

Application of Transmissibility Matrix Method to NVH Source Contribution Analysis

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Nomenclature

f	Frequency
t	Time
$\{X\}$	Vector of spectra of all sources
$\{Y\}$	Vector of spectra measured or calculated at the receiver positions
$\{V\}$	Vector of spectra measured at indicator positions
$[H_{xy}]$	Frequency Response Function matrix between all source and receiver positions
$[H_{xv}]$	Frequency Response Function matrix between all source and indicator positions
$[T_{vy,x}]$	Transmissibility matrix between responses at receiver positions and responses at indicator positions under the excitation at the source positions
C_{ij}	Contribution from the j -th source (path) to the i -th receiver
S_{ij}	Contribution from the j -th source (path) to the i -th receiver calculated by the Transmissibility method
$[Y_M], [V_M]$	Matrices of M operating measurements built of vectors $\{Y\}$ and $\{V\}$ respectively
$[]^+$	Pseudoinverse of a matrix
ε	A small bookkeeping parameter

Abstract

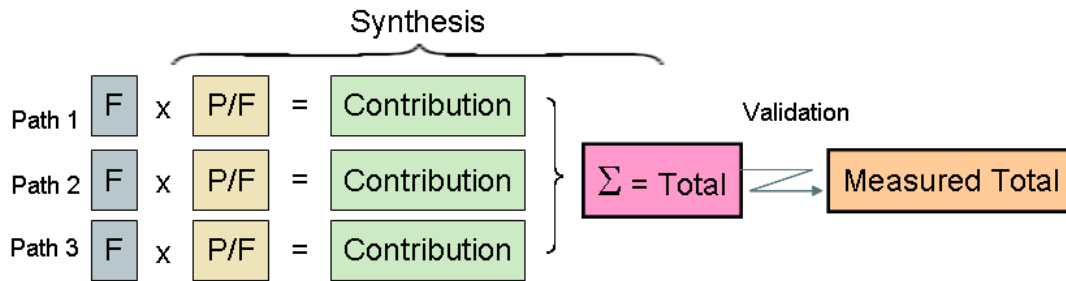
Analysis of contributions from different noise sources and paths is an important part of an automotive NVH evaluation process. Since the classical techniques which are based on measured transfer functions and estimated source strengths, are time consuming and error-prone, a search for new easier and more reliable methods keeps going on.

The current study continues investigation of the method based on the transmissibility matrix. The method is very attractive for NVH engineers since it extracts the transmissibility matrix from the operating measurements and therefore does not require measuring the transfer functions. The transmissibility matrix is later used for the contribution estimation. The study compares the results of air-borne contribution analysis obtained by the Transmissibility method with ones from the classical source substitution method; the experiments are conducted on a vehicle equipped with an Engine Noise Simulator.

1. Introduction

The main goal of Source Path Contribution (SPC) analysis is to find noise *contributions* from either different *sources* (i.e. source contributions) or noise approaching a receiver via different *paths* (i.e. noise path analysis). Obviously, the main customer of SPC (also known as Transfer Path Analysis (TPA) and Noise Path Analysis (NPA)) is automotive industry, where interior and exterior sound comfort plays important role. But other industries like aircraft industry and consumer goods industry are starting looking into SPC methods.

a)



b)

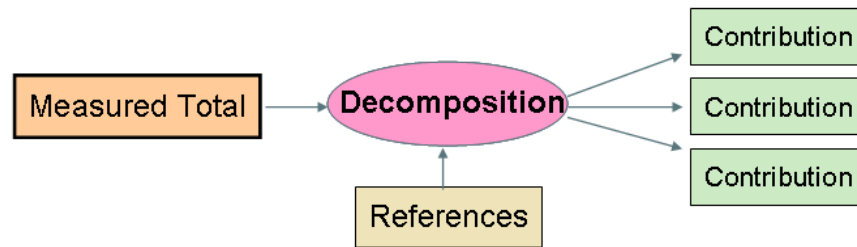


Figure 1. SPC methods: a) synthesis; b) decomposition.

