APPLICATION OF THE WAVELET TRANSFORM IN MACHINE-LEARNING

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The wide variety of waveform in EEG signals and the high non-stationary nature of many of them is one of the main difficulties to develop automatic detection system for them. In sleep stage classification a relevant transient wave is the K-complex. The present paper purposes the developing an algorithms in order to achieve an automatic K-complex detection from EEG raw data. The algorithm is based on a time-frequency analysis and two time-frequency techniques , the Continuous Wavelet Transform (CWT), are tested in order to find out which one is the best for our purpose, being of two wavelet functions to measure the capability of them to detect K-complex and to choose one to be employed in the algorithms. The algorithm is based on the energy distribution of the CWT detecting the spectral component of the K-complex.

Keywords: machine-learning, k-complex, wavelet-type core functions.

1. Introduction

The use of electroencephalography (EEG) – method used to explore brain's bioelectrical manifestations via electrodes applied on scalp - enabled knowledge about the changes in brain's electrical potential in both waking and during various stages of sleep. This method has become a valuable tool in the set of instruments used by physician in investigating the patient, brain's electrical potential being different in the healthy person than in persons with medical problems, but also an effective means of scientific research and exploration of the human brain[1]. Electroencephalography also made possible the knowledge about the internal organization of sleep, comprising two distinct stages: one stage of paradoxical sleep or with rapid eye movements and one without such movements.

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The application in this paper refers to the study of one of the slow transient waveforms, components of sleep's electroencephalogram microstructure, namely: *K-complex*. As in the case of other non-stationary phenomena occurring during deep sleep, such as *delta waves'* surges, the highlighting of the *K-complex* also requires methods of statistical pattern recognition.[2][3]

The detection of the *K-complex* is a relatively difficult problem, the specialized literature reporting no absolutely safe methods to highlight it.

The method for the investigation of the *K-complex* proposed herein allows the development of an automated system for the detection and classification of the aperiodic *K-complex*, occurring in EEG during the second stage of sleep, with a very good probability.

The good results reported in a number of works concerning the non-stationarity of sleep electroencephalogram justify the interest for the time-frequency and wavelet representations that may characterize the transient phenomena which occur in the early stages of sleep.

The time-frequency two-dimensional representations are a powerful tool for signal analysis which have the advantage that they allow highlighting of certain "hidden" properties of signals.

The analysis of signals at the lowest level possible, comparable with the noise made by the device that carries out the acquisition of EEG, is interesting from the standpoint of analysis system. That is why time-frequency analyses should be carried out on the signals affected by noise, the signal-to-noise ratio having a special significance in the evaluation of analyzed signal's parameters.

We resorted to this method of representation and analysis of transient signals to develop a *K-complex* detector. The proposed methodology for the synthesis of detectors with imposed structure has a wider applicability, for instance, in the processing of voice signal.

A number of works treating nonstationarity approach this topic in terms of the Fourier transform, which, beside the wavelet analysis, proves to be a particularly effective tool in the analysis of transient signals.[4]

The pattern recognition (machine learning) has become a growing area both theoretically and applicative, being, on one hand, an evolved form of information processing and, from another perspective, being a component of artificial intelligence. Many mathematical methods, grouped into two categories - statistical decision and syntactic-structural - were proposed to solve the pattern recognition issues. This separation is relative as certain inherent interlacing was noticed. For instance, the description of primary patterns requires statistical approaches, the same as representations of complex patterns or multiple-class classifications lead to the replacement of statistical methods with structural methods.

2. Electroencephalogram analysis

Electroencephalogram represents signals' bioelectrical activity triggered by brain's electrical activity. Brain oscillations are called brain waves. They have certain characteristics, including: amplitude ranging between 10-500 μV and frequency ranging between 0.5-40 Hz. The international standardized system called „International Federation 10-20 system" is used to measure brain waves. Conventionally, „slow-wave sleep " or the NREM (Non Rapid Eye Movement) stage groups stages 1, 2 and 3, while the „rapid – paradoxical sleep" or the REM (Rapid Eye Movement) stage stands out in stage 4.[6] [7]

