Application of reliability methods in geological engineering design

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ABSTRACT
Reliability methods, in conjunction with more traditional design methods, have begun to play an increasingly important role in geotechnical projects. Such analyses offer a more rational approach to quantify risk by incorporating uncertainty in the input variables and evaluating the probability of failure for the system. While these concepts are relatively new to geological engineering, they require little additional effort beyond conventional analyses and provide a more complete definition of risk and safety. This eliminates the need for overly conservative design methods and allows a more economic design to be selected. This paper summarizes the main reliability methods available for geotechnical projects and presents the findings of a reliability analysis for an underground pillar design in limestone.

RÉSUMÉ
Méthodes de fiabilité, en conjonction avec des méthodes plus traditionnelles de design, ont commencé à jouer un rôle de plus en plus important dans les projets géotechniques. Ces analyses offrent une approche plus rationnelle pour quantifier le risque par l'intégration de l'incertitude des variables d'entrée et l'évaluation de la probabilité de défaillance du système. Bien que ces concepts sont relativement nouveaux en génie géotechnique, ils demanderont peu d'efforts supplémentaires plus des analyses classiques et de fournir une définition plus complète des risques et de sécurité. Ceci élimine le besoin pour des méthodes trop conservatrice et permet une design plus économique pour être choisi. Ce document résume les principales méthodes de fiabilité disponibles pour des projets géotechniques et présente les résultats d'une analyse de fiabilité pour un design pilier de métro dans le calcaire.

1 INTRODUCTION

Since the development of standardized field and lab testing procedures for the study of geological materials in the early 20th century, it has been understood that uncertainty is prevalent in both material and stress parameters. Unfortunately, present design methods have yet to adopt a logical basis for describing this uncertainty and assessing its impact on performance. Instead, deterministic approaches, which consider only a single set of representative parameters, are typically used. Such approaches have significant shortcomings as they provide only a first-moment approximation of the mean response and can frequently miss the true failure mechanism (Sayed *et al.* 2010). To address this, conservative values are selected to ensure the structure can withstand a range of potential loading conditions. Such methods provide an inconsistent measure of risk and in many cases can result in unnecessary cost and schedule overruns.

Reliability methods, in conjunction with more traditional design methods, provide better insight into design performance by quantifying uncertainty in both the loads and resistances acting on a system. In doing so, the probability of failure p_f can be assessed with respect to a prescribed failure criterion or mode, leading to a greater understanding of risk and a more economic design. Despite the inherent benefits of these methods, they have yet to achieve widespread use in geological engineering. This is largely due to the use of unfamiliar statistical terms and the misconception that such analyses require considerably more effort than conventional design methods. Contrary to this belief, reliability analyses can be easily applied to geotechnical problems and provide invaluable information to the engineer regarding system performance. While reliability methods are still in their infancy, examples of their use in civil and geological engineering projects exist in the literature for retaining wall design (Basma *et al.* 2003, Sayed *et al.* 2010), slope stability assessments (El-Ramly *et al.* 2002, Xu and Low 2006, Kavvadas *et al.* 2009, Duncan 2000), tunnel design (Schweiger *et al.* 2003, Brinkman 2009, Mollon *et al.* 2009, Papaioannou *et al.* 2009) and underground pillar analyses (Griffiths *et al.* 2002). Baecher & Christian (2003) also provides an excellent overview of reliability methods and potential applications.

To perform a reliability analysis for a particular system, a performance function must be used to define the critical limit state between stable and failure conditions. For simple systems, the performance function can be defined explicitly and system performance (expressed as the reliability index β) can be determined easily. In complex systems, such as in underground applications where the rock is acting as both a load and a resistance, closed form solutions rarely exist. In these cases, the engineer must employ techniques that approximate the behaviour of the performance function to provide a reasonable estimate of system reliability. Monte Carlo simulations can also be used for direct calculation of the probability of failure where sufficient computer power is available.

This paper provides a brief description of the sources of uncertainty in geotechnical engineering and outlines a standard approach to reliability design. The major reliability-based methods that can be used in geological design are then presented and discussed. These methods are then applied to assess the performance of an underground pillar in limestone.

2 UNCERTAINTY IN GEOLOGICAL ENGINEERING

Given the complex history of formation and the difficulties associated with testing geomaterials, uncertainty plays a significant role in geotechnical engineering. In general, sources of uncertainty include the natural heterogeneity or in situ variability of the material, the limited availability of information about subsurface conditions and errors made during the measurement and testing phase (Schweiger *et al.* 2001). This uncertainty in the absolute value creates a distribution of possible values for each input parameter. To determine the likelihood of a certain value occurring, a probability density function (PDF) is defined for each parameter using a series of statistical moments, the most common of which are the mean μ (first moment) and variance σ^2 (second moment). For most geological parameters, it is suitable to use a normal or lognormal distribution (Baecher and Christian 2003), however other distributions may be used where appropriate.

