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MODELING METHODOLOGY

Applications of GCorr™ Macro: Risk Integration, Stress Testing, and Reverse Stress Testing

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Abstract

This research develops an approach to expand the Moody's Analytics Global Correlation Model (GCorr) to include macroeconomic variables. Within the context of this document, macroeconomic variables can include financial market variables, economic activity variables, and other risk factors.

The expanded correlation model, known as GCorr Macro, lends itself to several functions that facilitate a cohesive and holistic risk management practice. Using GCorr Macro allows for the ability to aggregate and allocate credit, market, and other risks using a factor based model. In addition to risk integration, using GCorr Macro facilitates stress testing and reverse stress testing. This approach addresses several economic needs as well as regulatory initiatives related to Solvency II and Comprehensive Capital Analysis and Review (CCAR).

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1 Introduction

Credit correlations are typically best described through factor models, with factors that characterize the credit environment.[1] Moody's Analytics GCorr Corporate, an example of such a model, uses factors based on firms' asset returns that are segmented by industry and country classifications. The model recognizes that the credit environment can be specific to those classifications. As an example, the recent crisis was particularly impactful for Europe and the U.S., but did not seem to impact China as much. Alternatively, the recent crisis hit financial institutions particularly hard, while the technology downturn in the early 2000s hit telecom and software severely. While useful in describing credit correlations, GCorr Corporate factors can be abstract and are not as intuitive as macroeconomic variables when communicating credit portfolio results throughout an organization. In this document, we describe applications for using GCorr Macro, a correlation model that includes both credit risk factors and macroeconomic variables.

Within the context of this document, macroeconomic variables can include financial market variables (for example, S&P returns, changes in 10 year interest rates), economic activity variables (for example, growth in GDP, changes in unemployment rate), as well as other risk factors (for example, market risk factors, operational risk factors). GCorr Macro lends itself to several functions that facilitate a cohesive and holistic risk management practice. GCorr Macro provides the ability to aggregate and allocate credit, market and other risks using a factor based model. In addition to risk integration, GCorr Macro facilitates stress testing and reverse stress testing. The approach addresses several economic needs as well as regulatory initiatives related to Solvency II and CCAR.[2]

GCorr is a multi-factor model that describes the correlation structure across a wide range of credit entities, including large corporates, sovereigns, Commercial Real Estate (CRE), private firms including Small to Medium Enterprises (SMEs), and retail.[3] Extending GCorr to include macroeconomic variables does not provide additional explanatory power as far as describing credit correlations, given that the credit factors are already designed to best describe systematic credit portfolio risk. GDP growth is a broad brush measure and does not provide insight regarding the nature of a credit crisis. This is particularly true when we recognize that the turn of the century recession was associated with, for example, the technology sector, and the more recent crisis was associated with the financial and retail sectors. That said, GDP growth is an intuitive measure that is useful as a communication vehicle. GCorr Macro is a flexible model that can be customized to meet client needs. More specifically, it can be calibrated to include variables from the CCAR stress testing study conducted by the Federal Reserve System, Moody's Analytics CMM® (Commercial Mortgage Metrics), Moody's Analytics Economic & Consumer Credit Analytics (ECCA), The Barrie and Hibbert Economic Scenario Generator (ESG), and other variables that an institution finds relevant.

Once the GCorr Macro includes the relevant market risk factors, you can link simulated credit losses from RiskFrontier with losses generated from market risk systems, ALM risk systems, or any risk system driven by a set of factors that overlap with the GCorr Macro model. This naturally allows you to aggregate capital across risk types (credit, market, etc.), and allocate capital at the instrument level.[4]

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[1] See "Factor Models for Portfolio Credit Risk" (Schönbucher, P. J., 2000).

[2] See "Comprehensive Capital Analysis and Review: Methodology and Results for Stress Scenario Projections (CCAR)" (Board of Governors of the Federal Reserve System 2012).

[3] For more information, see "Modeling Credit Correlations: An Overview of the Moody's Analytics GCorr Model" (Huang, J., M. Lanfranconi, N. Patel, and L. Pospisil, 2012)

[4] Using the framework described in this paper, capital can be allocated at the instrument level, even if the instrument loads to many industries.

In addition to risk aggregation and allocation, the GCorr Macro model lends itself to stress testing and reverse stress testing. In this document, we introduce the following two approaches to stress testing:

- The first approach is more involved and utilizes simulation output from Moody's Analytics RiskFrontier™. This approach describes the loss distribution conditional on a macroeconomic scenario defined by a set of variables, such as economic activity or financial market variables. While more involved, this approach characterizes the extent to which the scenario describes portfolio risks. The extent to which there is residual risk after conditioning on the scenario is an indication that the macro scenarios do not span the loss distribution. This is particularly relevant in CCAR, given the requirement that an institution consider factors specific to its portfolio, as well as the requirement to buffer for unaccounted risks.
- The second stress testing approach produces an expected loss (or term structure of expected loss as required by CCAR) conditional on a particular scenario. This approach leverages the GCorr Macro framework, which allows an analytic representation of stressed default probabilities and stressed losses given default. These stressed parameters depend on counterparty characteristics and collateral, along with the correlation between the macroeconomic scenario and the systematic credit risk factors to which the counterparty's credit quality and the collateral value are exposed.

In addition to stress testing, GCorr Macro lends itself to reverse stress testing, defined as a description of macroeconomic scenarios (either a distribution or expected value) conditional on a specified level of loss.

The remainder of the document is organized as follows:

- Section 2 provides an overview of the GCorr Macro model.
- Section 3 describes risk aggregation and allocation using the GCorr Macro model, and presents examples integrating market and credit risks.
- Section 4 describes stress testing and reverse stress testing analyses using a RiskFrontier simulation output, as well as the analytical stress testing approach.
- Section 5 concludes this document.
- Appendix A provides technical details of the analytical stress testing approach.

2 Using GCorr Macro within RiskFrontier

This section describes GCorr Macro in more detail and illustrates how it fits into the RiskFrontier credit portfolio modeling framework.

Figure 1 shows the RiskFrontier framework for credit portfolio modeling, including the expanded Gcorr model.

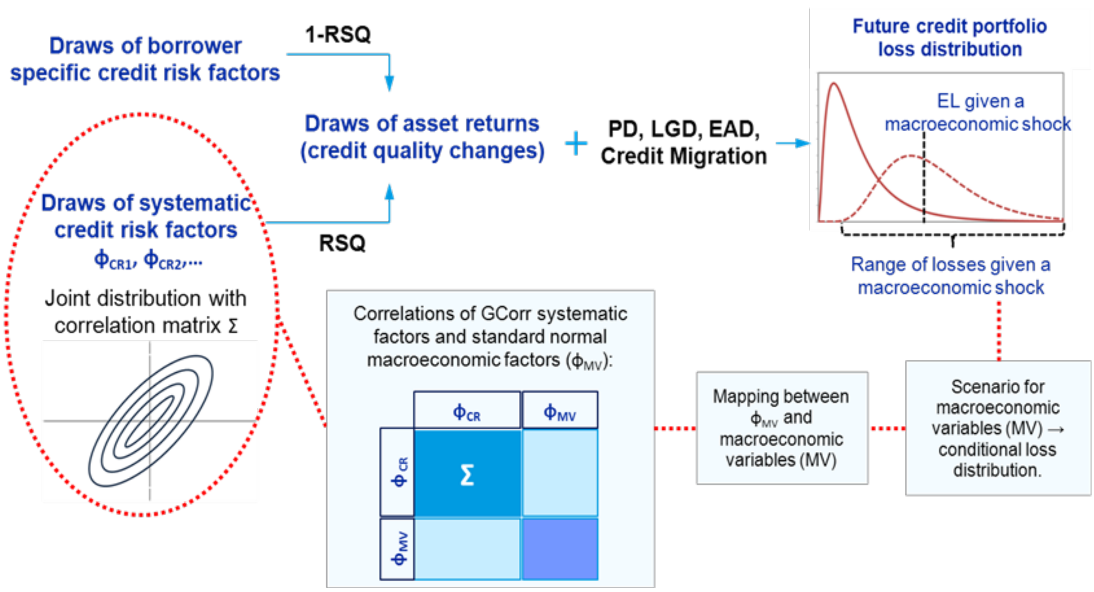

Figure 1 RiskFrontier framework for credit portfolio modeling

Let us briefly summarize the main components of this framework.[5] RiskFrontier employs a bottom-up approach to estimating portfolio value distribution at a future time horizon. Such an approach begins with modeling the credit quality of an individual borrower. A borrower's credit quality can change due to a systematic shock and an idiosyncratic (or borrower-specific) shock. A parameter called R-squared (RSQ) represents the proportion of the borrower's credit quality change attributable to the systematic shock. Systematic shocks reflect changes in the general economic conditions, while idiosyncratic shocks capture risks faced independently by each borrower. These shocks together establish a borrower's credit quality at horizon.

