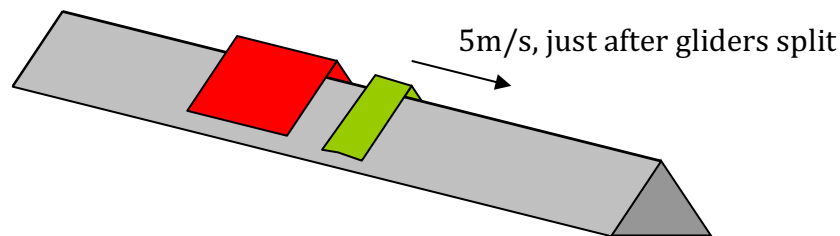This test covers momentum, impulse, conservation of momentum, elastic collisions, inelastic collisions, perfectly inelastic collisions, 2-D collisions, and center-of-mass, with some problems requiring a knowledge of basic calculus.

Part I. Multiple Choice

1.

5m/s, just after gliders split

An air track glider of mass M is built, consisting of two smaller connected gliders with a small explosive charge located between them. The glider is traveling along a frictionless rail at 2 m/s to the right when the charge is detonated, causing the smaller glider with mass $\frac{1}{4}M$, to move off to the right at 5 m/s. What is the final velocity of the second small glider?

 a. 4 m/s to the left
 b. 2 m/s to the left
 c. 1 m/s to the left
 d. 0 m/s
 e. 1 m/s to the right

2. A force acting on an object varies as a function of time according to the equation $\mathbf{F} = k\,t^2$, where k is a constant. If the object had an initial momentum of 0 at time $t = 0$, what is the momentum of the object at time t?

 a. $2t$

 b. $2kt$

 c. $\dfrac{1}{2}kt^3$

 d. $\dfrac{1}{3}kt^3$

 e. $\dfrac{1}{3}kt^2$

3. A student with mass M is standing on a wooden plank of mass m that is less than the mass of the student. The plank itself is resting on the frictionless surface of a frozen lake. The student then begins to walk with a speed v toward the nearby shore. What is the velocity of the plank, relative to the shore?
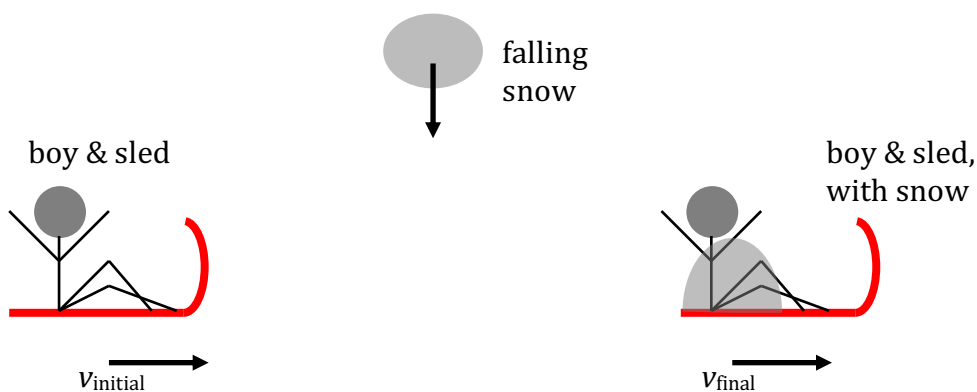   a. v, away from the shore
   b. Less than v, away from the shore
   c. Less than v, toward the shore
   d. More than v, away from the shore
   e. More than v, toward the shore

4.
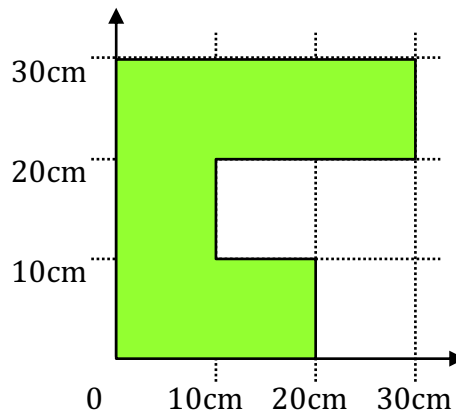
A flat piece of metal of uniform density has the shape and dimensions shown here. The center of mass for the piece of metal is located at:

