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APPENDIXES
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What Grade 7 Foundational Knowledge and Skills Are Associated with Missouri Students' Algebra I Achievement in Grade 8?

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See https://go.usa.gov/xwuzB for the full report.

Appendix A. Literature review

Success in advanced math courses provides students access to a wider variety of college and career options (National Mathematics Advisory Panel, 2008). To increase opportunities for students to take more advanced math courses in high school, many school districts enroll middle school students in Algebra I, a gateway course for advanced math (Star et al., 2015). Between 1990 and 2011, the proportion of students taking Algebra I or more advanced math courses in middle school doubled (Domina, 2014).

But students who take Algebra I in grade 8 and skip other math courses, such as grade 8 general math, might miss opportunities to develop the foundational knowledge and skills required for success in advanced math courses (Domina, McEachin, Penner, & Penner, 2015). Math is a sequential and cumulative discipline (Pillay, Wilss, & Boulton-Lewis, 1998): learning builds as students progress from grade to grade. Students typically begin with number concepts and skills and then move to more abstract ideas, such as working with generalized quantities (for example, functional relationships like $y = 2 + 3x$, in which variables do not represent one specific value). The repetition of concepts and skills that was common across grades in the past meant that only about 30 percent of the content in grade 8 textbooks was new (Flanders, 1987). Thus, students could be accelerated with little concern about what content they would skip or how their understanding of content would be advanced.

However, current standards are more developmental and linear in that they exhibit less overlap in content from grade to grade. Additionally, the content of standards has shifted considerably with regard to development of algebraic concepts and skills before students enter high school. For example, the Common Core State Standards Initiative (2011) includes a domain focused on operations and algebraic thinking that begins in kindergarten and progresses through grade 5. That domain leads directly into the domain of expressions and equations, which is a focus from grade 6 onward. In the expressions and equations domain, solving systems of linear equations is now a standard for all grade 8 students (Common Core State Standards Initiative, 2011). Students who skip grade 8 general math to take Algebra I might miss opportunities to develop an understanding of the process for solving systems of equations and other topics emphasized at that grade level. Therefore, educators face the challenge of determining which students can successfully skip grade 8 math to take Algebra I and which students will benefit from taking grade 8 math before Algebra I.

Factors that influence student readiness for Algebra I in grade 8

A large body of research has examined the types of experiences that students need in order to be successful in Algebra I (for example, Blanton, 2008; Blanton, Levi, Crites, Dougherty, & Zbiek, 2011; Carpenter, Levi, Berman, & Pligge, 2005; Carraher, Martinez, & Schliemann, 2008; Dougherty, 2008; Dougherty, Bryant, Bryant, Darrough, & Pfannenstiel, 2015). That research has identified three components that form the basis of algebraic thinking that should be incorporated into approaches to introducing content before Algebra I: generalizing math ideas; representing and justifying generalizations in multiple ways (natural language, tables, charts, graphs, symbols, and physical materials); and reasoning with generalizations (Blanton et al., 2011). Overall, the research focuses more on what instructional strategies could be used to prepare students for Algebra I and less on how to identify students who may be ready to take Algebra I in grade 8. Additionally, the math domains in these previous studies were identified by the researchers and chosen specifically to answer their research questions. A limitation in the research is thus that math domains are inconsistent across research studies, making it difficult to build on the knowledge base.

State assessment data are available for nearly all students and may be a valuable source of information in identifying students who are ready to take Algebra I in grade 8. Previous research provides evidence for the feasibility of using Algebra I readiness test scores to determine readiness (for example, Huang, Snipes, & Finkelstein, 2014). However, given concerns about the amount of time already devoted to student assessments, educators may be reluctant to administer additional tests. Instead, they may favor examining math domain scores from statewide assessments, such as the Missouri Assessment Program, which provide scale scores in multiple content areas, such as ratios and proportional relationships; the number system; expressions and equations; geometry; and statistics and probability.

Previous research on the association between the five math domains assessed by the Missouri Assessment Program and later Algebra I achievement has examined two of the five math domains—the number system domain and the expressions and equations domain—more frequently than the other three. One limitation of previous research in this area is that researchers have considered a wide variety of content domains that are inconsistent across studies and that rarely align with the domains assessed by the Missouri Assessment Program in grade 7.

The number system

Experts have long believed that students need to understand whole and rational numbers before progressing to more formal algebra (see box B1 for definitions of key terms). However, determining relative fraction magnitudes, rather than understanding whole numbers, has been found to be more predictive of students' improvement in encoding equation features and solving equations (Booth, Newton, & Twiss-Garrity, 2014). Student success with nonunit fractions may also predict success in Algebra I (Booth et al., 2014). Thus, the proportional reasoning skills required for correct placement of nonunit fractions could link students' knowledge of fractions with algebraic understandings.

Because fractions and decimals are different notations for rational numbers, an association between student performance on tasks with fractions and tasks with decimals would seem evident, with both areas affecting Algebra I achievement. However, when student responses to tasks that could be modeled with either fraction or decimal representations are compared, the contexts of problems have been found to influence student representations and performance (DeWolf, Bassok, & Holyoak, 2015). If the tasks involved discrete quantities, students were more likely to use fractions successfully. But if the problems involved continuous quantities, students more frequently used decimals in correct solutions. Both of these contexts are inherent in algebraic problems, and therefore facility with modeling contexts with fractions and decimals has an impact on algebraic understanding.