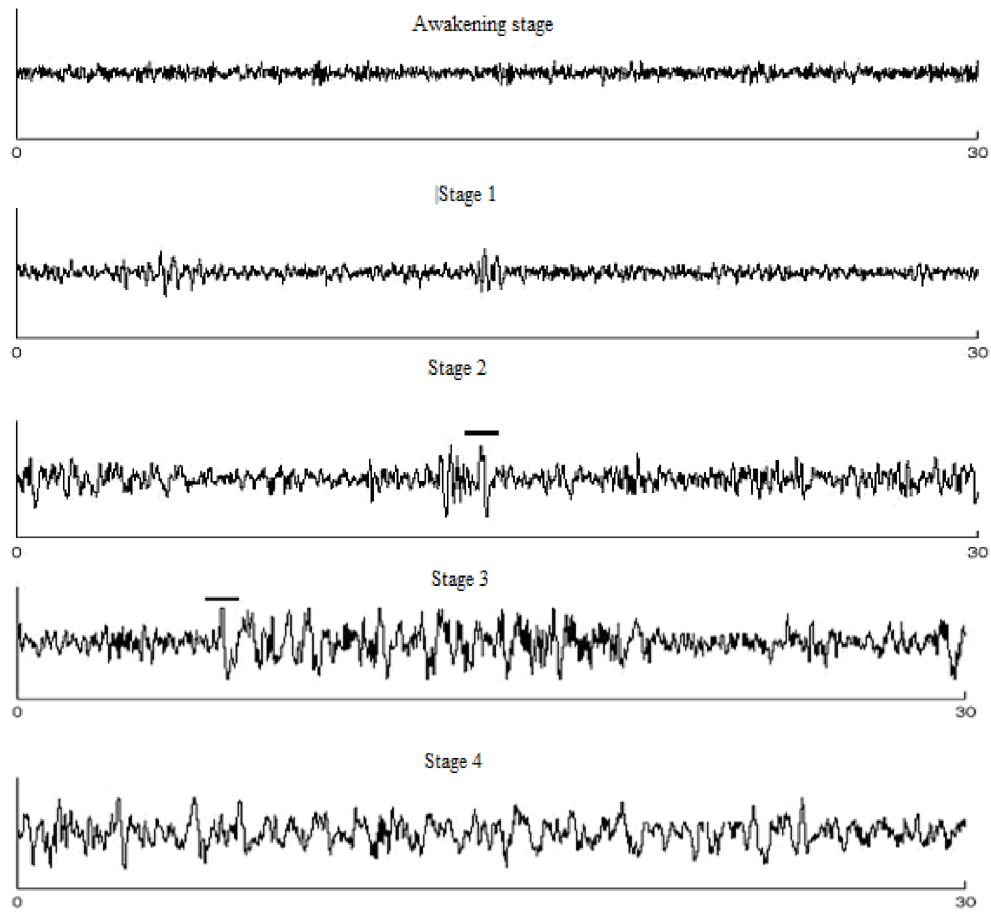

Fig. 1. Illustration of the four stages of sleep with the help of analysis sequences during the 30 seconds of EEG. The horizontal segments occurring in 2nd and 3rd stage sequences indicate the presence of the K-complex.

3. Electroencephalogram microstructure

This paragraph describes briefly the best-known phasic events of sleep electroencephalogram, without insisting on their physiological role, which is still under debate.

● Vertex points: As shown in figure 2, negative impulses occur in stage 1, during the sleep stage, whose amplitude increases once with deeper sleep (as sleep grows deeper). These phasic phenomena may occur in response to certain external stimuli and occur spontaneously.

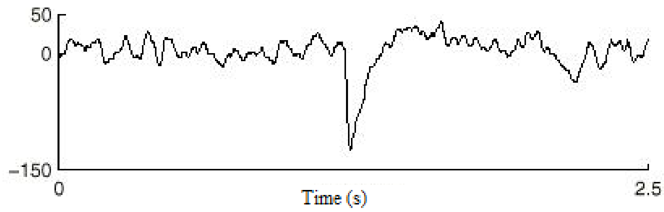

Fig. 2. Exemples of Vertex points

● Sleep spindle: These events are one of the criteria for definition of stage 2. They appear on EEG as quasi-sinusoidal transient signals, of frequency ranging between 12-14 Hz and variable duration between 0.5 and 1 second in adults. An example is shown in Figure 3. It is easily associated to a neurological mechanism that protects the body from sleep's external disturbances.

