I


## Figure II

6. A small ice cube with a mass $m$ slides on a horizontal frictionless surface to the right with velocity $\mathrm{v}_{0}$ towards a large inclined ramp of mass 5 m , which is initially at rest (See Figure I). The ramp is movable and can slide without friction on the same surface. When the cube slides up the ramp it reaches a maximum height $h$ and then slides down the incline. (See Figure II).
a. Find the speed of the ramp when the cube is at the highest point.
b. Find the maximum height $h$ to which the cube rises.
c. Find the final speed of the ramp when the cube returns back to the horizontal surface.
d. Find the final speed of the cube when it is on the horizontal surface.

7. A ball of mass 4 m is dropped from rest from a height $\mathrm{h}=10.0 \mathrm{~m}$ above the ground, as shown above. It undergoes a perfectly elastic collision with the ground and rebounds. At the instant that the ball rebounds, a small piece of clay of mass $m$ is released from rest from the original height h , directly above the ball, as shown above on the right. The clay, which is descending, eventually collides with the ball, which is ascending. Assume that $\mathrm{g}=$ $10 \mathrm{~m} / \mathrm{s}^{2}$, air resistance is negligible, and the collision process takes negligible time.
a. Determine the speed of the ball immediately before it hits the ground.
b. Determine the time after the piece of clay is released at which the collision takes place.
c. Determine the height above the ground at which the collision takes place.
d. Determine the speeds of the ball and the piece of clay immediately before the collision.
e. If the ball and the clay stick together on impact, what is the magnitude and direction of their velocity immediately after the collision?


## Figure II

8. A light spring with a force constant $k=500 \mathrm{~N} / \mathrm{m}$ is attached at its left end to a vertical wall, as shown in Figure I. Initially, block A of mass $m_{a}=6.0 \mathrm{~kg}$ and block B of mass $m_{B}=1.5$ kg rest on a horizontal surface with block A in contact with the spring (but not compressing it) and with block B in contact with block A. Block A is then moved to the left, compressing the spring a distance of 0.50 m , and held in place while block $B$ remains at rest as shown in Figure II. (Use g=10 m/s ${ }^{2}$.)
a. Determine the elastic energy stored in the compressed spring.

Block $A$ is then released and accelerates to the right, towards block $B$. The surface is rough and the coefficient of friction between each block and the surface is $\mu=0.3$. The two blocks collide instantaneously, stick together, and move to the right. Remember that the spring is not attached to block A. Determine each of the following:
b. The speed of block $A$ just before it collides with block $B$.
c. The speed of blocks $A$ and $B$ just after they collide.
d. The horizontal distance the blocks move before coming to rest.

9. A block of mass $M_{1}=2 \mathrm{~kg}$ moves toward block $\mathrm{M}_{2}$ which is attached to a light spring. The second end of the spring is fixed to the wall. The graph below shows the velocity of block $M_{1}$ as a function of time before and after the collision. Before the collision, block $M_{1}$ moves with a velocity $4 \mathrm{~m} / \mathrm{s}$ and after the inelastic collision the velocity drops down to $2 \mathrm{~m} / \mathrm{s}$.

a. Calculate the mass of block $\mathrm{M}_{2}$.
b. After the collision, the two blocks move with a velocity given by the following formula: $v=4 e^{-8 t}$. Find the stopping distance.
c. Before the collision, block $M_{1}$ was moving with a decreasing velocity because of a resistance force. The velocity was changing according to the following formula: $\frac{1}{V}=\frac{1}{5}+\alpha$, where $\alpha$ is a constant. Find the expression for the resistance force as a function of time and velocity.

## Removable Stopper

## 0.2 kg


10. A 2.5 kg block and a 7.5 kg block are both attached to an ideal spring (for which $\mathrm{k}=250$ $\mathrm{N} / \mathrm{m}$ ) and both are initially at rest on a horizontal frictionless surface, as shown in the diagram above.

In an initial experiment, a 0.2 kg piece of clay is thrown at the 2.5 kg block. The clay is moving horizontally with a speed v when it hits and sticks to the block. The 7.5 kg block is held still by a removable stopper. As a result, the spring compresses a maximum distance of 0.3 meters.
a. Calculate the energy stored in the spring at maximum compression.
b. Calculate the speed of the clay ball and 2.5 kg block immediately after the clay sticks to the block but before the spring compresses significantly.
c. Calculate the initial speed $v$ of the clay.