Box B1. Key terms

Concatenation. The joining of two strings into a single string (for example, "book" and "case" concatenate to form "bookcase"). In algebra, concatenation involves understanding, for example, that 3*n* represents 3 x *n*, so that when *n* is replaced by 2, the answer is 3 x 2 or 6, not 32.

Continuous quantity. A quantity that can take on any value between two other values (for example, distance or weight).

Discrete quantity. A quantity that is finite and can be counted (for example, the number of dimes or the results of an election).

Generalized quantity. A quantity that does not have one specific value and is often represented with a variable (for example, area *A*).

Number sets. In math, students work with different sets of numbers in different contexts:

- **Natural numbers.** The set of counting numbers, {1, 2, 3, …}. This set does not include zero, negative numbers, or fractions/decimals.
- **Whole numbers.** The set of natural numbers with the addition of zero, {0, 1, 2, 3, …}.
- **Integers.** The set of all whole numbers and their opposites, {…, –3, –2, –1, 0, 1, 2, 3, …}.
- **Rational numbers.** The set of all numbers that can be written as the fraction *p*/*q*, where both *p* and *q* are integers and *q* does not equal zero. This set contains all common fractions and decimals. This set includes repeating decimals (for example, 0.3333333… and 3.252525…) and terminating decimals (for example, 0.25 and 5.6).

Nonunit fraction. A fraction in which the numerator is not equal to 1 (for example, 2/3, 3/5, or 9/11).

Proportion. A statement showing that two ratios are equal (for example, 2/3 = 4/9).

Ratio. A statement comparing the relative sizes of two quantities (for example, 3 apples to 4 oranges, or 20 miles per hour). Often written as a fraction (3/4) or with a colon (3:4).

Expressions and equations

The second domain involves expressions, equations, and inequalities, which often define how teachers and students view the core or meaning of algebra. A variable can refer to a specific, yet unknown, quantity whose value can be found by using an algorithmic procedure (for example, finding the value of the variable in the equation $3x + 4 = 13$) (Malisani & Spagnolo, 2009). While this concept of a variable may appear to students to be specifically related to algebra, variables can also be used to generalize properties of arithmetic—such as the distributive property, illustrated by $a(b + c) = ab + ac$—or to represent other patterns. In these generalizations, the variables do not represent specific quantities but can refer to any number. Additionally, a variable has another role in functional relationships, in which it can take on a range of values (as in, for example, $y = –3x + 4$).

Understanding the roles that variables can play is vital to success in Algebra I. Students tend to focus on natural numbers when substituting for literal symbols (that is, variables), even when working in algebraic contexts in which they ought to consider continuous numbers, negative numbers, or zero (Christou & Vosniadou, 2012). As a consequence, if students consider primarily natural numbers, they might, for example, interpret the graphs of linear functions as a set of discrete points rather than as a continuous representation. Students often think of a variable as an object rather than as a quantity because teachers and textbooks often use mnemonic symbols, such as *d* for dollars or *c* for cake (McNeil et al., 2010). Thus, the interpretation of algebraic expressions is hindered. Understanding variables as generalized quantities allows students to think more generally about relationships represented through expressions or equations, and this ability is related to conceptual understanding (Kieran, 2013).

Concerning solving equations, Herscovics and Linchevski (1994) conducted a two-part study to identify a possible demarcation between arithmetic and algebra. The first part determined the extent to which middle school students understood equality, order of operations, cancellation (+3 – 3), and concatenation. As all the students seemed to have a solid grasp of these components, the second part dealt with the students' ability to solve equations involving natural numbers without previous instruction. It explored their understanding when varying, for example, the placement of the variable, the size of the numbers, and the placement of the unknown quantity. Although students could solve these equations, their solution methods did not involve algebraic thinking. Rather, the method they employed was primarily numerically based (using arithmetic) and involved substituting values until a correct one was found. Thus, the understanding of the math concepts from the first part of the study was not clearly connected to solving equations using algebraic thinking.

Other math domains assessed by the 2016/17 Missouri Assessment Program in grade 7

Fewer studies have documented associations between student performance in three of the five domains assessed by the 2016/17 Missouri Assessment Program in grade 7 (ratios and proportional relationships; geometry; and statistics and probability) and Algebra I achievement.

Huang et al. (2014) explored the association between Algebra I readiness and two assessments: the California Standards Test, which assesses general math achievement in grades 6 and 7, and an Algebra I readiness test, which scores seven math content areas. Separate analyses using each assessment were conducted to predict success in Algebra I, with the goal of comparing the accuracy of potential placement decisions made using the information from each assessment. The study found that scores on the California Standards Test could be used with 78 percent accuracy to identify students who are more likely to succeed in Algebra I in grade 8.

The study also found that some content areas in the Algebra I readiness test were more strongly associated with success in Algebra I than others. Specifically, five areas assessed were predictive of success in Algebra I: decimals, exponents and square roots, integers, literal symbols (variables) and equations, and fractions. Two areas did not predict success in Algebra I: data analysis, probability, and statistics; and geometric measurement and coordinate geometry. The study also examined how the assessments could be used in combination.

Although the findings showed that considering the results of assessments—such as the Algebra I readiness test—in placement decisions was unlikely to increase the percentage of students who were successful in Algebra I, these assessments can help identify areas of additional support for students placed in Algebra I to increase their probability of success.