As uncertainty is present in the in situ parameters, this logically means that uncertainty will also exist in the expected performance of the design. Unfortunately, the overall uncertainty is rarely quantified. Instead, deterministic analyses are performed using conservative input parameters to ensure the structure is robust enough to withstand all potential loads. As the definition of "conservative" is variable, there is no guarantee that the design will perform as expected. Additionally, using overly conservative values often leads to significant cost and schedule overruns. In situations where conservatism is not appropriate, the observational method is typically used. This process begins during the design phase where possible methods of unsatisfactory performance are considered and plans are developed to address these issues. Field measurements are then made during the construction and operations phases to establish whether these developments are occurring. This observed behavior is then used to update the design and construction process. While this method is used extensively, it has significant limitations as it does not consider the relative likelihood of the undesirable occurrences and cannot anticipate possible design deficiencies. It also requires continuous communication between the engineer and decision maker to be effective.

3 RELIABILITY-BASED DESIGN

3.1 Reliability Index and the Probability of Failure

Reliability analyses offer a more rational approach to quantify design risk than deterministic analyses by incorporating uncertainty in the input parameters in the analysis. In doing so, a probability of failure p_f can be established with respect to a specific failure mode, with "failure" defined as either the ultimate collapse of the structure or loss of serviceability. This provides a more consistent and complete measure of risk as the probability of failure is invariant to all mechanically equivalent definitions of safety and incorporates additional uncertainty information (Sayed *et al.* 2010).

To perform a reliability analysis, a performance function $G(\boldsymbol{X})$ must be defined that relates the resistances $R(\boldsymbol{X})$ and the loads $Q(\boldsymbol{X})$ acting on the system. This relationship is written as:

$$G(\mathbf{X}) = R(\mathbf{X}) - Q(\mathbf{X}) \qquad [1]$$

where $\boldsymbol{X}$ is the collection of random input variables. Based on this definition, stable conditions are anticipated when $G(\boldsymbol{X}) > 0$, while $G(\boldsymbol{X}) < 0$ implies failure. The surface created by $G(\boldsymbol{X}) = 0$ is referred to as the critical limit state as it defines the boundary between these two conditions. When considering the critical limit state for a simple system such as a block moving on a surface, the performance function would be expressed as the shear strength that resists sliding minus the shear force that initiates sliding. Such equations can be evaluated analytically with little additional effort. For more complex problems, such as an analysis of tunnel deformation in brittle or squeezing conditions, it is difficult to define the loads and resistances explicitly. Approximate methods of evaluation are therefore required.

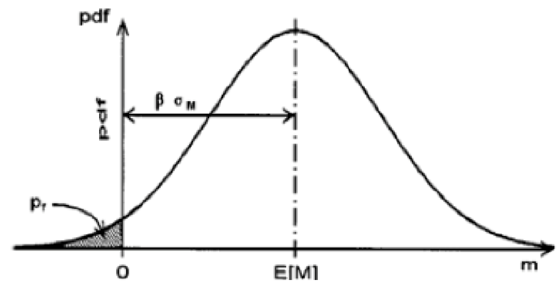

Figure 1. Probability Density Function (PDF) showing the reliability index β and probability of failure p_f for a performance function (Christian 2004).

To evaluate the reliability of the system, the distance between the mean value of the performance function and the critical limit state at $G(\boldsymbol{X}) = 0$ must be determined. When the distance between these two points is normalized with respect to the standard deviation (uncertainty) of the performance function, this is referred to as the reliability index β for the system (Figure 1). This is defined as:

$$\beta = \frac{\mu_G}{\sigma_G} \qquad [2]$$

where μ_G and σ_G are the mean value and standard deviation of the performance function, respectively. The disadvantage to Equation 2 is that to solve for the value of β, the exact shape of the performance function must be known, which is not always the case. A more versatile measurement of reliability is the Hasofer-Lind reliability index β_{HL}. This method, also referred to as the First Order Reliability Method (FORM), calculates the minimum distance in units of directional standard deviation from the mean value point of the multivariate distribution of the random variables to the boundary of the critical limit state (Figure 2). This provides a more consistent and invariant measure of reliability for the system and can also be easily calculated for correlated or uncorrelated variables using the approach outlined in Low and Tang (1997). The equation for β_{HL} can be defined as:

$$\beta_{HL} = min_{x \in F}\sqrt{(\boldsymbol{X}-\boldsymbol{\mu})^T \boldsymbol{C}^{-1}(\boldsymbol{X}-\boldsymbol{\mu})} \qquad [3]$$

where $\boldsymbol{X}$ is the vector of random variables, $\boldsymbol{\mu}$ is the vector of mean values of random variables and F defines the failure region of $G(\boldsymbol{X}) < 0$. The variable $\boldsymbol{C}$ defines the correlation matrix, which allows the user to establish either a positive or negative relationship between random variables. As an example, a positive correlation could be used to describe the relationship between the uniaxial compressive strength (UCS) and the Young's Modulus (E) of a material. For uncorrelated variables, the matrix $\boldsymbol{C}$ simplifies to a symmetric unit matrix. While matrix algebra may be unfamiliar to some, programs such as Microsoft Excel or MATLAB can be used to easily complete these calculations. One such method is summarized in Low and Tang (1997).

