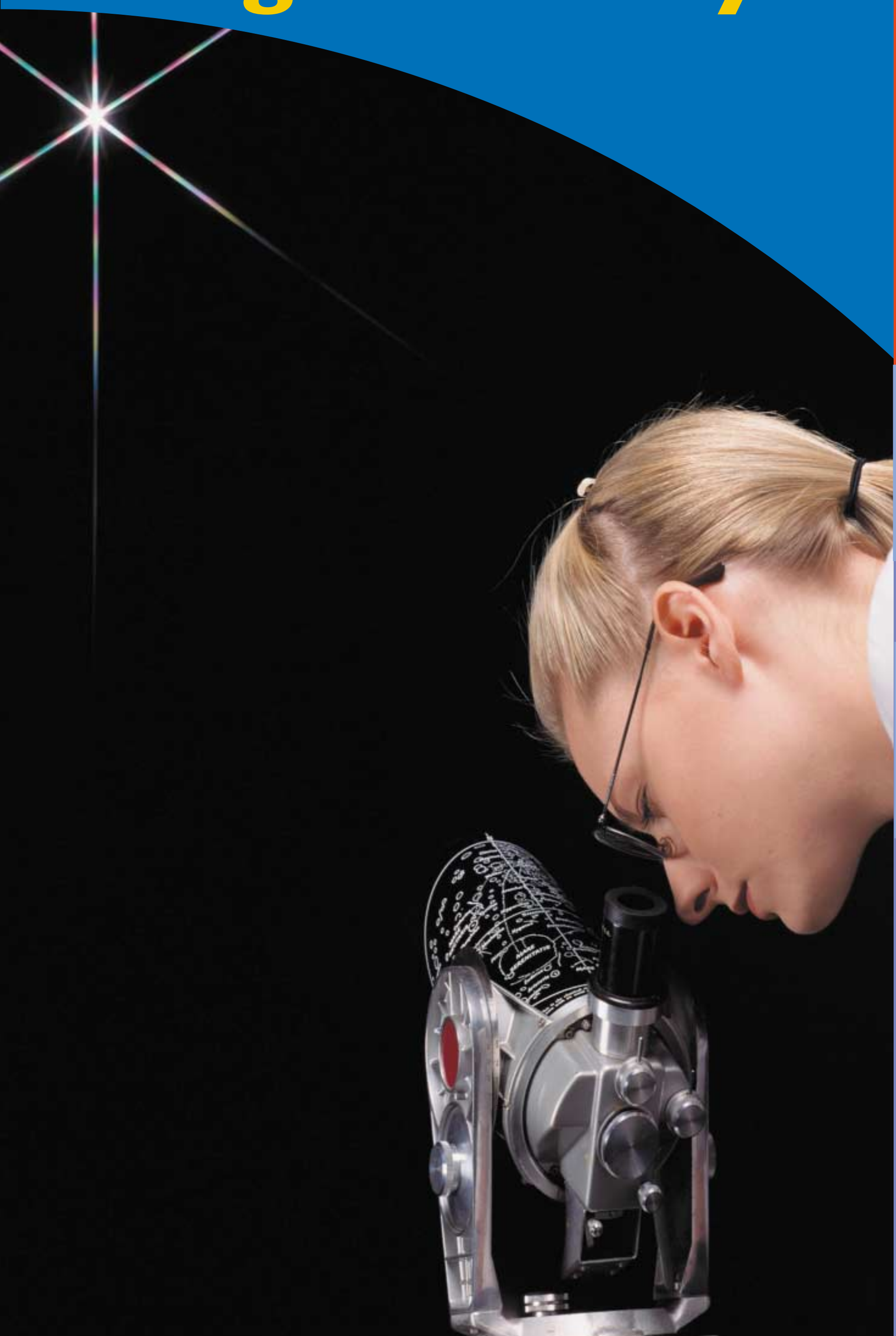
Applications of geometry and trigonometry

10



VCE coverage

Area of study

Units 3 & 4 • Geometry and trigonometry

In this chapter

Introduction

In the previous two chapters, the basic geometry and trigonometry skills and techniques were presented. In this chapter we shall examine some of the more complex applications in the real world, in particular, the application of geometry and trigonometry in navigation (for example, orienteering, sailing and so on) and surveying (location, area, contour maps and so on).

Angles

Angles are measured in degrees (°). In navigation, accuracy can be critical, so fractions of a degree are also used. For example, a cruise ship travelling 1000 kilometres on a course that is out by half a degree would miss its destination by almost 9 kilometres.

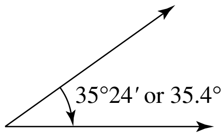
The common unit for a fraction of a degree is the minute (′), where 1 minute or $1' = \frac{1}{60}$ of a degree and 35°24′ is read as 35 degrees 24 minutes.

60 minutes = 1 degree

30 minutes = $\frac{1}{2}$ or 0.5 degree

15 minutes = $\frac{1}{4}$ or 0.25 degree

6 minutes = $\frac{1}{10}$ or 0.1 degree

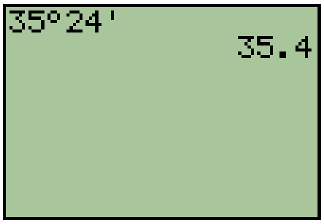


Converting angles

Converting angles from decimal form to degree–minute–second (DMS) form and vice versa can be done using in-built functions in calculators or manual techniques. Note that calculators will actually give angles in degrees, minutes and seconds, where 60 seconds equals 1 minute, as it does with time. For this course, however, we shall use only degrees and minutes to measure angles.

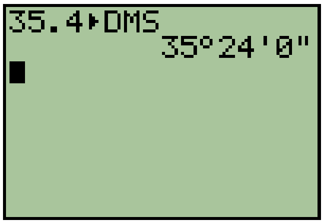
Graphics Calculator tip! Working with the ANGLE function

Entering angles in degrees and minutes



1. Set your calculator to degree mode by pressing MODE and ensuring that **Degree** is highlighted. Press 2nd [QUIT] to return to the home screen.
2. Enter the number of degrees, press 2nd [ANGLE] and choose **1: °**.
3. Enter the number of minutes, press 2nd [ANGLE] and choose **2: ′**.
4. If required, press ENTER to obtain the angle in degrees only, as a decimal.

Changing angles from decimal degrees only to degrees and minutes (and seconds)



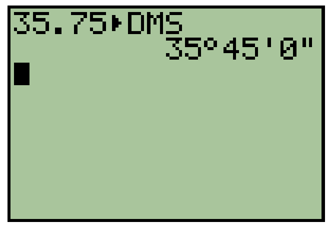
After entering the angle, press 2nd [ANGLE] and choose **4:▸ DMS** then press ENTER.

WORKED Example 1

Convert 35.75° to degrees and minutes.

THINK	WRITE/DISPLAY
1 Separate the whole and decimal portions.	35° and 0.75°
2 Multiply by 60 to convert the decimal portion to minutes.	$0.75° = 0.75 \times 60$ minutes $= 45$ minutes
3 Combine the two portions.	$35.75° = 35° + 45'$ $= 35°45'$

Alternatively:
On a graphics calculator, enter the angle 35.75, press 2nd [ANGLE], select 4:▸ DMS and press ENTER.

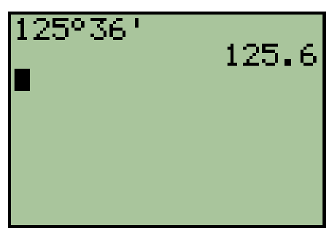


WORKED Example 2

Convert 125°36′ to its decimal form.

THINK	WRITE/DISPLAY
1 Separate the degrees and minutes portions.	125° and 36′
2 Divide by 60 to convert the minutes portion to degrees as a decimal.	$36' = \left(\frac{36}{60}\right)^{\circ}$ $= 0.6°$
3 Combine the two portions.	$125° + 0.6° = 125.6°$

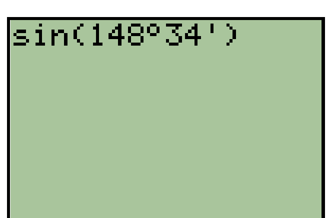
Alternatively:
On a graphics calculator, enter the angle using the ANGLE function as shown on page 460. Press ENTER to obtain the angle in degrees only.



WORKED Example 3

Find the value of the trigonometric ratio, sin 148°34′ (to 3 decimal places).

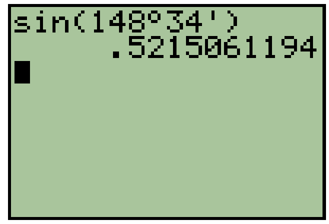
THINK	WRITE/DISPLAY
1 Enter the expression using the angle function (2nd [ANGLE]). (See page 460.)	



Continued over page

THINK | **WRITE/DISPLAY**

2. Press ENTER to calculate the result.



3. State your answer to three decimal places. sin 148°34′ = 0.522

Adding and subtracting angles

WORKED Example 4

a Add 46°37′ and 65°49′. **b** Subtract 16°55′ from 40°20′.

THINK | **WRITE**

a
1. Add the degrees and minutes portions separately.

$$\begin{array}{rr} & 46^\circ \\ + & 65^\circ \\ \hline = & 111^\circ \end{array} \qquad \begin{array}{rr} & 37' \\ + & 49' \\ \hline = & 86' \end{array}$$

2. As the minutes portion is greater than 60 minutes, convert to degrees.

$86' = 60' + 26'$
$= 1^\circ + 26'$

3. Combine the two portions.

$111^\circ + 1^\circ + 26' = 112^\circ 26'$

b
1. We cannot subtract 55′ from 20′ so change 1° to 60′ in the angle 40°20′.

$40^\circ 20' = 39^\circ + 60' + 20'$
$= 39^\circ 80'$

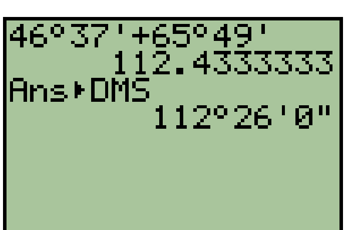
2. Subtract the degrees and minutes portions separtely.

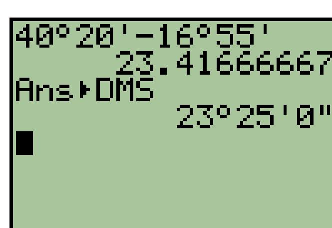
$$\begin{array}{r} 39^\circ \\ -\ 16^\circ \\ \hline = 23^\circ \end{array} \qquad \begin{array}{r} 80' \\ -\ 55' \\ \hline = 25' \end{array}$$

3. Combine the two potions.

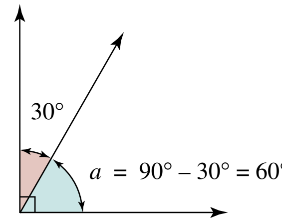
$40^\circ 20' - 16^\circ 55' = 23^\circ 25'$

For worked example 4, we can use a graphics calculator to enter the addition or subtraction of two angles. Use the **ANGLE** function and press ENTER. This gives the total in terms of degrees only, as a decimal. Change this to degrees and minutes (and seconds) by pressing 2nd **[ANGLE]** and selecting **4:▸DMS**. Then press ENTER.





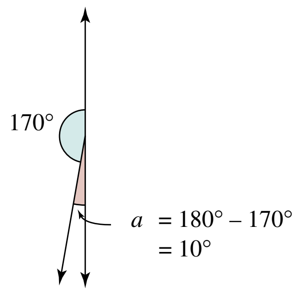
Some geometry (angle) laws

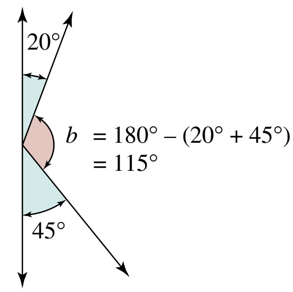


The following angle laws will be valuable when finding unknown values in the applications to be examined in this chapter. Often we will need the laws to convert given directional bearings into an angle in a triangle.

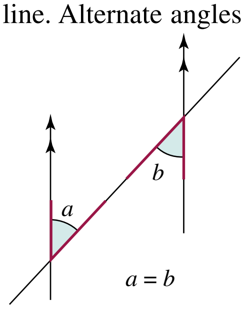
Two or more angles are *complementary* if they add up to 90°.

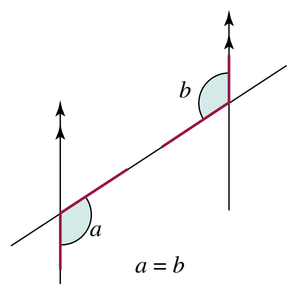
Two or more angles are *supplementary* if they add up to 180°. An angle of 180° is also called a *straight angle*.



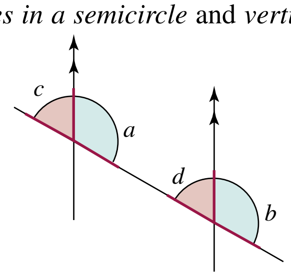


For *alternate* angles to exist we need a minimum of one pair of parallel lines and one transverse line. Alternate angles are *equal*.

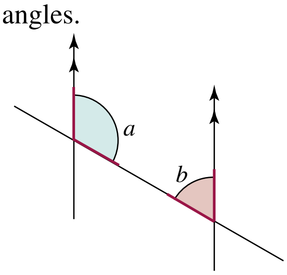




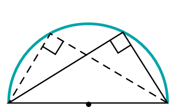
Other types of angles to be considered are *corresponding* angles, *co-interior* angles, *triangles in a semicircle* and *vertically opposite* angles.



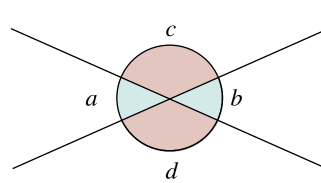
Corresponding angles are equal:
$a = b$
$c = d$



Co-interior angles are supplementary:
$a + b = 180°$



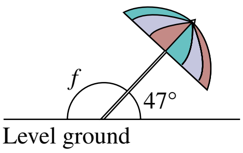
A triangle in a semicircle always gives a right-angled triangle



Vertically opposite angles are equal:
$a = b$
$c = d$

WORKED Example 5

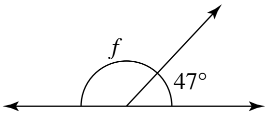
Find the value of the pronumeral, *f*, the angle a beach umbrella makes on a level beach.



THINK

1. Recognise that the horizontal line is a straight angle, or 180°.
2. To find the unknown angle, use the supplementary or straight angle law.

WRITE



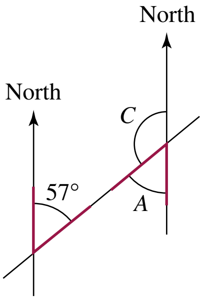
$180° = 47° + f$

$f = 180° - 47°$

$= 133°$

WORKED Example 6

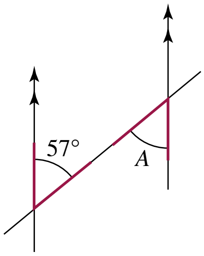
Find the value of the pronumerals *A* and *C* in the directions shown at right.



THINK

1. Recognise that the two vertical lines are parallel lines.
2. To find the unknown angle *A*, use the alternate angle law.
3. To find the unknown angle *C*, use the straight angle law. Alternatively, the co-interior angle law could be used with the same solution.

WRITE



$A = 57°$

$180° = 57° + C$

$C = 180° - 57°$

$= 123°$

remember

1. 60 minutes = 1 degree
2. Symbols: Degree (°) Minutes (′)
3. Complementary angles add up to 90°.
4. Supplementary angles and co-interior angles add up to 180°.
5. Alternate angles, corresponding angles and vertically opposite angles are equal.