	x_{CM} (cm)	y_{CM} (cm)
a.	$\dfrac{65}{6}$	$\dfrac{95}{6}$
b.	$\dfrac{70}{6}$	$\dfrac{95}{6}$
c.	$\dfrac{65}{6}$	$\dfrac{100}{6}$
d.	$\dfrac{55}{6}$	$\dfrac{80}{6}$
e.	$\dfrac{70}{6}$	$\dfrac{100}{6}$

falling
snow

boy & sled

boy & sled,
with snow

$v_{initial}$

v_{final}

5. A boy on a sled is sliding with negligible friction along an icy, horizontal surface. As the sled passes underneath a tree, a large mass of snow falls vertically and lands on the moving sled. Which of the following statements is *false*?

    a.   The snow collides inelastically with the sled.
    b.   The sled will slow down when the snow hits it.
    c.   Conservation of kinetic energy cannot be used to find the final velocity of the sled.
    d.   Conservation of mechanical energy cannot be used to find the final velocity of the sled.
    e.   Conservation of linear momentum cannot be used to find the final velocity of the sled.

6.
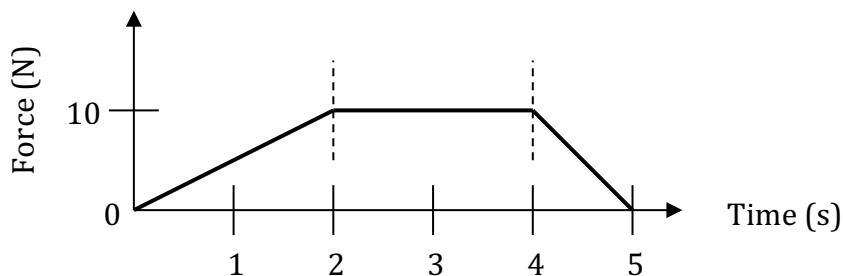
An object of mass $m = 2.0$kg experiences a force in Newtons according to the *Force* vs. *time* graph shown here. For the time interval shown, what is the total change in momentum of the object?

    a.   35 kg•m/s
    b.   70 kg•m/s.
    c.   -35 kg•m/s
    d.   -70 kg•m/s
    e.   none of these.

Part II. Free Response

7.

A 500-gram cart rolls with negligible friction along a straight flat track until it collides with a 750-gram cart that was initially at rest. Position-time data for the 500-gram cart before it hits the other cart is recorded in the data table below.

x-position of 500-gram cart (cm)	0	10	20	30	40
time (s)	0	0.5	1	1.5	2

a. At time $t = 2$ seconds, the two carts collide in a perfectly inelastic collision. Calculate the final velocity of the 500-gram cart after this collision.

b. The collision takes place over a time period of 0.5 seconds. Draw and label a graph of the velocity of *both* carts as a function of time for the time period $t = 0$ to 4 seconds.
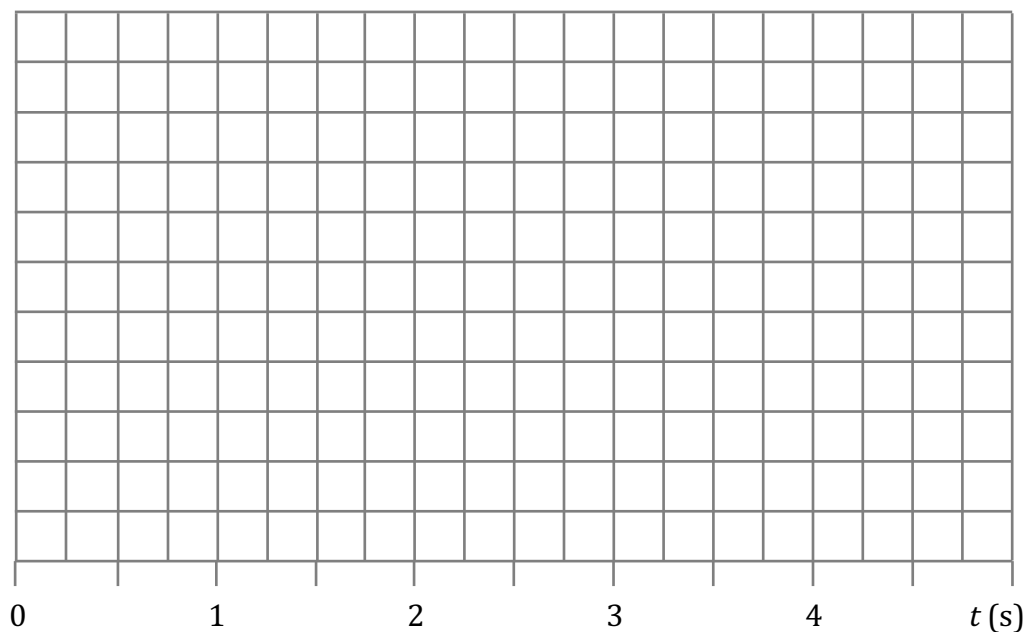
c.   Determine the impulse imparted to the 750g cart.

