B **B3B037** **Total Pages:3**

Reg. No.____________ Name:________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION, JANUARY 2017

Course Code**: CS 201**

Course Name**: DISCRETE COMPUTATIONAL STRUCTURES (CS, IT)**

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each Question carries 3 marks

1. Show that (A-B) – C=A-(B U C)
2. Show that the set of integers of positive, negative and zero are denumerable.
3. Show that if any five integers from 1 to 8 are chosen, then atleast two of them will have a sum 9.
4. Define:Partition,antisymmetric, Semigroup homomorphism.

PART B

Answer any two questions. Each Question carries 9 marks.

5. a. Prove that every equivalence relation on a set generates a unique partition of the set and the blocks of this partition corresponds to R-equivalence classes. (4.5)

   b.Let X = {1, 2 ……7 }and R = {< X,Y> / X-Y is divisible by 3}. Show that R is an equivalence relation. Draw the graph R. (4.5)

6. a. In how many ways can the letters of the word MONDAY be arranged? How many of them begin with M and end with Y?How many of them do not begin with M but end with Y? (4)

   b. Solve $a_{n+2} – 4 a_{n+1} + 4a_n = 2^n$, $a_0=1$,$a_1=1$ (5)

7. a. Draw Hasse diagram for D_{100}. Find GLB and LUB for B={10, 20}B={5,10,20,25 } (3)

   b. Let X= {1,2,3} and f,g,h be function from X to X given by f ={(1,2) , (2,3) , (3,1)} g = {(1,2),(2,1),(3,3)} h = { (1,1), (2,2),(3,1) }. Find f og, goh, f o h o g. (3)

   c. In a class of 25 students, 12 have taken Mathematics, 8 have taken Mathematics but not Biology. Find the number of students who have taken Mathematics and Biology and those who have taken Biology but not Mathematics. (3)

PART C

Answer All Questions. Each Question carries 3 marks.

8. Show that inverse of an element a in the group is unique.
9. Show that $(G,+_6)$ is acyclic group where G = {0,1,2,3,4,5}
10. A= {2, 3, 4,6,12,18,24,36} with partial order of divisibility. Determine the POSET is a lattice.
11. Consider the lattice D_{20} and D_{30} of all positive integer divisors of 20 and 30 respectively, under the partial order of divisibility. Show that is a Boolean algebra.

PART D

Answer any two Questions. Each Question carries 9 marks

12. a. Prove that the order of each subgroup of a finite group G is a divisor of the order of the group G. (4.5)

    b. Show that the set{ 0, 1, 2,3,4,5 }is a group under addition and multiplication modulo 6. (4.5)

13. a. Prove that every finite integral domain is a field. (4.5)

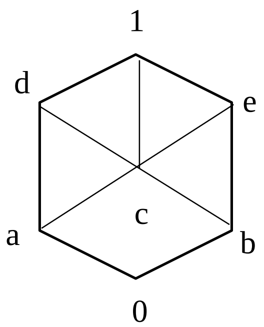
    b. Show that (Z, θ, Θ) is a ring where a θ b = a+b-1 and a Θ b = a+b–ab (4.5)

14. a. Consider the Boolean algebra D_{30}.Determine the following:

    i) All the Boolean sub-algebra of D_{30}.

    ii) All Boolean algebras which are not Boolean sub-algebras of D_{30} having atleast four elements. (4.5)

    b. Consider the Lattice L in the figure. Find the L is distributive and complemented lattice. Also find the complement of a,b,c. (4.5)



PART E

Answer any four Questions. Each Question carries 10 marks

15. a. Without using truth tables, prove the following $(\neg P \vee Q) \wedge (P \wedge (P \wedge Q)) \equiv P \wedge Q$

    b. Show that $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$ is a tautology.

16. a. Convert the given formula to an equivalent form which contains the connectives $\neg$ and $\wedge$ only: $\neg (P \leftrightarrow (Q \rightarrow (R \vee P)))$

    b. Show that $\neg P \wedge (\neg Q \wedge R) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$.

17. a. Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$

    b. Prove the validity of the following argument "If I get the job and work hard, then I will get promoted. If I get promoted, then I will be happy. I will not be happy. Therefore, either I will not get the job or I will not work hard.

18. a. Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)(P(x) \wedge (\exists x)(Q(x)$

    b. Consider the statement "Given any positive integer, there is a greater positive integer". Symbolize this statement with and without using the set of positive integers as the universe of discourse.