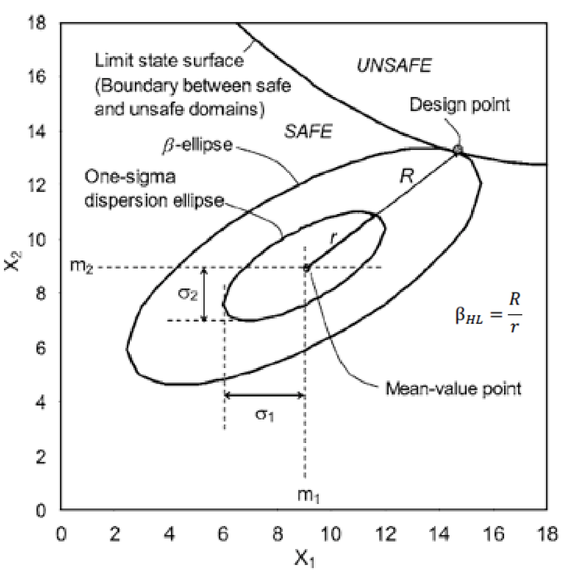

Figure 2. Design point, mean value point and reliability index in plane for a FORM analysis (Xu and Low 2006).

Once the reliability index has been determined, the probability of failure p_f for the system can be found by calculating the probability of $G(\boldsymbol{X}) < 0$. This is related to the reliability index using the following equation:

$$p_f = \int_{-\infty}^{0} G(\boldsymbol{X}) = \Phi(\beta) \qquad [4]$$

where Φ is the cumulative distribution function (CDF) for the performance function evaluated at 0 with a unit standard deviation and a mean β. As mentioned earlier, the shape of the distribution is rarely known and therefore must be assumed. In most cases, a normal distribution is reasonable, however a truncated or lognormal distribution may be more appropriate when the performance function depends on positive functions, such as factor of safety, extent of yield in a material or displacements.

From Equation 2 it is clear that for a constant mean value, as the reliability index increases, the uncertainty in the estimate of the performance function decreases (Figure 3). This results in a more narrow distribution for the performance function and a decrease in the probability of failure for the system (Table 1). The relationship between the reliability index and the probability of failure for a system is demonstrated graphically in Figure 4.

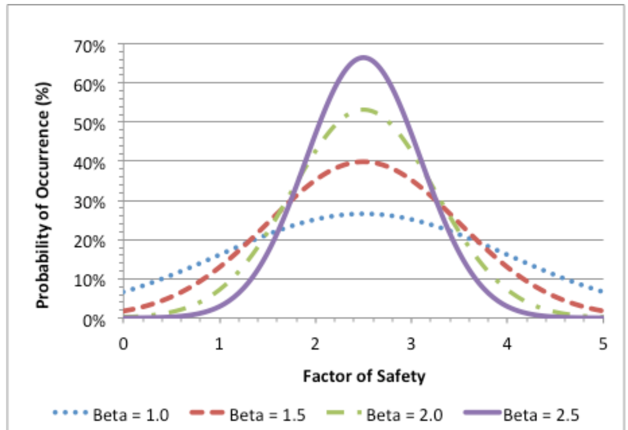

Figure 3. Effect of changing the reliability index on the uncertainty of a system with a mean factor of safety of 2.5, assuming the function is normally distributed.

Table 1. Effect of changing the reliability index on the probability of failure for a system. Reliability indices correspond to the PDFs in Figure 3.

Design	Reliability Index (β)	Probability of Failure (p_f)
Option 1	1.0	15.9%
Option 2	1.5	6.7%
Option 3	2.0	2.3%
Option 4	2.5	0.6%

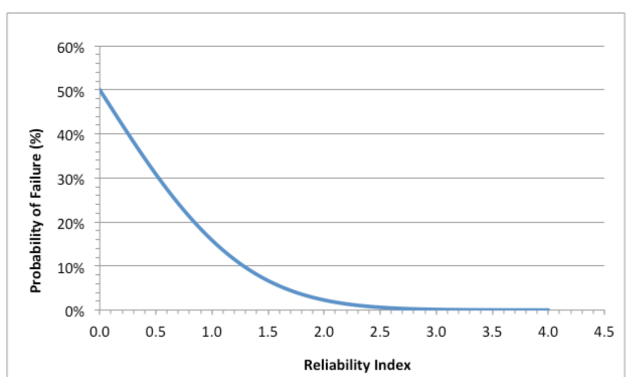

Figure 4. Relationship between the reliability index and the probability of failure for a system, assuming the performance function is normally distributed.