d.   Determine how much mechanical energy was converted to heat in the collision.

e.   Determine the magnitude of the average force acting on the 500-g cart during the collision.

f.   Calculate the velocity of the center-of-mass of the two cart system *before* the collision occurred.

8.
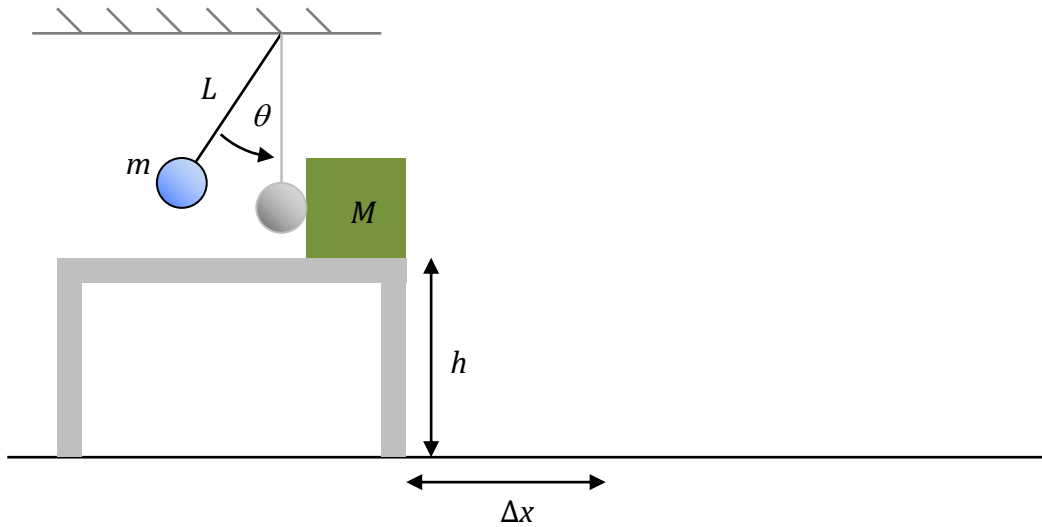
A pendulum of length $L = 1.0$ meter and bob with mass $m = 1.0$ kg is released from rest at an angle $\theta = 30°$ from the vertical. When the pendulum reaches the vertical position, the bob strikes a mass $M = 3.0$ kg that is resting on a frictionless table that has a height $h = 0.85$m.

    a.   When the pendulum reaches the vertical position, calculate the speed of the bob just before it strikes the box.

    b.   Calculate the speed of the bob and the box just after they collide elastically.

c. Determine the impulse acting on the box during the collision.