During the 15 years of development, several SPC methods were worked out [1]. Most of these methods belong to the *synthesis* family (Figure 1a), where a contribution is calculated as a product of path (or source) *strength* and the *transfer function* from the path (or source) to the receiver [2]. These methods are time consuming since they require measurement of transfer function between source (or path) and receiver. The methods are also error-prone since the path (or source) strengths cannot be measured directly but have to be estimated using one or another method (e.g. Mount Stiffness method, Matrix method, Source Substitution method, etc.) that adds extra uncertainties to the results.

Another family of methods is *decomposition* methods. Obviously, the noise (or vibration) measured at a receiver position is a mix of contributions from different noise sources and paths. The decomposition methods try to *split* the mix to a number of components according to one or other criteria, based on some reference signals. Then, each component is considered as a *contribution* from the path or source. For example, the Multiple Coherence method assumes the noise sources are uncorrelated and splits the measured mix using so called *reference* signals which are recorded in a vicinity of sources. A typical application of Multiple Coherence method is analysis of road noise where wheels of a vehicle are considered to be uncorrelated noise sources, and therefore the contribution from each wheel can be estimated.

Few years ago Noumura and Yoshida suggested a new method for contribution estimation [3, 4]. The method is based on the use of *transmissibility matrix*. As defined in e.g. [5], transmissibility is a ratio of two responses (either vibration or noise) measured at two locations of a structure. Originally, transmissibility function was defined for single input systems but later, in the works of Ribeiro and Maia [6, 7, 8], it was generalized for multiple input systems.

The method suggested by Noumura and Yoshida estimates the contributions without knowledge of the transfer functions, and it skips the calculations of the source (and/or path) strength; the method is based on operating measurements only, which makes it very practical and therefore highly attractive for many (and especially for automotive) applications.

Method attractiveness and the lack of detailed information about it initiated active research in order to validate the method, compare its results with the results of classical SPC techniques and define its applicability. E.g. [9, 10, 11] give a critical review of the method where three critical limitations were highlighted and examined, namely: (i) effect of path cross-coupling; (ii) numerical conditioning problems related to estimation of the transmissibility matrix and (iii) the potential errors due to missing paths in the analysis. The abovementioned works were mostly focused on the structure borne paths of engine noise. In contrast, in [12] the same method was examined focusing on air-borne noise. The following issues were addressed: (i) the effect of source correlation; (ii) ways of

collecting data for transmissibility matrix calculation; (iii) the problem of sources which are not accounted for; (iv) separation of strong and weak sources; (v) ways of dealing with distributed sources.

The current work continues the examination started in [12]. The method is applied to the data recorded on a real car equipped with Engine Noise Simulator (ENS). ENS allows full control of the noise radiation from the different engine faces, so it ensures perfect conditions for method validation: calculated contributions from different faces of the ENS can be directly compared with exact contributions, which can be easily measured by switching ENS faces on and off.

The paper is organized as following: in Section 2 a brief theoretical introduction to Transmissibility method is given, Section 3 describes the practical implementation of the method, Section 4 describes the application of the method to the data measured on a real vehicle.

2. Transmissibility method and its application to SPC

There are different ways explaining application of the multiple input transmissibility functions to the SPC problem. E.g. [12] refers to the classical approach similar to the original works of Ribeiro and Maia [6-8], while [11] uses parallels between calculation of transmissibility matrix and classical H_1 estimator known from modal analysis. In this work we will use the similarities between the Transmissibility method and the *Matrix method* widely used in SPC.

Following the standard synthesis SPC methods, the response at the receiver position is being presented as a product of source (path) strength and transfer function, in frequency domain (Figure 2a),

$$\{Y\} = [H_{xy}] \{X\} . \quad (1)$$

where $\{Y\}$ is a vector of spectra at the receiver positions, $\{X\}$ – vector of spectra of source (path) strengths, $[H_{xy}]$ is a matrix of FRFs measured between source (path) positions and receiver positions.

Since the strengths of sources (paths) cannot be measured directly, they are typically estimated using Matrix method (Figure 2b):

$$\{V\} = [H_{xv}] \{X\} , \quad (2a)$$

$$\{X\} = [H_{xv}]^+ \{V\} , \quad (2b)$$

where $\{V\}$ is a vector of spectra measured at so-called *indicator* positions (the blue circles, Figure 2b), $[H_{xv}]$ is a matrix of FRFs measured between source (path) positions and indicator positions and the superscript $+$ denotes matrix pseudoinverse.