EXERCISE 10A Angles

WORKED Example 1

SkillSHEET 10.1

1 Convert the following angles to degrees and minutes.

a 43.5° b 12.75° c 28.3° d 106.27°

e 273.872° f $56\frac{1}{3}$°

WORKED Example 2

2 Convert the following angles to their decimal form (to 2 decimal places).

a 40°15′ b 122°20′ c 82°6′ d 16°49′

e 247°30′ f 76°50′

WORKED Example 3

3 On your calculator, find the values of the following trigonometric ratios to three decimal places.

a sin 40°15′ b cos 122°20′ c tan 82°6′

d cos 16°49′ e sin 47°30′ f tan 76°50′

g sin 32°41′ h tan 27°28′

WORKED Example 4

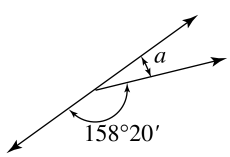
4 Add and then subtract the pairs of angles.

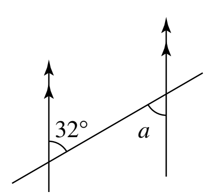
a 40°15′, 28°5′ b 122°20′, 79°35′ c 82°6′, 100°55′ d 16°49′, 40°15′

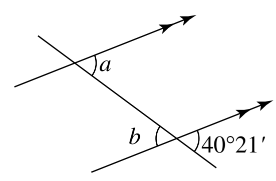
e 247°30′, 140°32′ f 76°50′, 76°20′ g 346°37′, 176°52′ h 212°33′, 6°33′

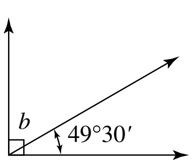
WORKED Example 5, 6

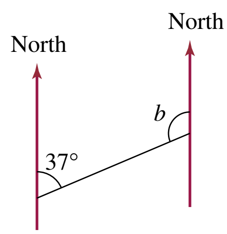
5 Find the values of the pronumerals.

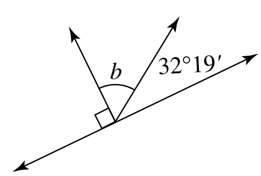
a


b


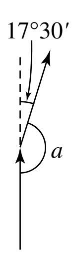
c


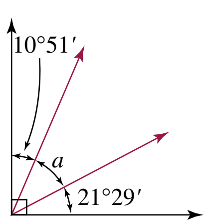
d


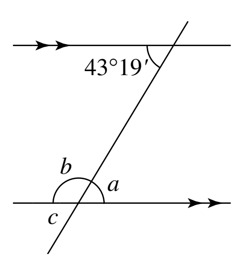
e


f


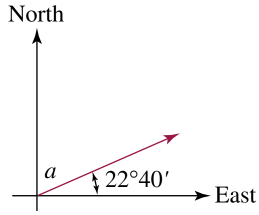
6 Find the values of the pronumerals.

a


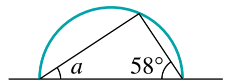
b


c


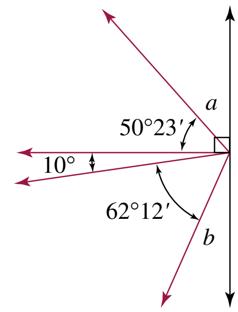
d



e

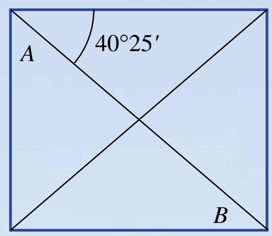


f



7 **multiple choice**

A plan for a rectangular farm gate is shown below.



a The value of angle *A* is:

A 40°25′ **B** 49.417° **C** 49°35′ **D** 50° **E** 90°

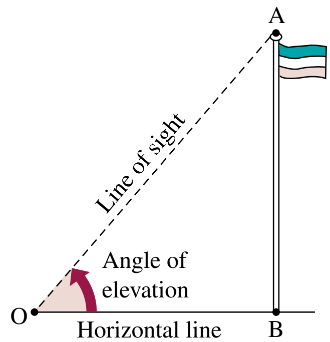
b The value of angle *B* is:

A 40°25′ **B** 49°35′ **C** 49.538° **D** 50° **E** 139°35′

Angles of elevation and depression

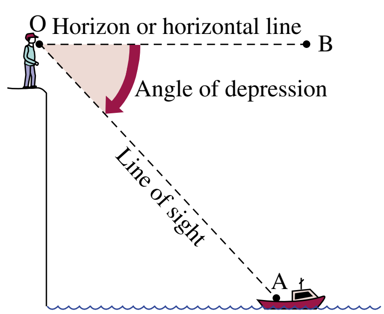
One method for locating an object in the real world is by its position above or below a horizontal plane or reference line.

The *angle of elevation* is the angle *above* the horizon or horizontal line.



Looking up at the top of the flagpole from position O, the angle of elevation, ∠AOB, is the angle between the horizontal line OB and the line of sight OA.

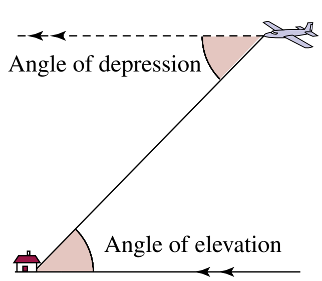
The *angle of depression* is the angle *below* the horizon or horizontal line.



Looking down at the boat from position O, the angle of depression, ∠AOB, is the angle between the horizontal line OB and the line of sight OA.

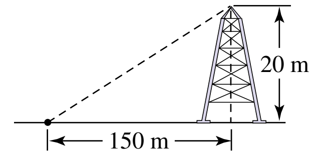
Angles of elevation and depression are in a *vertical plane*.

We can see from the diagram below that the angle of depression given from one location can give us the angle of elevation from the other position using the alternate angle law.



WORKED Example 7

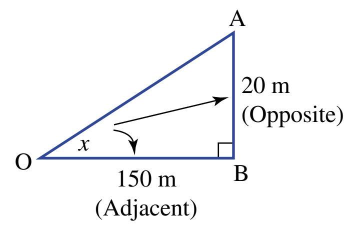
Find the angle of elevation (in degrees and minutes) of the tower measured from the road as given in the diagram.



THINK

1. The angle of elevation is ∠AOB. Use △AOB and trigonometry to solve the problem.
2. The problem requires the tangent ratio. Substitute the values and simplify.
3. Evaluate x and convert to degrees and minutes.
4. Write the answer in correct units.

WRITE



$$\tan\,\theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$= \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan x = \frac{20}{150}$$

$$\tan x = 0.133\,33$$

$$x = \tan^{-1}(0.133333)$$

$$x = 7.5946° = 7°36'$$

From the road the angle of elevation to the tower is 7°36′.

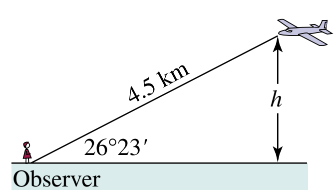
WORKED Example 8

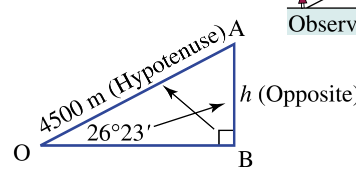
Find the altitude of a plane (to the nearest metre) if the plane is sighted 4.5 km directly away from an observer who measures its angle of elevation as 26°23′.

THINK

1. Draw a suitable diagram. Change distance to metres.
2. Use the sine ratio and simplify.
3. Evaluate.
4. Write the answer in correct units.

WRITE





$$\sin\,\theta = \frac{\text{length of opposite side}}{\text{length of hypotenuse side}}$$

$$= \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 26°23' = \frac{h}{4500}$$

$$h = 4500 \sin 26°23'$$

$$h = 1999.6857$$

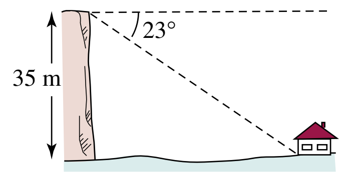
The plane is flying at an altitude of 2000 m.

WORKED Example 9

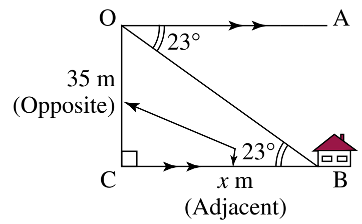
The angle of depression from the top of a 35-metre cliff to a house at the bottom is 23°. How far from the base of the cliff is the house (to the nearest metre)?

THINK | **WRITE**

1. Draw a suitable diagram.



2. Angle of depression is ∠AOB. Use the alternate angle law to give the angle of elevation ∠CBO.



3. Use the tangent ratio. Substitute into the formula and evaluate.

$$\tan\,\theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$= \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 23° = \frac{35}{x}$$

$$\frac{1}{\tan 23°} = \frac{x}{35}$$

$$x = \frac{35}{\tan 23°}$$

$$x = 82.4548\ldots$$

4. Write the answer in correct units.

The distance from the house to the base of the cliff is 82 metres.

remember

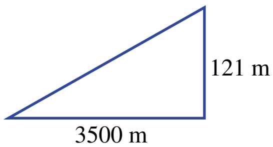
1. The angle of elevation is above the horizon or horizontal line.
2. The angle of depression is below the horizon or horizontal line.
3. These angles are in a vertical plane.
4. $\sin\,\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos\,\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\,\theta = \frac{\text{opposite}}{\text{adjacent}}$

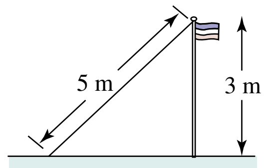
EXERCISE 10B Angles of elevation and depression

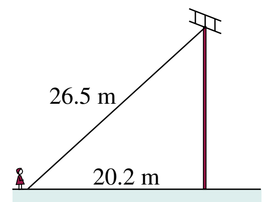
SkillSHEET 10.2

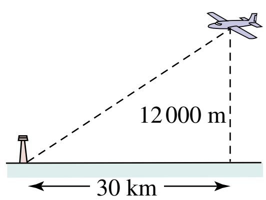
WORKED Example 7

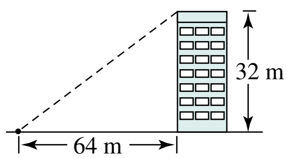
1 Find the angle of elevation (in degrees and minutes) in the following situations.

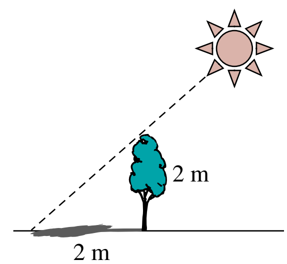
a


b


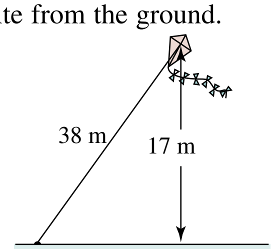
c


d


e


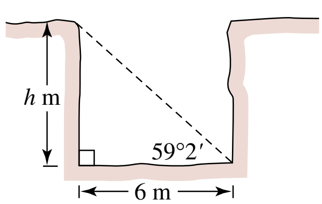
f


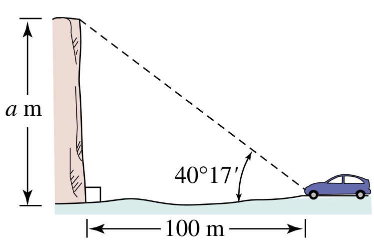
2 A kite is flying 17 metres above the ground on a taut line that is 38 metres long. Find the angle of elevation of the kite from the ground.

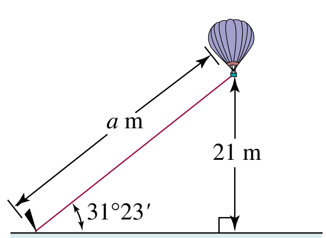


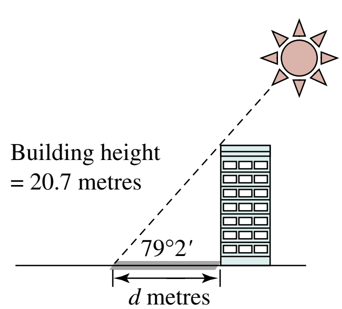
WORKED Example 8

3 Find the values of the pronumerals (to the nearest metre).

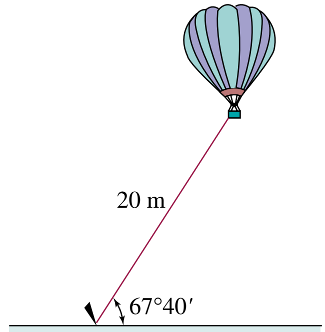
a


b


c


d


4 A taut rope is used to tether a hot-air balloon. If the angle of elevation of the rope is 67°40′, and the rope is 20 metres long, how far off the ground is the balloon?

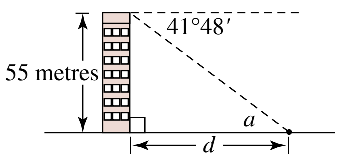


5 The angle of elevation of the sun at a particular time of the day was 49°. What is the length of a shadow cast by a 30-metre tall tower?

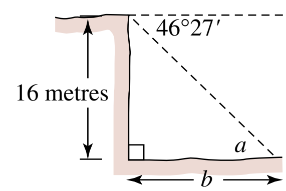
WORKED Example 9

6 Find the values of these pronumerals (in degrees and minutes or nearest metre).

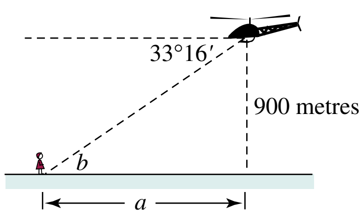
a



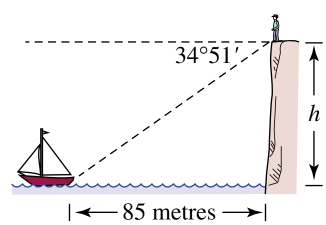
b



c

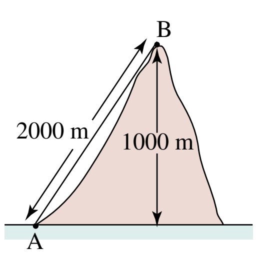


d

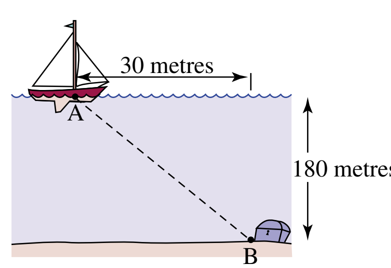


7 Find the angle of elevation or depression from observer position A to object B in each situation shown below, to the nearest degrees and minutes. State clearly whether it is an angle of depression or elevation.

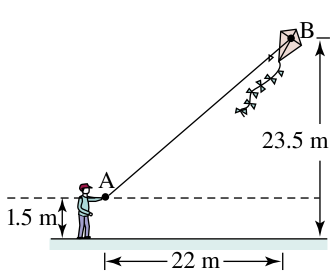
a



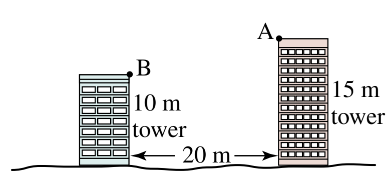
b



c



d



8 A hole has a diameter of 4 metres and is 3.5 metres deep. What is the angle of depression from the top of the hole to the bottom opposite side of the hole?

9 **multiple choice**

The angle of elevation of the top of a tree from a point 15.2 m from the base of the tree is 52°11′. The height of the tree is closest to:

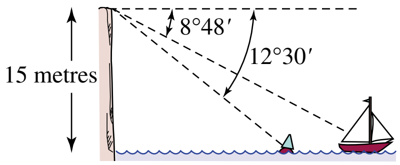
A 12 m **B** 15 m **C** 19 m **D** 20 m **E** 25 m

10 **multiple choice**

A supporting wire for a 16 m high radio tower is 23.3 m long and is attached at ground level and to the top of the tower. The angle of depression of the wire from the top of the tower is:

A 34°29′ **B** 43°22′ **C** 46°38′ **D** 55°29′ **E** 58°22′

11 The angle of depression to a buoy from the top of a 15-metre cliff is 12°30′. A boat is observed to be directly behind but with an angle of depression of 8°48′.



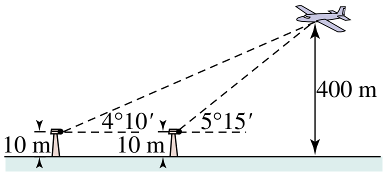
Find (to the nearest metre):

a the distance to the buoy from the base of the cliff
b the distance between the boat and the buoy.