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Appendix B. Methods

This appendix provides further details about the sample, data, and analysis methods used to conduct the study as well as about the interpretation of standardized regression coefficients.

Sample

The sample for the study included all Missouri students who took the Algebra I End-of-Course Assessment in grade 8 at the end of the 2017/18 school year and who had data available from the Missouri Assessment Program in grade 7 for the 2016/17 school year. In 2017/18, 11,601 grade 8 students took the Algebra I End-of-Course Assessment; 303 of those students (2.6 percent) had not taken the Missouri Assessment Program in grade 7 in 2016/17. Thus, the final sample included 11,298 students. About 78 percent of students in the sample were White, 9 percent were Black, 6 percent were Hispanic, and 4 percent were Asian or Hawaiian Native/Pacific Islander. About 53 percent of students were female. About 2 percent of students were English learner students, and 2 percent were receiving special education services. Three English learner students also received special education services. About 27 percent of students in the sample were eligible for the national school lunch program, an indicator of socioeconomic disadvantage (table B1).

Table B1. Characteristics of the study sample, 2016/17 and 2017/18

Characteristic	Number	Percent
Race/ethnicity		
American Indian/Alaska Native	31	0.3
Asian or Hawaiian Native/Pacific Islander	488	4.3
Black	1,009	8.9
Hispanic	617	5.5
White	8,798	77.9
Multiple races/ethnicities	355	3.1
Female	5,953	52.7
English learner students	228	2.0
Students receiving special education services	170	1.5
English learner students who were receiving special education services	3	0.0
Students eligible for the national school lunch program	3,040	26.9

Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

Data

The study used four types of administrative data from the Missouri Department of Elementary and Secondary Education:

- *Scores in the five domains of the 2016/17 Missouri Assessment Program in grade 7.* Scores were provided for each of the five math content domains: ratios and proportional relationships; the number system; expressions and equations; geometry; and statistics and probability. Scores were provided as the percentage of points earned.[1] The number of possible points for each domain varied from 7 to 13 (table B2).

[1] In the released practice form for this assessment (Missouri Department of Elementary and Secondary Education, 2018, n.d.), which is representative of the actual assessment, nearly all items are worth one point. Two items (4 percent) are worth 2 points; both of these items required students to provide multipart answers (for example, to plot five points on a graph).

Table B2. Number of possible points for math domains on the Missouri Assessment Program in grade 7, 2016/17

Domain	Possible points
Ratios and proportional relationships	10
The number system	8
Expressions and equations	13
Geometry	7
Statistics and probability	8

Source: Authors' construction.

- *Scale scores on the Algebra I End-of-Course Assessment in grade 8.* Scale scores on the Algebra I End-of-Course Assessment were used as the measure of Algebra I achievement. Students are expected to take the assessment after they complete the course content for Algebra I—that is, after completing a self-contained Algebra I course or after completing the same content spread across several courses (Questar Assessment, 2017). The Missouri Department of Elementary and Secondary Education converts these scale scores into proficiency levels. Scores below 389 are considered below basic, scores from 389 to 399 are considered basic, scores from 400 to 408 are considered proficient, and scores above 408 are considered advanced (Missouri Department of Elementary and Secondary Education, 2016).
- *Student background characteristics.* The Missouri Department of Elementary and Secondary Education provided information about five background characteristics for all students in the study: gender, race/ethnicity, eligibility for the national school lunch program, English learner status, and special education status. Missouri requires local education agencies to provide complete information on these variables, so no demographic data were missing.
- *District and school identifiers.* Identifiers were provided for all students in the study. These variables were used in the statistical analyses to adjust for the nesting of students within schools and districts.

Analysis methods

Due to the large sample size, the models had high statistical power to detect small effects that might be of little practical significance. Consequently, discussion of results focused on those that were significant at $p < .01$.

Preliminary analyses. Preliminary analyses were conducted to describe student performance on the 2016/17 Missouri Assessment Program in grade 7 and on the Algebra I End-of-Course Assessment in grade 8. Correlation coefficients were also computed to examine the extent to which scores in the five math domains in grade 7 were associated with one another.

Additional preliminary analyses examined the bivariate associations between each math domain score in grade 7 and Algebra I achievement in grade 8 by fitting five three-level bivariate regressions using hierarchical linear modeling software (Raudenbush, Bryk, & Congdon, 2013). Each regression included one domain score as the independent variable and the scale score on the Algebra I End-of-Course Assessment as the dependent variable. A three-level model was fit to account for the nesting of students within schools and districts. All variables were standardized to produce standardized regression coefficients by subtracting the mean of the variable from each student's score and then dividing the result by the standard deviation of the variable. Standardized regression coefficients ensure that the size of the coefficients describing the association between the domain score and the scale score on the Algebra I End-of-Course Assessment is not confounded with the variability of the domain score, which makes it easier to compare the size of the coefficients for the different predictors in the model. The results of these preliminary analyses are in appendix C.