d. Determine how far away from the bottom edge of the table, Δx, the box will the box strike the floor.
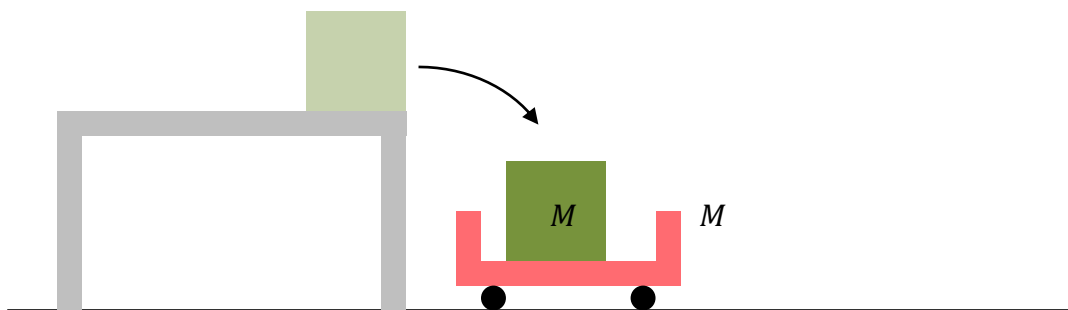
At the location where the box would have struck the floor, now a small cart of mass $M = 3.0$ kg and negligible height is placed. The box lands in the cart and sticks to the cart in a perfectly inelastic collision.

e. Calculate the horizontal velocity of the cart just after the box lands in it.

9.
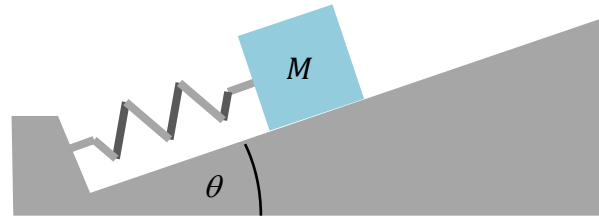
A light, ideal spring with a spring constant $k = 100$ N/m and uncompressed length $L = 0.30$ m is mounted to the fixed end of a frictionless plane inclined at an angle $\theta = 30°$ as shown above. Then a mass M 0.300 kg is affixed to the end of the spring and allowed to slowly slide down the plane until it is resting in an equilibrium position. The dimensions of the masses in this problem are insignificant. Give answers to all questions in terms of stated variables and fundamental constants.

    a.   Draw a free-body diagram of the forces acting on the mass M at this equilibrium position, x_0. (Do *not* include components in your diagram.)

    b.   Determine the distance Δx that the spring compressed as M was lowered down the ramp.

    c.   How much energy is stored in the spring relative to its original uncompressed length L?
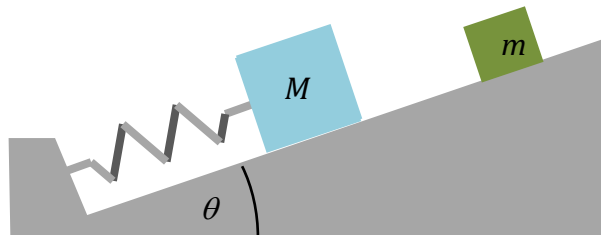
Now, at a distance $d = 0.30$ m above x_0, a second mass $m = 0.200$ kg is now placed, and released so that it slides down the ramp to collide with mass M.

    d.   Calculate the speed of m just before it strikes mass M.

    e.   Determine the speed of mass M and m just after they stick together in a perfectly inelastic collision.

    f.   Determine the maximum compression x' of the spring relative to x_0 when the two masses momentarily come to rest.
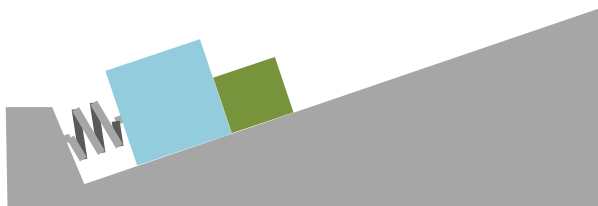
In a new experiment with the same equipment, mass M is used to compress the spring to the same position as in (f) above, and mass m is placed in contact against M as shown. When the spring is released, M remains attached to the spring, and m is launched up the ramp.

    g. Will mass m reach a position that is *higher*, *lower*, or *equal to* the position that it was released from in the previous experiment? Explain your answer, briefly.

    h. Calculate the position relative to x_0 at which the two masses will separate from each other during the launch.