Because all borrowers are exposed to a set of correlated factors, the credit quality changes across borrowers are correlated. A Monte Carlo simulation engine generates random draws of these correlated credit quality changes. In each simulation trial, a valuation framework is applied to determine the value of every instrument based on the credit quality of the corresponding borrower at horizon. The value depends on several input parameters, such as probability of default (PD), loss given default (LGD), credit migration matrix, etc. A portfolio value at horizon is given by the sum of the instrument values. Therefore, a distribution of the portfolio values can be estimated by running a large number of these simulations and calculations.

Figure 1 also indicates the role of GCorr Macro. The model captures the relationship between GCorr systematic credit risk factors ϕ_{CR} (*CR*–credit risk) and macroeconomic variables MV in two steps:

- The Monte Carlo engine simultaneously generates standard normal variables representing GCorr systematic factors ϕ_{CR} and standard normal macroeconomic factors ϕ_{MV}. These two sets of variables are linked by a Gaussian copula model with a correlation matrix also displayed in Figure 1.

[5] For details, see "An Overview of Modeling Credit Portfolios" (Levy, A., 2008).

- A mapping translates draws of the standard normal macroeconomic factors ϕ_{MV} to values of observable macroeconomic variables MV.

GCorr Macro can be used to determine the expected loss and the loss distribution given certain values or ranges of macroeconomic variables (conditional expected loss and conditional loss distribution). The conditional loss distribution is shown using the dashed line in the right-hand chart of Figure 1.

As pointed out above, GCorr Macro does not change borrowers' asset return loadings (credit quality changes) to systematic and idiosyncratic GCorr credit risk factors. As a consequence, if a user does not specify a macroeconomic scenario in the expanded model, the resulting portfolio loss distribution will be exactly the same as in the case of the GCorr model without macroeconomic variables.

Let us discuss how a stress testing analysis with GCorr Macro can be understood from a theoretical perspective. We assume that a given macroeconomic scenario was translated to conditions on standard normal macroeconomic factors ϕ_{MV}. By focusing on the trials in which ϕ_{MV} met certain conditions (for example, attained a certain value or fell in a certain range), we specify a conditional distribution of GCorr systematic credit risk factors ϕ_{CR}.

Figure 2 illustrates this with an example of the U.S. oil industry systematic credit risk factor $\phi_{US,Oil}$ (one of the ϕ_{CR} factors) and a factor representing oil price changes $\phi_{OilPrice}$ (one of the ϕ_{MV} factors). The unconditional distribution of $\phi_{US,Oil}$ is a standard normal distribution. If the correlation between the two factors equals 41%, then a two standard deviation drop in the oil price leads to a conditional normal distribution of $\phi_{US,Oil}$ with the mean of – 0.82 and the standard deviation of 0.91. The interpretation is that given a positive correlation between $\phi_{US,Oil}$ and $\phi_{OilPrice}$, a drop in the oil price will be associated with a negative shock to the credit qualities of firms in the U.S. oil industry.

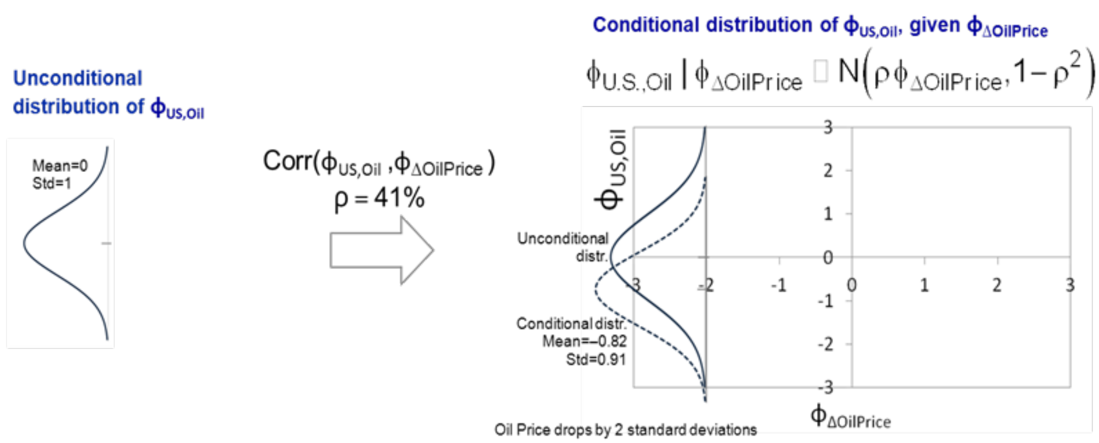

Figure 2 Conditional GCorr factor distribution (U.S. Oil industry factor), given a two standard deviation drop in the standard normal macroeconomic factor representing oil price changes.

This negative shock means that debt instruments issued by those oil industry firms or loans provided to those firms will decline in value, and more defaults will occur than would have occurred without any oil price drop. Therefore, we can expect higher losses on a portfolio of exposures to the U.S. oil industry. In mathematical terms, the conditional distribution of $\phi_{US,Oil}$ implies a conditional *PD* that is higher than the unconditional *PD.* Te conditional *PD* is also called the stressed *PD.*[6]

Figure 3 plots the impact on portfolio loss distribution. The left-hand chart depicts the unconditional and conditional distributions of the credit risk systematic factor $\phi_{US,Oil}$. If the portfolio is large and the idiosyncratic risks are diversified away, a return on $\phi_{US,Oil}$ implies a portfolio loss rate L. Therefore, the distributions of $\phi_{US,Oil}$ can be translated into

[6] See Section 4 for a discussion of a stressed LGD.

distributions of L, as we show in the right-hand chart of Figure 3. The chart illustrates how the expected loss increases from 1%, in the case of no scenario, to 2.3% under the scenario of the oil price drop. The probability of large losses also increases.

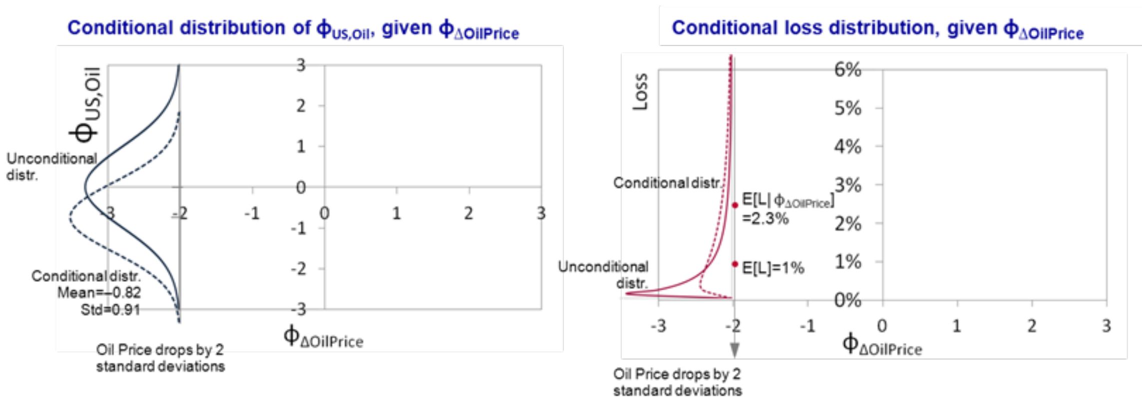

Figure 3 Transforming conditional factor distribution into conditional loss distribution, given a two standard deviation drop in the standard normal macroeconomic factor representing oil price changes.