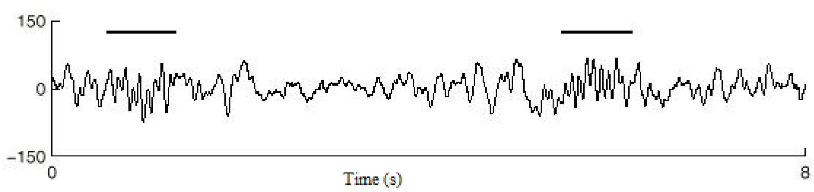

Fig3. Examples of sleep spindles

● K-complex: The K-complex is, together with the sleep cycles, one of the main "markers" of the beginning of sleep, since it appears from stage 2. It is defined by a wave with both polarities having 100 µVvv minimum amplitude, duration between 0.5 - 1 second, preceded and followed by low amplitude activity, of no more than 50 µVvv for at least 2 seconds.

The K-complex is shown in Figure 4. The K-complex may occur both spontaneously and under the influence of external stimuli. The K-complex has a standard frequency between 0.5 - 1.5 Hz for the first peak and 5 - 10 Hz for the second peak and it stands out by the large amplitude (65 µV) of the waveform from the background EEG in stage 2, as shown in Figure 1. At the same time, it is very difficult to be isolated in stages 2 and 3 due to pronounced similarities with other non-stationary phenomena noticed during deep, for example the surges of delta waves shown in figures 1. The importance of detecting the K-complex is owed to its significance in prognosis and diagnosis, but mainly to its occurrence in moments of human thinking, especially when the human subject tries to solve complex problems. The specialized literature refers to the K-complex, in terms of its occurrence in parapsychology activities.[8]

This justifies the interest in statistical methods, with a view to trying to get a satisfactory answer to these problems of detection.

The images below illustrate a series of K-complex's waveforms.

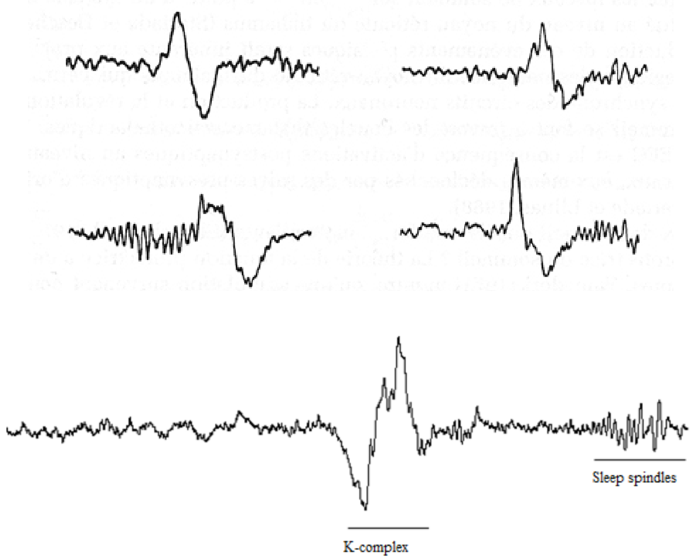

Fig. 4. Waveforms of the K-complex

4. Analysis of short-time Fourier transform (STFT)

The easiest way to transform locally a signal is to crop signal's section of interest and turn it into a Fourier. This is the basic idea of the Short-Time Fourier Transform. The name of the transformation comes from the fact that cropping is done by multiplying the signal with a well time-located function, called the "window". The Short-Time Fourier Transform or window is known in the English-language literature as *Windowed Fourier Transform* or *Short Time Fourier Transform*. Hereinafter, we shall refer to signals' *Short-Time Fourier Transform* by using the acronym ***STFT***. By definition, Short-Time Fourier Transform of a signal *u(t)*, in relation to the "window" *g(t)*, is given by the following equation:

$$STFT_u^{(g)}(\tau, f) = \int u(t) g^*(t-\tau) e^{-j2\pi ft} dt = \langle u(t); g(t-\tau) e^{j2\pi ft} \rangle \quad \tau, f \in R . (1)$$

The Short-Time Fourier Transform of a signal is not unique. It depends on the used function g(t). g(t) must be a local function (the largest part of the area it limits should be in a compact area of the definition scope) in order to have a good analysis resolution. An example of local function is Gaussian which, although it has infinite support, concentrates over 99% of the area it limits, in the interval $[\mu - 3\sigma; \mu + 3\sigma]$, where μ and σ are Gaussian's average, respectively, dispersion.