Knowing $\{X\}$ from (2b), the individual contributions C_{ij} (a contribution from the j -th source (path) to the i -th receiver) are calculated as

$$C_{ij} = [H_{xy}]_{ij} \{x\}_j , \quad (3)$$

and the total noise at the i -th receiver can be estimated as a sum of all contributions:

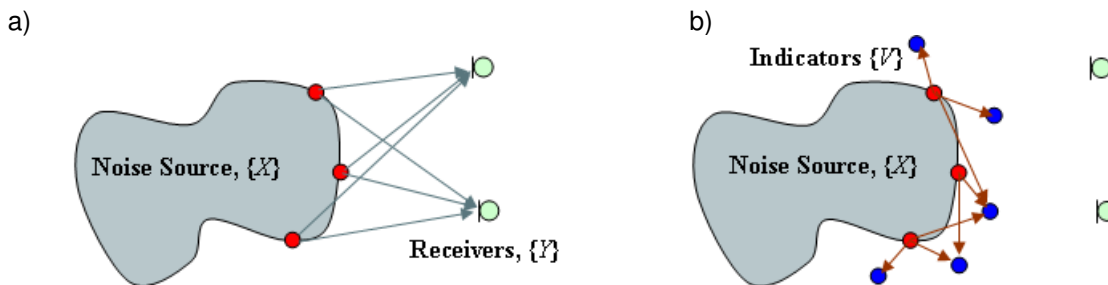


Figure 2. Illustration of the matrix method, arrows represent the FRF functions: a) expression (1); b) expression (2a).

$$y_i = \sum_j [H_{xy}]_{ij} \{x\}_j = \sum_j C_{ij} \cdot \quad (4)$$

Substituting expression (2b) into (1) yields to

$$\{Y\} = [H_{xy}] [H_{vx}]^+ \{V\}, \quad (5)$$

which actually can be read as a relation between two groups of responses: responses $\{Y\}$ measured at the receiver position and the responses $\{V\}$ measured at the indicator positions, or according to Ribiero *et al* [7], is a definition of multiple input, multiple output *transmissibility matrix*:

$$\{Y\} = [T_{vy.x}] \{V\}, \quad (6a)$$

where

$$[T_{vy.x}] = [H_{xy}] [H_{vx}]^+ \quad (6b)$$

is a transmissibility matrix between responses at receiver positions and responses at indicator positions *under the excitation at the source positions*.

Expression (6a) looks very similar to (1) which allows one to suggest that if elements of vector $\{V\}$ are closely related to the corresponding sources, then

$$S_{ij} = [T_{vy.x}]_{ij} \{v\}_j, \quad (7)$$

will be closely related to contributions as defined by (3). Then the total noise measured at the i -th receiver can be presented as

$$y_i = \sum_j [T_{vy.x}]_{ij} \{v\}_j = \sum_j S_{ij}, \quad (8)$$

(compare to (4)).

As it was correctly mentioned in [9-12], S_{ij} are not true contributions but rather ‘pseudo-contributions’ since they do not represent a causal relationship between the source of noise (i.e. excitation) and the response (the resulting noise) but only relationship between responses. However, in many practical cases the values according to (7) are close to true contributions and can be used for making engineering decisions.

The idea of the Transmissibility method is to estimate the transmissibility matrix $[T_{vy.x}]$ using *operating measurements only*. Indeed, providing the spectra $\{Y\}$ and $\{V\}$ are measured for M different operating conditions, one can form the following matrices:

$$[Y_M] = [\{Y\}^{(1)} \{Y\}^{(2)} \dots \{Y\}^{(M)}], \quad (9a)$$

$$[V_M] = [\{V\}^{(1)} \{V\}^{(2)} \dots \{V\}^{(M)}], \quad (9b)$$

and, according to (6a), matrix $[T_{vy.x}]$ can be estimated from

$$[T_{vy.x}] = [Y_M] [V_M]^+. \quad (10)$$

There are two conditions for using expression (10): (i) the excitation should be applied at the same set of position for all measurements; (ii) the matrix $[V_M]$ should be invertible.

Practically, the first condition means that the all the sources active during all M operating measurements should be accounted for by placing one or several indicators close to them. This naturally implies that the number of indicators N_V is greater or equals the number of sources N_S . The effect of active sources which are not accounted for by indicators has been examined in [11, 12]. Considering (6b), one can make a practical conclusion that placing the indicators as close as possible to the sources is better since this makes matrix $[H_{vx}]$ better conditioned.