Figure 2 and Figure 3 depict a stress testing analysis for a single GCorr systematic factor and a single macroeconomic variable. In practice, such an analysis can be carried out with an arbitrary set of macroeconomic variables and applied to a portfolio diversified across industries and countries. However, the principle remains the same—macroeconomic variables and their correlations to the GCorr credit risk factors provide a conditional distribution of the systematic factors, which in turn implies a conditional loss distribution.

Consider a large portfolio where idiosyncratic risks are diversified away. Many stress testing frameworks, such as CCAR, are concerned primarily with the conditional expected loss. It is important to realize that even if we specify a macroeconomic scenario, losses can still differ from the conditional expected loss when the macroeconomic variables do not completely explain the systematic credit risk of the exposures in the portfolio. This is shown in Figure 2 and Figure 3, where the correlation between $\phi_{US,Oil}$ and $\phi_{OilPrice}$ is less than one, and therefore the conditional loss distribution is dispersed around the conditional expected loss given a two standard deviation drop in the oil price. In other words, movement in the GCorr factor is not completely explained by the oil price changes. The special case when the variation in GCorr systematic factors is completely described by a set of macroeconomic variables would imply a conditional loss distribution concentrated in one point: the conditional expected loss.

From another perspective, Figure 2 provides an economic interpretation for the GCorr systematic factors. Decreases or increases in macroeconomic variables imply conditional distributions of GCorr systematic factors. Conversely, a decrease or increase in a GCorr systematic factor can be associated with certain economic scenarios defined by macroeconomic variables.

3 Risk Integration with the GCorr Macro Model

This section describes risk aggregation and allocation using the GCorr Macro model, and presents examples integrating market and credit risks.

3.1 Integrating Market and Credit Risk Using a Top-Down Framework

In this section, we discuss a top-down risk integration framework based on the GCorr Macro model in a multi-factor setting.[7] Figure 4 provides an overview of the framework. For exposition, we consider two types of portfolios containing financial instruments: credit risk sensitive portfolios and market risk sensitive portfolios. Our objective is to estimate the joint distribution of losses on all of these portfolios through their exposure to credit risk factors and market risk factors. The GCorr Macro model links the factors across risk types. Having the estimated joint distribution allows us to determine the distribution of total losses across all portfolios.

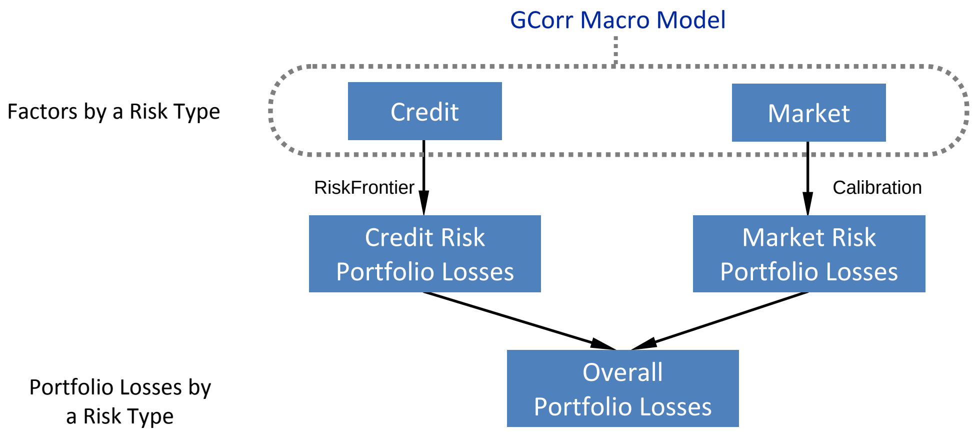

Figure 4 Overview of the risk integration framework with the GCorr Macro model

As explained in the previous section, GCorr Macro provides a correlation matrix that links the GCorr credit risk factors ϕ_{CR} and macroeconomic variables ϕ_{MV}. Suppose a subset of the macroeconomic variables ϕ_{MV} are the relevant market risk factors which are divided into factor sets f_m . A set f_m containing F_m factors: $f_m = \left\{ f_m^j,\ \ j = 1, \ldots, F_m \right\}$. While each market portfolio is exposed to one factor set, there can be an overlap between two factor sets if two market risk portfolios are exposed to the same market factor. The joint distribution of losses on the credit portfolios can be estimated using RiskFrontier. Meanwhile, market risk is analyzed through a market risk system with factors that overlap with GCorr Macro. We denote losses on credit portfolio m as L_m^{Credit} and denote losses on market portfolio m as L_m^{Market}.

The two risk systems can be linked though the following steps. First, a market risk platform is used to simultaneously generate draws of losses on a market risk portfolio and returns on the corresponding factor set. Second, one can use regression techniques to estimate parameters for a polynomial representation in equation *(1)*:

$$L_{m,i}^{Market} = \beta_m^0 + \sum_{j=1}^{F_m} \sum_{p=1}^{N_p} \beta_m^{j,p} \left(f_{m,i}^j \right)^p + \sum_{j=1}^{F_m} \sum_{k=1}^{F_m} \beta_{m,\,Cross}^{j,k} \left(f_{m,i}^j f_{m,i}^k \right) + \sqrt{1 - R_m^2}\, \sigma_m \, \varepsilon_{m,i} \qquad (1)$$

where N_p is the degree of the polynomial function.

[7] Various top-down and bottom-up risk integration approaches are presented in "Risk Integration: New Top-down Approaches and Correlation Calibration" (Chen, N., A. Kaplin, A. Levy, and Y. Wang, 2010).

In equation (1), $f_{m,i}^{j}$ is the realization of factor j from factor set f_m used in construction of the loss distribution for market portfolio m during trial i. Parameter R_m^2 represents the R-squared of the polynomial regression and σ_m the standard deviation of the loss distribution. Variable $\varepsilon_{m,i}$ is the idiosyncratic portion of the loss for trial i, with the expected value 0 and standard deviation 1. Once the parameters from equation (1) are estimated, we can determine the calibrated loss on a market portfolio, given the market factors f_m, as follows:

$$\hat{L}_m^{Market}\left(f_m\right)=\hat{\beta}_m^0+\sum_{j=1}^{F_m}\sum_{p=1}^{N_p}\hat{\beta}_m^{j,p}\left(f_m^j\right)^p+\sum_{j=1}^{F_m}\sum_{k=1}^{F_m}\hat{\beta}_{m,Cross}^{j,k}\left(f_m^j f_m^k\right)+\sqrt{1-\hat{R}_m^2}\,\hat{\sigma}_m\,\varepsilon_m^* \qquad (2)$$

Symbols $\hat{\beta}_m, \hat{R}_m^2, \hat{\sigma}_m$ denote the estimated parameters, while ε_m^* stands for a random draw from a standard normal distribution, uncorrelated with the factors.[8]

A model of type *(1)* can be considered suitable for our risk integration framework if R_m^2 is high, which means that the factors f_m explain most of the variation in the portfolio losses. If R_m^2 is low, the model can still be utilized, but only in the case when ε_m is uncorrelated with other idiosyncratic factors. For example, gains and losses on a portfolio of U.S. treasury securities can be explained by several factors representing movements of several points on the treasury yield curve. In this case, the R_m^2 value can be expected to be high.