5. Continuous Wavelet Transform (CWT)

It is defined as the sum over all the time of the signal multiplied by scaled, shifted versions of the wavelet function g.. Given a finite energy signal x(t) and a normalized sampling period , Ts = 1 we can present a discrete wavelet analysis of the sampled sequence $x[n] = x(t)\big|_{t=nT_s}$, $n \in Z$ as follows:

$$\int_{-\infty}^{\infty} \psi(t) dt = 0 , \ \psi \in L^2(R) \tag{2}$$

The discrete synthesis operation can be presented as follows:

$$CWT_{\Psi,f}(a,b) = \Psi_{\Psi,f}(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \psi^* \left(\frac{t-b}{a} dt \right) \tag{3}$$

where, $\Psi_{l,k}(a,b) = \langle f, \psi_{a,b}(t) \rangle$

Table 1

Qualities required for wavelet transforms and methods for processing of wavelet coefficients depending on the type of application

Application	Desired transformation	Processing method
Signal analysis	Time-frequency localization	Highlighting and comparison
Data compression	Appropriate rate-distortion feature	Quantization and encoding
Noise reduction and statistical estimation	Separation of signal from noise	Change of wavelet coefficients
Detection	Concentration of the signal	Statistical detection

6. Supervised statistical classification

The main goal of pattern recognition (PR) is to determine which category a given observation vector may belong to. PR mainly uses special mathematical methods of statistical decision theory and this approach is called *statistical pattern recognition*. Statistical methods are essential for pattern recognition, because inputs are vectors of observations, therefore sets of random sizes or, in mathematical language, random vectors relating to the same probability field for actual PR systems. In statistical recognition, a pattern is represented by a n-dimensional vector of characteristics (having pattern samples as elements) and the classification decision relies on a measure of similarity, which is expressed as a measure for distance or discriminant function.

Patterns' statistical recognition systems consist of two components:

a) selection of features
b) statistical classifier.

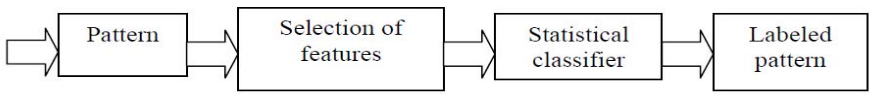

Fig. 5. Statistical recognition systems of patterns

The feature selection strongly affects performances of statistical classification, which is why it is considered one of the main stages of the statistical recognition process of patterns. The feature selection aims the transformation of features' original space (n- dimensional) into a space with fewer dimensions, $m(m << n)$ so that the transformation preserves the significant information for classification. The need to use this transformation emerges from the fact that some classification algorithms which are effective in a space with a small number of dimensions can become unpractical in a space with more dimensions. In order to achieve the transition from initial observation space to a space with fewer dimensions, we shall use the *Karhunen- Loėve transformation* ("Karhunen- Loėve Transformation"= KLT or "Principal Component Analysis"= PCA)**.**

7. Time-frequency analysis with wavelet-type core functions

We use the JTFA libraries in the LabVIEW program to make the time-frequency analysis. The JTFA method involves the selection of the closest kernel functions that correspond to the fundamental waveform that describes the K-complex. Thus, we must choose the kernel pattern based on peak values (location) and amplitude of a "witness".

The resolution in the frequency corresponding to the spectrum analysis, which varies in time, is equal to the Nyquist frequency divided by 2^n (n = 8). The time resolution is 2^n ms (n = 4), being imposed by the applied method.

The kernel function is as follows:

$$C(t,w) = \frac{1}{4\pi^2}\iiint s^*\left(u - \frac{1}{2}u\right)s\left(u + \frac{1}{2}\right)\rho(\theta,\tau)e^{-j\theta t - j\tau t + j\theta u}\,du\,d\tau\,d\theta \qquad (4)$$

where $\rho(\theta,\tau)$ is a two-dimensional function called kernel function.