3.2 Reliability-Based Design Approach

Based on the general description of reliability theory in the previous section, a set of steps can be defined for a general reliability analysis. These are:

1. Develop a performance function by defining the failure conditions for the system. When considering the ultimate limit state of a structure, this will typically involve a minimum factor of safety. For serviceability analyses, the definition may involve critical displacements, failure of support elements or maximum convergence.
2. Determine the statistical moments of relevant input parameters including the mean and variance. Select an appropriate PDF based on the data set.
3. Calculate the statistical moments of the performance function. In simple cases these values can be determined analytically, while approximations are needed in more complex cases. For geological engineering problems, finite element models are typically required.
4. Calculate the reliability index β directly (Equation 2) or by using a FORM analysis (Equation 3).
5. Calculate the probability of failure p_f. If a sufficiently large number of evaluations have been performed, this can be calculated directly. For other cases, an assumption must be made about the shape of the performance function.

As many of the reliability methods are based on approximations, it is expected that different methods will produce different results. It is important that results from different methods are compared to obtain a more accurate understanding of system performance.

4 DIRECT RELIABILITY METHODS

For complex problems, the performance function cannot be stated explicitly. In such cases, reliability methods are typically coupled with the finite element method (FEM) to evaluate the performance function at a series of discrete points and approximate the overall behaviour. The number of evaluations required and what input parameters are selected depends on the reliability method used. The following section describes four methods that can be used to approximate the statistical moments of the performance function.

4.1 First Order Second Moment Method

In cases where the performance function is smooth and regular, the mean and variance can be calculated using the first terms of a Taylor series expansion according to the First Order, Second Moment (FOSM) method. This method assumes that the expected value of the performance function is approximately equal to the value of the function calculated with the mean values of all variables. The variance is then determined by calculating the partial derivatives of the performance function with respect to each of the uncertain variables. For N uncorrelated random variables, the variance of the performance function is defined as:

$$\sigma_G^2 \approx \sum_{i=1}^{N} \left(\frac{\partial G}{\partial X_i}\right)^2 {\sigma_{X_i}}^2 \quad [5]$$

where X_i refers to the N random variables. As the performance function cannot be explicitly stated in most geotechnical applications, a linear approximation of the partial derivative is required. This is accomplished by changing each variable by a small amount (ΔX_i) while all other variables are kept at their mean value. The change in the performance function (ΔG) that results is then divided by the difference in the input. To maintain a consistent level of uncertainty, the input variables are typically chosen at the mean plus and minus one standard deviation. Equation 5 can then be rewritten as:

$$\sigma_G^2 \approx \sum_{i=1}^{N} \left(\frac{\Delta G}{\Delta X_i}\right)^2 \sigma_{X_i}^2 \quad [6]$$

For this case, $2N+1$ approximations are required. With the moments of the performance function known, the reliability index and probability of failure can be calculated using Equations 2 and 4, assuming a normal distribution. A straightforward application of this method to a slope stability calculation can be found in Duncan (2000).

The FOSM approach is considered a fairly simple method of analysis when compared to other direct reliability methods as only the statistical moments of the input variables are required, rather than complete knowledge of the distribution. The shortcoming of this method is that the accuracy of the results depends upon the particular values of the variables at which the partial derivatives are approximated or calculated (Christian 2004). The assumption of a linear approximation also creates difficulties when the behaviour of the performance function changes significantly over ΔX. One such example of this would be where the lower value results in failure, and the upper value results in a stable condition. In this case, an estimate of 50% failure would be produced due to the assumed linear behaviour between the results.

4.2 Point Estimate Method

Originally proposed in Rosenblueth (1978, 1981), the Point Estimate Method (PEM) is a numerical procedure that approximates the moments of a function by evaluating it at a series of specifically chosen, discrete points. Evaluation points are chosen at the mean plus and minus one standard deviation for each variable, resulting in 2^N evaluations for N random variables (Figure 5). A weighting value ρ is used at each evaluation point to ensure the moments of the input variables are recovered. If all evaluation points are weighted equally, this value is simply $1/N$ for each variable. The statistical moments of the performance function for the PEM are defined as:

$$\mu_G = \sum_{i=1}^{2^N} \rho_i G(X_i) \quad [7]$$

$$\sigma_G^2 = \sum_{i=1}^{2^N} \rho_i (G(X_i))^2 - \left(\sum_{i=1}^{2^N} \rho_i G(X_i)\right)^2 \quad [8]$$

While the Rosenblueth PEM is considered a robust method when the coefficients of variation for input variables are small, the number of evaluations can be significant when a large number of random variables are considered. Methods have been developed by several authors to reduce the number of evaluations required, however the user must be mindful of the assumptions made. These methods are summarized further in Tsai and Franceschini (2005).