19. a. Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and $\neg M$.

    b. Prove by mathematical induction that $6^{n+2}+7^{2n+1}$ is divisible by 43 for each positive integer n.

20. Discuss indirect method of Proof. Show that the following premises are inconsistent.

    (i) If Jack misses many classes through illness, then he fails high school.

    (ii) If Jack fails high School, then he is uneducated.

    (iii) If Jack reads a lot of books, then he is not uneducated.

    (iv) Jack misses many classes through illness and reads a lot of books.

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER BTECH DEGREE EXAMINATION JULY 2017

Course Code**: CS 201**

Course Name**: DISCRETE COMPUTATIONAL STRUCTURES (CS, IT)**

Max Marks: 100 Duration: 3 hrs

PART A

Answer all questions, 3marks each.

1. Draw the Hasse diagram for the divisibility relation on the set A ={2,3,6,12,24,36}
2. Define equivalence relation? Give an example of a relation that is not an equivalence relation?
3. In how many different ways can the letters of the word 'MATHEMATICS' be arranged such that the vowels must always come together?
4. Define homomorhism and isomorphism?

PART B

Answer any 2 Questions, 9 marks each.

5. (a) Consider f,g and h are functions on integers $f(n) = n^2$, $g(n) = n + 1$, $h(n) = n-1$. Determine i) fogoh ii) gofoh iii) hofog (6)

   (b) State Pigeonhole Principle. Prove that at least two of the children were born on the same day of the week. (3)
6. (a) Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n$ (6)
   (b) Show that set of all integers is countable (3)
7. (a) Find the number of integers between 1 and 250 both inclusive that are not divisible by any of integers 2,3,5 and 7. (5)
   (b) If * is the binary operation on the set R of real numbers defined by a*b=a+b+2ab. Find if { R, *} is a semigroup. Is it commutative? (4)

PART C

Answer all questions, 3marks each.

8. Show that the set {1,2,3,4,5} is not a group under addition modulo 6.
9. Prove that every distributive lattice is modular.
10. Show that the set Q+ of all positive rational numbers forms an abelian group under the operation * defined by a*b= ½ ab; a,b ε Q+
11. Simplify the Boolean expression $a'b'c + ab'c + a'b'c'$

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PART D

Answer any 2 Questions, 9 marks each.

12. For the set $I_4=\{0,1,2,3\}$,show that modulo 4 system is a ring. (9)
13. a) Prove that the order of each sub group of a finite group G is a divisor of the order of group G. (6)
    b) If A = (1 2 3 4 5) and B = (2 3)(4 5). Find product of permutation AB. (3)
14. If (L, ≤) is a lattice, then for any a,b,c εL, the following properties hold. If b ≤ c, then i) a v b ≤ a v c ii) a ^ b ≤ a ^ c (9)

PART E

Answer any 4 Questions, 10 marks each.

15. (a) Show that (P→Q) ∧ (Q→P) is logically equivalent to P↔Q. (5)
    (b) Suppose x is a real number. Consider the statement "If $x^2 = 4$, then $x = 2$." Construct the converse, inverse, and contrapositive (5)
16. (a) Prove that (P∧Q) → (P↔Q) is a tautology. (5)
    (b) Show that (a v b) follows logically from the premises $p \vee q, (p \vee q) \rightarrow \neg r, \neg r \rightarrow (s \wedge \neg t)$ and $(s \wedge \neg t) \rightarrow (a \vee b)$ (5)
17. (a) Represent the following sentence in predicate logic using quantifiers i) All men are mortal. ii) Every apple is red iii) Any integer is either positive or negative. (6)
    (b) Use the truth table to determine whether p →(q∧˥q) and ˥p are logically equivalent. (4)
18. Show that the premises "one student in this class knows how to write programs in JAVA" and "Everyone who knows how to write programs in JAVA can get a high paying job" imply the conclusion "Someone in this class can get a high paying job" (10)
19. (a) Prove the following statement by contraposition:
    If a product of two positive real numbers is greater than 100, then at least one of the numbers is greater than 10. (6)
    (b) Express the negation of the following statement in English using quantifiers
    i) If the teacher is absent, then some students do not keep quiet ii) All students keep quiet and teacher is present. (4)
20. (a) Prove that √2 is irrational using proof by contradiction. (6)
    (b) Find the truth table (~ Q ⇒~P) ⇒ (P ⇒ Q) (4)