Now that we have introduced all components of the risk integration framework, we can combine them to estimate the joint distribution of losses on all portfolios as well as the total loss distribution. The credit and market risk factors are jointly simulated from a Gaussian copula within RiskFrontier. For a given draw of credit risk and market risk factors, we can determine the credit portfolio losses using RiskFrontier and the market portfolio losses using equation *(2)*. The total loss is given as the sum of losses across all portfolios:

$$L^{Total}=\sum_c L_c^{Credit}+\sum_m \hat{L}_m^{Market}\left(f_m\right) \qquad (3)$$

We summarize the estimation process and its output in Table 1.

Table 1 Estimating the joint distribution of portfolio losses and the total loss distribution

GCorr Macro Model
Gaussian Copula

Sum

Trial	Simulated GCorr Credit Risk Factors	Simulated Market Risk Factor Sets	Losses on Credit Risk Portfolios	Losses on Market Risk Portfolios	Aggregate Losses
1	$\phi_{k,1}, k=1,\ldots,N_K$	$f_{m,1}, m=1,\ldots,N_M$	$L_{c,1}^{Credit},\ c=1,\ldots,N_C$	$\hat{L}_m^{Market}\left(f_{m,1}\right),\ m=1,\ldots,N_M$	L_1^{Total}
2	$\phi_{k,2}, k=1,\ldots,N_K$	$f_{m,2}, m=1,\ldots,N_M$	$L_{c,2}^{Credit},\ c=1,\ldots,N_C$	$\hat{L}_m^{Market}\left(f_{m,2}\right),\ m=1,\ldots,N_M$	L_2^{Total}
…	…	…	…	…	…
N	$\phi_{k,N}, k=1,\ldots,N_K$	$f_{m,N}, m=1,\ldots,N_M$	$L_{c,N}^{Credit},\ c=1,\ldots,N_C$	$\hat{L}_m^{Market}\left(f_{m,N}\right),\ m=1,\ldots,N_M$	L_N^{Total}

RiskFrontier

Calibrated Model, Equation *(2)*

In Section 3.2 we present portfolio analyses which utilize the output from Table 1 and, more generally, the multi-factor risk integration framework from Figure 4.

[8] Alternatively ε_m^* can be sampled by bootstrapping the regression standard errors.

We conclude this section by discussing the features of the framework that distinguish it from other risk integration approaches. The framework requires estimation of two inputs: the GCorr Macro model linking factors across risk types and calibrated models relating market portfolio losses to market risk factors. Compared to the traditional copula risk integration approaches, our framework requires estimation of more input parameters. However, the parameters have intuitive interpretations, whether it is the correlation between a shock to the credit quality of corporates within the U.S. air transportation industry and the U.S. stock market return, or the sensitivities of a U.S. Treasury securities portfolio to points on the treasury yield curve. Moreover, the rich and flexible factor structure of our framework allows for a more accurate description of correlations and concentrations across credit and market portfolios compared to simpler approaches relying on few parameters.

Note that there are other methodologies that allow combination of credit risk and market risk scenarios produced by different systems. For example, the paper by Morrison (2013) uses GCorr Macro to simulate credit and market risk scenarios using reordering techniques.[9] The main idea behind this method is the existence of one (or a few) underlying risk factor(s) common between different risk systems that will allow one to describe the effects of interaction between the full sets of underlying risk factors in each system while maintain the stand-alone loss distributions unchanged. While this reordering approach might be relatively simple to implement, it also induces certain dependencies across underlying risk factors for various systems rather than specifying them directly and, therefore, requires careful consideration and validation of a particular reordering procedure.

3.2 Economic Capital Aggregation and Allocation

In this section we discuss how the simulation output with calibrated market portfolio losses described earlier can be used to do the following:

- Determine aggregate capital
- Allocate the aggregate capital to individual portfolios based on their Risk Contribution (RC)/Tail Risk Contribution (TRC)

The analysis accounts for correlations and concentrations across risk types, geographies, sectors, etc., through factor correlations implied by GCorr Macro.

Financial organizations are subject to various risk sources and are typically required to assess overall risk. At the same time, the management of various risk sources is often siloed, and the respective organization units often have sophisticated risk systems in place to assess those risk sources separately. For example, a unit responsible for the trading book is likely to have a good idea of its market risk exposure, but their system will not typically account for the credit risk of the banking book. The top-down approach discussed earlier offers a solution where risk sources are analyzed separately and combined in order to arrive at the overall risk picture.

To illustrate, in this section we assume that the organization has four units managing four separate portfolios that capture the following four risk sources: U.S. Credit Risk, UK Credit Risk, U.S. Market Risk, and UK Market Risk. Furthermore, market portfolio losses can be approximated using a quadratic representation.

$$\begin{aligned} Loss_{US}^{Market} = {} & \beta_0^{US} + \beta_1^{US}\mathrm{S\&P500} + \beta_2^{US}\mathrm{USRate} + \beta_3^{US}\left(\mathrm{S\&P500}\right)^2 + \beta_4^{US}\left(\mathrm{USRate}\right)^2 \\ & + \beta_5^{US}\left(\mathrm{S\&P500}\cdot\mathrm{USRate}\right) + \varepsilon_{US}^{Market} \end{aligned}$$

and

$$\begin{aligned} Loss_{UK}^{Market} = {} & \beta_0^{UK} + \beta_1^{UK}\mathrm{FTSE100} + \beta_2^{UK}\mathrm{UKRate} + \beta_3^{UK}\left(\mathrm{FTSE100}\right)^2 + \beta_4^{UK}\left(\mathrm{UKRate}\right)^2 \\ & + \beta_5^{UK}\left(\mathrm{FTSE100}\cdot\mathrm{UKRate}\right) + \varepsilon_{UK}^{Market} \end{aligned}$$

[9] See “Aggregation of market and credit risk capital requirements via integrated scenarios” (Morrison, 2013).

To parameterize the coefficients in these calibrations, we make the following assumptions which are based on intuitive relationships.

With β_0 = 0 we assume that losses are in excess of expected loss and the factors are normalized to have zero means. We also assume that the market portfolio has positive exposure to the equity markets and thus β_1 is negative as we are modeling losses. Thirdly, β_2 is the measure related to the modified duration of the market portfolio and we would expect it to be positive (value goes down as the rates go up leading to increase in losses or decrease in gains). β_3 is assumed to represent instruments which tend to have concave sensitivity to changes in market returns and is expected to be negative. β_4 is related to the convexity measure and we would expect it to be positive as well. β_5 captures the cross-moment effects which we expect to be negative to compensate for the correlation effects between these two factors. The coefficients used in the subsequent examples are presented in Table 2.

Table 2 Parameterization of the calibrated loss functions used for the illustration exercise

Coefficient	Value	Coefficient	Value
β_1^{US}	-0.3	β_1^{UK}	-0.4
β_2^{US}	0.2	β_2^{UK}	0.15
β_3^{US}	-0.7	β_3^{UK}	-0.8
β_4^{US}	0.4	β_4^{UK}	0.3
β_5^{US}	-0.2	β_5^{UK}	-0.3

We can now use these parameterizations to calculate market portfolio loss distributions that are consistent with the credit portfolio loss distributions simulated by RiskFrontier with GCorr Macro[10]. Figure 5 demonstrates the stand-alone simulated and calibrated loss distributions in our exercise. Note that the tail of the loss distribution is much more pronounced for the credit portfolios compared to the market portfolios.

[10] The credit portfolios used in the analysis are subsets of the IACPM portfolio, which is a diversified portfolio consisting of 3000 borrowers across seven developed countries and 60 industries. Specifically, the U.S. credit portfolio consists of 1,133 borrowers and the UK credit portfolio consists of 359 borrowers.