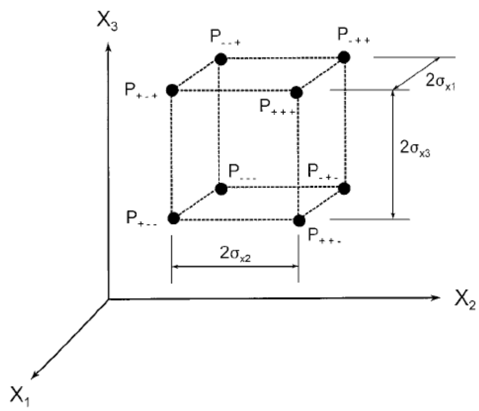

Figure 5. Evaluation points for a system of three variables using the Rosenblueth point estimate method (Christian and Baecher 1999).

4.3 Response Surface Method and First Order Reliability Method

The Response Surface Method (RSM) is commonly used in civil engineering design to approximate the mechanical response of a structure. In reliability methods, the RSM can be used to approximate the performance function by relating the input and output parameters for a system by a simple mathematical expression. For civil geotechnical systems, it has been shown that quadratic polynomials are suitable for localized approximation (Sayed *et al.* 2010). The exact limit state function $G(\boldsymbol{X})$ can therefore be approximated by a polynomial function $G'(X)$:

$$G'(X) = l + \sum_{i=1}^{N} m_i X_i + \sum_{i=1}^{N} n_i X_i^2 \quad [9]$$

where X_i refers to the N random variables and l, m_i and n_i are coefficients that must be determined. To properly evaluate the number of unknowns in the quadratic equation, $2N+1$ evaluations are required (Bucher and Bourgund 1990). Once the approximate limit state function has been established, the FORM (Equation 3) is used to determine the reliability index directly. This is more accurate than the FOSM method as it uses geometric interpretations to determine the reliability index rather than determining statistical moments through a linear extrapolation of mean input values. The advantage to the combined RSM/FORM method is that it can be used for correlated and non-normal input variables and is suitable for any linear limit state surface. One disadvantage is the assumption that the inputs and outputs are related through a quadratic equation, which may not be valid in all situations.

4.4 Monte Carlo Simulation

In cases where the behaviour of the performance function is difficult to evaluate, the probability of failure for a certain limit state can be evaluated directly using Monte Carlo simulations. In this method, large sets of randomly selected input values are generated according to their PDF. Each set of parameters is then used in the analytical model to determine the behaviour of the system. The frequency of each outcome can then be plotted to directly calculate the probability of failure. Several examples of the application of this method can be found in the literature including Griffiths et al (2002), which combines it with the random finite element method (RFEM).

The lack of approximations makes the Monte Carlo method an ideal benchmark to compare the other reliability methods to. While the method is straightforward to apply, it can be computationally intensive and time consuming. Additionally, as each set is randomly generated, the relative contributions of each variable to the overall uncertainty cannot be calculated.

5 CASE STUDY: PILLAR ANALYSIS IN SPALLING GROUND

To demonstrate the applicability of reliability methods, a case study is examined for an underground pillar analysis based on conditions similar to those observed at the Norton Barberton mine in Ohio. Pillar stability analyses are relevant to several engineering applications, such as pump storage design, expansion of existing mine works and nuclear waste storage in an underground repository.

5.1 Finite Element Analysis

A 13 m wide, unreinforced limestone pillar at a depth of 700 m has been considered. The pillar separates room excavations measuring 8.5 m wide and 7 m high. Stress conditions at depth are considered isotropic with a ratio of horizontal stress to vertical stress (K) equal to 1.5. Given the high quality and strength of the limestone and the stress conditions at depth, spalling and brittle failure were considered the most likely failure mechanisms.

Expected values and standard deviations were determined for each parameter by analyzing geotechnical test results. All variables were determined to exhibit approximate normal behaviour and are assumed to be uncorrelated. The statistical moments for each variable are shown in Table 2. To reduce the number of random variables considered, a parametric study was completed to determine which parameters had a substantial effect on pillar failure. From this, it was concluded that only the UCS and crack initiation ratio have engineering significance and need to be treated as random variables. All other parameters were treated deterministically and their expected (mean) values were used.

Hoek-Brown peak and residual strength parameters were calculated according to the spalling criterion proposed in Diederichs (2007) and are shown in Table 3.

As these strength parameters are functions of the in situ parameters, variances were determined using error propagation techniques. Using this method resulted in coefficients of variation in excess of 40% for the parameters, indicating a significant increase in uncertainty. In the case of the residual *m* parameter, a coefficient of variation of 60% was calculated, which was considered unreasonably high given a range of 7 to 10 is expected (Diederichs 2007). As such, a standard deviation of 0.5 was used as this ensures 99% of values will fall within the expected range. This approach, referred to as the 3-sigma rule, is considered to be an acceptable alternative method to error propagation techniques for determining uncertainty in spalling parameters where unreasonable coefficients of variation are determined.

Table 2. Mean μ, standard deviation σ and coefficient of variation COV for random variables.