The second condition (provided the first condition is fulfilled) means that $[V_M]$ is a full rank matrix; the necessary condition for this is the number of its columns M is greater or equal the number rows N_V ; however at least N_V of its columns must be linearly independent. Practically this implies that the M operating conditions used to form the matrices (9a,b) should be as different as possible. The selection of operating conditions was discussed in [12], the

problem of invertibility of the matrix when using excitation by engine orders during vehicle run-up was discussed in [11].

It is important to note that expression (7) can be re-written in time domain: the transmissibility functions $[T_{vy,x}]_{ij}$ can be converted into FIR-filters, which then are being convolved with time series recorded at indicator positions:

$$S_{ij}(t) = [T_{vy,x}(t)]_{ij} \otimes \{v(t)\}_j. \quad (11)$$

The result is a “contribution” in time domain. This technique can be applied to non-stationary operating conditions e.g. to vehicle run-up. The resulting contributions can be post-processed in order to obtain e.g. contribution overall profile, which can be useful in pass-by tests. The example of such applications is shown in Section 4.

3. Practical implementation of the method

Practically, the method is applied as follows:

1. Data acquisition part:
 - a) Determine the sources active during the target operating conditions;
 - b) Place indicator sensors as close as possible to the sources;
 - c) Place receiver sensors at the target receiver positions;
 - d) Record the signals from the indicator and receiver sensors during different operating conditions (the choice of operating conditions and problems related to this were addressed in [11, 12]).
2. Calculation part:
 - a) For each operating condition measurement, check degree of correlation between the sources. As described in [12], perform Principal Component Decomposition (PCD) if the sources are uncorrelated or partly correlated;
 - b) For each principal component calculate phase assign spectra of both receiver and indicator signals w.r.t. some selected reference signal;
 - c) Form the $[Y_M]$ and $[V_M]$ matrices. If more then one principal components are detected, the number of the columns in these matrices can be increased as suggested in [12, expression (18)];
 - d) Estimate the transmissibility matrix according to (10);
 - e) For the selected *target* operating conditions, calculate the contributions according to (7). For validation, the sum of contributions (8) can be compared with the measured signal at the receiver position, but as it was found in [11, 12] and explained later in this study, this comparison always gives matching results and has to be used with care.

4. Application of the method to a real vehicle

Since the transmissibility method is relatively new, a set of experiments was planned to validate the method, examine its practical feasibility and define its applicability. In [12] the method was applied to simulated data resembling data from a real vehicle. This study continues the investigation by applying the method to the data measured on a real vehicle. Since the main goal of this study is to compare the contributions obtained by the method with exact contributions, we decided to use a vehicle equipped with Engine Noise Simulator (ENS).

ENS (Figure 3) is a wooden box approximately resembling the geometry of the real engine. ENS is mounted on the car frame same way as the real engine is. The box has 7 faces; each (except the bottom one) is equipped by two loudspeakers. The bottom face is additionally equipped with one extra loudspeaker with bigger diameter. The loudspeakers are supposed to model the sound radiation of working engine. To model the engine vibration, the ENS has also three mini-shakers mounted inside, which are able to excite the box in three directions.

In this study, the method was applied to air-borne engine noise only. In order to avoid the structure borne contributions, the use of an external receiver was chosen. This resembles a measurement scenario which is quite typical for automotive NVH program: an in-door pass-by test combined with noise contributions identification. The schematic measurement setup is shown on Figure 4a. Figure 4b shows the indicator microphones mounted on