The selection of the kernel function for various time-frequency distributions of the EEG was studied by Devedeux and Duchêne. They defined the kernel function (ρ_c) as follows:

$$\begin{aligned} \rho_c(t,\tau) &= \xi(\tau) \text{ for u } |\tau| \geq 2|t| \\ \rho_c(t,\tau) &= 0 \quad \text{for the others} \end{aligned} \tag{5}$$

The kernel function defined by Zhao [Shi96], is mathematicaly expressed as:

$$\rho(\theta,\tau) = \xi(\tau)\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-j\theta t}\,dt = 2\xi(\tau)\frac{\sin\left(\frac{\theta\tau}{2}\right)}{\theta} \tag{6}$$

for which:

$$\begin{aligned} \xi(\tau) &= \frac{1}{\tau}e^{-\alpha\tau^2} \\ \rho(\theta,\tau) &= \frac{\sin\left(\frac{\theta\tau}{2}\right)}{\frac{\theta\tau}{2}}e^{-\alpha\tau^2} \end{aligned} \tag{7}$$

The simplified diagram for analysis of the wavelet kernel function from the Cohen category is:

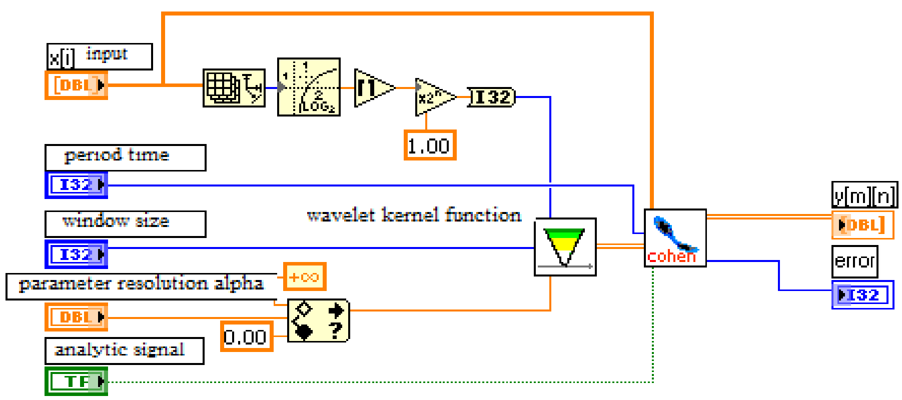

Fig. 6. Simplified diagram of the kernel function (cone-shape kernels)

In mathematics and numerical analysis, the Ricker wavelet is the negative normalized second derivative of a Gaussian function, i.e., up to scale and normalization, the second Hermite function. It is a special case of the family of continuous wavelets(wavelets used in a continuous wavelet transform) known as Hermitian wavelets:

$$\Psi(t) = \frac{2}{\sqrt{3\sigma\pi^{\frac{1}{4}}}}\left(1 - \frac{t^2}{\sigma^2}\right) * e^{-\frac{t^2}{2\sigma^2}} \quad (8)$$

The Morlet wavelet(or Gabor wavelet) is a wavelet composed of a complex exponential(carrier) multiplied by a Gaussian window.

The execution of the approximation kernel function using the "Mexican hat"- type and Morlet-type wavelet transformants is shown in the following figure:

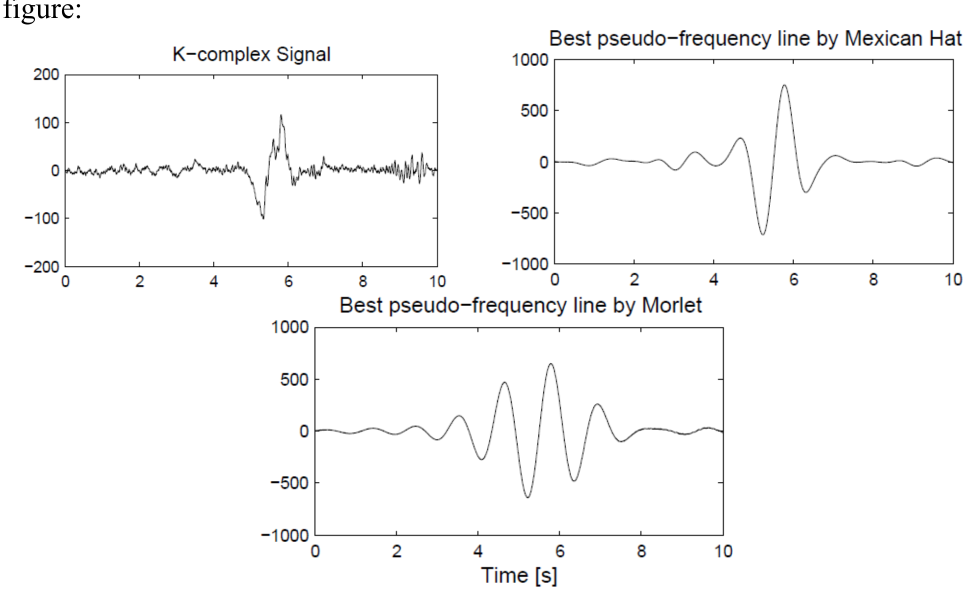

Fig. 7. Representation of the K-complex by approximations with Mexican hat- type and Morlet-type wavelet transformants

The results of the approximations presented in the previous figure underlie the selection of K-complex's waveform features required to achieve a supervised statistical classification (machine learning).