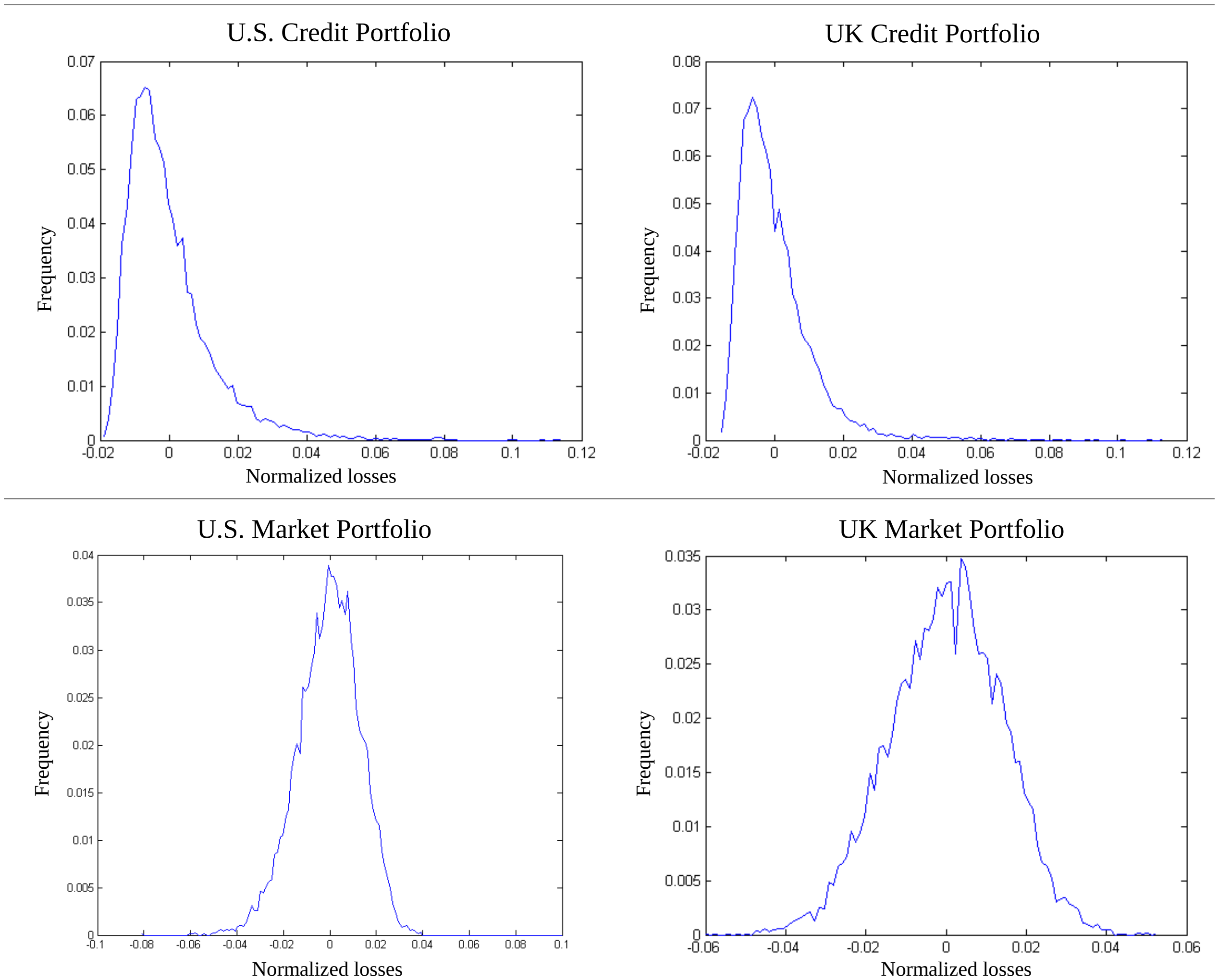

Figure 5 Stand-alone loss distributions simulated by RiskFrontier (U.S. Credit and UK Credit) and calculated using the calibrated loss functions based on simulated underlying factors (U.S. Market and UK Market)

These stand-alone loss distributions are thus constructed consistently and, therefore, the Aggregated Loss Distribution can be calculated by adding the loss realizations for the four loss distributions for each trial,

$$Loss_i^{Aggregated} = Loss_{US,i}^{Credit} + Loss_{UK,i}^{Credit} + Loss_{US,i}^{Market} + Loss_{UK,i}^{Market}$$

guaranteeing that the underlying correlation structure is described by GCorr Macro. Figure 6 displays the aggregated loss distribution.

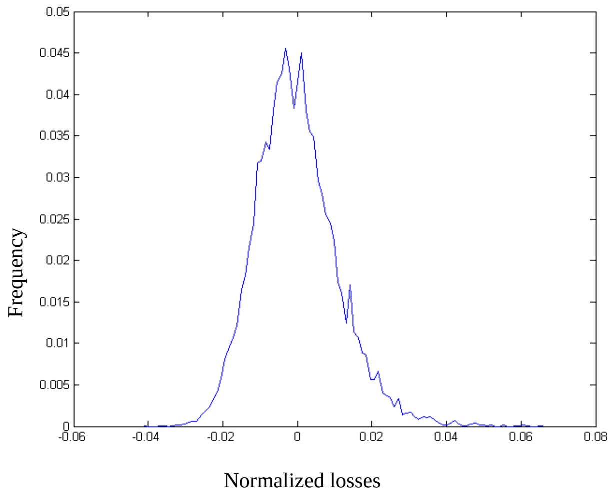

Figure 6 Aggregate loss distribution–combining market and credit risk

As soon as we know the aggregate loss, how much of that overall loss should be attributed to each sub-portfolio? To calculate the allocations one can use either Risk Contributions that measures the contribution of a particular portfolio to the aggregated portfolio's Unexpected Loss (or Standard Deviation of losses)

$$RC^{RiskSource} = \frac{Cov\left(\tilde{L}oss^{RiskSource}, \tilde{L}oss^{Aggregated}\right)}{UL^{Aggregated}}$$

or Tail Risk Contributions that measure the effect of various portfolios on the tail of the aggregated loss distribution.[11]

$$TRC^{RiskSource} = E\left[\tilde{L}oss^{RiskSource} \middle| LowerBound \leq \tilde{L}oss^{Aggregated} \leq UpperBound\right]$$

Because financial institutions often focus on tails (or extreme losses) of their portfolios, for this illustration we use Tail Risk Contributions that are calculated over the worst 1% of losses of the aggregated portfolio. Table 3 presents the standalone and allocated aggregated capitals for each of the overall portfolios. It also supplies the reductions experienced by every portfolio.

Table 3 Standalone and reallocated capital

Portfolio	Portfolio MTM	Standalone Capital	Reallocated Capital	Reduction
U.S. Credit	36.284 B	7.93%	7.62%	3.91%
UK Credit	8.357 B	7.83%	4.46%	43.04%
U.S. Market	30.000 B	3.58%	2.86%	20.11%
UK Market	10.000 B	4.10%	3.65%	10.98%
Total	84.641 B	5.92%	5.15%	13.01%

[11] Tail Risk Contribution is usually calculated as contribution to capital, which represents discounted loss. In this case, the discount rate can be assumed to be zero.

Notice that the diversification effects are very different across sub-portfolios. For example, the U.S. Credit portfolio is the largest, and the dynamics of the aggregated portfolio are driven to a large extent by the dynamics of that portfolio. It is not surprising that this portfolio demonstrates the smallest diversification benefit. Meanwhile, the UK Credit portfolio exhibits the largest reduction. While it has relatively high stand-alone capital, the aggregated portfolio is dominated by the U.S. exposures with which it has correlation far from perfect positive correlation.