Analyses to address the research questions. To address the first research question on the extent to which scores in the five math domains of the 2016/17 Missouri Assessment Program in grade 7 are associated with Algebra I achievement in grade 8, hierarchical linear models with multiple predictors were fit, again accounting for the nesting of students within schools and districts. The dependent variable of interest was the scale score on the Algebra I End-of-Course Assessment in grade 8. The model included scores in all five domains in grade 7 as independent variables. Parameter estimates were examined for each domain score to determine which domain scores had the strongest association with Algebra I achievement in grade 8. The results from the model with all five domains supplement the information gleaned from the first set of bivariate regressions. That is, while the preliminary set of bivariate regressions provide information on the strength of the association between each domain and Algebra I achievement in grade 8, the model with all five domains indicates the strength of the association between each domain score and Algebra I achievement in grade 8, above and beyond all other domains.

The modeling process helped identify which math domains were associated with Algebra I achievement for the general student population. To address the second and third research questions on whether the associations vary by English learner status and special education status, the hierarchical linear model described above was fit two additional times. The first model included being an English learner student and interaction terms for each domain with being an English learner student. The second model included receiving special education services and interaction terms for each domain with receiving special education services.

Interpretation of standardized regression coefficients

The analysis was conducted with standardized variables to allow for easier comparison of the size of coefficients for the five math domains (results are summarized in table 1 of the main report). To interpret the findings, it is helpful to consider the variables in their unstandardized forms. To convert back to the original, unstandardized metric for the variables, a few calculations are necessary. This section illustrates these calculations using the coefficient for the expressions and equations domain in table 1 as an example.

The coefficient for the expressions and equations domain was 0.21, indicating that if the score in this math domain increased by one standard deviation and scores in the other four math domains stayed the same, a student's scale score on the Algebra I End-of-Course Assessment is estimated to increase by about a fifth of a standard deviation. The standard deviation for the expressions and equations domain, which is scored as a percentage of possible points earned, was 21.7. Getting an additional 3 items correct of this domain's 13 items[2] would raise the percentage of correct answers by 23.1 points (because 3 is 23.1% of 13), which is slightly larger than the standard deviation of 21.7 (106 percent of 21.7). To understand the increase in the raw scale score on the Algebra I End-of-Course Assessment, it is necessary to consider that variable's standard deviation (12.7). Because the coefficient for the expressions and equations domain was 0.21, getting an additional three items correct (a standard deviation increase of 106 percent) is associated with a standard deviation increase of 0.22 (1.06 × 0.21) in scale score on the Algebra I End-of-Course Assessment. A 0.22 standard deviation increase is equal to a 2.8 point change in scale score (0.22 × 12.7). To put the increase in perspective, a scale score between 400 and 408 is considered proficient. So, the analysis indicates that, if scores in the other four math domains stay the same, getting an additional three items correct in the expressions and equations domain is associated with an increase in the Algebra I End-of-Course scale score that is equivalent to about a third of the range of scores in the proficient achievement level.

[2] The vast majority (96 percent) of the released items were worth 1 point.

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Appendix C. Supporting analysis

This appendix presents the results of preliminary analyses as well as the full model results for the hierarchical linear models with interaction terms (see tables 2 and 3 in the main report).

Preliminary analyses

Preliminary analyses included describing the math performance of the study sample on the 2016/17 Missouri Assessment Program in grade 7 and the Algebra I End-of-Course Assessment in grade 8, examining the associations between scores in the five math domains of the 2016/17 Missouri Assessment Program in grade 7, and examining the bivariate associations between the score in each domain in grade 7 and Algebra I achievement in grade 8.

Student performance on math assessments. On average, students earned 52–76 percent of the possible points in the math domains of the 2016/17 Missouri Assessment Program in grade 7 (table C1). The highest percentage of possible points earned was in the ratios and proportional relationships domain, and the lowest was in the statistics and probability domain. The average scale score on the Algebra I End-of-Course Assessment in grade 8 was about 412, which falls into the point range of the advanced achievement level.

Table C1. Descriptive statistics for students' scores on the Missouri Assessment Program in grade 7 and the Algebra I End-of-Course Assessment in grade 8, 2016/17 and 2017/18

Assessment and domain	Mean	Standard deviation	Range	Skewness	Kurtosis
Missouri Assessment Program in grade 7 (percent correct)					
Ratios and proportional relationships	76.3	18.7	0–100	–1.0	0.9
The number system	54.5	23.7	0–100	–0.2	–0.6
Expressions and equations	55.9	21.7	0–100	–0.1	–0.6
Geometry	61.5	23.5	0–100	–0.2	–0.6
Statistics and probability	52.4	22.4	0–100	–0.1	–0.6
Algebra I End-of-Course Assessment (scale score)	411.9	12.7	181–471	–0.7	12.5

Note: *n* = 11,298.
Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

English learner students scored significantly lower, on average, than non–English learner students on both the Missouri Assessment Program in grade 7 and the Algebra I End of Course Assessment in grade 8 (table C2). On average, English learner students earned 37–63 percent of the possible points on the Missouri Assessment Program in grade 7. As with the sample as a whole, the highest percentage of possible points earned by English leaner students was in the ratios and proportional relationships domain and the lowest was in the statistics and probability domain. The average scale score on the Algebra I End-of-Course Assessment was about 406, which falls into the point range of the proficient achievement level.