The results of the time - frequency analysis with the Cohen class to detect the K-complex and the delta wave are shown in Fig. 8.

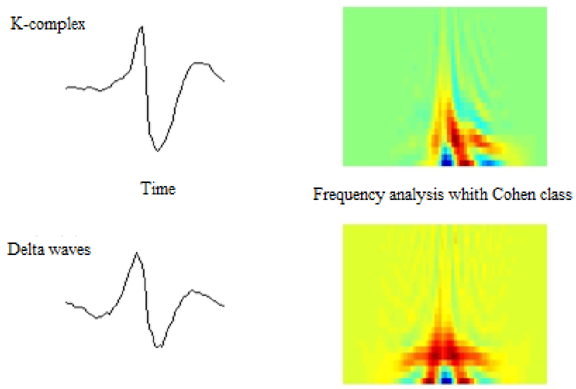

Fig. 8. Time-frequency analysis for K-complex and delta waves

The *figure* shows the time-frequency comparative analysis using Mexican hat-type and Morlet-type wavelet approximation, between the 2nd sleep stage (stage in which K-complex transient waves and sleep spindles occur), paradoxical sleep (Rapid Eye Movement) and awakening stage.

The analysis of results shown in Figs. 8 and 9 reveals the *energy impression characteristic to the K-complex.*

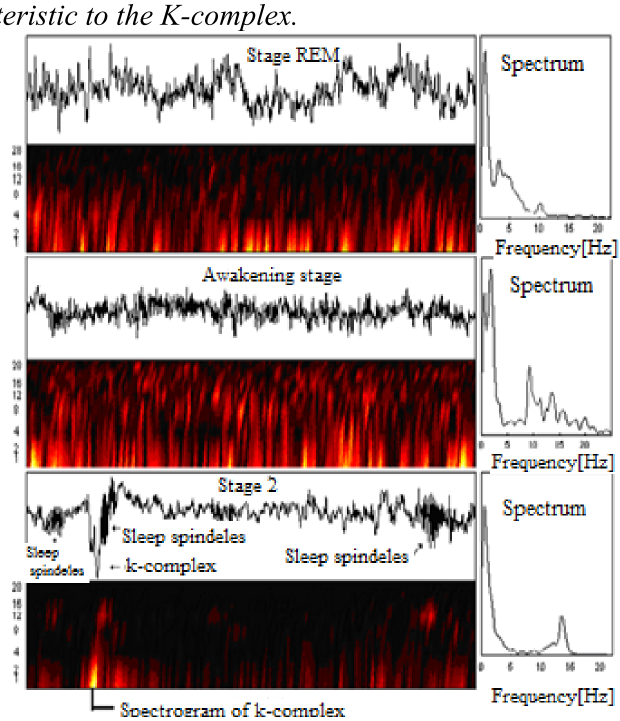

Fig. 9.. Spectrogram resulting from Cohen class time-frequency analysis for various stages of sleep

8. Conclusions

At least two methods can be considered to solve a detection problem. The first method refers to the free structure detection for which the relation of the detection test results from the application of a decision criterion and from the knowledge of observation probability laws.

The second method, called detection with imposed structure, requires prior definition of a detection test class before establishing the optimum test for a given criterion.

The material was dedicated to the definition of a methodology for the elaboration of detectors with imposed structure. It was mainly shown that there are at least two arguments which plead for one choice in favor of statistical classifiers.

The first argument is based on the performance guarantees that the statistical classification represents. The second argument, more practical, is based on the search for a detector in a class that is tantamount to linear discriminant, method used in analysis of pattern recognition.

The supervised learning method presented in this paper enabled the response to this choice, leading to determination of the best decision criterion for the analyzed issue, namely obtaining a detector with minimum error probability.

The proposed experimental results confirm, on one hand, the theory of proposed algorithms, while the increase of learning is important in relation to the level of detection statistics.

Acknowledgements

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