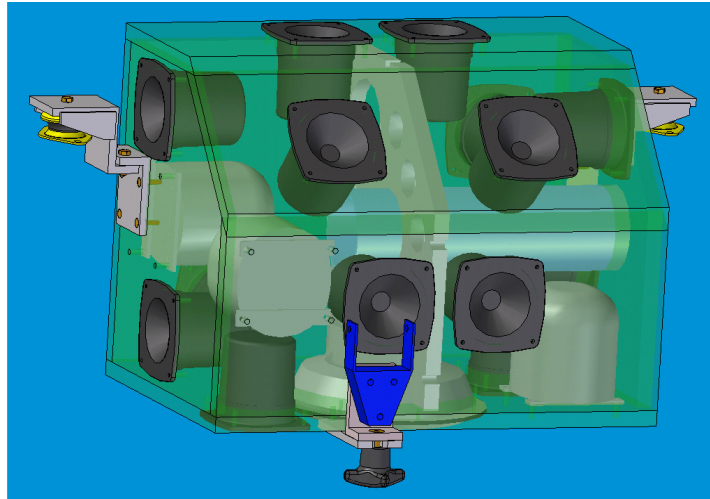


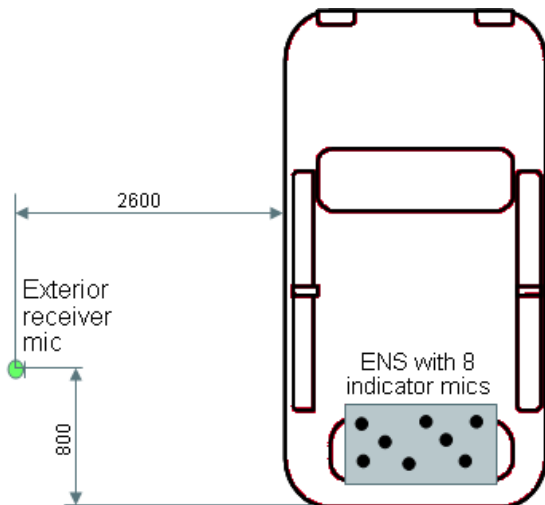
Figure 3. Engine Noise Simulator (ENS). Rear view.

the surface of the ENS. In total, 8 indicator microphones were used for the measurements: 2 mics per face for Left, Right, Bottom and Rear faces. The Front, Top and Skewed faces were not instrumented and kept switched off during the measurements.

A set of constant speed operating conditions was selected for transmissibility matrix estimation. As it was demonstrated in [12], using sets of stationary operating conditions produces better estimations of the transmissibility matrix. We fed the same signal to all loudspeakers; the different vehicle speeds were modeled by varying the level of signals. The signal we used was a signal previously recorded on the similar car by a microphone located inside the engine bay. In total, 14 operating measurements were conducted that corresponds to 14 columns in $[Y_M]$ and $[V_M]$ matrices. Based on these measurements, the $[T_{vy,x}]$ matrix was estimated. No PCD was performed since only one principal component was obviously present.

The next step was evaluation of contributions for selected *target* operating conditions. Two types of target

a)



b)

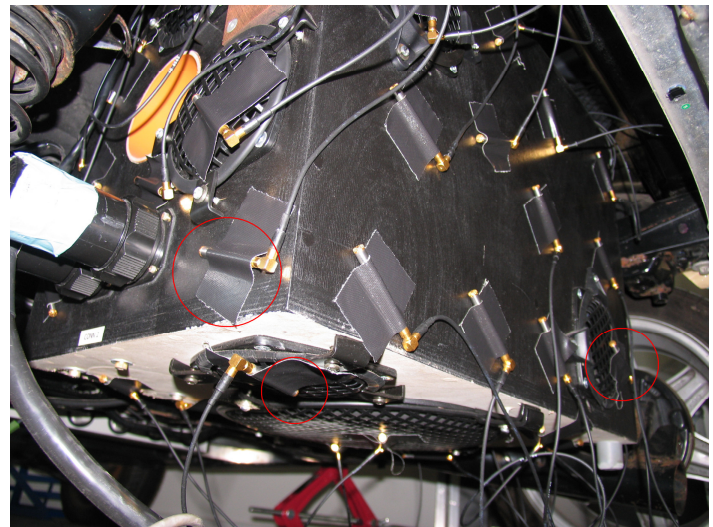


Figure 4. a) Schema of the measurement setup; b) Indicator microphones (circled) mounted on the ENS surface. Note, there are other microphones which were not used for these measurements. Left, Bottom and Rear faces are shown.

operating conditions were chosen: the first test was designed to validate the separation abilities of the method; the second one was to evaluate the applicability of the method in time domain in order to obtain contributions of engine faces during vehicle run-up/down test.