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Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: CS201

Course Name: DISCRETE COMPUTATIONAL STRUCTURES (CS, IT)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks Marks

1 Let $X = \{1,2,3,4\}$ and $R = \{<x,y> \mid x > y\}$. Draw the graph of R and also give its matrix. (3)

2 Define countable and uncountable set. Prove that set of real numbers are uncountable. (3)

3 State Pigeonhole principle. A school has 550 students. Show that at least two of them were born on the same day of the year. (3)

4 How many 4-digit numbers can be formed from six digits 1, 2, 3, 5, 7, 8. Also find how many numbers are less than 4500. (3)

PART B

Answer any two full questions, each carries 9 marks

5 a) Let Z be the set of integers and R be the relation called congruence modulo 3 defined by $R = \{<x,y> \mid$ x and y are elements in Z and (x-y) is divisible by 3$\}$. Determine the equivalence classes generated by the elements of Z. (5)

b) Let A be the set of factors of a particular positive integer m and let <= be the relation divides, ie relation <= be such that x<=y if x divides y. Draw the Hasse diagrams for m= 30 and m= 45. (4)

6 a) Let f(x) = x+2, g(x) = x-2 and h(x) =3x for x is in R, where R is the set of real numbers. Find gof, fog, (foh)og , hog . (4)

b) Among 100 students, 32 study mathematics, 20 study physics, 45 study biology, 15 study mathematics and biology, 7 study mathematics and physics, 10 study physics and biology and 30 do not study any of the three subjects. (5)

i) Find the number of students studying all three subjects.

ii) Find the number of students studying exactly one of three subjects.

7 a) Solve the recurrence equation $a_r + 5 a_{r-1} + 6 a_{r-2} = 42\ 4^r$ where $a_2 = 278$ and $a_3 = 962$. (4)

b) Define Monoid. Show that the algebraic systems $<Z_m , +_m>$ and $<Z_m , *_m>$ are monoids where m = 6. (5)

PART C

Answer all questions, each carries 3 marks

8 Define Abelian group. Prove that the algebraic structure $<Q^+, *>$ is an abelian group. * defined on Q^+ by a * b = (ab)/2. (3)

9 Define Cosets and Lagrange's theorem. (3)

10 Draw the diagram of lattices <Sn, D> for n = 15 and n = 45. Where Sn be the set (3)

of all divisors of n and D denote the relation 'divides'.

11 Define sub Boolean algebra. Give one example. (3)

PART D

Answer any two full questions, each carries 9 marks

12 a) Show that the set{ 0, 1, 2, 3, 4, 5 } under addition and multiplication modulo 6 is group or not. (5)

b) Find all the subgroups of $\langle Z_{12}, +_{12} \rangle$ (4

13 a) Define ring and field. Give one example to each. (5)

b) A= {2, 3, 4, 6, 12, 18, 24, 36} with partial order of divisibility. Determine whether the POSET is a lattice or not. (4)

14 a) Show that the lattice < Sn, D > for n = 216 is isomorphic to the direct product of lattices n = 8 and n = 27. (5)

b) Define complemented lattice and distributive lattice. Give one example to each. (4)

PART E

Answer any four full questions, each carries 10 marks

15 a) Prove that p v (q ^ r) and (p v q)^(p v r) are logically equivalent (5)

b) Show that (~p ^ (~q ^ r)) v (q ^ r) v (p ^ r) <=> r (5)

16 a) Show that s v r is tautologically implied by (p v q) ^ (p -> r) ^ (q -> s) (5)

b) Show that r ^ (p v q) is a valid conclusion from the premises p v q, q -> r, p -> m, and ~m (5)

17 a) "If there are meeting, then traveling was difficult. If they arrived on time, then traveling was not difficult. They arrived on time. There was no meeting". Show that the statements constitute a valid argument. (6)

b) Construct truth table for ~ (p ^ q) <-> (~p v ~q). Determine whether it is tautology or not. (4)

18 a) Show that (x) (P(x) -> Q(x)) ^ (x) (Q(x) -> R(x)) => (x) (P(x) -> R (x)) (5)

b) Prove that (∃x) (P(x) ^ Q(x)) => (∃x) P(x) ^ (∃x) Q(x) (5)

19 a) Symbolize the statements: (4)

i) All the world loves a loverii) All men are giants.

b) Show that (∃x) M(x) follows logically from the premises (x) (H(x) -> M(x)) and (∃x) H(x) (6)