Random Variable	μ	σ	COV
Density (Mg/m^3)	2.69	0.02	1%
UCS (MPa)	111	24	21%
Crack Initiation Ratio	41.6%	4.3%	10%
Tensile Strength (MPa)	4.3	1.2	27%
Intact Young's Modulus (GPa)	37.3	8.8	24%
Poisson's Ratio	0.31	0.09	28%

Table 3. Mean μ, standard deviation σ and coefficient of variation COV for Hoek-Brown spalling parameters.

Random Variable	μ	σ	COV
Peak (spalling) m	0.773	0.415	54%
Peak (spalling) s	0.030	0.012	41%
Peak (spalling) a	0.250	-	-
Residual m	8.643	0.50	6%
Residual s	0.000	-	-
Residual a	0.750	-	-

Pillar yield was estimated using the finite element analysis program 'Phase2' developed by Rocscience Incorporated (www.rocscience.com). Given the symmetry of the room and pillar design, the model was centred about the pillar with the edges through the centre of the adjacent rooms to reduce computational requirements (Figure 6). The analysis was carried out in three stages. In the first stage, the self-weight of the rockmass is applied. Boundary conditions are used to restrain the walls of the model in the lateral direction and the bottom of the model in the vertical direction. An equivalent field stress is applied to the top of the model to simulate loading from the column of rock above. In the second stage, the room to the left of the pillar is fully excavated, while the room to the right of the pillar is fully excavated in the third stage. The finite element mesh used for the analysis consists of 4006 nodes and 7778 3-noded triangular elements. A maximum of 500 iterations is used for each load step.

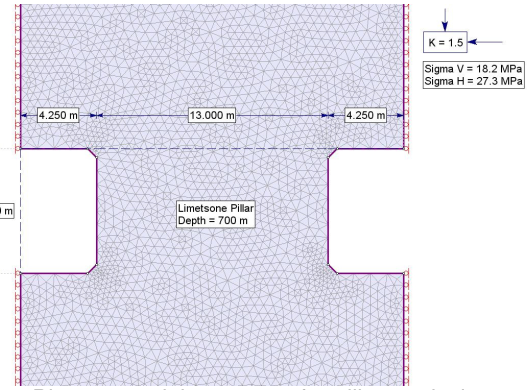

Figure 6. Phase2 model geometry for pillar analysis.

5.2 Results and Discussion

For this analysis, the performance function is defined by the amount of pillar damage (yield) sustained. The performance function is therefore defined as:

$$G(\mathbf{X}) = D_{max} - D(\mathbf{X}) \qquad [10]$$

where D_{max} is the serviceability limit that defines the maximum amount of pillar damage permitted and $D(\mathbf{X})$ is an expression of the pillar damage as a function of the input variables (measured along the mid-height of the pillar). Each of the four reliability methods described in Section 4 was used to evaluate the probability of failure for the pillar. Examples of pillar damage in the models are shown in Figures 7 and 8.

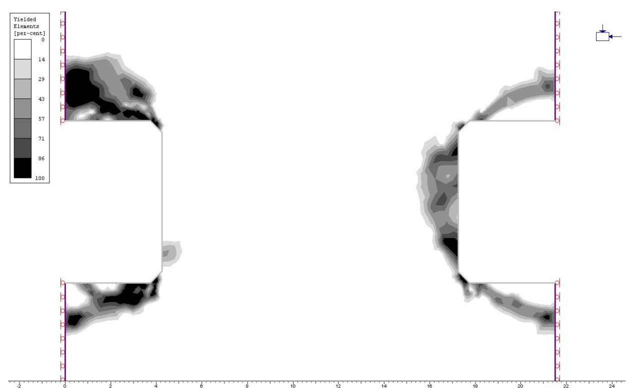

Figure 7. Phase2 yielded element output showing 16% pillar damage (yield zone shown in black).

Results for the reliability analyses are shown in Table 4 for a D_{max} value of 40%. While no failure is predicted for the expected (mean) case, the probability of pillar damage exceeding the serviceability limit state is predicted to be between 5 and 13%, with an expected value of 9%. The variability in the estimates of probability of failure demonstrates the importance of using more than one reliability method in these analyses. At least two methods should always be considered to ensure the probability is not seriously under or overestimated.

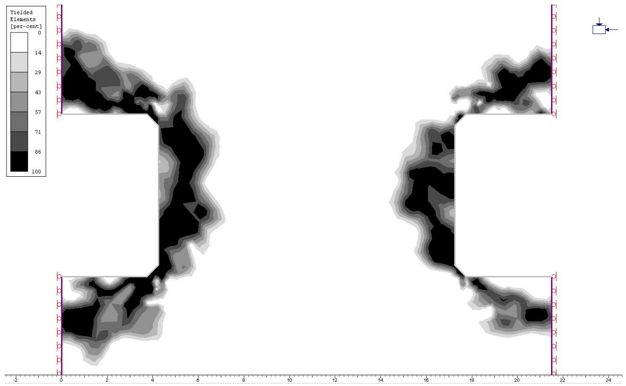

Figure 8. Phase2 yielded element output showing 36% pillar damage (yield zone shown in black).