1. The correct answer is e. This is a conservation of momentum problem, in which the total momentum of the glider at the beginning of the problem is equal to the sum of the momenta of the individual gliders at the end of the problem.

$$Mv = m_1 v_1' + m_2 v_2'$$

$$Mv = \tfrac{1}{4} M v_1' + \tfrac{3}{4} M v_2'$$

$$v = \tfrac{1}{4} v_1' + \tfrac{3}{4} v_2'$$

$$v_2' = \tfrac{4}{3}(v - \tfrac{1}{4} v_1')$$

$$v_2' = \tfrac{4}{3}(2 - \tfrac{1}{4} 5') = 1 m/s$$

2. The correct answer is d. The relationship between Force and change in momentum is described by the equation $\Delta p = \int_{t_i}^{t_f} F \bullet dt$.

$$\Delta p = \int_{t_i}^{t_f} F \bullet dt$$

$$p_f - p_i = \int_0^t kt^2 \bullet dt$$

$$p_f - 0 = \frac{1}{3} kt^3$$

3. The correct answer is **d**. The problem can be analyzed using $F_{net} = ma$ or conservation of momentum. Using a momentum analysis, consider the total momentum of the system to be zero at the beginning of the problem.

$$m_{student} v_{student} + m_{plank} v_{plank} = m_{student} v_{student}' + m_{plank} v_{plank}'$$

$$M(0) + m(0) = M(v_{student}) + m(v_{plank})$$

$$v_{plank} = -\frac{M}{m} v_{student}$$

Here, the negative sign indicates that the plank is moving in a direction opposite that of the student (ie. away from the shore), and the fact that M>m means that the velocity of the plank is going to be greater than the velocity of the student.

4. The correct answer is e. The center of mass, in both x- and y-directions, is calculated

using $x_{cm} = \dfrac{\sum x_i m_i}{\sum m_i}$. Although the object can be thought of as six small pieces, it's faster
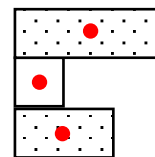
and easier to calculate the center of mass based on dividing the figure up into 3 symmetric pieces as shown, and using the center of mass of each of those pieces in the calculation:

$$x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3} = \frac{3m(15cm) + 1m(5cm) + 2m(10cm)}{6m} = \frac{70}{6}cm$$

$$y_{cm} = \frac{3m(25cm) + 1m(15cm) + 2m(5cm)}{6m} = \frac{100}{6}cm$$

5. The correct answer is **e**. Conservation of linear momentum *can* be used to solve this problem. The linear momentum of the sled in the horizontal direction remains constant, because no net force is applied in that direction.

$$F_x t = \Delta p_x$$

$$0 = \Delta p_x$$

$$m_i v_i = m_f v_f$$

$$m_{sled\,\&\,boy} v_i = (m_{sled\,\&\,boy} + m_{snow})v_f$$

$$v_f = \frac{m_{sled\,\&\,boy}}{m_{sled\,\&\,boy} + m_{snow}} v_i$$

All the other statements in the problem are true. The snow *does* collide inelastically with the sled, the sled *does* slow down when the snow hits it (as shown in our analysis), and conservation of kinetic and mechanical energies cannot be used to solve for velocity because energy is converted to heat in the collision between the snow and the sled.

6. The correct answer is **a**. This is an impulse problem, and requires using the equation $Ft = m\Delta v$. Although we don't know the change in velocity to calculate change in momentum, we can easily determine the impulse by adding up the total area under the curve of the *Force-time* graph.

For the first line segment, $Area = \frac{1}{2}(10N \cdot 2s) = 10N \cdot s$
For the second line segment, $Area = (10N \cdot 2s) = 20N \cdot s$
For the third line segment, $Area = \frac{1}{2}(10N \cdot 1s) = 5N \cdot s$

The total change in momentum is the sum of these areas, or 35N•s.