20 a) Prove by contradiction that if n^2 is an even integer then n is even. (5)

b) Prove that $23^n - 1$ is divisible by 11 for all positive integers n. (5)

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Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017

Course Code: CS201

Course Name: DISCRETE COMPUTATIONAL STRUCTURES (CS, IT)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks. Marks

1		Assume A={1,2,3} and ρ(A) be its power set. Let ⊆ be the subset relation on the power ser. Draw the Hasse diagram of (ρ(A),⊆)	(3)
2		Let R denote a relation on the set of ordered pairs of positive integers such that (x, y)R(u, v) iff xv = yu. Show that R is an equivalence relation	(3)
3		Prove that in any group of six people, at least three must be mutual friends or at least three must be mutual strangers.	(3)
4		Define GLB and LUB for a partially ordered set. Give an example	(3)

PART B

Answer any two full questions, each carries 9 marks.

5	a)	Suppose f(x)=x+2,g(x)= x-2 and h(x)=3x for x ϵ R,where R is the set of real numbers. Find g o f ,f o g, f o f, g o g, f o h, h o g,h o h and (f o h) o g	(4)
	b)	Prove that every equivalence relation on a set generates a unique partition of the set with the blocks as R-equivalence classes	(5)
6	a)	Show that the set N of natural numbers is a semigroup under the operation x*y=max(x,y). Is it a monoid?	(3)
	b)	Solve the recurrence relation $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 - 2r + 1$	(6)
7	a)	Show that for any commutative monoid <M,*>, the set of idempotent elements of M forms a submonoid.	(5)
	b)	Define subsemigroups and submonoids.	(4)

PART C

Answer all questions, each carries 3 marks.

8		Show that, for an abelian group, $(a * b)^{-1}= a^{-1} * b^{-1}$	(3)
9		Show that every chain is a distributive lattice.	(3)
10		Simplify the Boolean expression a’b’c+ab’c+a’b’c’	(3)
11		Let G = {1, a, a^2, a^3} (a^4 = 1) be a group and H = {1, a^2} is a subgroup of G under multiplication. Find all cosets of H.	(3)

PART D

Answer any two full questions, each carries 9 marks.

12 a) Show that the order of a subgroup of a finite group divides the order of the group. (6)

b) Define ring homomorphism. (3)

13 Show that (I,Θ,ʘ) is acommutative ring with identity, where the operations Θ and ʘ are defined, for any a,b∊ I, as a Θ b = a+b-1 and a ʘ b = a+b-ab. (9)

14 a) Let (L,≤) be a lattice and a,b,c,d∊ L. Prove that if $a \le c$ and $b \le d$, then (5)

(i) $a \vee b \le c \vee d$

(ii) $a \wedge b \le c \wedge d$

b) Show that in a Boolean algebra,for any a,b,c (4)

$$(a \wedge b \wedge c) \vee (b \wedge c) = b \wedge c$$

PART E

Answer any four full questions, each carries 10 marks.

15 a) a) Construct truth table for $(\sim p \wedge (\sim q \wedge r)) \vee ((q \wedge r) \vee (p \wedge r))$ (6)

b) Explain proof by Contrapositive with example. (4)

16 Prove the following implication (10)

(x)(P(x) ∨Q(x))==> (x) P(x) ∧ (Ǝx) Q(x)

17 a) Represent the following sentences in predicate logic using quantifiers (6)

(i) "x is the father of the mother of y"

(ii) "Everybody loves a lover"

b) Determine whether the conclusion C follows logically from the premises (4)

H_1: ~p∨q, H_2: ~(q∧~r), H_3: ~r C: ~p

18 a) Without using truth table prove p → (q → p) <=> ~p → (p → q) (4)

b) Determine the validity of the following statements using rule CP. (6)

"my father praises me only if I can be proud of myself. Either I do well in sports or I can't be proud of myself. If I study hard, then I can't do well in sports. Therefore if my father praises me then I do not study well"

19 a) Show that r → s can be derived from the premises p →(q →s), ~r ∨ p , q (4)

b) Prove, by Mathematical Induction, that n(n + 1)(n + 2)(n + 3) is divisible by 24, for all natural numbers n (6)

20 a) "If there are meeting, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. There was no meeting". Show that these statements constitute a valid argument. (6)

b) Show that $2^n < n!$ For $n \ge 4$ (4)

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