Table 4. Reliability analysis results for underground pillar analysis (for D_{max} value of 40%).

Method (Distribution)	Number of Models	Reliability Index (β)	Probability of Failure (p_f)
FOSM (Normal)	9	1.62	5.3%
PEM (Normal)	17	1.22	11.1%
PEM (Lognormal)	17	1.29	9.8%
RSM (Normal)	9	1.13	12.9%
Monte Carlo Simulation	100	-	6.0%

Selecting an appropriate distribution to represent the performance function and calculate the probability of failure from the reliability index was a consistent issue for the FOSM, PEM and RSM analyses. As can be seen in the Monte Carlo simulation results, 40% of the evaluations show no pillar damage while the remaining 60% show between 0 and 50% pillar damage (Figure 9). While an exponential function has a similar shape as these results, pillar damage due to spalling does not logically follow such behaviour. Exponential functions model processes in which events occur continuously and independently at a constant average rate, while spalling failure is better described as an instantaneous event that occurs when a critical stress condition is exceeded. Given this reasoning and in the interests of reducing complexity, normal and lognormal distributions were used to express the performance function for each analysis.

The probability of failure was determined for each method for a D_{max} value of 0 to 100% in 10% increments. The results indicate that the reliability methods tend to approximate the behaviour of high (>40%) damage events well, but either underestimate or overestimate the probability of low (0-40%) damage events (Figure 10). The RSM proved to be the least accurate of the methods while the PEM estimates the probability of failure of high damage events the most accurately. At lower critical damage values, Monte Carlo simulations would be required to accurately determine the probability of failure. It is worth noting that by averaging the PEM and FOSM results, the resultant curve has the least amount of error when compared to the Monte Carlo simulation results.

A significant limitation to the reliability methods as proposed involves the limited range of input parameters that are sampled during analysis. In risk analyses, relatively rare events, which occur at the tails of the distribution, can have extreme consequences and need to be considered. While these points are sampled in a Monte Carlo simulation, they are not considered by the FOSM, PEM or RSM methods. When using approximate methods, further analyses should be performed to evaluate system behaviour at two and three standard deviations from the mean. These points could then be used to refine the proposed distribution of the performance function. While this will add to the number of evaluations being performed, it will still be less computationally intensive than a Monte Carlo simulation, which would ideally involve thousands of runs.

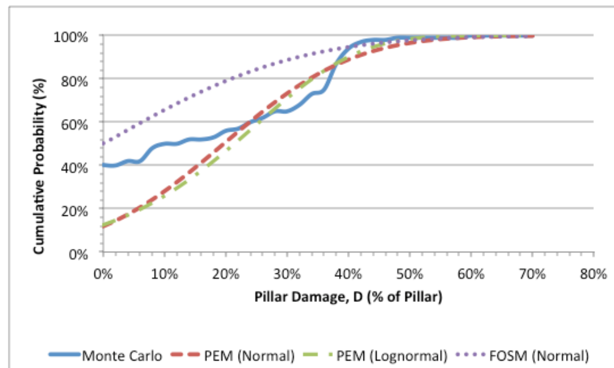

Figure 9. Pillar damage comparison between results obtained using a Monte Carlo simulation and the expected normal distributions developed using the FOSM and Rosenblueth PEM.

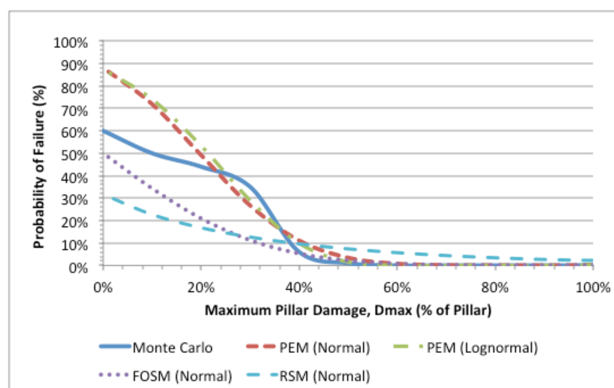

Figure 10. Probability of exceeding a variety of maximum pillar damage values (D_{max}) for each reliability method.

6 CONCLUSIONS

Uncertainty is inherent in geological engineering problems and can have a significant impact on design performance if not properly accounted for. Currently, the reliability-based design approach is the only methodology that quantifies uncertainty and provides a consistent measure of safety by determining the probability of failure for a system. While these methods may appear complicated, they require little additional effort when compared to conventional design methods and can be logically applied to a variety of engineering problems. In the case of complex situations where the rock acts as both a load and a resistance, finite element methods should be used in conjunction with reliability methods to approximate the behaviour of the system. As the results will differ depending on the method selected, two or more reliability approaches should be used to gain an appreciation of the errors involved. For approximate methods, additional sampling should occur at the tails of the distribution to ensure that low probability and high consequence events are assessed. While this will result in additional evaluations of the performance function, thus degrading the advantage of approximate methods over Monte Carlo simulations, the number of required evaluations will still be far less.