12 Two buildings are 50 metres apart. Building *A* is 110 metres high. Building B is 40 metres high.

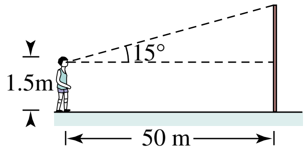
a Find the angle of elevation from the bottom of building A to the top of building B.
b Find the angle of depression from the top of building A to the bottom of building B.
c Find the angle of depression from the top of building B to the bottom of building A.

13 Watchers in two 10-metre observation towers each spot an aircraft at an altitude of 400 metres. The angles of elevation from the two towers are shown in the diagram. (Assume all three objects are in a direct line).



a What is the horizontal distance between the nearest tower and the aircraft (to the nearest 10 metres)?
b How far apart are the two towers from each other (to the nearest 100 metres)?

14 A boy standing 1.5 metres tall measures the angle of elevation of the goalpost using a clinometer.



a If the angle was 15° when measured 50 metres from the base of the goalpost, how tall is the goalpost?
b If the angle of elevation to the top of the goalpost is now 55°30′, how far is the boy from the base of the goalpost?
c The angle of elevation is measured at ground level and is found to be 45°. Find the distance from the base of the goalpost to where the measurement was made.
d The result in part **c** is the same as the height of the goalpost. Explain why.

15 A plane goes from an altitude of 30 000 metres to 10 000 metres over a horizontal distance of 200 kilometres. What was the angle of depression of its descent?

Bearings

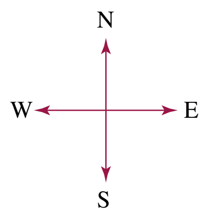
Bearings are used to locate the position of objects or the direction of a journey on a two-dimensional horizontal plane. Bearings or directions are straight lines from one point to another. A compass rose should be drawn centred on the point from where the bearing measurement is taken.

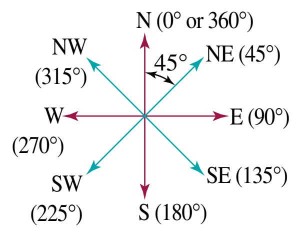
There are three main ways of specifying bearings or direction:

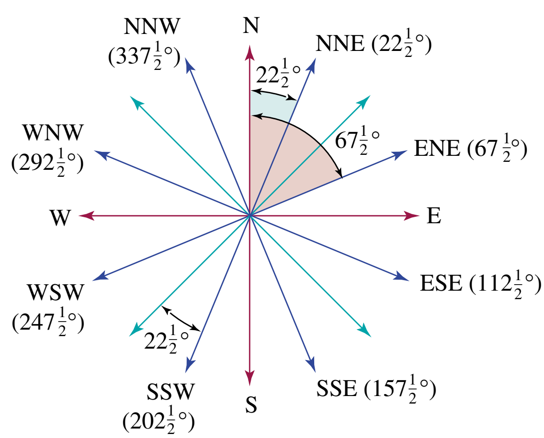
1. standard compass bearings (for example, N, SW, NE)
2. other compass bearings (for example, N10°W, S30°E, N45°37′E)
3. true bearings (for example, 100°T, 297°T, 045°T, 056°T)

Standard compass bearings

There are 16 main standard bearings as shown in the diagrams below. The N, S, E and W standard bearings are called *cardinal points*.

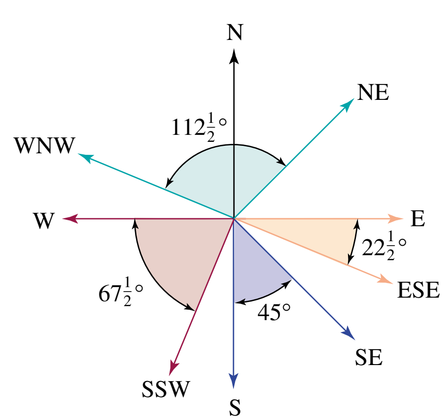






It is important to consider the angles between any two bearings. For example, the angles from north (N) to all 16 bearings are shown in brackets in the diagrams above.

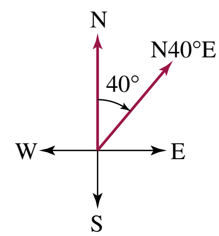
It can be seen that the angle between two adjacent bearings is $22\frac{1}{2}$°. Some other angles that will need to be considered are shown at right.



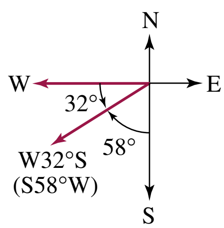
Other compass bearings

Often the direction required is not one of the 16 standard bearings. To specify other bearings the following approach is taken.

1. Start from north (N) or south (S).
2. Turn through the angle specified towards east (E) or west (W).

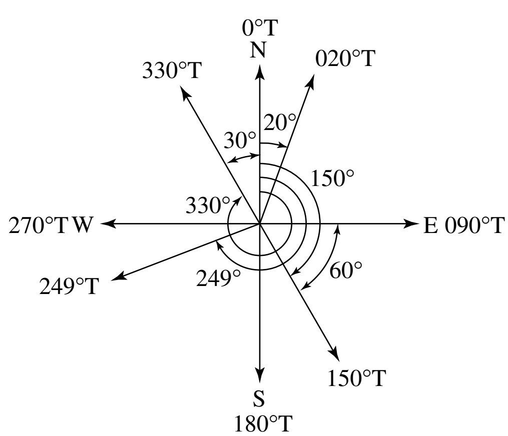


Sometimes the direction may be specified unconventionally, for example, starting from east or west as given by the example W32°S. This bearing is equivalent to S58°W.



True bearings

True bearings is another method for specifying directions and is commonly used in navigation.

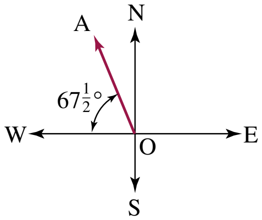


To specify true bearings, first consider the following:

1. the angle is measured from north
2. the angle is measured in a clockwise direction to the bearing line
3. the angle of rotation may take any value from 0° to 360°
4. the symbol T is used to indicate it is a true bearing, for example, 125°T, 270°T
5. for bearings less than 100°T, use three digits with the first digit being a zero to indicate it is a bearing, for example, 045°T, 078°T.

WORKED Example 10

Specify the direction in the figure at right as:
a **a standard compass bearing**
b **a compass bearing**
c **a true bearing.**



THINK

a 1 Find the angle between the bearing line and north, that is, ∠AON.

2 Since the angle is $22\frac{1}{2}°$, the bearing is a standard bearing. Refer to the standard bearing diagram.

b The bearing lies towards the north and the west. The angle between north and the bearing line is $22\frac{1}{2}°$.

c Find the angle from north to the bearing line in a clockwise direction. The bearing of west is 270°T.

WRITE

a $\angle AON = 22\frac{1}{2}°$

The standard bearing is NNW.

b The compass bearing is N$22\frac{1}{2}°$W.

c Angle required $= 270° + 67\frac{1}{2}°$
$= 337\frac{1}{2}°$

The true bearing is $337\frac{1}{2}°$T.

WORKED Example 11

Draw a suitable diagram to represent the following directions.
a **S17°E** **b** **252°T**

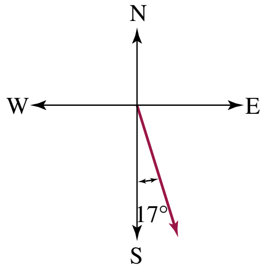
THINK

a Draw the 4 main standard bearings. A compass bearing of S17°E means start from south; turn 17° towards east. Draw a bearing line at 17°. Mark in an angle of 17°.

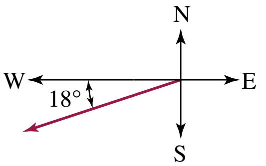
b A true bearing of 252°T is more than 180° and less than 270°, so the direction lies between south and west. Find the difference between the bearing and west (or south). Draw the 4 main standard bearings and add the bearing line. Add the angle from west (or south).

WRITE

a



b Difference from west $= 270° - 252°$
$= 18°$



WORKED Example 12

Convert:

a **the true bearing, 137°T, to a compass bearing**

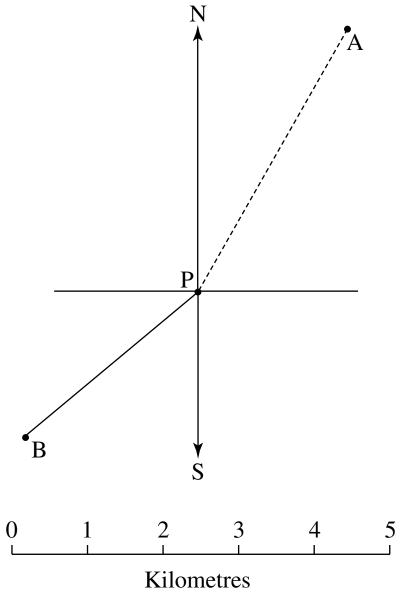
b **the compass bearing, N25°W, to a true bearing.**

THINK	WRITE
a 1 The true bearing 137°T means the direction is between south and east. Find the angle from south to the bearing line.	**a** Angle required = 180° − 137° = 43°
2 Write the compass bearing.	Compass bearing is S43°E
b 1 Find the angle between the bearing line and west.	**b** Angle from west = 90° − 25° = 65°
2 Find the angle from north to the bearing line in a clockwise direction. The angle from north clockwise to west is 270°.	Angle required = 270° + 65° = 335°
3 Write the true bearing.	True bearing is 335°T.

WORKED Example 13

Use your protractor and ruler to specify the locations of points A and B from location P. State the directions as true bearings and as compass bearings and write the distances to the nearest kilometre.

THINK	WRITE
1 Find ∠NPA and write as a true bearing and as a compass bearing.	∠NPA = 30° True bearing is 030° T Compass bearing is N30°E
2 Measure PA and convert the scale length to kilometres.	PA = 4 cm PA represents 4 km
3 Specify the location of A.	A is 4 km on a bearing of 030° T or N30°E from P.
4 Repeat steps 1–3 above for location B, this time with reference to south.	∠SPB = 50° True bearing is 230°T. Compass bearing is S50°W. PB = 3 cm which represents 3 km. B is 3 km on a bearing of 230°T or S50°W from P.



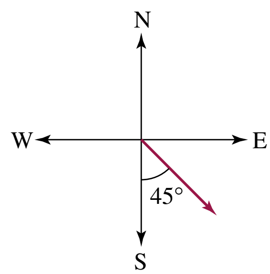
remember

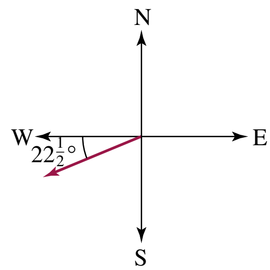
1. Draw a compass rose at the point from where the direction is measured.
2. The 3 types of bearings are:
   (i) standard compass bearings (for example N, SW, NE)
   (ii) other compass bearings (for example, N10°W, S30°E, N45°37′E)
   (iii) true bearings (for example, 100°T, 297°T, 045°T, 056°T).

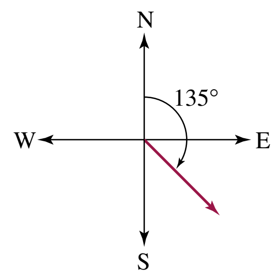
EXERCISE 10C Bearings

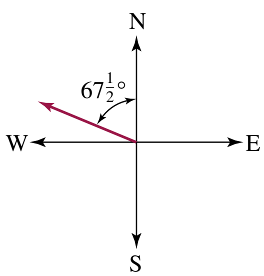
WORKED Example 10a

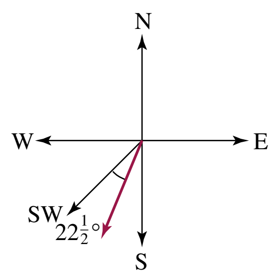
1 Specify the following directions as standard compass bearings.

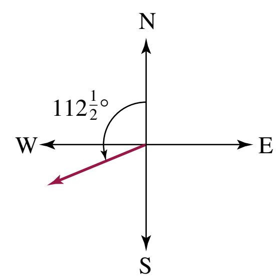
a


b


c


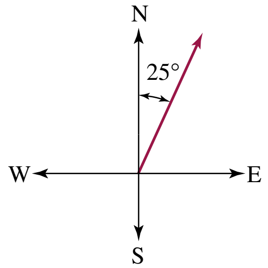
d


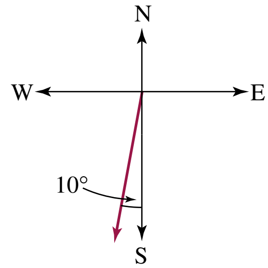
e


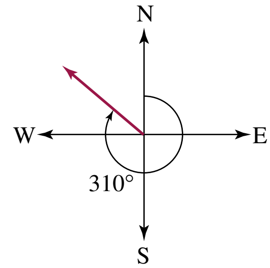
f


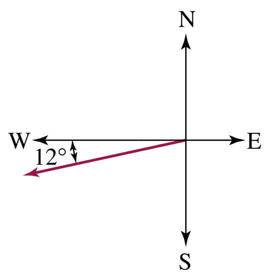
WORKED Example 10b, c

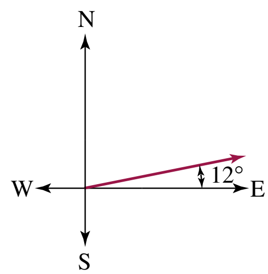
2 Specify the following directions as compass bearings and true bearings.

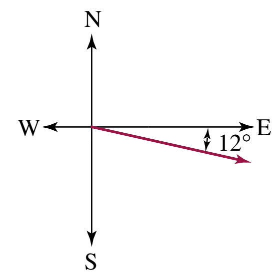
a


b


c


d


e


f


Cabri Geometry — True bearings

WORKED Example 11

3 Draw suitable diagrams to represent the following directions.

a N45°E b S20°W c 028°T d 106°T
e 270°T f S60°E g 080°T h N70°W

WORKED Example 12a

4 Convert the following true bearings to compass bearings.

a 040°T b $022\frac{1}{2}$°T c 180°T d 350°T
e 147°T f 67°30′T g 120°T h 135°T

WORKED Example 12b

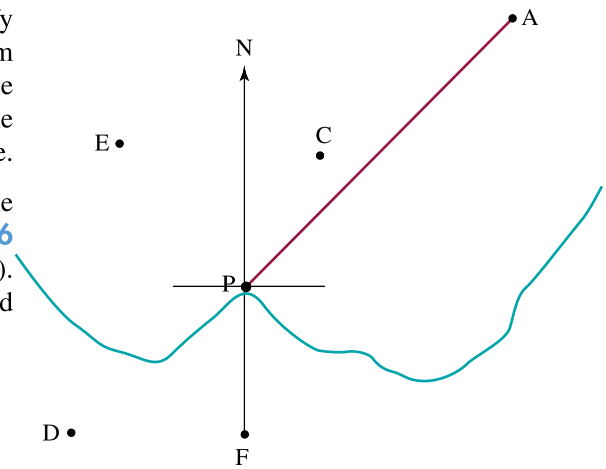
5 Convert the following compass bearings to true bearings.

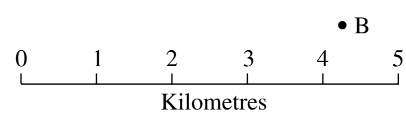
a N45°W b S$40\frac{1}{2}$°W c S d S35°E
e N47°E f S67°30′W g NNW h S5°E

WORKED Example 13

6 Use your protractor and ruler to specify the location of each of the points from location P. State the directions as true bearings and compass bearings, and the distance to the nearest half of a kilometre.

7 Now find the location of each of the points in the diagram from question 6 from location B (as compass bearings). Also include the location from B to P and compare it to the direction from P to B.