7.
   a. Use the data table to determine that the $v_{initial}$ of the 500 g cart is

   $$v = \frac{\Delta x}{\Delta t} = \frac{40cm}{2s} = 20cm/s = 0.20m/s$$

   The collision is perfectly inelastic, so
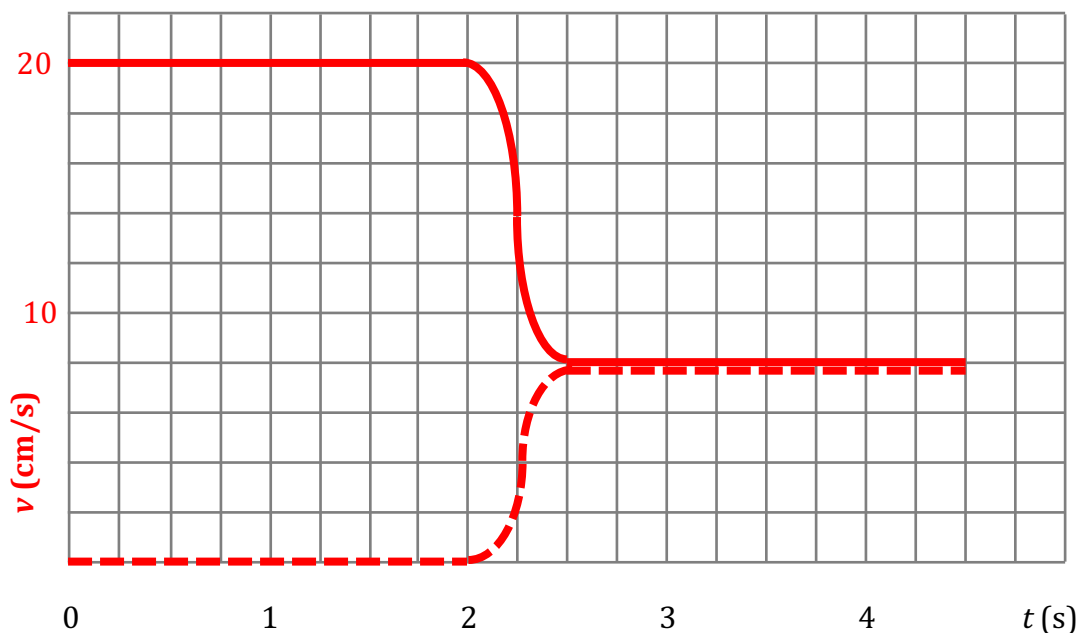
   $$p_{500} + p_{750} = p'_{500} + p'_{750}$$

   $$mv_{500} + mv_{750} = (m_{500} + m_{750})v'$$

   $$(.5kg)(.20m/s) + 0 = (1.25kg)v'$$

   $$v' = 0.08m/s, \text{ to the right}$$

b.

c. The impulse acting on the cart could be determined by looking at the area under the curve of a Force-time graph, but we don't have that here. Impulses produce a change in momentum, however, and we do have that information.

$$\text{Impulse } J = Ft = m\Delta v$$

$$J = m(v_f - v_i) = (0.750kg)(0.08m/s - 0)$$

$$J = 0.06kg \bullet m/s \text{ or } 0.06N \bullet s$$

Notice that I've chosen to *kg* and *m/s* for the units here, because those are the standard SI base units for mass and velocity, and I'll need to be using those units if I plan on calculating Joules or Newtons.

d. Mechanical energy refers to kinetic and potential energies. Here, the only energies of concern are kinetic, both before and after the collision.

$$\sum K_{initial} = \sum K_{final}$$

$$K_{500} - \Delta E_{internal} = K_{500+750}$$

$$\Delta E_{internal} = K_{500} - K_{500+750}$$

$$\Delta E_{internal} = \frac{1}{2}m_{500}v_i^2 - \frac{1}{2}m_{500+750}v_f^2$$

$$\Delta E_{internal} = \frac{1}{2}(0.5kg)(0.20m/s)^2 - \frac{1}{2}(1.25kg)0.8m/s)^2$$

$$\Delta E_{internal} = 0.006J$$

e. The average Force can be determined using the Impulse and the time of contact, where the Impulse on the 500-gram cart is equal to the impulse on the 750-gram cart (which we've already determined in (c) above).

$$\text{Impulse } J = Ft$$

$$0.06N \bullet s = F(0.5s)$$

$$F_{avg} = 0.12N$$

f. The easy way to solve this is to realize that the velocity of the center of mass *before* and *after* the collision is the same, as long as there aren't any external forces acting on the system. The velocity of the center of mass *after* the collision is simply the velocity of the two carts: 0.08 m/s.