In a second experiment, an identical piece of clay is thrown at another identical 2.5 kg block, but this time the stop is removed so that the 7.5 kg block is free to move.
d. State whether the maximum compression of the spring will be greater than, equal to, or less than 0.3 meters. Explain briefly.

11. A 0.3 kg frame is suspended from a spring with the second end of the spring attached to a ceiling. The frame stretches the spring by 0.5 m . A piece of clay with a mass of 0.4 kg is dropped onto the frame from a height of 0.4 m .
a. Find the velocity of the clay-frame system after the collision.
b. Find the energy loss due to the collision.
c. Find the maximum distance the frame moves down from its initial position.
d. Find the period of oscillations.

12. A heavy block B of mass 4.5 kg is initially at rest at the edge of a 1.2 m high frictionless table. The table has a length $L=2 \mathrm{~m}$. A light block $A$ of mass 0.4 kg slides across the table at a speed of $25 \mathrm{~m} / \mathrm{s}$ and strikes block B, causing block B to leave the table in the direction in which block A was moving. The figure below shows a graph of the force exerted on block $B$ by block $A$ as a function of time.

a. Determine the total impulse given to block B.
b. Determine the horizontal velocity of block B immediately after the collision.
c. Determine the following for block A immediately after the collision:
I. Its speed
II. Its direction of travel (right or left), if moving
d. Determine the kinetic energy dissipated in the collision.
e. Determine the distance between the two points of impact of the blocks with the floor.

## Answer-key:

1. a. $\frac{2 m v_{o}}{3 M}$
b. $\frac{2 m v_{o}{ }^{2}(m-2 M)}{9 M}$
c. $\frac{-3 m v_{o}{ }^{2}}{8}$
d. $4 \mathrm{~L} / 3$
e. In (b) block M is given Kinetic Energy, lowering the speed of the bullet. In (c) all of the Kinetic energy goes to bullet, making its velocity greater than what was in (b).
2. a. $3.1 \mathrm{~m} / \mathrm{s}$
b. $347.54 \mathrm{~m} / \mathrm{s}$
c. 538.68 J is lost
d. 0.36 s
3. a. $\frac{x}{\sqrt{2 h / g}}$
b. $\frac{(M+m) x}{m \sqrt{2 h / g}}$
C. $\frac{m+M}{m}$
d. $\frac{x^{2}}{4 H L}$
4. a. $\sqrt{2 g L(1-\cos \theta)}$
b. $\frac{(m+M) \sqrt{2 g L(1-\cos \theta)}}{m}$
c. $\frac{m+M}{m}$
d. $(m+M) g *(3-2 \cos \theta)$
5. a. $8 \sqrt{g R}$
b. $4 \sqrt{5 g R}$
c. 24 mg
6. a. $\frac{v_{0}}{6}$
b. $\frac{5 v_{o}{ }^{2}}{12 g}$
c. $\frac{v_{0}}{3}$
d. $\frac{-2 v_{o}}{3}$
7. a. $14 \mathrm{~m} / \mathrm{s}$
b. $0.71 \mathrm{~m} / \mathrm{s}$
c. $7.53 \mathrm{~m} / \mathrm{s}$
d. Ball $=6.96 \mathrm{~m} / \mathrm{s} ;$ Clay $=-6.96 \mathrm{~m} / \mathrm{s}$
e. $4.18 \mathrm{~m} / \mathrm{s}$; up
8. a. 62.5 J
b. $4.23 \mathrm{~m} / \mathrm{s}$
c. $3.38 \mathrm{~m} / \mathrm{s}$
d. 1.94 m
9. a. 2 kg
b. $\frac{1}{2}$ meters
c. $-M_{1} \alpha v^{2}$
10. a. 11.25J
b. $2.89 \mathrm{~m} / \mathrm{s}$
c. $39.02 \mathrm{~m} / \mathrm{s}$
d. Less. The initial Kinetic Energy id not only transferred into the 2.5kg block, like in the first scenario. Now, the initial Kinetic Energy is split in both the first and second block.
11. a. $-1.62 \mathrm{~m} / \mathrm{s}$
b. 0.68 J is lost
c. 0.55 m down
d. 2.15 s
12. a. $12 \mathrm{~N}-\mathrm{s}$
b. $2.67 \mathrm{~m} / \mathrm{s}$
c. i. $5.04 \mathrm{~m} / \mathrm{s}$
ii. Left
d. 103.88 J
e. 5.79 m