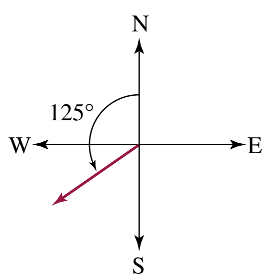




8 **multiple choice**

The direction shown in the diagram is:

A N125°W
B S35°W
C WSW
D 235°T
E 125°T



9 **multiple choice**

An unknown direction — given that a second direction, 335°T, makes a straight angle with it — is:

A S15°E B SSE C S25°E D 235°T E 135°T

10 **multiple choice**

The direction of a boat trip from Sydney directly to Auckland was S20°E. The direction of the return trip would be:

A S20°W B NNW C N20°E D 235°T E 340°T

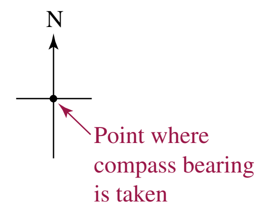
11 **multiple choice**

The direction of the first leg of a hiking trip was S40°W. For the second leg, the hikers turn 40° right. The new direction for the second leg of the hike is:

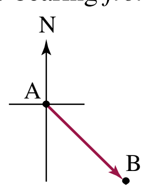
A W B S C S80°W D N40°E E N80°W

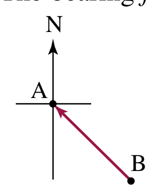
Navigation and specification of locations

In most cases when you are asked to solve problems, a carefully drawn sketch of the situation will be given. When a problem is described in words only, very careful sketches of the situation are required. Furthermore, these sketches of the situation need to be converted to triangles with angles and lengths of sides added. This is so that Pythagoras' theorem, trigonometric ratios, areas of triangles, similarity and sine or cosine rules may be used.



Hints:
1. Carefully follow given instructions.
2. Always draw the compass rose at the starting point of the direction requested.
3. Key words are *from* and *to*. For example:
   The bearing *from* A *to* B is very different from The bearing *from* B *to* A.



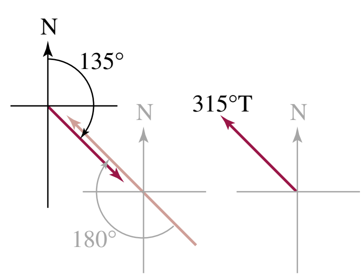


4. When you are asked to determine the direction to return directly back to an initial starting point, it is a 180° rotation or difference. For example, to return directly back after heading north, we need to change the direction to head south.

   Other examples are:

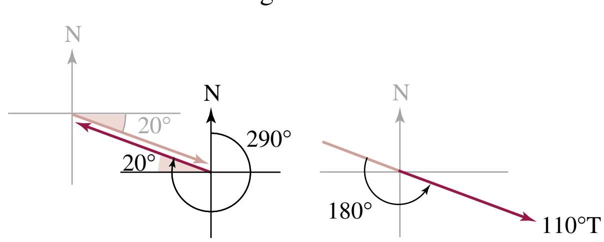
   Returning directly back after heading 135°T

   $$\text{New heading} = 135° + 180° = 315°\text{T}$$



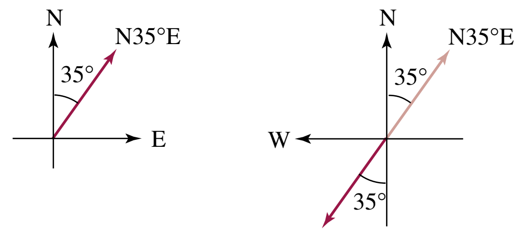
   Returning directly back after heading 290°T

   $$\text{New heading} = 290° - 180° = 110°\text{T}$$



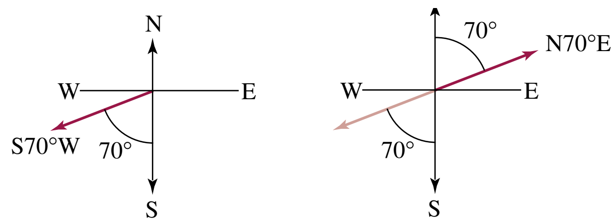
Returning directly back after heading N35°E

New heading = N35°E + 180° = S35°W



Returning directly back after heading S70°W

New heading = N70°E



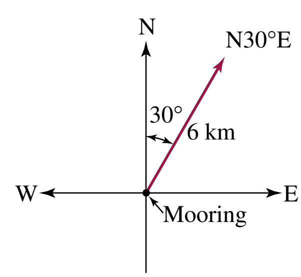
or simply use the opposite compass direction. North becomes south and east becomes west and vice versa.

WORKED Example 14

A ship leaves port, heading N30°E for 6 kilometres as shown.

a How far north or south is the ship from its starting point (to 1 decimal place)?

b How far east or west is the ship from its starting point (to 1 decimal place)?



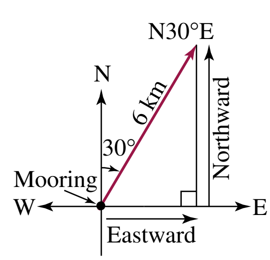
THINK

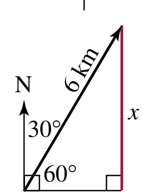
a 1 Draw a diagram of the journey and indicate or superimpose a suitable triangle.

2 Identify the side of the triangle to be found. Redraw a simple triangle with most important information provided. Use the bearing given to establish the angle in the triangle, that is, use the complementary angle law.

WRITE

a

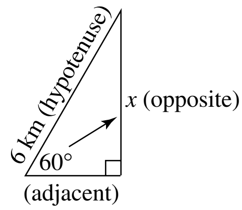




$90° - 30° = 60°$

THINK | **WRITE**

3. Identify the need to use a trigonometric ratio, namely the sine ratio, to find the distance north.



4. Substitute and evaluate.

$$\sin\theta = \frac{\text{length of opposite side}}{\text{length of hypotenuse side}}$$
$$= \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 60° = \frac{x}{6}$$
$$x = 6 \times \sin 60°$$
$$x = 6 \times 0.8660$$
$$= 5.196$$

5. State the answer to the required number of decimal places.

The ship is 5.2 km north of its starting point.

b 1. Use the same approach as in part **a**. This time the trigonometric ratio is cosine to find the distance east, using the same angle evaluated.

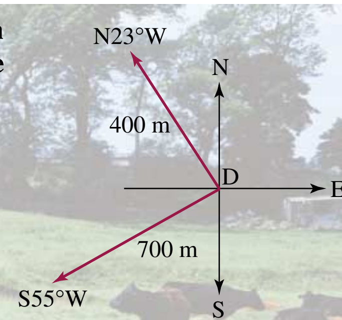
b
$$\cos\theta = \frac{\text{Length of adjacent side}}{\text{Length of hypotenuse side}}$$
$$= \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 60° = \frac{y}{6}$$
$$y = 6 \times \cos 60°$$
$$y = 6 \times 0.5$$
$$= 3.0$$

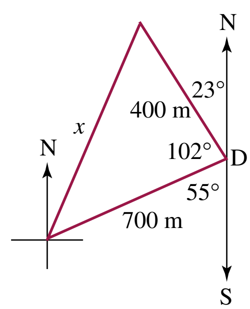
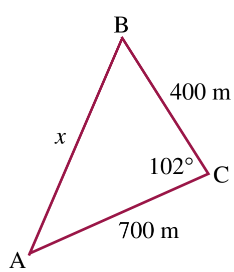
2. Answer in correct units and to the required level of accuracy.

The ship is 3.0 km east of its starting point.

WORKED Example 15

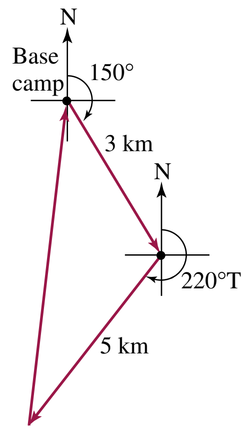
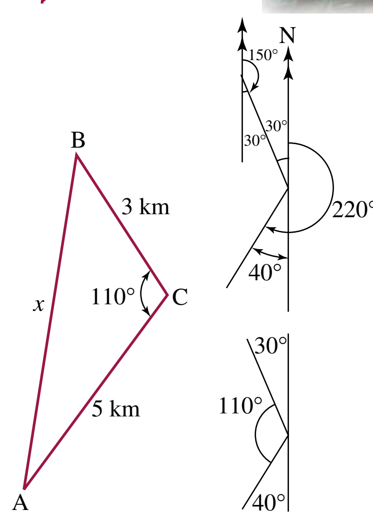
A triangular paddock has two complete fences. From location D, one fence line is on a bearing of N23°W for 400 metres. The other fence line is S55°W for 700 metres.
Find the length of fencing (to the nearest metre) required to complete the enclosure of the triangular paddock.



THINK	WRITE
1 Identify the side of the triangle to be found. Redraw a simple triangle with the most important information provided.	
2 Use the bearings given to establish the angle in the triangle, that is, use the supplementary angle law.	
3 Identify the need to use the cosine rule, as two sides and the included angle are given.	 $a = 400$ m $b = 700$ m $C = 102°$ $c = x$ m
4 Substitute and evaluate.	$c^2 = a^2 + b^2 - 2ab \times \cos C$ $x^2 = 400^2 + 700^2 - 2 \times 400 \times 700 \times \cos 102°$ $x^2 = 650\,000 - 560\,000 \times -0.207\,91$ $x^2 = 766\,430.55$ $x = \sqrt{766\,430.55}$ $= 875.46$
5 Answer in correct units and to the required level of accuracy.	The new fence section is to be 875 metres long.

WORKED Example 16

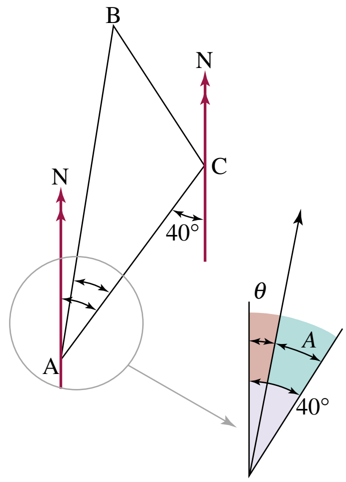
Soldiers on a reconnaissance set off on a return journey from their base camp. The journey consists of three legs. The first leg is on a bearing of 150°T for 3 km; the second is on a bearing of 220°T for 5 km.
Find the direction and distance of the third leg by which the group returns to its base camp.

THINK	WRITE
1 Draw a diagram of the journey and indicate or superimpose a suitable triangle.	
2 Identify the side of the triangle to be found. Redraw a simple triangle with most important information provided.	
3 Identify that the problem requires the use of the cosine rule, as you are given two sides and the angle in between.	$a = 3$ km $b = 5$ km $C = 110°$ $c = x$ km
4 Substitute the known values into the cosine rule and evaluate.	$c^2 = a^2 + b^2 - 2ab \times \cos C$ $x^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos 110°$ $x^2 = 44.260\,604$ $x = \sqrt{44.260\,604}$ $= 6.65$

Continued over page

THINK / **WRITE**

5. For direction, we need to find the angle between the direction of the second and third legs using the sine or cosine rules.



$a = 3 \quad b = 5 \quad c = 6.65 \text{ or } \sqrt{44.260\,604}$

6. Substitute the known values into the rearranged cosine rule.
Note: Use the most accurate form of the length of side *c*.

$$\cos A = \frac{b^2 + c^2 - a^2}{2 \times b \times c}$$

$$\cos A = \frac{5^2 + 44.260\,604 - 3^2}{2 \times 5 \times \sqrt{44.260\,604}}$$

$$\cos A = 0.9058$$

$$A = 25.07° = 25°4'$$

7. Calculate the angle of the turn from the north bearing.

$$\theta = 40° - 25°4' = 14°56'$$

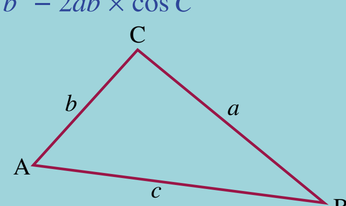
Bearing is N14°56′E

8. Write the answer in correct units and to the required level of accuracy.

The distance covered in the final leg is 6.65 km on a bearing of N14°56′E.

remember

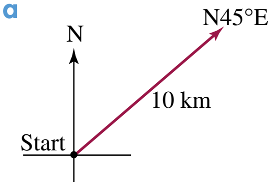
1. The bearings are in a horizontal plane.
2. Bearings are directions, not angles. From bearings important angles in a triangle can be found.
3. In most cases you will need to consider laws such as the alternate, complementary and supplementary angle laws.
4. Carefully read the specification of direction, especially for the words *from* and *to*.
5. Cosine rule: $c^2 = a^2 + b^2 - 2ab \times \cos C$

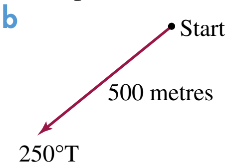


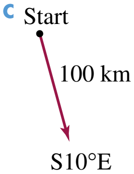
EXERCISE 10D Navigation and specification of locations

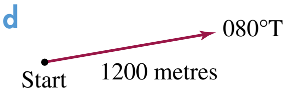
WORKED Example 14

1 For the following, find how far north or south and east or west the end point is from the starting point (to 1 decimal place).

a


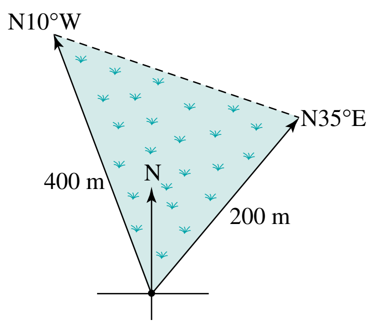
b


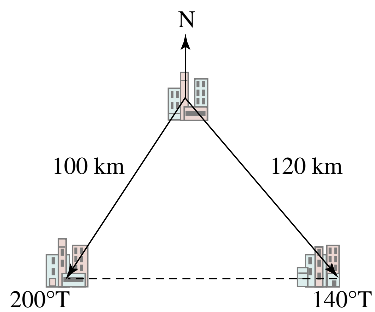
c


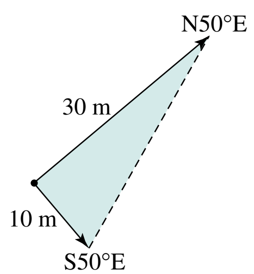
d


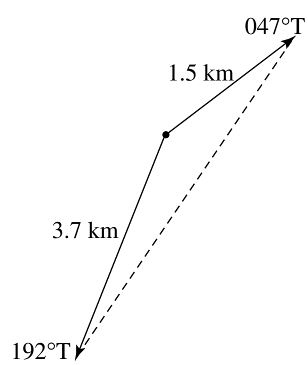
WORKED Example 15

2 Find the length of the unknown side of each of the shapes given (to the nearest unit).

a


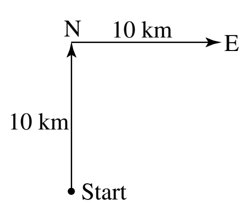
b


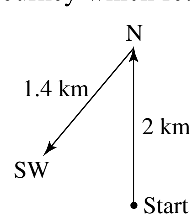
c


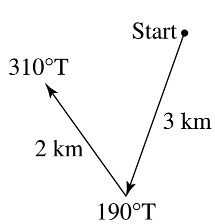
d


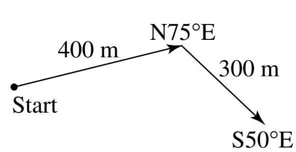
WORKED Example 16

3 In each of the diagrams below, the first two legs of a journey are shown. Find the direction and distance of the third leg of the journey which returns to the start.

a


b


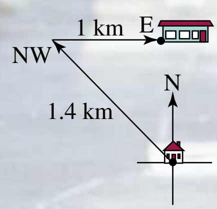
c


d


4 A student walks from home to school heading NW for 1.4 km and then east for 1 km.

a How far is the school from home (to the nearest metre)?

b What is the direction from the school to the student's home (in degrees and minutes)?