To demonstrate the value of reliability analyses, a case study was performed for a pillar design in limestone at depth. While the mean case showed no pillar damage, a 9 ± 3% probability of failure was calculated for a serviceability limit state of 40% damage using reliability methods. Had only the mean case been considered, as would have been the case with a deterministic analysis, no pillar damage would have been anticipated and the design risk would have been significantly underestimated. Issues in selecting an appropriate distribution shape to calculate the probability of failure were present, however averaging the results from multiple methods provided an appropriate measure of reliability for the system for high damage levels.

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REFERENCES

Baecher, G.B., and Christian, J.T. 2003. *Reliability and Statistics in Geotechnical Engineering*. John Wiley & Sons Ltd., Chichester, West Sussex, UK.

Basma, A.A., Barakat, S.A., and Omar, M.T. 2003. Reliability based risk index for the design of reinforced earth structures, *Geotechnical and Geological Engineering*, 21(3): 225-242.

Brinkman, J. 2009. Probabilistic approach on soil deformations for shield tunnelling at Groene Hart tunnel, *2nd International Conference on Computational Methods in Tunnelling*, Bochum, Germany: 579-584.

Bucher, C.G., and Bourgund, U. 1990. A fast and efficient response surface approach for structural reliability problems, *Structural Safety*, 7(1): 57-66.

Christian, J.T. 2004. Geotechnical engineering reliability: how well do we know what we are doing? *Journal of Geotechnical and Geoenvironmental Engineering*, 130(10): 985-1003.

Christian, J.T., and Baecher, G.B. 1999. Point estimate method as numerical quadrature, *Journal of Geotechnical and Geoenvironmental Engineering*, 125(9): 779-786.

Diederichs, M. 2007. The 2003 Canadian Geotechnical Colloquium: Mechanistic interpretation and practical application of damage and spalling prediction criteria for deep tunneling, *Canadian Geotechnical Journal*, 44(9): 1082-1116.

Duncan, J.M. 2000. Factors of safety and reliability in geotechnical engineering, *Geotechnical and Geoenvironmental Engineering*, 126(4): 307-316.

El-Ramly, H., Morgenstern, N.R., and Cruden, D.M. 2002. Probabilistic slope stability analysis for practice, *Canadian Geotechnical Journal*, 39(3): 665-683.

Griffiths, D.V., Fenton, G.A., and Lemons, C.B. 2002. Probabilistic analysis of underground pillar stability, *International Journal for Numerical and Analytical Methods in Geomechanics*, 26, 775-791.

Kavvadas, M., Karlaftis, M., Fortsakis, P., and Stylianidi, E. 1999. Probabilistic analysis in slope stability, *17th International Conference on Soil mechanics and Geotechnical Engineering*, Alexandria, Egypt, Vol. 2: 1650-1653.

Low, B.K., and Tang, W.H. 1997. Efficient reliability evaluation using spreadsheet, *Journal of Engineering Mechanics*, 123(7): 749-752.

Mollon, G., Dias, D., and Soubra, A. 2009. Probabilistic analysis of circular tunnels in homogeneous soil using response surface methodology, *Journal of Geotechnical and Geoenvironmental Engineering*, 135(9): 1314-1325.

Papaioannou, I., Heidkamp, H., Düster, A., Rank, E., and Katz, C. 2009. Random field reliability analysis as a means for risk assessment in tunneling, *2nd International Conference on Computational Methods in Tunnelling*, Bochum, Germany: 585-592.

Rosenblueth, E. 1975. Point estimates for probability moments, *Proceedings of the National Academy of Sciences of the United States of America*, 72(10): 3812-3814.

Rosenblueth, E. 1981. Two-point estimates in probabilities, *Applied Mathematical Modelling*, 5(5): 329-335.

Sayed, S., Dodagoudara, G.R., and Rajagopala, K. 2010. Finite element reliability analysis of reinforced retaining walls, *Geomechanics and Geoengineering*, 5(3): 187-197.

Schweiger, H.F., Thurner, R., and Pöttler, R. 2001. Reliability analysis in geotechnics with deterministic finite elements, *International Journal of Geomechanics*, 1(4): 389-413.

Tsai, C.W., and Franceschini, S. 2005. Evaluation of probabilistic point estimate methods in uncertainty analysis for environmental engineering applications, *Journal of Environmental Engineering*, 131(3): 387-395.

Xu, B., and Low, B.K. 2006. Probabilistic stability analyses of embankments based on finite-element method, *Journal of Geotechnical and Geoenvironmental Engineering*, 132(11): 1444-1454.