During the first test, white band-pass filtered noise was applied to the faces of ENS: the Right face produced the noise at 200-1200 Hz frequency band, the Left and Bottom faces produced noise at 1000-2200 Hz, the Rear face produced noise at 2000-3000 Hz. The calculated contributions were compared with exact contributions obtained by switching off all faces except the face of interest. The results are shown on Figure 5.

As it can be seen, the method provides quite good separation for the most powerful source - Left + Bottom (Figure 5c) while the contributions from the other two faces are not well separated. Consider for example the Right face (Figure 5a). At 200-1200 Hz range, the contribution level is almost correct. However at 1000-2200 Hz it is overestimated. This is because the two Right face indicator microphones pick the noise from the Left and Bottom faces which radiate noise at this frequency range, and are the most powerful sources. Further up in frequency, at 2000-3000 Hz range, the calculated contribution is overestimated again but less pronounced. This can be explained the same way: the Right face indicators pick the noise coming from the Rear face, which radiates the noise at this frequency range. Since the Right face is relatively weak source, the degree of overestimation is less.

These observations together with examination of expression (7) allow us to make a conclusion that the Transmissibility method is inevitably prone to fail when separating sources. The only case the method separates contributions correctly is when the indicators pick the noise *only* from the corresponding sources, i.e. they are not subjected to any influence from other sources. In mathematical terms it means the $[H_{xv}]$ is diagonal. This also helps to understand the applicability of the method: the closer the $[H_{xv}]$ matrix is to diagonal, the better the separation results are expected to be. In practice this means that good results can be anticipated when calculating contributions from distant sources, e.g. separating air-borne contributions from engine and exhaust; the results can be improved by placing the indicator sensors closer to the sources. In contrast, the application of the method should be avoided (or the results should not be trusted) when the presence of significant nondiagonal elements is unavoidable, e.g. at resonance frequencies of the car frame when applied to structure borne SPC.

In the Appendix, considering a simple 2 receivers x 2 sources x 2 indicators system as an example, is demonstrated why the method tends to overestimate the contributions in the frequency ranges where the contributions should be zero. It also explains the observation why the sum of contributions (8) is almost always in a good agreement with the total noise measured at the receiver position.

The second target operating conditions was a vehicle run up/down. The contribution calculations were performed in time domain according to (11). The sound recorded by a microphone placed in the engine bay of a similar car during car's run up/down was played via all ENS loudspeakers. The overall profile (overall sound pressure vs. time) was calculated as required by pass-by standard [13], namely using the A-weighted sound pressure level and time weighting F ('Fast'). The computed contributions were compared with exact contributions obtained by switching off irrelevant ENS faces. The results are shown on Figure 6.

Here, the calculated contributions from the Right face and from the Bottom + Left faces match quite well with exact contributions obtained by switching off irrelevant faces. The contribution from the Rear face is underestimated (the authors cannot find a reasonable explanation to this; it could simply be a measurement error). Again, the total noise level is predicted quite well.

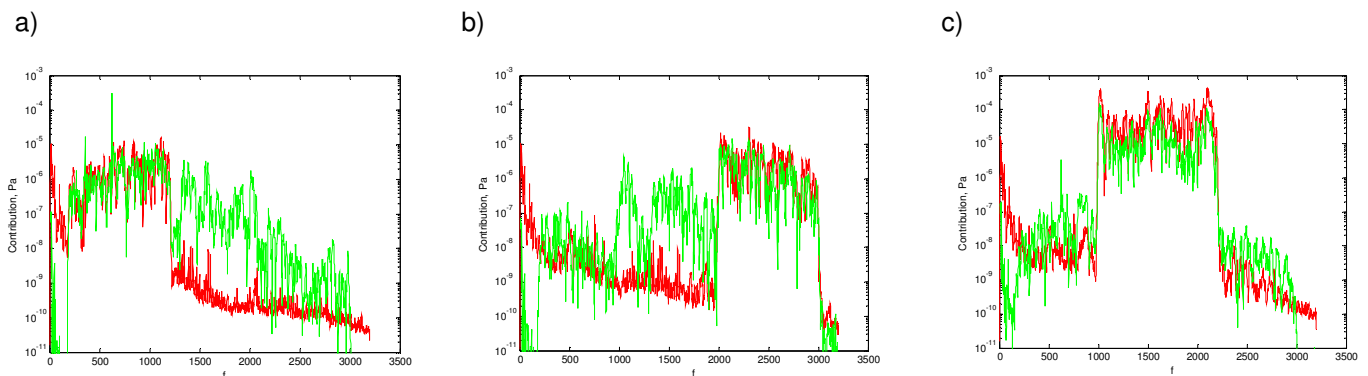


Figure 5. Calculated contributions (green) vs. exact ones (red). a) Right face; b) Rear face; c) Left and Bottom faces.