5 Draw a diagram to represent each of the directions specified below and give the direction required *to return* to the starting point:

a from A to B on a bearing of N40°W

b from C to E on a bearing of 157°T

c to F from G on a bearing of S35°W

d from B to A on a bearing of 237°T.

6 **multiple choice**

A boat sails from port A for 15 km on a bearing of N15°E before turning and sailing for 21 km in a direction of S75°E to port B.

a The distance between ports A and B is closest to:

A 15 km B 18 km C 21 km D 26 km E 36 km

b The bearing of port B from port A is:

A N69°28′E B N54°28′E C N20°32′E D S20°32′E E 054°28′T

7 **multiple choice**

In a pigeon race the birds start from the same place. In one race, pigeon A flew 35 km on a bearing of N65°W to get home, while pigeon B flew 26 km on a bearing of 174°T.

a The distance between the two pigeons' homes is closest to:

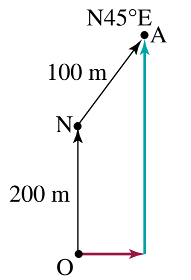
A 13 km B 18 km C 44 km D 50 km E 53 km

b The bearing of pigeon A's home from pigeon B's home is closest to:

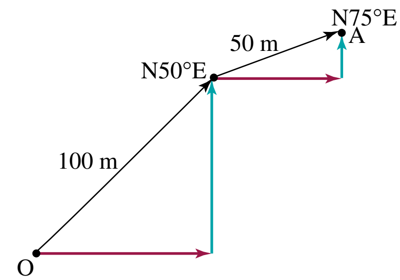
A N28°16′W B N34°16′W C N40°16′W D 208°16′T E 220°16′T

8 For each of the following, find how far north/south and east/west position A is from position O.

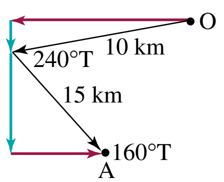
a



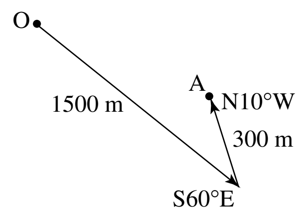
b



c



d



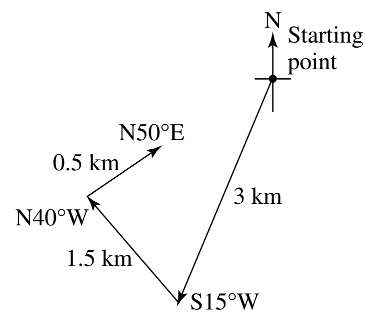
9 For the hiking trip shown in the diagram, find (to the nearest metre):

a how far south the hiker is from the starting point

b how far west the hiker is from the starting point

c the distance from the starting point

d the direction of the final leg to return to the starting point.



10 Captain Cook sailed from Cook Island on a bearing of N10°E for 100 km. He then changed direction and sailed for a further 50 km on a bearing of SE to reach a deserted island.

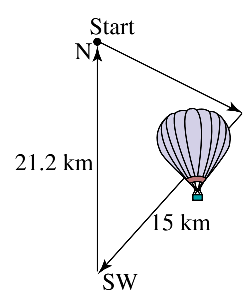
a How far from Cook Island is Captain Cook's ship (to the nearest kilometre)?

b Which direction would have been the most direct route from Cook Island to the deserted island (in degrees and minutes)?

c How much shorter would the trip have been using the direct route?

d What would have been the bearing of the shortest route?

11 A journey by a hot-air balloon is shown. The balloonist did not initially record the first leg of the journey. Find the direction and distance for the first leg of the balloonist's journey.

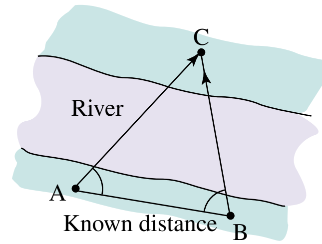


12 A golfer is teeing off on the 1st hole. The distance and direction to the green is 450 metres on a bearing of 190°T. If the tee shot of the player was 210 metres on a bearing of 220°T, how far away from the green is the ball and in what direction should she aim to land the ball on the green with her second shot? (Give the distance to the nearest metre and the direction to the nearest degree.)

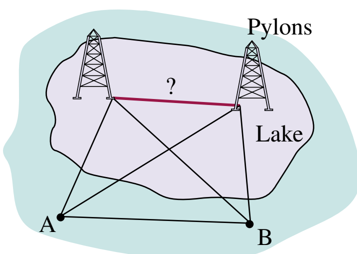
WorkSHEET 10.1

Triangulation — cosine and sine rules

In many situations, certain geographical or topographical features are not accessible to a survey. To find important locations or features, triangulation is used. This technique requires the coordination of bearings from two known locations, for example, fire spotting towers, to a third inaccessible location, the fire (see worked example 17).

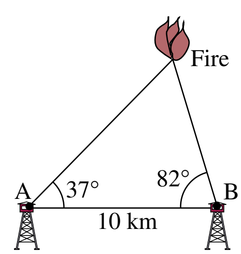


1. Triangulation should be used when:
  (a) the distance between two locations is given and
  (b) the direction from each of these two locations to the third inaccessible location is known.
2. For triangulation:
  (a) the sine rule is used to find distances from the known locations to the inaccessible one
  (b) the cosine rule may be used occasionally for locating a fourth inaccessible location.



WORKED Example 17

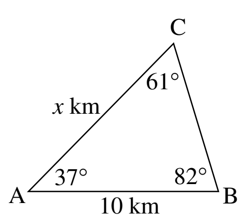
How far (to 1 decimal place) is the fire from Tower A?



THINK

1. Draw a triangle and identify it as a non-right-angled triangle with a given length and two known angles. Determine the value of the third angle and label appropriately for the sine rule.
2. Substitute into the formula and evaluate.
3. Write the answer in the correct units and to the required level of accuracy.

WRITE



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where

$c = 10$ km $C = 180° - (37° + 82°) = 61°$

$b = x$ $B = 82°$

$$\frac{x}{\sin 82°} = \frac{10}{\sin 61°}$$

$$x = \frac{10 \times \sin 82°}{\sin 61°}$$

$$x = 11.3$$

The fire is 11.3 km from Tower A.

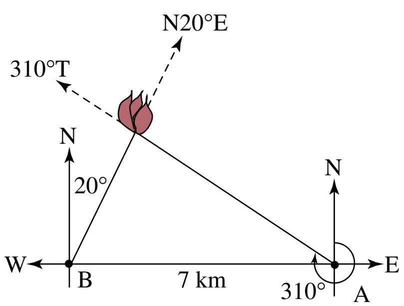
WORKED Example 18

Two fire-spotting towers are 7 kilometres apart on an east–west line. From Tower A a fire is seen on a bearing of 310°T. From Tower B the same fire is spotted on a bearing of N20°E. Which tower is closest to the fire and how far is that tower from the fire?

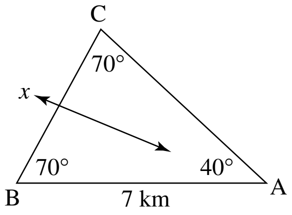
THINK

WRITE

1. Draw a suitable sketch of the situation described. It is necessary to determine whether Tower A is east or west of Tower B.



2. Identify the known values of the triangle and label appropriately for the sine rule.
Remember: The *shortest side* of a triangle is *opposite* the *smallest angle*!



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where

$a = x \qquad A = 40°$

$c = 7$ km $\quad C = 180° - (70° + 40°) = 70°$

3. Substitute into the formula and evaluate.
Note: △ABC is an isosceles triangle, so Tower A is 7km from the fire.

$$\frac{x}{\sin 40°} = \frac{7}{\sin 70°}$$

$$x = \frac{7 \times \sin 40°}{\sin 70°}$$

$x = 4.788\,282$ km

4. Write the answer in the correct units.

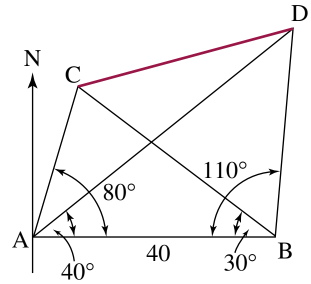
Tower B is closest to the fire at a distance of 4.8 km.

WORKED Example 19

From the diagram at right find:

a **the length of $\overline{CD}$**

b **the bearing from C to D.**

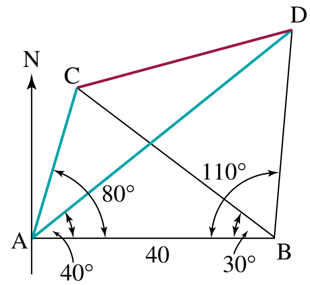


THINK

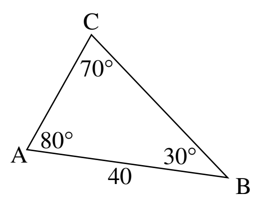
WRITE

a 1 To evaluate $\overline{CD}$, we need to first determine the lengths of $\overline{AC}$ and $\overline{AD}$. (Alternatively, we can find the lengths of $\overline{BC}$ and $\overline{BD}$.)

a



2 Label $\triangle ABC$ for the sine rule and evaluate $\overline{AC}$.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$b = \overline{AC}$ $\quad B = 30°$

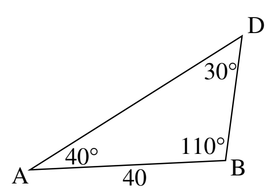
$c = 40$ $\quad C = 70°$

$$\frac{\overline{AC}}{\sin 30°} = \frac{40}{\sin 70°}$$

$$\overline{AC} = \frac{40 \times \sin 30°}{\sin 70°}$$

$$= 21.283\,555$$

3 Label $\triangle ABD$ for the sine rule and evaluate $\overline{AD}$.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{d}{\sin D}$$

$b = \overline{AD}$ $\quad B = 110°$

$d = 40$ $\quad D = 30°$

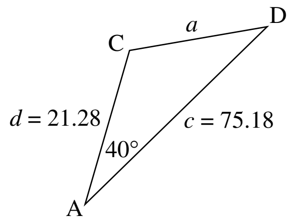
$$\frac{\overline{AD}}{\sin 110°} = \frac{40}{\sin 30°}$$

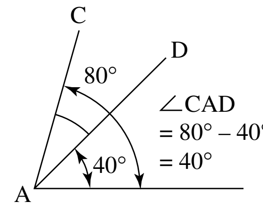
$$\overline{AD} = \frac{40 \times \sin 110°}{\sin 30°}$$

$$= 75.175\,41$$

THINK | **WRITE**

4 Draw $\triangle$ACD, which is needed to find $\overline{CD}$. Use the two given angles to find the angle ∠CAD. Now label it appropriately for the cosine rule.





$a = \overline{CD}$ $\quad A = 40°$

$d = 21.283\,55$ $\quad c = 75.175\,41$

5 Substitute into the formula and evaluate.

$a^2 = c^2 + d^2 - 2cd \times \cos A$

$\overline{CD}^2 = 75.175\,41^2 + 21.283\,555^2 - 2 \times 75.175\,41 \times 21.283\,555 \times \cos 40°$

$\overline{CD} = \sqrt{3652.9898}$

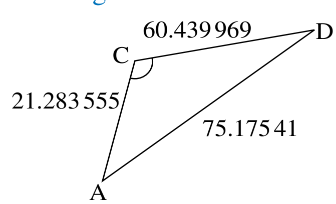
$\overline{CD} = 60.439\,969$

6 Write the answer in the correct units.

The length of $\overline{CD}$ is 60.4 units.

b 1 Redraw $\triangle$ACD and label it with the known information.

b



2 Bearing required is taken *from* C, so find ∠ACD by using the cosine rule.

$a = 60.439\,969$ $\quad c = 75.175\,41$

$d = 21.283\,555$

3 Substitute into the rearranged cosine rule and evaluate C.

$$\cos C = \frac{a^2 + d^2 - c^2}{2 \times a \times d}$$

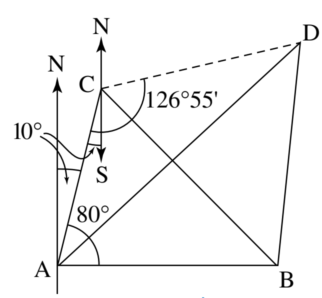
$$\cos C = \frac{(60.439\,969)^2 + (21.283\,555)^2 - (75.175\,41)^2}{2 \times 60.439\,969 \times 21.283\,555}$$

$\cos C = -0.6007$

$C = 126.92°$

$= 126°55'$

4 Redraw the initial diagram (from the question) with known angles at point C in order to find the actual bearing angle.



5 Determine the angle from south to the line $\overline{CD}$.

$\angle SCD = 126°55' - 10°$

$= 116°55'$

6 Determine the bearing angle.

$\angle NCD = 180° - 116°55'$

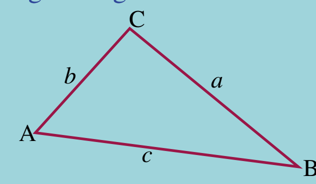
$= 63°05'$

7 Write the bearing of D from C.

The bearing of D from C is N63°05′E.

remember

1. It helps to find all available angles. The third angle in a triangle, C, can be calculated when the two other angles (A and B) have been given. That is $C = 180° - (A + B)$.
2. Use the sine rule if two sides and a non-included angle are given:
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
3. Use the cosine rule if two sides and the angle in between or all three sides are given:
$c^2 = a^2 + b^2 - 2ab \times \cos C$.



EXERCISE 10E Triangulation — cosine and sine rules

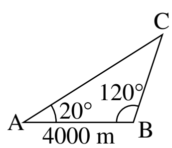
Mathcad Sine rule

Mathcad Cosine rule

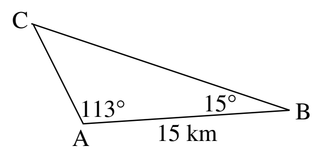
WORKED Example 17

1 Find the distance from A to C $(\overline{AC})$ in each case below (to 1 decimal place).

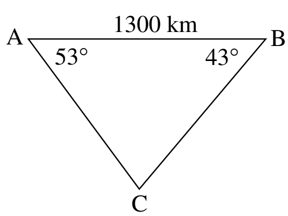
a



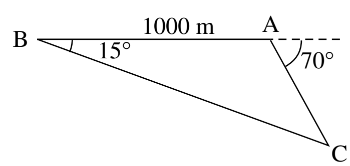
b



c



d



WORKED Example 18

2 **a** Two fire-spotting towers are 17 kilometres apart on an east–west line. From Tower A a fire is seen on a bearing of 130°T. From Tower B the same fire is spotted on a bearing of S20°W. Which tower is closest to the fire and how far is that tower from the fire?

b Two fire-spotting towers are 25 kilometres apart on a north–south line. From Tower A a fire is reported on a bearing of 082°T. Spotters in Tower B see the same fire on a bearing of 165°T. Which tower is closest to the fire and how far is that tower from the fire?

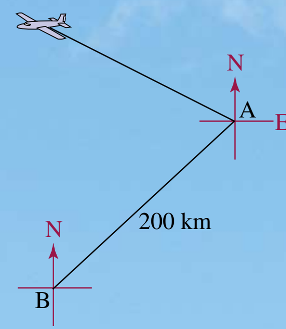
c Two lighthouses are 33 km apart on an east–west line. The keeper in lighthouse P sees a ship on a bearing of N63°E, while the keeper at lighthouse Q reports the same ship on a bearing of 290°T. How far away from the nearest lighthouse is the ship? Which lighthouse is this?

3 Two lighthouses are 20 km apart on a north–south line. The northern lighthouse spots a ship on a bearing of S80°E. The southern lighthouse spots the same ship on a bearing of 040°T.

a Find the distance from the northern lighthouse to the ship.

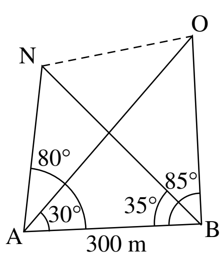
b Find the distance from the southern lighthouse to the ship.