Table C2. Descriptive statistics for students' scores on the Missouri Assessment Program in grade 7 and the Algebra I End-of-Course Assessment in grade 8, by English learner status, 2016/17 and 2017/18

	English learner students (n = 228)		Non–English learner students (n = 11,070)		
Assessment and domain	Mean	Standard deviation	Mean	Standard deviation	*t*-statistic
Missouri Assessment Program in grade 7 (percent correct)					
Ratios and proportional relationships	62.5	22.7	76.6	18.5	9.3***
The number system	40.1	24.7	54.8	23.6	9.3***
Expressions and equations	39.8	21.2	56.2	21.5	11.4***
Geometry	49.9	23.6	61.7	23.5	7.5***
Statistics and probability	37.3	22.2	52.7	22.3	10.3***
Algebra I End-of-Course Assessment (scale score)	406.0	14.0	412.0	12.7	6.4***

*** Significant at $p < .001$.

Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

On both the Missouri Assessment Program in grade 7 and the Algebra I End-of-Course Assessment in grade 8, students who were receiving special education services scored significantly lower, on average, than students who were not receiving special education services (table C3). On average, students who were receiving special education services earned 40–59 percent of the possible points on the Missouri Assessment Program in grade 7. As with the sample as a whole, the highest percentage of possible points earned by students who were receiving special education services was in the ratios and proportional relationships domain and the lowest was in the statistics and probability domain. The average scale score on the Algebra I End-of-Course Assessment was about 402, which falls into the point range for the proficient achievement level.

Table C3. Descriptive statistics for students' scores on the Missouri Assessment Program in grade 7 and the Algebra I End-of-Course Assessment in grade 8, by special education status, 2016/17 and 2017/18

	Students who were receiving special education services (n = 170)		Students who were not receiving special education services (n = 11,128)		
Assessment and domain	Mean	Standard deviation	Mean	Standard deviation	*t*-statistic
Missouri Assessment Program in grade 7 (percent correct)					
Ratios and proportional relationships	59.4	30.2	76.6	18.4	12.0***
The number system	41.6	27.0	54.7	23.6	7.2***
Expressions and equations	42.4	27.7	56.1	21.5	8.2***
Geometry	47.5	28.6	61.7	23.4	7.8***
Statistics and probability	39.6	26.0	52.6	22.3	7.5***
Algebra I End-of-Course Assessment (scale score)	401.8	17.4	412.0	12.6	10.5***

*** Significant at $p < .001$.

Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

Associations between math domain scores in grade 7. Correlation coefficients between the five math domain scores in grade 7 ranged from .47 for the association between the geometry domain and the ratios and proportional relationships domain to .65 for the association between the expressions and equations domain and the number system domain (table C4).

Table C4. Correlations between math domain scores on the Missouri Assessment Program in grade 7, 2016/17

Domain	Ratios and proportional relationships	The number system	Expressions and equations	Geometry	Statistics and probability
Ratios and proportional relationships	—	.54***	.59***	.47***	.51***
The number system		—	.65***	.51***	.54***
Expressions and equations			—	.57***	.61***
Geometry				—	.51***
Statistics and probability					—

*** Significant at $p < .001$.

Note: $n = 11{,}298$.

Source: Authors' analysis of 2016/17 data from the Missouri Department of Elementary and Secondary Education.

Bivariate associations between math domain scores in grade 7 and Algebra I achievement in grade 8. The strongest association with Algebra I achievement in grade 8 was for the expressions and equations domain (.48; table C5). The weakest association, although still moderately strong, was for the ratios and proportional relationships domain (.38).

Table C5. Bivariate associations between math domain scores on the Missouri Assessment Program in grade 7 and scale scores on the Algebra I End-of-Course Assessment in grade 8, 2016/17 and 2017/18

Domain	Coefficient
Ratios and proportional relationships	.38***
The number system	.43***
Expressions and equations	.48***
Geometry	.40***
Statistics and probability	.41***

*** Significant at $p < .001$

Note: $n = 11{,}298$. Coefficients are from bivariate three-level regression models to account for nesting of students within schools and districts. Assessment scores were standardized to allow for comparison of coefficients across predictors with different standard deviations.

Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

Complete results from hierarchical linear models

The complete results from the hierarchical linear model (summarized in table 1 of the main report) are in table C6. Follow-up analyses using the multivariate hypothesis tests feature in the hierarchical linear modeling software (Raudenbush, Bryk, & Congdon, 2013) were conducted to test whether the coefficients from the model were significantly different from one another. The results of these analyses indicated that the coefficient for the expressions and equations domain was significantly larger than all the other coefficients and that the coefficient for the ratios and proportional relationships domain was significantly smaller than all the other coefficients (table C7).

Table C6. Complete results for the hierarchical linear model examining the association between math domain scores on the Missouri Assessment Program in grade 7 and scale scores on the Algebra I End-of-Course Assessment in grade 8, 2016/17 and 2017/18

Variable	Coefficient
Intercept	–0.18*** (0.04)
Ratios and proportional relationships	0.10*** (0.01)
The number system	0.16*** (0.01)
Expressions and equations	0.21*** (0.01)
Geometry	0.15*** (0.01)
Statistics and probability	0.14*** (0.01)

*** Significant at $p < .001$.

Note: n = 11,298. Numbers in parentheses are standard errors. Coefficients are from a three-level multiple regression model that accounted for the nesting of students within schools and districts. Assessment scores were standardized to allow for comparison of coefficients across predictors with different standard deviations.

Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

Table C7. Results of tests to determine whether pairs of coefficients from the hierarchical linear model examining the association between math domain scores on the Missouri Assessment Program in grade 7 and scale scores on the Algebra I End-of-Course Assessment in grade 8 were significantly different, 2016/17 and 2017/18

Domain	Ratios and proportional relationships	The number system	Expressions and equations	Geometry	Statistics and probability
Ratios and proportional relationships	—				
The number system	19.66***	—			
Expressions and equations	46.38***	19.24***	—		
Geometry	11.91***	0.71	20.92***	—	
Statistics and probability	9.24***	1.96	29.81***	0.22	—

*** Significant at $p < .001$.

Note: n = 11,298. For domains without any asterisks the difference in coefficients was not statistically significant at $p < .01$. Chi-square tests with one degree of freedom were conducted using the multivariate hypothesis tests feature in the hierarchical linear modeling software (Raudenbush et al., 2013) to examine whether pairs of coefficients from the model reported in table C6 were significantly different. The Benjamini-Hochberg procedure was applied to adjust for multiple comparisons.

Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

The complete results from the hierarchical linear models used to address research question 2 are in tables C8–C10. The results for the model for English learner students are in table C8. The results for the model for non–English learner students are in table C9. An additional model was fit with all students to test for differences in the strength of the association between the math domain scores and Algebra I achievement for the two groups (table C10). That model included a dummy variable, coded 1 for English learner students and 0 for non–English learner students, as well as interaction terms between English learner status and the five domain scores.

Table C8. Complete results for the hierarchical linear model examining the association between math domain scores on the Missouri Assessment Program in grade 7 and scale scores on the Algebra I End-of-Course Assessment in grade 8 for English learner students, 2016/17 and 2017/18

Variable	Coefficient
Intercept	0.01 (0.06)
Ratios and proportional relationships	0.21*** (0.05)
The number system	0.29*** (0.05)
Expressions and equations	0.19*** (0.05)
Geometry	0.05 (0.05)
Statistics and probability	0.09 (0.04)

*** Significant at $p < .001$.

Note: $n = 228$. Numbers in parentheses are standard errors. For domains without any asterisks the coefficient is not statistically significant at $p < .01$. Coefficients are from a three-level multiple regression model that accounted for the nesting of students within schools and districts. Assessment scores were standardized to allow for comparison of coefficients across predictors with different standard deviations.

Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

Table C9. Complete results for the hierarchical linear model examining the association between math domain scores on the Missouri Assessment Program in grade 7 and scale scores on the Algebra I End-of-Course Assessment in grade 8 for non–English learner students, 2016/17 and 2017/18

Variable	Coefficient
Intercept	–0.18*** (0.04)
Ratios and proportional relationships	0.10*** (0.01)
The number system	0.15*** (0.01)
Expressions and equations	0.21*** (0.01)
Geometry	0.15*** (0.01)
Statistics and probability	0.14*** (0.01)

*** Significant at $p < .001$.

Note: $n = 11{,}070$. Numbers in parentheses are standard errors. Coefficients are from a three-level multiple regression model that accounted for the nesting of students within schools and districts. Assessment scores were standardized to allow for comparison of coefficients across predictors with different standard deviations.

Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

Table C10. Complete results for the hierarchical linear model with interaction terms to test for differences by English learner status in the strength of the association between the math domain scores and Algebra I achievement, 2016/17 and 2017/18

Variable	Coefficient
Intercept	–0.18*** (0.04)
English learner status	0.20*** (0.06)
Ratios and proportional relationships	0.10*** (0.01)
The number system	0.15*** (0.01)
Expressions and equations	0.21*** (0.01)
Geometry	0.15*** (0.01)
Statistics and probability	0.14*** (0.01)
Ratios and proportional relationships × English learner status	0.10 (0.05)
The number system × English learner status	0.16** (0.06)
Expressions and equations × English learner status	–0.03 (0.06)
Geometry × English learner status	–0.13 (0.06)
Statistics and probability × English learner status	–0.02 (0.06)

** Significant at $p < .01$; *** significant at $p < .001$.

Note: n = 11,298. Numbers in parentheses are standard errors. Coefficients are from bivariate three-level regression models to account for nesting of students within schools and districts. Assessment scores were standardized to allow for comparison of coefficients across predictors with different standard deviations.

Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

The complete results from the hierarchical linear models used to address research question 3 are in tables C11–C13. The results for the model for students who were receiving special education services are in table C11. The results for the model for students who were not receiving special education services are in table C12. An additional model was fit with all students to test for differences in the strength of the association between the math domain scores and Algebra I achievement for the two groups (table C13). That model included a dummy variable, coded 1 for students who were receiving special education services and 0 for students who were not receiving special education services, as well as interaction terms between special education status and the five domain scores.

Table C11. Complete results for the hierarchical linear model examining the association between math domain scores on the Missouri Assessment Program in grade 7 and scale scores on the Algebra I End-of-Course Assessment in grade 8 for students who were receiving special education services, 2016/17 and 2017/18

Variable	Coefficient
Intercept	0.03 (0.06)
Ratios and proportional relationships	0.16 (0.07)
The number system	0.21*** (0.07)
Expressions and equations	0.11 (0.08)
Geometry	0.18*** (0.06)
Statistics and probability	0.12 (0.06)

*** Significant at $p < .001$.

Note: $n = 170$. Numbers in parentheses are standard errors. For domains without any asterisks the coefficient is not statistically significant at $p < .01$. Coefficients are from a three-level multiple regression model that accounted for the nesting of students within schools and districts. Assessment scores were standardized to allow for comparison of coefficients across predictors with different standard deviations.

Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

Table C12. Complete results for the hierarchical linear model examining the association between math domain scores on the Missouri Assessment Program in grade 7 and scale scores on the Algebra I End-of-Course Assessment in grade 8 for students who were not receiving special education services, 2016/17 and 2017/18

Variable	Coefficient
Intercept	–0.18*** (0.04)
Ratios and proportional relationships	0.10*** (0.01)
The number system	0.16*** (0.01)
Expressions and equations	0.21*** (0.01)
Geometry	0.15*** (0.01)
Statistics and probability	0.14*** (0.01)

** Significant at $p < .001$.

Note: $n = 11{,}128$. Numbers in parentheses are standard errors. Coefficients are from a three-level multiple regression model that accounted for the nesting of students within schools and districts. Assessment scores were standardized to allow for comparison of coefficients across predictors with different standard deviations.

Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

Table C13. Complete results for the hierarchical linear model with interaction terms to test for differences by special education status in the strength of the association between the math domain scores and Algebra I achievement, 2016/17 and 2017/18

Variable	Coefficient
Intercept	–0.17*** (0.04)
Receiving special education services	–0.10 (0.06)
Ratios and proportional relationships	0.10*** (0.01)
The number system	0.16*** (0.01)
Expressions and equations	0.21*** (0.01)
Geometry	0.14*** (0.01)
Statistics and probability	0.14*** (0.01)
Ratios and proportional relationships × receiving special education services	0.08 (0.05)
The number system × receiving special education services	0.07 (0.08)
Expressions and equations × receiving special education services	–0.10 (0.08)
Geometry × receiving special education services	0.06 (0.06)
Statistics and probability × receiving special education services	0.02 (0.07)

*** Significant at $p < .001$.

Note: $n = 11{,}298$. Numbers in parentheses are standard errors. For domains without any asterisks the coefficient is not statistically significant at $p < .01$. Coefficients are from bivariate three-level regression models to account for nesting of students within schools and districts. Assessment scores were standardized to allow for comparison of coefficients across predictors with different standard deviations.

Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

Reference

Raudenbush, S. W., Bryk, A. S, & Congdon, R. (2013). HLM 7.01 for Windows [Computer software]. Skokie, IL: Scientific Software International.

Appendix D. Other analyses using proficiency groups as the dependent variable

The primary analyses to address the research questions used the scale score on the Algebra I End-of-Course Assessment as the dependent variable. An alternative approach is to divide students into two proficiency groups—those who scored basic or below basic and those who scored proficient or advanced—and use this dichotomous variable as the dependent variable.

The scale score was chosen as the dependent variable in the primary analyses because dividing a continuous variable into two groups results in the loss of information that might be useful for answering the research questions. That is, there might be meaningful differences between students who perform very poorly on the Algebra I End-of-Course Assessment and those who score just below the cutoff for proficiency. Likewise, there may be meaningful differences between students who score just above the cutoff for proficiency and those who perform very well on the assessment. When a dichotomous proficiency variable is used as the dependent variable, the differences within proficiency groups do not factor into the estimation of the association between math domain scores in grade 7 and Algebra I achievement in grade 8. However, given the policy relevance of students meeting the threshold for proficiency, a second set of analyses was conducted using the dichotomous proficiency variable as the dependent variable. This appendix presents the results of those exploratory models.

Because the models include a dichotomous dependent variable, they are fit as three-level logistic regression models using hierarchical linear modeling software (Raudenbush, Bryk, & Congdon, 2013). The interpretation of logistic models is different from the interpretation of the linear regression models presented in the main report. Logistic models present odds ratios rather than regression coefficients. Here, odds ratios describe the change in the odds of scoring proficient on the Algebra I End-of-Course Assessment that is associated with a one-unit change in math domain score. Because all the domain scores were standardized before conducting the analyses, a one-unit change in standardized domain score is the same as a one standard deviation change in the corresponding unstandardized domain score.

After the four other math domain scores were controlled for, the expressions and equations domain had the strongest association with Algebra I proficiency in grade 8

Each of the five math domain scores in grade 7 was independently associated with Algebra I proficiency in grade 8, above and beyond the associations of the other four domain scores (table D1). The strongest association was for the expressions and equations domain. The odds ratio for this domain was 1.79, indicating that a one standard deviation increase in this domain score was associated with a 79 percent increase in the odds of scoring proficient or advanced on the Algebra I End-of-Course Assessment.

Table D1. Coefficients and odds ratios for the hierarchical linear model examining the association between math domain scores on the Missouri Assessment Program in grade 7 and proficiency on the Algebra I End-of-Course Assessment in grade 8, 2016/17 and 2017/18

Variable	Coefficient	Odds ratio
Intercept	2.61*** (0.11)	13.6
Ratios and proportional relationships	0.38*** (0.04)	1.5
The number system	0.46*** (0.05)	1.7
Expressions and equations	0.58*** (0.06)	1.8
Geometry	0.45*** (0.05)	1.6
Statistics and probability	0.31*** (0.05)	1.4

*** Significant at $p < .001$.

Note: $n = 11{,}298$. Numbers in parentheses are standard errors. Coefficients are from a three-level multiple logistic regression model that accounted for the nesting of students within schools and districts. Reported coefficients and odds ratios are derived from unit-specific regression estimates. For more about the distinction between unit-specific and population average estimates in nonlinear regression models, see Raudenbush and Bryk (2002). Domain scores in grade 7 were standardized, so a one-unit change in standardized domain score is the same as a one standard deviation change in the corresponding unstandardized domain score.

Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

The associations between the five math domain scores in grade 7 and Algebra I proficiency in grade 8 did not differ significantly by English learner status

When the strength of the associations between math domain scores in grade 7 and Algebra I proficiency in grade 8 were compared for English learner students and non–English learner students, none of the differences were statistically significant (table D2).

Table D2. Coefficients and odds ratios for the hierarchical linear model with interaction terms to test for differences by English learner status in the strength of the association between the math domain scores on the Missouri Assessment Program in grade 7 and proficiency on the Algebra I End-of-Course Assessment in grade 8, 2016/17 and 2017/18

Variable	Coefficient	Odds ratio
Intercept	2.60*** (0.11)	13.4
English learner status	1.18 (0.62)	3.3
Ratios and proportional relationships	0.37*** (0.04)	1.4
The number system	0.45*** (0.05)	1.6
Expressions and equations	0.58*** (0.06)	1.8
Geometry	0.46*** (0.05)	1.6
Statistics and probability	0.33*** (0.05)	1.4
Ratios and proportional relationships × English learner status	0.51 (0.28)	1.7
The number system × English learner status	0.57 (0.35)	1.8
Expressions and equations × English learner status	0.41 (0.38)	1.5
Geometry × English learner status	–0.51 (0.35)	0.6
Statistics and probability × English learner status	–0.52 (0.34)	0.6

*** Significant at $p < .001$.

Note: n = 11,298. Numbers in parentheses are standard errors. For domains without any asterisks the coefficient is not statistically significant at $p < .01$. Coefficients are from a three-level multiple logistic regression model that accounted for the nesting of students within schools and districts. Reported coefficients and odds ratios are derived from unit-specific regression estimates. For more about the distinction between unit-specific and population average estimates in nonlinear regression models, see Raudenbush and Bryk (2002). Domain scores in grade 7 were standardized, so a one-unit change in standardized domain score is the same as a one standard deviation change in the corresponding unstandardized domain score.

Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

The associations between the five math domain scores in grade 7 and Algebra I proficiency in grade 8 did not differ significantly by special education status

When the strength of the associations between math domain scores in grade 7 and Algebra I proficiency in grade 8 was compared for students who were receiving special education services and students who were not, none of the differences were statistically significant (table D3).

Table D3. Coefficients and odds ratios for the hierarchical linear model with interaction terms to test for differences by special education status in the strength of the association between the math domain scores on the Missouri Assessment Program in grade 7 and proficiency on the Algebra I End-of-Course Assessment in grade 8, 2016/17 and 2017/18

Variable	Coefficient	Odds ratio
Intercept	2.62*** (.11)	13.7
Receiving special education services	–0.55 (.45)	0.6
Ratios and proportional relationships	0.37*** (.04)	1.4
The number system	0.45*** (.05)	1.6
Expressions and equations	0.59*** (.06)	1.8
Geometry	0.45*** (.05)	1.6
Statistics and probability	0.31*** (.05)	1.4
Ratios and proportional relationships × receiving special education services	0.20 (.28)	1.2
The number system × receiving special education services	0.73 (.49)	2.1
Expressions and equations × receiving special education services	–0.29 (.43)	0.8
Geometry × receiving special education services	–0.22 (.34)	0.8
Statistics and probability × receiving special education services	–0.23 (.36)	0.8

*** Significant at $p < .001$.

Note: $n = 11,298$. Numbers in parentheses are standard errors. For domains without any asterisks the coefficient is not statistically significant at $p < .01$. Coefficients are from a three-level multiple logistic regression model that accounted for the nesting of students within schools and districts. Reported coefficients and odds ratios are derived from unit-specific regression estimates. For more about the distinction between unit-specific and population average estimates in nonlinear regression models, see Raudenbush and Bryk (2002). Domain scores in grade 7 were standardized, so a one-unit change in standardized domain score is the same as a one standard deviation change in the corresponding unstandardized domain score.

Source: Authors' analysis of 2016/17 and 2017/18 data from the Missouri Department of Elementary and Secondary Education.

Comparison of results from primary and exploratory models

Overall, the results of these exploratory models were similar to those of the primary models, which used scale scores on the Algebra I End-of-Course Assessment as the dependent variable. In both sets of models the expressions and equations domain was the strongest predictor of performance on the Algebra I End-of-Course Assessment. Results for both sets of models also revealed that the strength of associations between math domain scores in grade 7 and performance on the Algebra I End-of-Course Assessment in grade 8 did not differ by special education status. The only case in which the two sets of models differed was for the models examining whether the association between domain scores in grade 7 and Algebra I achievement in grade 8 differed by English learner status. In the primary models the association between the number system domain and Algebra I achievement was stronger for English learner students than for non–English learner students. The difference was not significant in the models using the dichotomous proficiency variable as the dependent variable. The difference in the results of the two models could be due to information being lost when the continuous variable is changed into a dichotomous variable.

References

Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models: Applications and data analysis methods* (2nd ed.). Thousand Oaks, CA: Sage.

Raudenbush, S. W., Bryk, A. S, & Congdon, R. (2013). HLM 7.01 for Windows [Computer software]. Skokie, IL: Scientific Software International.