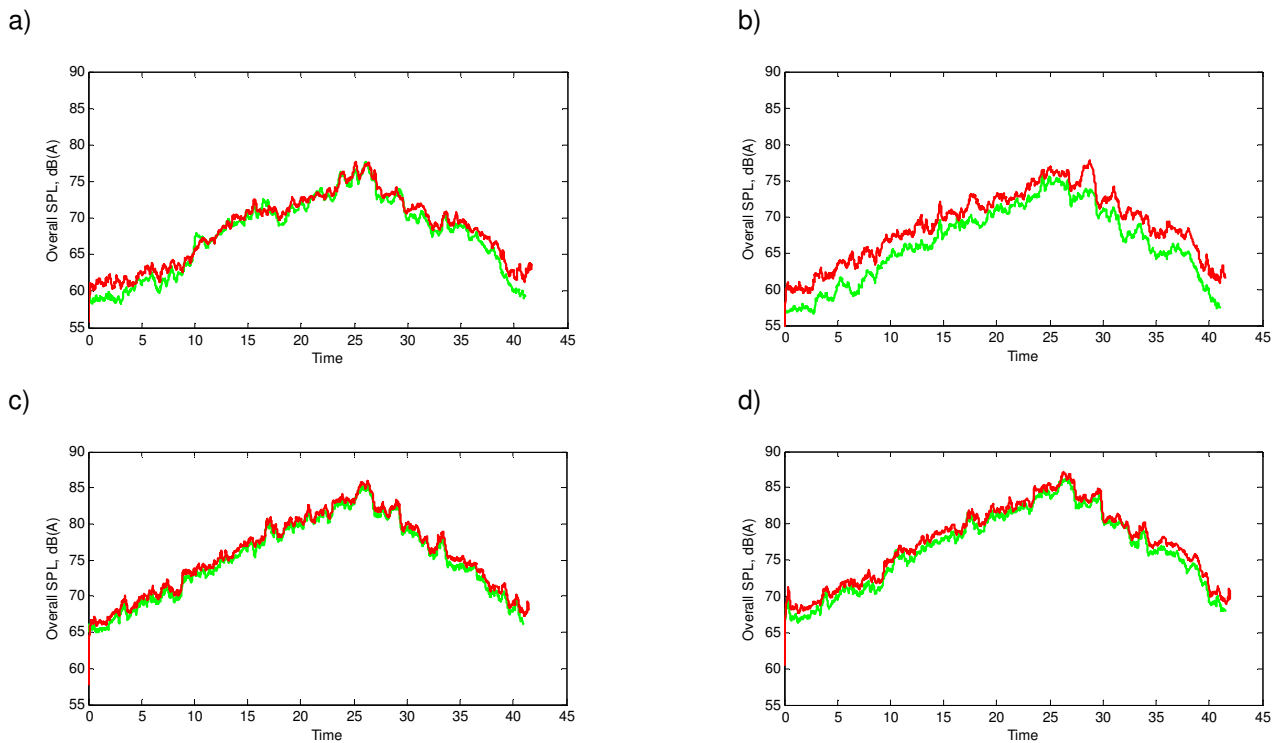


Figure 6. Calculated contributions (green) vs. exact contributions. Overall profile (overall sound pressure level vs. time) for: a) Right face, b) Rear face c) Left and Bottom faces; d) Measured total vs. sum of contributions.

Conclusion

The present study concerns the application of Transmissibility method to automotive NVH source path contribution problems, specially focusing on the air-borne applications. Continuing the previous study, the present one addresses the validation of the method on the data measured on a real car equipped with Engine Noise Simulator.

A brief theoretical introduction to the method is done where the method is compared with Matrix Inversion method which is well-known for NVH engineers. It is shown that the quality of the results from the Transmissibility method is very much depend on a matrix defining FRFs between sources and indicators, and therefore, the calculated contributions are very much influenced by the position of indicator sensors. It is also shown why the sum of contributions calculated by the method is always in a good agreement with the measured noise at the receiver position.