The hard way to solve this is to calculate the velocity of the center of mass based on the sum of the individual momenta of the two carts:

$$p_{cm} = \sum p_i m_i$$

$$Mv_{cm} = m_{500}v_{500} + m_{750}v_{750}$$

$$v_{cm} = \frac{m_{500}v_{500} + m_{750}v_{750}}{M}$$

$$v_{cm} = \frac{(0.5)(0.2) + (0.75)(0)}{1.25} = 0.08m/s$$

8.

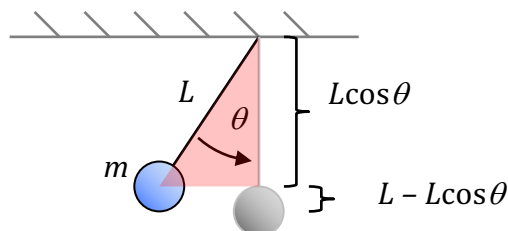a. The potential energy of the pendulum is converted to kinetic energy:

$$U_i = K_f$$

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$h = L - L\cos\theta$$

$$v = \sqrt{2 \bullet 9.8 \bullet (1 - 1\cos 30)} = 1.62m/s$$

b. In an elastic collision, both momentum and kinetic energy (p and K) are conserved. One of the shortcuts to solving one-dimensional elastic collisions, derived in most physics textbooks, is this equation: $v_{1i} + v_{1f} = v_{2i} + v_{2f}$

We can use this equation along with the conservation of momentum equation to determine the velocities of both objects just after the collision:

$$m_{ball}v_{ball} + m_{block}v_{block} = m_{ball}v'_{ball} + m_{block}v'_{block}$$

$$(1)(1.62) + 0 = 1(v'_{ball}) + 3(v'_{block})$$

$$v_{ball} + v'_{ball} = v_{block} + v'_{block}$$

$$1.62 + v'_{ball} = 0 + v'_{block}$$

$$v'_{block} = 0.81m/s$$

$$v'_{ball} = -0.81m/s$$

c. The impulse acting on the box during the collision can be calculated by getting the change in momentum of the box during the collision.

$$\text{Impulse } J = Ft = m(v_f - v_i)$$

$$J = (3kg)(0.81m/s - 0) = 2.43kg \bullet m/s, \text{ or } 2.43\text{N}\bullet\text{s}$$

d. This is a projectile problem, with no initial vertical velocity for the box as it leaves the table with a horizontal velocity as calculated in (b) above.

Vertically:

$$\Delta y = v_i t + \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(-0.85m)}{-9.8m/s^2}} = 0.416s$$

Horizontally:
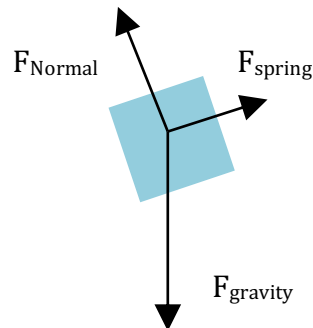
$$\Delta x = v_x t$$

$$\Delta x = (0.81m/s)(0.42s) = 0.34m$$

e. Use conservation of momentum in the *x*-direction, based on the component of the block's velocity in the *x*-direction:

$$m_{block}v_{block} + m_{cart}v_{cart} = (m_{block} + m_{cart})v'$$

$$v' = \frac{m_{block}v_{block} + 0}{m_{block} + m_{cart}} = \frac{(3kg)(0.81m/s)}{3kg + 3kg} = 0.41m/s$$
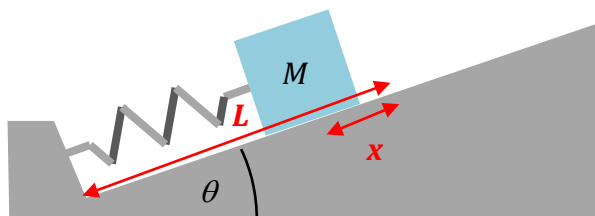
9.

a.

F_{Normal} F_{spring}

$F_{gravity}$

b. The force of gravity acting down the ramp ($F_{\text{g-parallel}}$) pulls down on the mass until the force of the spring is pushing up with an equal amount of force. (See diagram next page.)

$$F_{spring} = F_{g-parallel}$$

$$kx = mg\sin\theta$$

$$x = \frac{mg\sin\theta}{k} = \frac{(0.3)g\sin(30)}{100} = 0.0147m$$

c. The potential energy stored in the spring is calculated based on the displacement from its unstretched position.