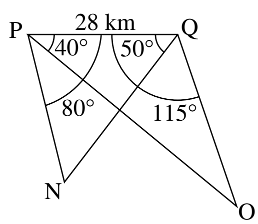
4 Two air traffic control towers detect a glider that has strayed into a major air corridor. Tower A has the glider on a bearing of 315°T. Tower B has the glider on a bearing of north. The two towers are 200 kilometres apart on a NE line as shown. To which tower is the glider closer? What is the distance?

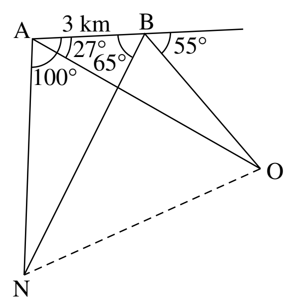


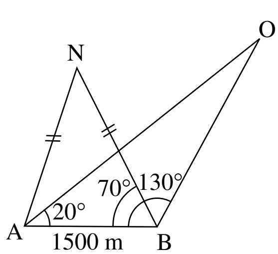
WORKED Example 19a

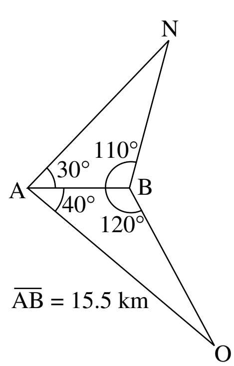
5 Find the value of line segment $\overline{NO}$ in each case below (to 1 decimal place).

a


b


c


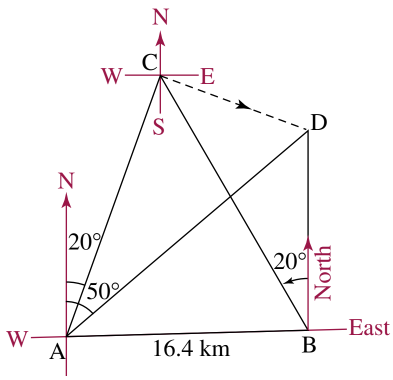
d


e


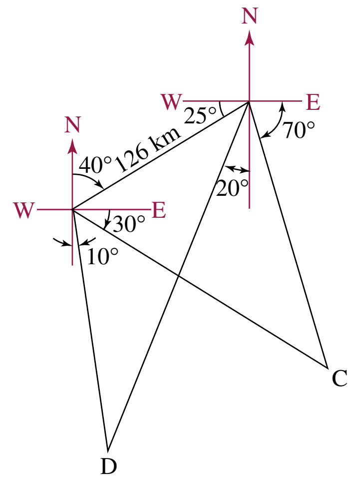
WORKED Example 19

6 Find the distance (to 1 decimal place) and bearing from C to D (in degrees and minutes).

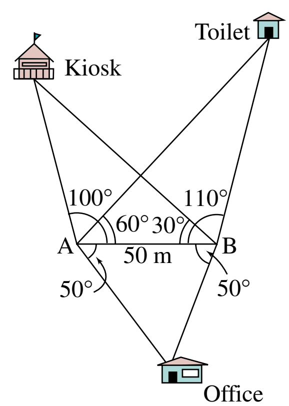
a



b



7 A student surveys her school grounds and makes the necessary measurements to 3 key locations as shown in the diagram.



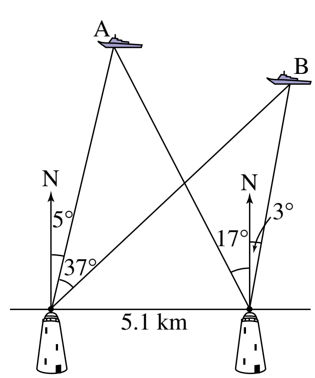
a Find the distance to the kiosk from:
- **i** location A
- **ii** location B

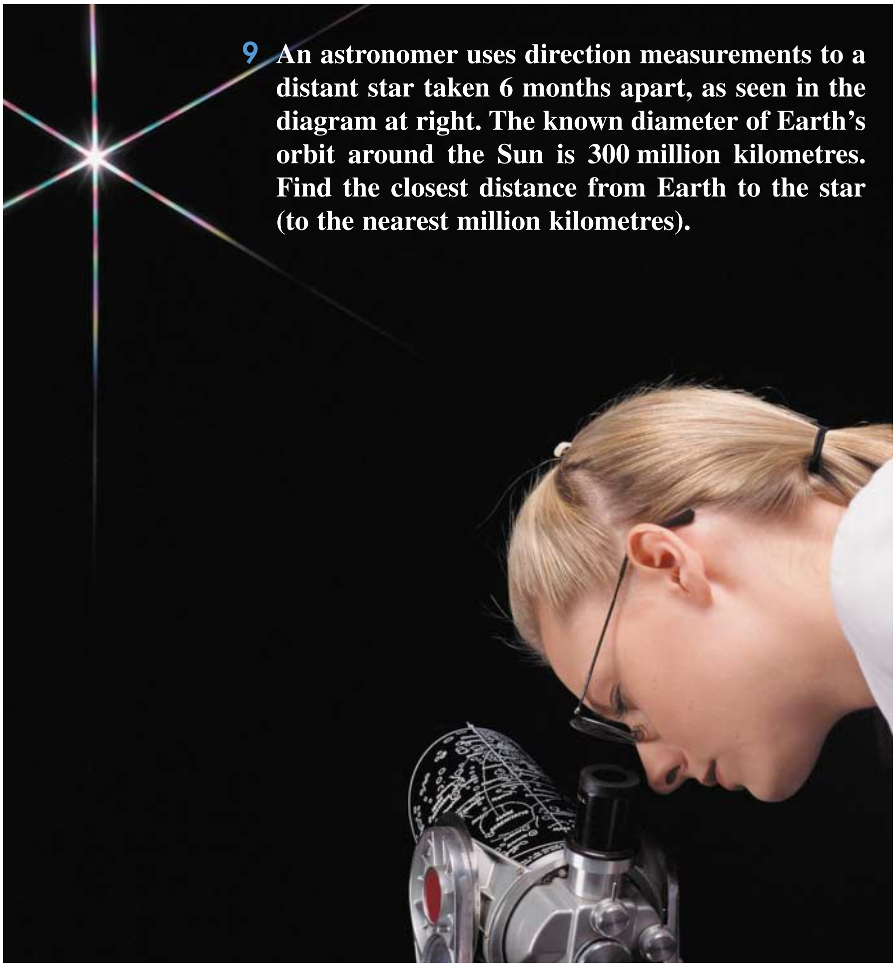
b Find the distance to the toilets from:
- **i** location A
- **ii** location B.

c Find the distance from the toilet to the kiosk.

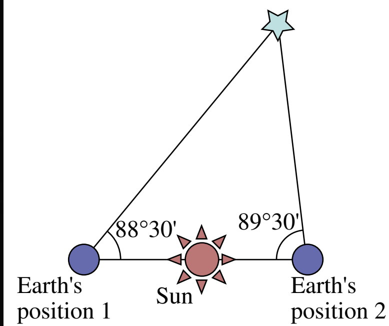
d Find the distance from the office to location A.

8 From the diagram below find the distance between the two ships and the bearing from Ship A to Ship B.



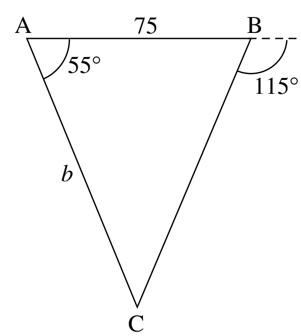


9 **An astronomer uses direction measurements to a distant star taken 6 months apart, as seen in the diagram at right. The known diameter of Earth's orbit around the Sun is 300 million kilometres. Find the closest distance from Earth to the star (to the nearest million kilometres).**



10 **multiple choice**

In the triangle at right the length of side b can be found by using:



A $\dfrac{b}{\sin 115°} = \dfrac{75}{\sin 60°}$ B $\dfrac{b}{\sin 55°} = \dfrac{75}{\sin 65°}$

C $\dfrac{b}{\sin 65°} = \dfrac{75}{\sin 60°}$ D $\dfrac{b}{\sin 115°} = \dfrac{75}{\sin 55°}$

E $\dfrac{b}{\sin 130°} = \dfrac{7}{\sin 30°}$

11 **multiple choice**

Two girls walk 100 metres from a landmark. One girl heads on a bearing of S44°E, while the other is on a bearing of N32°E. After their walk, the distance between the two girls, to the nearest metre, is closest to:

A 123 m B 158 m C 126 m D 185 m E 200 m

12 **multiple choice**

Two ships leave the same port and sail the same distance, one ship on a bearing of NW and the other on SSE. If they are 200 kilometres apart, what was the distance sailed by each ship?

A 100 km B 101 km C 102 km D 202 km E 204 km

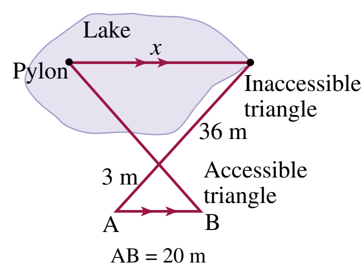
Triangulation — similarity

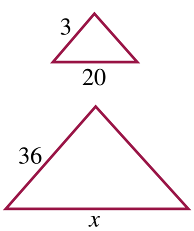
Another method of solving triangulation problems is by using similar triangles. There are situations where a triangle can be constructed in an area that is accessible so as to determine the dimensions of a similar triangle in an inaccessible region.

1. **We need two corresponding lengths to establish the scale factor between the two similar triangles. A second accessible side will be used to scale up or down to the corresponding inaccessible side.**
2. **For similar triangles use the following rules as proof:**
   (a) **AAA — all corresponding angles are the same**
   (b) **SSS — all corresponding sides are in the same ratio**
   (c) **SAS — two corresponding sides are in the same ratio with the same included angle.**

WORKED Example 20

Find the unknown length, x, from the pylon to the edge of the lake.



THINK	WRITE
1 Identify that the two triangles are similar (proof: AAA rule).	
2 Draw the triangles separately, highlighting the corresponding sides.	
3 Identify the scale factor: $\dfrac{\text{length of image}}{\text{length of original}}$	Scale factor $= \dfrac{3}{36} = \dfrac{20}{x}$
4 Transpose the equation to get the unknown by itself and evaluate.	$x = \dfrac{20 \times 36}{3}$ $= 240$ metres
5 Write the length and include units with the answer.	The distance from the edge of the lake to the pylon is 240 metres.

remember

For similar triangles use the following rules as proof:
1. AAA — all corresponding angles are the same
2. SSS — all corresponding sides are in the same ratio
3. SAS — two corresponding sides are in the same ratio with the same included angle.

EXERCISE 10F Triangulation — similarity

WORKED Example 20

1 In figure 1 below, find the length of the proposed bridge, AB.

2 In figure 2 below, find the length of the base of a hillside from C to D.

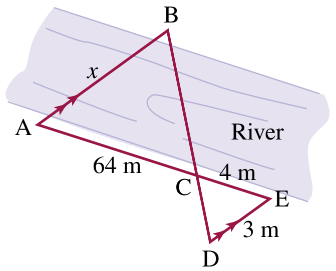


Figure 1

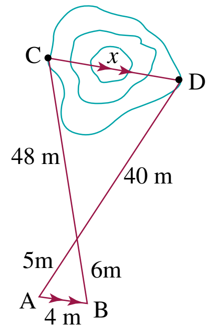


Figure 2

3 In figure 3 below, find the perpendicular gap between the two city buildings.

4 In figure 4 below, find the distance between the two lighthouses.

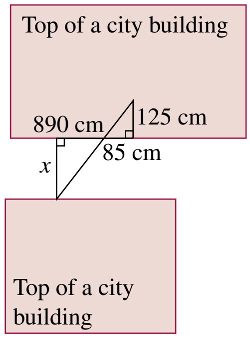


Figure 3

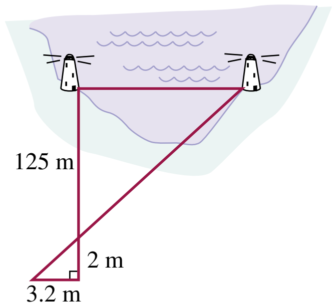


Figure 4

5 In figure 5 below, find the distance across the lake.

6 In figure 6 below, find the height of the cyprus tree.

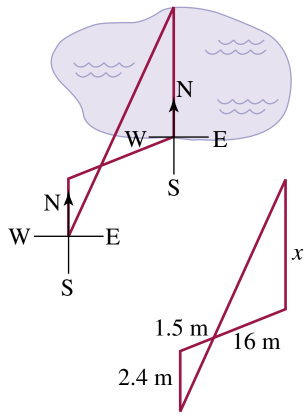


Figure 5

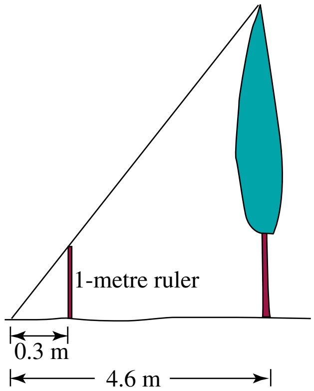
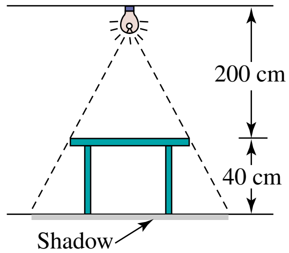


Figure 6

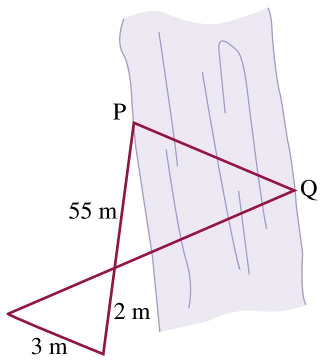
7 Find the width (to the nearest centimetre) of the shadow under the round table which has a diameter of 115 centimetres.



8 **multiple choice**

The distance across a river is to be determined using the measurements outlined at right.
The width from P to Q is closest to:

A 37 m
B 60 m
C 83 m
D 113 m
E 330 m



9 **multiple choice**

The shadow formed on the ground by a person who is 2 m in height was 5 m. At the same time a nearby tower formed a shadow 44 m long. The height of the tower to the nearest metre is:

A 18 m **B** 20 m **C** 51 m **D** 110 m **E** 123 m

10 Find the height (to the nearest centimetre) of the person being photographed (figure 7).

11 Find the minimum distance from the tree to the camera, x metres, so that the tree is completely in the photo (figure 8).

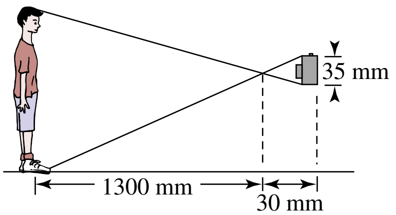


Figure 7

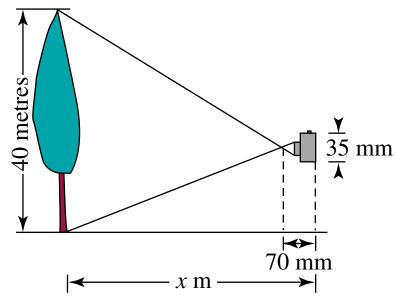


Figure 8

12 A girl is looking through her window.

a She is standing 2 metres from the window which is 2.4 metres wide. What is the width of her view:
  i 300 metres from the window (to the nearest metre)
  ii 1.5 kilometres from the window (to the nearest 100 metres)
  iii 6 kilometres from the window (to the nearest kilometre)?

b She is now standing 1 metre from the window. What is the width of her view:
  i 300 metres from the window (to the nearest metre)
  ii 1.5 kilometres from the window (to the nearest 100 metres)
  iii 6 kilometres from the window (to the nearest kilometre)?