It is also shown that if the indicator sensors are placed very close to the expected sources, the method can produce good results, both from source separation view point and overall source strength view point. This was demonstrated for two sets of target operating measurements and two calculation domains.

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Appendix

Consider a simple system with two sources, two indicators placed in the vicinity of the sources, and two receivers. Let us:

$$[H_{xy}] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, [H_{xv}] = \begin{bmatrix} h_{11} & \varepsilon h_{12} \\ \varepsilon h_{21} & h_{22} \end{bmatrix}, \{X\} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}, \{Y\} = \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} \text{ and } \{V\} = \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}, \quad (\text{A.1})$$

where ε is a bookkeeping parameter. Since we consider the indicator sensors being placed close to the correspondent sources, we can expect ε being a small parameter, so $h_{ii} \gg \varepsilon h_{ij}$.

For the considered 2x2 case, the inverse of $[H_{xv}]$ can be calculated analytically, so

$$[H_{xv}]^{-1} = \frac{1}{\det([H_{xv}])} \begin{bmatrix} h_{22} & -\varepsilon h_{12} \\ -\varepsilon h_{21} & h_{11} \end{bmatrix}, \quad (\text{A.2})$$

and the determinant of $[H_{xv}]$ is

$$\det([H_{xv}]) = h_{11} h_{22} - \varepsilon^2 h_{12} h_{21} = h_{11} h_{22} + O(\varepsilon^2). \quad (\text{A.3})$$

Let us calculate the contributions to, e.g. y_1 , using two methods: a standard SPC method (1)-(4) and the transmissibility method (5)-(8).

According to (4),

$$y_1 = C_{11} + C_{12}, \quad (\text{A.4})$$

where, according to (3),

$$C_{11} = a_{11}x_1 \text{ and } C_{12} = a_{12}x_2. \quad (\text{A.5})$$

Following (8),

$$y_1 = S_{11} + S_{12}, \quad (\text{A.6})$$

and applying some algebra, one obtains:

$$\begin{aligned} S_{11} &= a_{11}x_1 + \varepsilon \left(\frac{a_{11}h_{12}}{h_{11}}x_2 - \frac{a_{12}h_{21}}{h_{22}}x_1 \right) + O(\varepsilon^2) \\ S_{12} &= a_{12}x_2 + \varepsilon \left(\frac{a_{12}h_{21}}{h_{22}}x_1 - \frac{a_{11}h_{12}}{h_{11}}x_2 \right) + O(\varepsilon^2) \end{aligned}, \quad (\text{A.7})$$

It can be seen (*cf.* (A.5)) that

$$\begin{aligned} S_{11} &= C_{11} + \varepsilon \left(\frac{a_{11}h_{12}}{h_{11}}x_2 - \frac{a_{12}h_{21}}{h_{22}}x_1 \right) + O(\varepsilon^2) = C_{11} + O(\varepsilon) \\ S_{12} &= C_{12} + \varepsilon \left(\frac{a_{12}h_{21}}{h_{22}}x_1 - \frac{a_{11}h_{12}}{h_{11}}x_2 \right) + O(\varepsilon^2) = C_{12} + O(\varepsilon) \end{aligned}, \quad (\text{A.8})$$

so S_{ij} is equal to C_{ij} plus a term of order of ε . The contributions coincide if $\varepsilon = 0$, i.e. there is no cross-talk between the sources.

Expressions (A.7) can also explain the overestimation observed on Figure 5: Let us consider for example the first expression in (A.7). For the frequency ranges where $x_1 = 0$, the contribution $C_{11} = 0$ but $S_{11} > 0$ due to the presence of the second term. The discrepancy is of order of ε .

It is also interesting to note that the sum of contributions calculated via the Transmissibility method is equal to the sum of contribution calculated via classical SPC method (the difference is a small term of order of ε^2):

$$y_1 = S_{11} + S_{12} = C_{11} + C_{12} + O(\varepsilon^2), \quad (\text{A.9})$$

since the second terms in (A.8) cancels out. This explains the observation made in previous studies [9-12], where it was noticed that the sum of contribution computed using the Transmissibility method is always in a very good match with measured total noise at the receiver positions.

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