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(100)(0.0147)^2 = 0.0108J$$

Note that although this is energy stored in the spring relative to its uncompressed length, under these conditions, it's common to consider the mass's current position as x_0, because this is where the mass-spring system is in equilibrium. This emphasizes the point that it is *changes* in potential energy that are significant–one cannot calculate a potential energy for something without specifying a reference point at which the potential energy is 0.

d. The mass slides down the ramp without friction. Use kinematics or energy to determine the velocity:

$$U_g = K_f$$

$$mgh = \frac{1}{2}mv^2$$
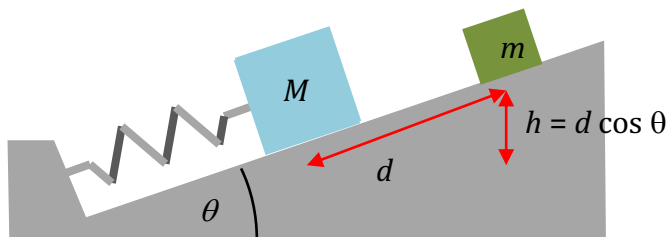
$$v = \sqrt{2gh}, \text{ where } h = x\sin\theta$$

$$v = \sqrt{2(9.8)(0.3\sin 30)} = 1.71m/s$$

or

$F_{net} = ma$ for block sliding down

$$a = \frac{F_{net}}{m} = \frac{F_{parallel}}{m} = \frac{mg\sin\theta}{m} = g\sin\theta$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$v_f = \sqrt{2ax} = \sqrt{2(9.8\sin 30)(0.3)} = 1.71m/s$$

e. By conservation of momentum:

$$m_{0.2}v_{0.2} + m_{0.3}v_{0.3} = (m_{0.2} + m_{0.3})v_{final}$$

$$(0.2)(1.71) + 0 = (0.2 + 0.3)v_{final}$$

$$v_{final} = 0.684m/s$$

f. Let's use conservation of energy after the impact to figure out how far the spring compresses. The two masses are slowing down *and* losing height as they move down the ramp–that energy goes into elastic potential energy stored in the spring. There is a subtlety here, though. When the spring was compressed by mass M, the spring more or less nullified gravity as M came to equilibrium. Pushing M farther gives it spring potential energy, but not gravitational potential energy. On the other hand, the small mass m isn't a part of that system, so you have to keep track of its gravitational potential energy. With all of that in mind, we can write:

$$K_i + U_{g-f} + U_{spring-i} = K_f + U_{g-f} + U_{spring-f}$$

$$\frac{1}{2}m_{total}v^2 + m_1gh + 0 = 0 + 0 + \frac{1}{2}kx^2; \ h = x\sin\theta$$

$$\frac{1}{2}(0.5)(0.684)^2 + (0.2)(9.8)(x'\sin 30) = \frac{1}{2}(100)x'^2$$

Solve quadratic equation to get

$$x' = 0.059 \text{ m}$$

g. *m* will reach a *lower* position than it was originally released from. In the first experiment when the mass was released from high on the ramp, some of its energy was converted to heat in the inelastic collision, so the *K* of the *mM* blocks as they began to compress the spring was less than it would have been otherwise. In launching the blocks the *K* of the system at equilibrium is the same as it was before, but this will not be sufficient to allow *m* to reach its original height. Also, some of U_{spring} has gone into giving M velocity up the ramp–there is less energy, then, for *m* to be able to return to its original position.

h. As the masses are pushed back up the ramp by the spring, they experience less acceleration–the combined weight of the masses produces a greater $F_{g-parallel}$, and thus the net Force acting on the masses is less. Still, the spring will continue to push up on the masses until *M* reaches the old equilibrium position, at which point the spring begins to pull back on *M*. Now *M* has two forces pulling down on it–$F_{g-parallel}$ and F_{spring}–leaving mass *m* free to continue moving up the ramp, with only $F_{g-parallel}$ impeding its motion. Thus, the two masses separate at x_0.