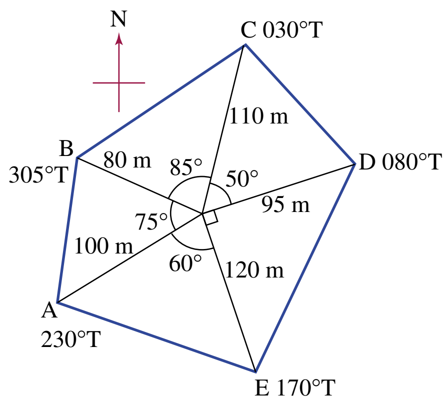
WorkSHEET 10.2

Traverse surveying

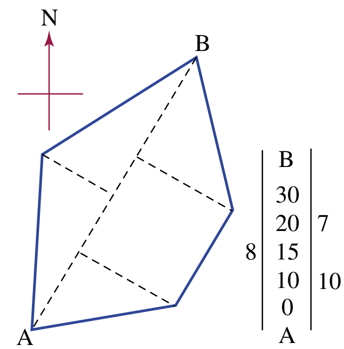
A surveyor is often required to find:

- the perimeter of a block or piece of land (for example, for fencing purposes)
- the area of a block or piece of land
- the distances and bearings between two diagonally opposite vertices or corners.

When the blocks of land or regions are irregular shapes as shown below, traverse or radial surveying can be used.



Radial surveying



Traverse surveying

In a *radial survey* of an area a central point is established and bearings of several points on the perimeter of the area are taken. Distances from the central point are also measured. The cosine rule is used to find the lengths of boundary sections and the formula $\frac{1}{2}ab\sin C$ is used to find the areas of the triangular sections formed (see worked example 25).

In a *traverse survey* of an area Pythagoras' theorem and trigonometry can be applied to find directions and lengths of sides or distances between vertices of an irregular shaped region. (See the steps below.)

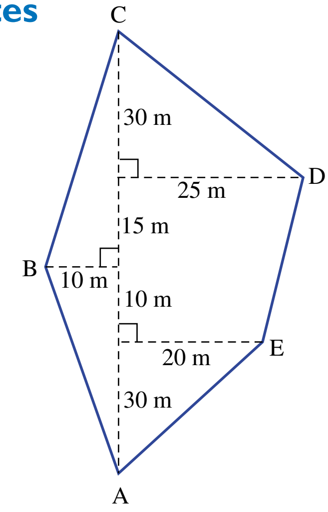
Traverse surveying and surveyor's notes

Consider the area at right which is bounded by the points A, B, C, D and E. That is, its perimeter is ABCDE.

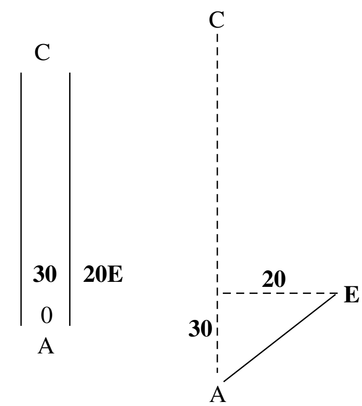
To survey an area such as this a surveyor follows these steps:

Step 1. A diagonal (usually the longest) is chosen ($\overline{AC}$). Along this *traverse line* measurements are made.

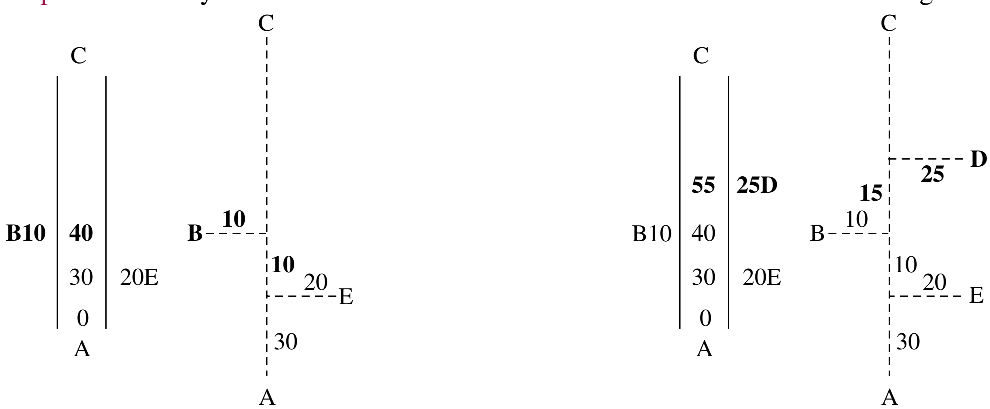
Step 2. The surveyor starts at one end (say A) and moves along the diagonal until a *vertex* or *corner* (E), perpendicular to the diagonal, is found.



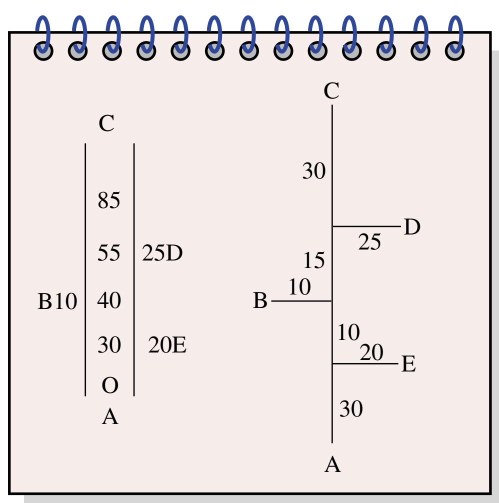
Step 3. The surveyor records the distance, from the end point, along the diagonal as well as the perpendicular distance (or *offset*) from the diagonal to the vertex or corner.



Step 4. The surveyor continues this until he or she reaches the other end of the diagonal.



Step 5. The surveyor returns from the field with the completed notes. The notes are converted into a drawing for performing calculations.



Surveyor's notes

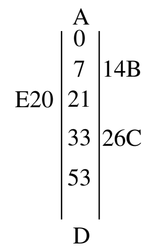
Alternative surveyor's notes

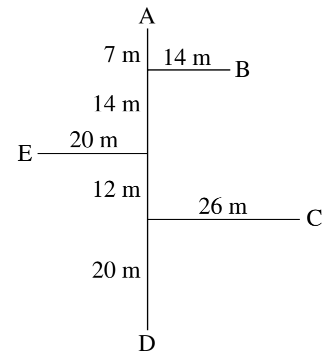
Note: In the figure at right, two different sets of notes made by a surveyor in the field are shown. Those on the left-hand side of each figure represent the usual type of notes taken by the method described in the five steps. Those on the right-hand side are alternative surveyor's notes. These notes are similar but they have offset lines shown (all are the same length) and the lengths along the traverse line are not added cumulatively.

It is important to note that the area drawn is an accurate *scale drawing* of the region surveyed. From this drawing, distances can be determined. However, *in this chapter we shall not draw scale diagrams or perform calculations in this way*. Instead, a suitable sketch of the region will be drawn and the techniques developed in the previous two chapters will be used to perform calculations.

WORKED Example 21

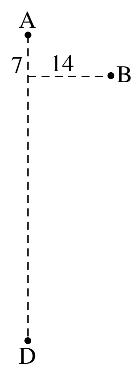
From the surveyor notes provided, draw a suitable sketch of the irregular shaped region.



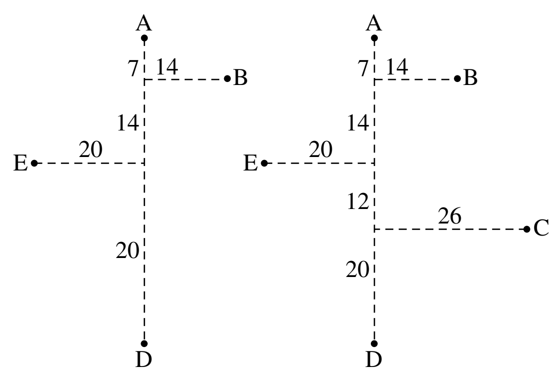


THINK / **WRITE**

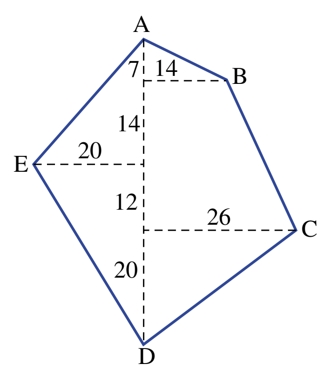
1. Draw a dashed line, say 4cm, to represent the diagonal $\overline{AD}$ which is 53 m long.
   Estimate a suitable length along the diagonal for the first offset. Draw a perpendicular line to the diagonal line $\overline{AD}$ of appropriate length on the corresponding side. Write the dimensions of each of the line segments and name the vertex.



2. Repeat this for each of the corners or vertices and add to the diagram.

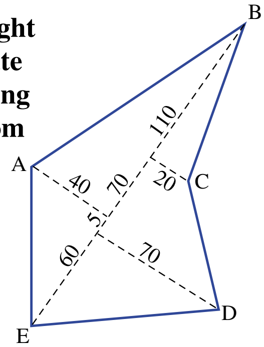


3. Now join the corners to form an outside closed boundary.



WORKED Example 22

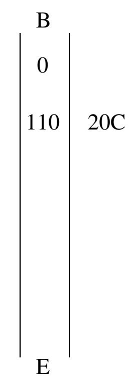
From the plan at right write the appropriate surveyor's notes using the traverse line from B to E.

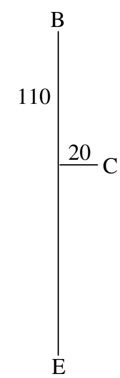


THINK

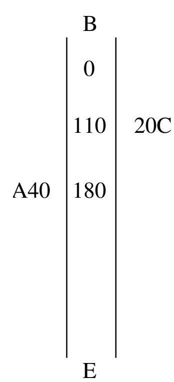
WRITE

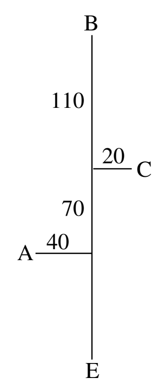
1. *Note:* Alongside each diagram is the alternative method for recording the various lengths.
   Start from one end of the traverse line $\overline{BE}$, say B, and begin at a distance of 0. Add the distance along the traverse line to the first offset and record. Now record on the corresponding side the length of the offset line or distance to the vertex, and the vertex.



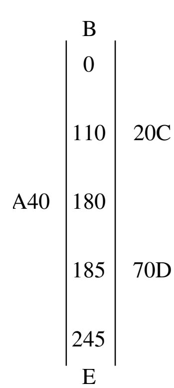


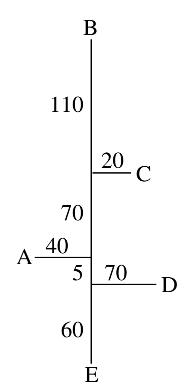
2. Add the distance along the traverse line $\overline{BE}$ to the second offset and record. Also record the length of the offset and the vertex on the corresponding side.





3. Repeat this for the final corner or vertex, D, and until you reach the other end of the traverse line at E.



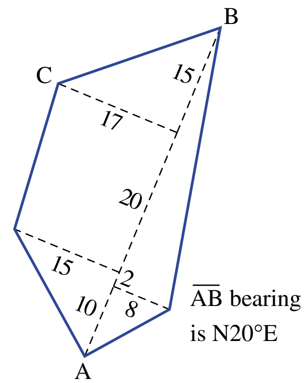


4. Check: The final figure should be the total length of the traverse line, $\overline{BE}$.

$$\text{Length} = 110 + 70 + 5 + 60$$
$$= 245$$

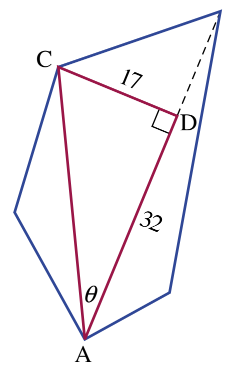
WORKED Example 23

Calculate the distance (to the nearest metre) and the bearing (to the nearest degree) from A to C in the plan shown. (Measurements are in metres).



THINK | **WRITE**

1. Identify and label the right-angled triangle that contains $\overline{AC}$ and is part of the traverse line.



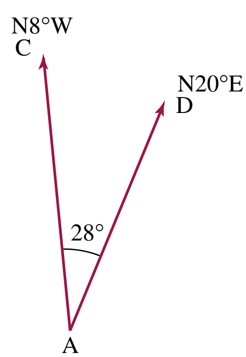
2. Calculate the distance using Pythagoras' theorem.

$c^2 = a^2 + b^2$
$c^2 = 17^2 + 32^2$
$c^2 = 1313$
$c = \sqrt{1313}$
$= 36.235\,34$ m

3. Calculate ∠CAD using the tangent ratio.

$\tan\theta = \frac{\text{opp}}{\text{adj}}$
$\tan\theta = \frac{17}{32}$
$= 0.531\,25$
$\theta = 27.9794° \approx 28°$

4. Convert the angle into a bearing.



Bearing $=$ N20°E $-$ 28°
$=$ N8°W

5. Write your answer clearly.

The vertex, C, is 36 metres on a bearing of N8°W from location A.

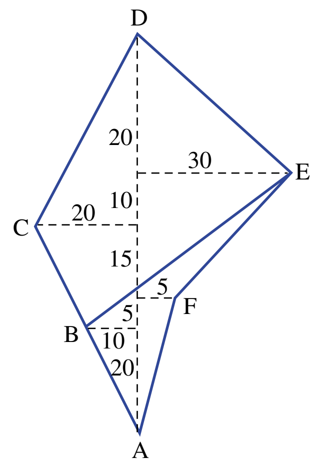
WORKED Example 24

Find the length of the line joining the two vertices B and E from the surveyor's notes given at right. (Measurements are in kilometres.)

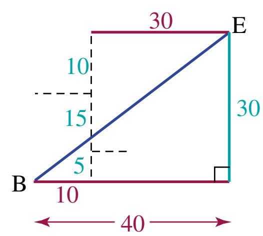
	D	
	70	
	50	30E
C20	40	
	25	5F
B10	20	
	0	
	A	

THINK | **WRITE**

1. Draw a suitable sketch of the irregular block.



2. Identify and label the right-angled triangle.



3. Calculate the distance using Pythagoras' theorem (or recognise it as a multiple of the 3, 4, 5 Pythagorean triad).

$c^2 = a^2 + b^2$

$c^2 = 30^2 + 40^2$

$c = 50$

4. Write your answer and include units.

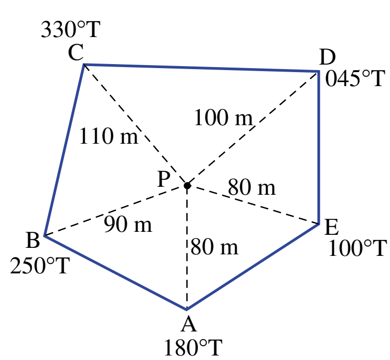
The length from B to E is 50 km.

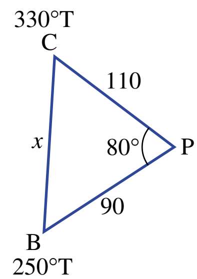
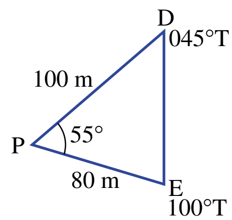
WORKED Example 25

For the radial survey sketch given, find:

a **the length of the line joining the two vertices B and C (to 1 decimal place)**

b **the area of the region bounded by the vertices D, E and P, to the nearest 10 m^2.**



THINK	WRITE
a 1 Identify $\triangle BPC$ with given details.	**a** 
2 Calculate the length using the cosine rule and write it in the relevant form for this triangle. Substitute values from the triangle to find x.	$p^2 = b^2 + c^2 - 2bc \times \cos P$ $x^2 = 110^2 + 90^2 - 2 \times 110 \times 90 \times \cos 80°$ $= 16\,761.766\,08$ $x = 129.467\,24$ The length of BC is 129.5 metres.
b 1 Identify and label $\triangle DPE$.	**b** 
2 Calculate the area using $\frac{1}{2}ab\sin C$ or in this case $\frac{1}{2}de\sin P$.	Area $\triangle DPE = \frac{1}{2}de\sin P$ $= \frac{1}{2} \times 80 \times 100 \times \sin 55°$ $= 3276.608\,177$ $\approx 3280\text{ m}^2$ The area of $\triangle DPE$ is approximately 3280 m^2.

remember

1. A surveyor may use traverse or radial techniques when surveying an area.
2. In traverse surveying a traverse line is drawn between two diagonally opposite points on the boundary of the area. Offset distances are drawn at right-angles to the traverse line to other points on the perimeter of the area.
3. Surveyor's notes are used to summarise the lengths of these offset lines and their position along the traverse line.
4. In a radial survey of an area a central point is established and bearings of, and distances to, several points on the perimeter of the area are taken.