4 Stress Testing and Reverse Stress Testing

In this section, we illustrate how the GCorr Macro model can be used for various stress testing and reverse stress testing analyses. All of the analyses are designed to provide insights into relationships between credit portfolio losses and macroeconomic variables.

The stress testing and reverse stress testing analyses are based on a vector of stationary macroeconomic variables $MV=\{MV_l\ l=1\ldots N_{MV}\}$. As explained in Section 2, GCorr Macro expands the existing GCorr credit risk factors to include the correlations of these factors with macroeconomic variables. The macroeconomic variables can refer to the same period (contemporaneous), but may also refer to past periods (lags). This feature allows us to capture various persistency patterns;for example, when a stock market crash affects defaults not only over the same period, but also over future periods. Many macroeconomic variables MV, in practice, do not have a normal distribution; therefore we need to introduce a concept of mapping, another component of the GCorr Macro model. A mapping for a variable MV_l is a monotonous function F_l, which translates the variable into a macroeconomic factor $\phi_{MV,l}$ that has a standard normal distribution: $\phi_{MV,l} = F_l(MV_l)$.

In Section 4.1 and Section 4.2, we use the simulation output from Figure 4 for stress testing and reverse stress testing analyses. Section 4.3 presents a method for stress testing credit and market portfolios that avoids Monte Carlo simulation. Although this method does not provide the same range of insights as the simulation-based approach, it still produces stressed expected losses. Importantly, the method is easier to implement in practice, especially for a multi-period stress testing exercise.

4.1 Simulation-Based Stress Testing Method

Several types of stress tests can be conducted with the Monte Carlo simulation output, and we can examine relationships between losses and macroeconomic variables. In particular, we are interested in the directions and strengths of the relationships. Note that we can map the simulated macroeconomic factors to the corresponding marginal distribution using $MV_l = F_l^{-1}(\phi_{MV,l})$, where F_l^{-1} represents the inverse empirical cumulative distribution function.

Figure 8 provides examples of univariate analyses. It shows relationships between losses on a U.S. credit portfolio and a UK credit portfolio and an S&P 500 return.[12] In line with economic intuition, the trials with a large negative S&P 500 return tend to be associated with higher losses on both portfolios. This relationship is stronger for the U.S. portfolio, as expected; the S&P 500 represents the U.S. stock market index. The black dots on the figure represent the conditional expected loss conditional on the S&P 500 return. The dispersion of trial level losses around the conditional expected loss reflects the fact that S&P 500 returns do not completely capture the systematic credit risk of the portfolios.

[12] The portfolio used in the analysis is the IACPM portfolio which is a diversified portfolio consisting of 3000 borrowers across seven developed countries and 60 industries.

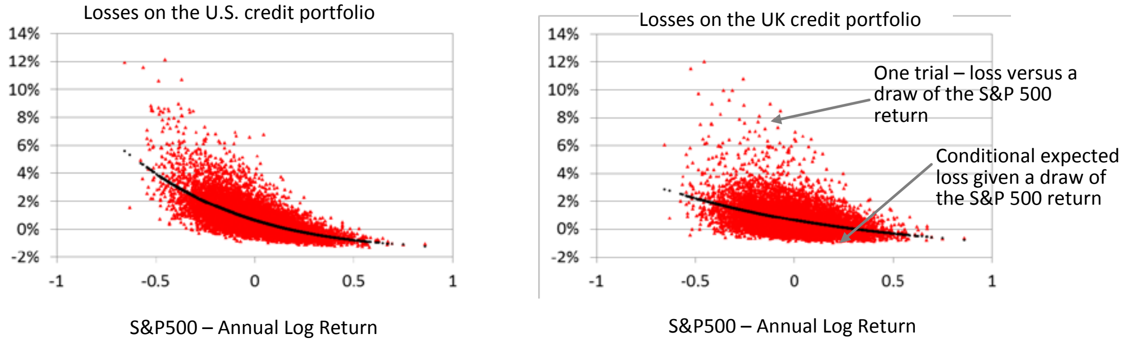

Figure 7 Relationships between losses on credit portfolios and S&P 500 returns

While the univariate analyses provide insights into impacts of individual macroeconomic variables, a typical stress testing exercise requires characterization of losses given a scenario for multiple macroeconomic variables. For this exercise, we need to describe the relationship between portfolio losses and a set of macroeconomic variables. We can achieve this by estimating a regression model across trials, with a portfolio loss *L* as the dependent variable and a set of macroeconomic variables *MV* as independent variables. The regression models provide a link between *MV* and the conditional expected loss. Equation *(4)* presents examples of the link for credit and market portfolios:[13]

$$E\left[L_c^{Credit}\middle|MV\right]=N\left(\gamma_{c,0}+\sum_l \gamma_{c,l}F_l\left(MV_l\right)\right)$$
$$E\left[L_m^{Market}\left(f_m\right)\middle|MV\right]=\delta_m^0+\sum_l\sum_{p=1}^{N_p}\delta_m^{l,p}\left(F_l\left(MV_l\right)\right)^p+\sum_k\sum_l\delta_{m,Cross}^{k,l}\left(F_k\left(MV_k\right).F_l\left(MV_l\right)\right) \quad (4)$$

After the model *(4)* is estimated, macroeconomic scenarios can be specified to obtain conditional expected losses (stressed expected losses).

The fit of model *(4)* provides an indication to the extent to which the select macroeconomic variables span the systematic risks of the portfolios. If the model fit is strong, the variables explain most of the portfolio risk, and vice versa.

An alternative to fitting the models would to take the simulation output file, select only the trials in which the macroeconomic variables met the conditions of the scenarios,[14] and analyze losses across these select trials, including the calculation of the stressed expected loss by averaging the losses. The advantage of the model based approach *(4)* is that it uses information across all trials and can be applied to scenarios for which the simulation output does not provide a sufficient number of trials. On the other hand, the trial selection method may be suitable for scenarios that are associated with a relatively large number of trials. In this case, we do not need to specify a model for the conditional expected loss.

Let us highlight two features of the simulation-based approach:

- Moody's Analytics has developed a PD-LGD correlation module which can be used when running the RiskFrontier Monte Carlo simulation engine. The module introduces a dependence of LGD draws on the systematic credit risk factors—negative shocks to those factors lead to higher LGD values. As a result, adverse macroeconomic scenarios will impact credit portfolio losses via two channels: more downgrades and defaults, but also higher LGD values.

[13] The credit conditional loss in equation *(4)* is specified in such a way that if the portfolio loads to a single systematic factor, the formula matches the analytical expression in equation *(8)*. That expression can be understood as the conditional expected loss on a credit portfolio of instruments with only default/no-default valuation on a horizon and *LGD = 100%*. The market conditional loss is based on formula *(2)*. Let us emphasize that the equation *(4)* serves as an example–the models can be specified in a different way.
[14] In practice, a scenario may prescribe that, for example, the GDP declines by 5% (ΔGDP=– 5%). In that case, we select the trials, in which the draws of ΔGDP fall into a narrow bandwidth around 5%.

- The simulation-based method uses RiskFrontier to calculate instrument losses at horizon of analysis date. This fully utilizes the instrument valuation methods that are available within RiskFrontier, including dynamics coming from the likes of credit migration or prepayments based on improving credit qualities.

4.2 Reverse Stress Testing

In addition to stress testing, we can use the Monte Carlo simulation output to characterize macroeconomic scenarios associated with certain levels of portfolio losses, typically extreme losses. This type of analysis is called reverse stress testing.

We define losses as extreme if they exceed a threshold: $L \geq Thr$. The threshold may represent a $1-\alpha$ percentile of the loss distribution. We conduct a reverse stress testing exercise by selecting only those trials from the Monte Carlo simulation output in which the losses exceeded the threshold. Then we can link the macroeconomic variable distribution across the select trials with the scenario types associated with extreme losses.

Figure 8 shows an example of a reverse stress testing analysis which describes the S&P 500 returns corresponding to extreme total losses.[15] We select the threshold as the 99th percentile of the total loss distribution. In other words, we focus on the worst 1% of the total losses. The figure provides the conditional distribution of the S&P 500 returns given the extreme losses, and compares it with the unconditional distribution. Both distributions are estimated from the Monte Carlo simulation output. As the result suggests, the extreme losses tend to be associated with large negative S&P 500 returns, which is consistent with economic intuition.

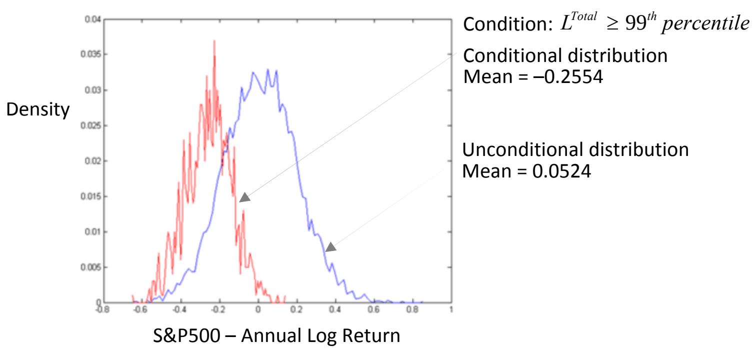

Figure 8 Reverse stress testing: conditional distribution of S&P 500 annual log returns, given extreme total losses.

4.3 Analytical Stress Testing Method

The simulation-based stress testing method described in Section 4.1 provides various insights into relationships between portfolio losses and macroeconomic variables. It accounts for Mark-to-Market losses from credit migration, and allows for a full use of RiskFrontier valuation methodology for credit instruments. However, financial institutions must often conduct a stress testing analysis over multiple periods.[16] In that case, the simulation-based method may not be suitable due to the substantial growth in computational time with every additional period. Therefore, we introduce an analytical approach to stress testing which avoids the Monte Carlo simulation and can be easily applied to a multi-period scenario.

[15] The portfolio used in the analysis is the portfolio as described in Section 3.
[16] An example is the stress testing exercise described in "Comprehensive Capital Analysis and Review: Methodology and Results for Stress Scenario Projections (CCAR)" (Board of Governors of the Federal Reserve System 2012)."

The analytical approach requires the same inputs as the simulation-based method, and produces instrument and portfolio level stressed expected losses for each period.[17] For credit portfolios, the losses account only for defaults, not for instrument value changes resulting from changes in credit quality.[18]

Let us explain the idea behind the analytical approach. A macroeconomic scenario specifies values of macroeconomic variables over several future periods. Normal transformations of the scenario can be translated to values of macroeconomic factors using the mapping functions F. In the first step, we can analytically determine the conditional distribution of the credit and market factors, given values of macroeconomic factors specified by the scenario. The conditional distribution is normal, which makes further calculations tractable. In the second step, we can use this conditional distribution to analytically determine the stressed expected loss. For credit portfolios, we can analytically convert the conditional distribution of the credit risk factors to the stressed instrument level credit risk parameters: probabilities of default and losses given default. These stressed parameters imply stressed expected losses on the credit portfolios.

To outline the analytics of the stressed expected loss calculation, the future periods t are defined as quarters. The calculation is carried out as of the analysis date, which is the last day before the beginning of quarter 1. Variable L_t represents the loss on a portfolio over a quarter t. A scenario is a set of quarterly conditions on stationary macroeconomic variables MV_t. A scenario may hypothesize, for example, that the real U.S. GDP will drop by 5% at an annual rate from the first to the second quarter after the analysis date. We assume that such conditions are given for quarters $1,\ldots,T$. In our notation, $Sc_{1,t}$ represents the conditions for quarters 1 through t.

In the analytical approach, the credit portfolios consist of instruments with possible losses coming from defaults. The stressed expected loss on an instrument k in a portfolio c over a future quarter t depends on the stressed default probability $PD_{c,k,t}\left(Sc_{1,t}\right)$ and the stressed loss given default $LGD_{c,k,t}\left(Sc_{1,t}\right)$ over that quarter. Both stressed parameters can be determined analytically because we know the conditional distribution of credit risk factors under the scenario and are able to link these two parameters to the credit risk factors. For the relationship between loss given default and the macroeconomic scenario, we employ the Moody's Analytics PD-LGD framework.[19] We describe the calculation of the stressed parameters in more detail in Appendix A. Our method does not explicitly model dependence of the commitment amount, Cmt, and the usage given default, UGD, on macroeconomic variables. Rather, we assume that values of these parameters under the scenario are already available.

The stressed expected loss on a credit portfolio c can be calculated as follows:

$$E\left[L_{c,t}^{Credit}\middle|Sc_{1,t}\right]=\sum_{k=1}^{NI_c}E\left[L_{c,k,t}^{Credit}\middle|Sc_{1,t}\right]=\sum_{k=1}^{NI_c}Cmt_{c,k,t}\times UGD_{c,k,t}\times PD_{c,k,t}\left(Sc_{1,t}\right)\times LGD_{c,k,t}\left(Sc_{1,t}\right) \quad (5)$$

Cmt should reflect the financial institution's assumptions about the dynamics of the portfolio volume and composition under the scenario. If the institution's credit risk strategy prescribes, for example, reduction of the credit limit on a revolving line of credit k in a future quarter t under the scenario, parameter $Cmt_{c,k,t}$ should reflect this through inequality: $Cmt_{c,k,t} < Cmt_{c,k,1}$. On the other hand, if the institution assumes it will add new volume to an amortizing loan k in the future to keep the commitment constant, then $Cmt_{c,k,t} = Cmt_{c,k,1}$. The UGD parameter can represent the financial institution's assumption about usage given default under the scenario for a revolving line of credit.

[17] The term "stressed" stands for the phrase "conditional on a macroeconomic scenario" throughout this section.

[18] Note that credit migration dynamics are included to affect an instrument's future probability of default. However, we only compute losses based on defaults not credit quality downgrades.

[19] See "Incorporating Systematic Risk in Recovery: Theory and Evidence" (Levy, A. and Z. Hu, 2007), and "Implications of PD-LGD Correlation in a Portfolio Setting" (Meng, Q., A. Levy, A. Kaplin, Y. Wang and Z.Hu, 2010).

5 Conclusion

This document introduces GCorr Macro, a model that links the Moody's Analytics credit portfolio modeling framework to macroeconomic variables. Furthermore, we describe how GCorr Macro can facilitate holistic risk management practices. Specifically, we describe how GCorr Macro can be used for risk integration including aggregation and allocation of capital across market and credit portfolios. We also introduce analytical and simulation-based methods for conducting stress testing analysis that leverage a portfolio view of credit risk. The approaches described in this paper provide a framework to conduct multi-period analysis, which is required under the CCAR regulatory stress testing exercise developed by the Federal Reserve System. CCAR specifies an economic scenario over several quarters, and requires financial institutions to estimate their portfolio losses under the scenario. Finally, we described how GCorr Macro can be used for reverse stress testing.

In addition to the applications described above, GCorr Macro provides additional insights. While current GCorr factors are sufficient to explain systematic credit risk in a portfolio, these factors are latent. By expanding GCorr to include macroeconomic variables and other market risk factors, we can now use more intuitive factors to describe credit portfolio dynamics.

Appendix A Technical Details for Analytical Stress Testing

In this appendix, we provide technical details for the analytical stress testing approach outlined in Section 4.3.

First, we derive the conditional distribution of all risk factors in GCorr Macro given a macroeconomic scenario. Typically, the macroeconomic scenario will only specify values for a subset of the variables found within GCorr Macro. For all other risk factors not specified by the macroeconomic scenario (credit, market, other macroeconomic variable not in the scenario), we will derive the conditional distribution based on macroeconomic scenario.

Let ϕ_t be returns of the risk factor over a future quarter t. The vectors of stationary macroeconomic variables and of the corresponding returns on macroeconomic factors over t will be denoted as MV_t and $\phi_{MV,t}$. As mentioned in the introduction to Section 4, MV_t may contain macroeconomic variables referring to the same quarter as the credit factors, which is t, but also lagged variables referring to previous quarters, for example t–1. Including such lagged variables makes economic sense if there is a significant correlation between the macroeconomic factors corresponding to the lagged variables and the credit risk factors for quarter t.

The correlation matrix of factors ϕ_t and macroeconomic factors $\phi_{MV,t}$ is given in Table 4.

Table 4 Correlation matrix of risk factors and macroeconomic factors specified in the macroeconomic scenario

	Risk Factors not Specified in Scenario ϕ	Macroeconomic Factors Specified in Scenario ϕ_{MV}
Risk Factors not Specified in Scenario ϕ	Σ_{11}	Σ_{12}
Macroeconomic Factors Specified in Scenario ϕ_{MV}	Σ_{21}	Σ_{22}

We note that the correlation matrix is identical for all quarters t. Due to the joint normality of ϕ_t and $\phi_{MV,t}$, the conditional distribution of the credit and market factors, given the macroeconomic factors, is normal with the following parameters:

$$\phi_t \middle| \phi_{MV,t} \sim N\left(\Sigma_{12} \cdot \Sigma_{22}^{-1} \cdot \phi_{MV,t} \;,\; \Sigma_{11} - \Sigma_{12} \cdot \Sigma_{22}^{-1} \cdot \Sigma_{21} \right) \tag{6}$$

An important feature of the conditional distribution is that, while its mean depends on the values of the macroeconomic factors $\phi_{MV,t}$, the conditional covariance matrix does not. The values of $\phi_{MV,t}$ are obtained by converting the scenario values of MV_t using the mapping functions F.

Next, we use the conditional distribution *(6)* to derive analytical expressions of stressed credit risk parameters. We begin by calculating the stressed default probability for an instrument k and the first quarter after the analysis date, t=1. If the counterparty is exposed to the credit risk factor $\phi_{k,1}$, its conditional default probability, given $\phi_{k,1}$, can be expressed as follows:[20]

$$PD_{c,k,1}\left(\phi_{k,1}\right) = N\left(\frac{N^{-1}\left(PD_{c,k,1}\right) - \sqrt{RSQ_{c,k}} \cdot \phi_{k,1}}{\sqrt{1 - RSQ_{c,k}}} \right) \tag{7}$$

[20] For a derivation of formula *(7)*, see "Loan Portfolio Value," (Vasicek, O., 2002, *Risk*).

$PD_{c,k,1}$ is the unconditional default probability of the counterparty for the first quarter and $RSQ_{c,k}$ measures the sensitivity of the counterparty's credit quality changes to the credit risk factor $\phi_{k,1}$.

Since the scenario specifies values of macroeconomic factors, we need to determine the conditional default probability given $\phi_{MV,1}$. We denote the conditional mean and the conditional variance of the credit risk factor $\phi_{k,1}$, given the macroeconomic factors $\phi_{MV,1}$, as $\mu_{c,k}\left(\phi_{MV,1}\right)$ and $1-\rho^2_{c,k}$, respectively. These parameters come from the conditional distribution *(6)*. The normal density function with these parameters will be denoted by $\varphi_{c,k,\phi_{MV,1}}$. The stressed default probability over the first quarter is:

$$
\begin{aligned}
PD_{c,k,1}\left(Sc_{1,1}\right) &= PD_{c,k,1}\left(\phi_{MV,1}\right) = \int_{-\infty}^{\infty} PD_{c,k,1}\left(\phi_{k,1}\right)\cdot\varphi_{c,k,\phi_{MV,1}}\left(\phi_{k,1}\right)d\phi_{k,1} \\
&= N\left(\frac{N^{-1}\left(PD_{c,k,1}\right)-\sqrt{RSQ_{c,k}}\cdot\mu_{c,k}\left(\phi_{MV,1}\right)}{\sqrt{1-RSQ_{c,k}\cdot\rho^2_{c,k}}}\right)
\end{aligned} \tag{8}
$$

To determine the stressed default probability over a future quarter t, we need to derive stressed cumulative default probabilities. We denote the stressed cumulative default probability over quarters 1 through t as $CPD_{c,k,t}\left(Sc_{1,t}\right)$ If $t=1$, this probability is given by formula *(8)*. For $t>1$, the stressed cumulative default probability can be decomposed into the stressed cumulative default probability over the first t-1 quarters and the stressed forward default probability for quarter t, $FPD_{c,k,t}\left(Sc_{1,t}\right)$. The stressed forward default probability is the conditional default probability over quarter t, given the scenario and given no default prior to t. Once the stressed cumulative default probabilities have been specified for $t=1,\dots,T$, the differences in the successive values of their term structure represent the stressed quarterly default probabilities. The calculations are summarized in formula *(9)*.

$$
\begin{aligned}
&CPD_{c,k,1}\left(Sc_{1,1}\right) = PD_{c,k,1}\left(Sc_{1,1}\right) \\
&CPD_{c,k,t}\left(Sc_{1,t}\right) = 1-\left(1-CPD_{c,k,t-1}\left(Sc_{1,t-1}\right)\right)\cdot\left(1-FPD_{c,k,t}\left(Sc_{1,t}\right)\right), \quad t=2,\dots,T \\
&PD_{c,k,t}\left(Sc_{1,t}\right) = CPD_{c,k,t}\left(Sc_{1,t}\right)-CPD_{c,k,t-1}\left(Sc_{1,t-1}\right), \quad t=2,\dots,T
\end{aligned} \tag{9}
$$

The main question is how to calculate the stressed forward default probability $FPD_{c,k,t}\left(Sc_{1,t}\right)$. Under simple assumptions, one can use a version of equation *(8),* with the initial forward default probability as the unconditional parameter. However, if the result is to account for the fact that the credit quality of the counterparty may deteriorate faster under an adverse scenario than in the unconditional case, the formula for $FPD_{c,k,t}\left(Sc_{1,t}\right)$ would have to be more general and include stressed credit migration probabilities between the analysis date and the beginning of quarter t.

We determine the stressed loss given default using the Moody's Analytics PD-LGD correlation model. The model defines a link between a loss given default and a recovery factor return, which is correlated with credit quality change of the corresponding counterparty. Specifically, the counterparty's credit quality deterioration will tend to be associated with negative shocks to the recovery return which in turn implies higher loss given default. Within such a framework, the stressed loss given default can be expressed as follows:

$$LGD_{c,k,t}\left(S_{1,t}\right)=\int_{-\infty}^{\infty} g\left(z,\kappa,LGD_{c,k,t}\right)\times p_{c,k,\phi_{MV,t}}\left(z\right)dz \tag{10}$$

Variable z represents a recovery factor in a normal space and function g converts z to a Beta distribution variable using the unconditional expected loss given default, LGD, and parameter κ related to variance of the Beta distribution.

$$g\left(z,\kappa,LGD\right)=Beta^{-1}\left(1-N\left((z-a)/b\right),\ (\kappa-1)\cdot LGD,\ (\kappa-1)\cdot(1-LGD)\right) \tag{11}$$

$Beta^{-1}$ is the inverse cumulative distribution function of a Beta distribution and N is the cumulative distribution function of a standard normal distribution. Parameters a and b are given by the PD-LGD correlation model. Function p is the density of the recovery factor conditional on default and on the macroeconomic factors for quarter t.

In practice, the integral in formula (10) needs to be evaluated by utilizing numerical techniques.

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