EXERCISE 10G Traverse surveying

WORKED Example 21

1 From the surveyor's notes provided, draw a suitable sketch of the irregular shaped region in each case below.

a

	A	
	0	
	20	10B
	40	10C
E10	60	
	80	
	D	

b

	D	
	70	
	50	20C
E15	40	
F15	32	
	27	15B
G5	20	
	0	
	A	

c

	A	
		5
15		13
15		22
20		18
	B	

d

	B	
		20
		15
		5
30		30
	A	

e

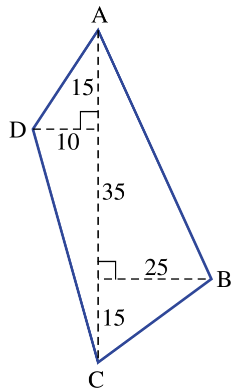
	A	
	0	5
10	12	
	28	20
15	36	
	50	
	B	

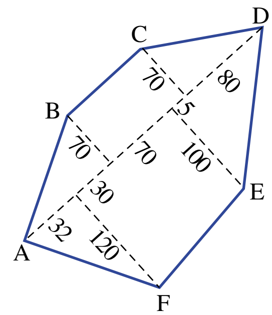
f

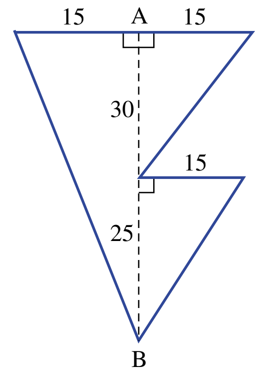
	A	
	70	22
	50	0
20	35	
	15	15
	0	
	B	

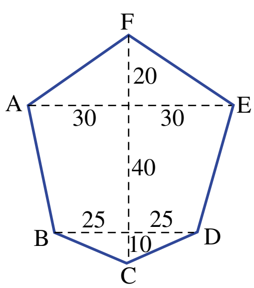
WORKED Example 22

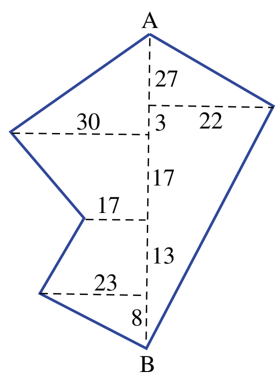
2 From the plans of the irregular blocks below, give an appropriate set of surveyor's notes in each case.

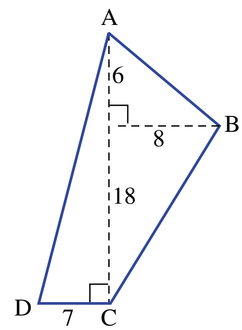
a


b


c


d


e


f


WORKED Example 23

3 From the diagram at right, calculate the distance (to the nearest metre) and bearing (to the nearest degree):

a from A to C

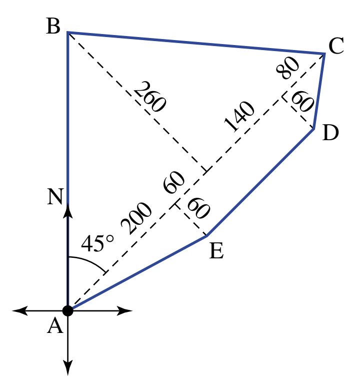
b from A to B

c from A to E

d from A to D.

e from D to E

Distance measurements are in metres.



WORKED Example 24

4 From the given surveyor's notes, with measurements in metres, find the length of the line (to the nearest metre) joining the two vertices:

a A and F
b A and E
c B and F
d B and E
e C and F
f C and E

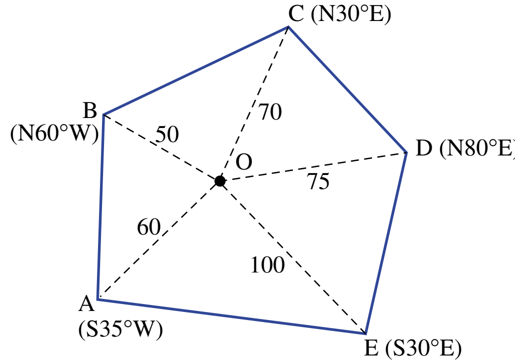
	G	
	0	
A5	10	
B10	25	
	40	10F
	60	15E
C15	80	
	90	
	D	

5 Using the surveyor's notes provided in question **4** find the length of the line (to the nearest metre) joining the two vertices:

a A and B **b** A and C **c** B and C **d** E and F

WORKED Example 25

6 From the diagram below, calculate: (measurements are in metres).



a the distance (to the nearest metre) from:
 i A to B **ii** B to C **iii** C to D **iv** D to E **v** A to E

b the perimeter of the shape

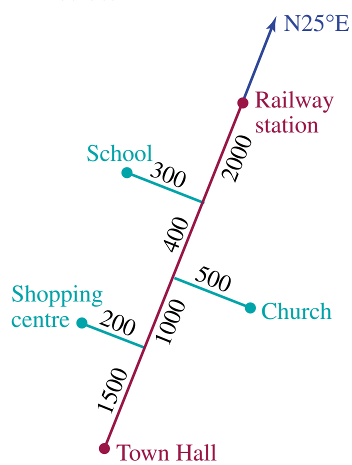
c the area (to the nearest 10 m^2) bounded by the vertices:
 i A, O and E **ii** D, O and E **iii** D, O and C **iv** C, O and B **v** A, O and B

d the area bounded by A, B, C and O.

7 Consider the surveyed area provided in question **6**. Calculate the distance (to the nearest metre) from:

a A to C **b** A to D **c** B to D **d** B to E **e** C to E

8 The diagram below is a sketch of the major attractions in a regional city, where distance measurements are in metres.



a Find the distance and direction from the town hall to:
 i the shopping centre
 ii the church
 iii the school.

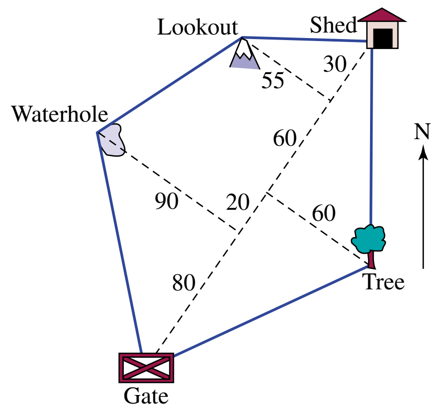
b Find the distance and direction from the railway station to:
 i the shopping centre
 ii the church
 iii the school.

9 multiple choice

The results of a traverse survey of an area that will enclose deer are shown at right, with measurements in metres.

The length of fencing that will be required from the waterhole to the lookout is closest to:

A 87 m
B 97 m
C 120 m
D 150 m
E 166 m



10 multiple choice

A traverse survey has just been completed for a marine reserve. The surveyor's notes are shown below with the tunnel lying directly North of the wreck.

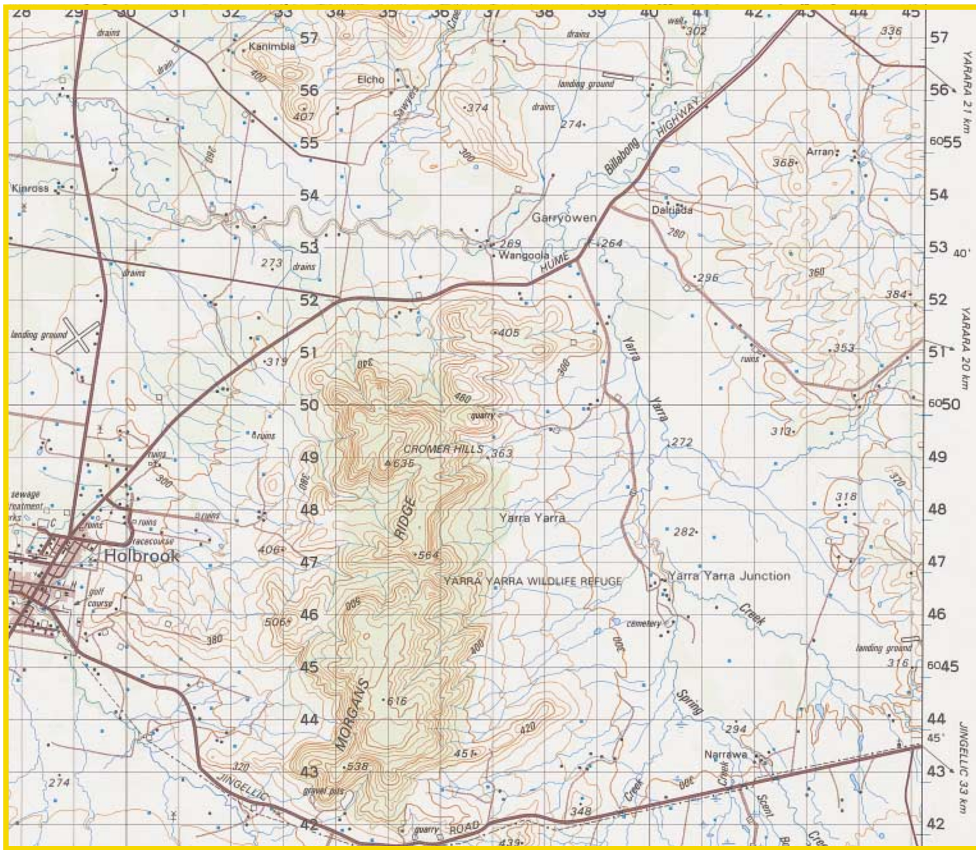
	Wreck	
	0	
	10	30 Kelp forest
Rock wall 35	50	
	60	40 Sand patch
	80	
	Tunnel	

If a scuba diver is looking at crayfish in the rock wall, the bearing that she would need to follow to go from the rock wall to the kelp forest would be:

A S36°52′W **B** N58°24′E **C** N31°36′E **D** S58°24′W **E** S31°36′W

Contour maps

Contour maps are used to represent the shape of the undulation or terrain of a region. That is, they indicate whether the land surface goes up or down between two points and how steeply the land slopes.



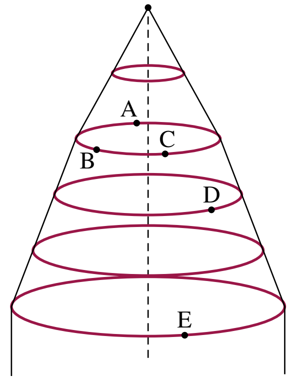
Such maps are used:

1. by cross-country hikers in the sport of orienteering
2. by civil engineers planning new developments such as road constructions
3. as tourist information guides for local walks.

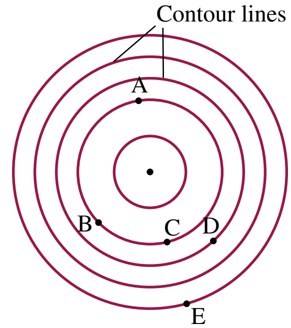
These contour or topographic maps are a two-dimensional overhead view of a region. Contour lines along with a map scale provide information about the region in a concise manner.

Contour lines and intervals

A contour line is defined as a line that *joins places* that are at the *same height* above sea level or a reference point.



3-dimensional representation of a region



Contour map of the region

In the above figures, points A, B and C lie on the same contour line and hence are at the same height above sea level.

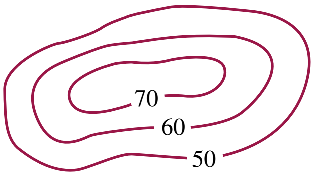
The distance between contour lines indicates the steepness of the slope.

The closer the lines are, the steeper the slope.

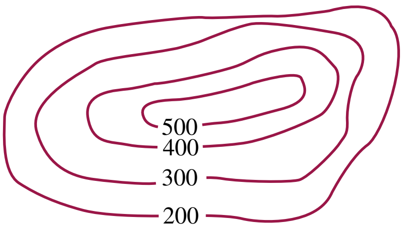
The further apart the contour lines are, the flatter the slope.

Look at the 3-dimensional representation at the top of the page. Compare the slope in the lower section of the shape (where the contour lines are close together) with the slope in the upper section (where the contour lines are further apart).

Intervals are indicated on the line to show the relative difference in height or altitude.

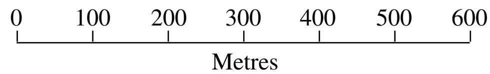


Intervals change in regular multiples of 10s or 100s of metres.



Map scales

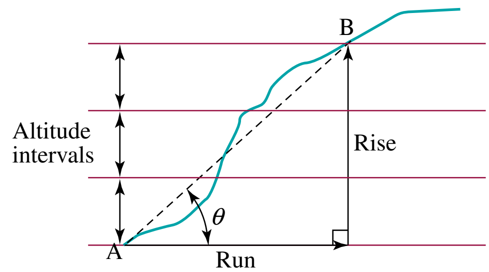
Map scales are given as a ratio, for example 1 : 25 000 or a linear scale (below).



Average slope

The average slope of land between two points A and B is given by:

$$\begin{aligned}\text{Average slope} &= \text{gradient}\\ &= \frac{\text{rise}}{\text{run}}\\ &= \tan\,\theta\end{aligned}$$

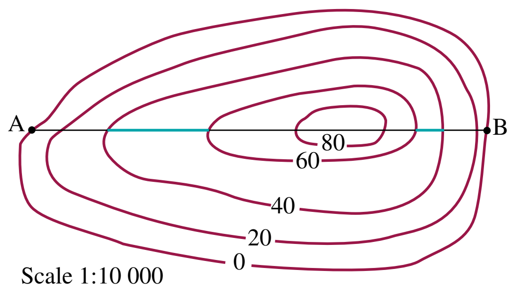


From the average slope we can determine the angle of elevation, θ, of B from A, which is equal to the angle of depression of A from B.

A contour map can be used to draw a profile of the terrain. A profile is a side view of the land surface between two points, as shown above.

WORKED Example 26

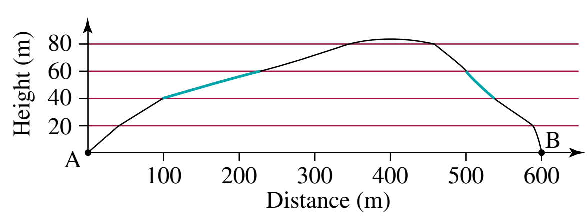
For the contour map, give an appropriate profile along the cross-sectional line $\overline{AB}$.



THINK

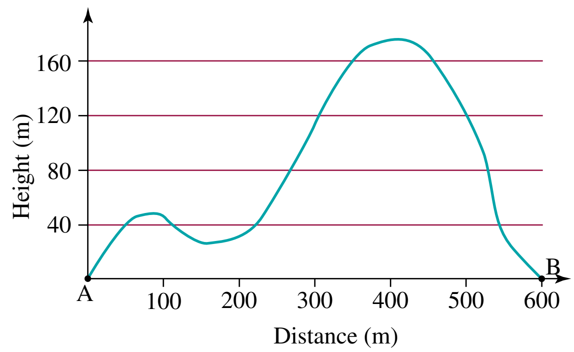
1. Draw a horizontal line of the same length as from A to B. Use the scale provided to add units to this line.
2. Find the maximum and minimum heights from the contour lines that $\overline{AB}$ crosses and draw a vertical axis from A. This axis should be to scale.
3. Locate the points intersected by the line $\overline{AB}$ on the cross-sectional grid at their appropriate height.
4. Join the points together. The exact profile in between the intervals can only be guessed.

WRITE



WORKED Example 27

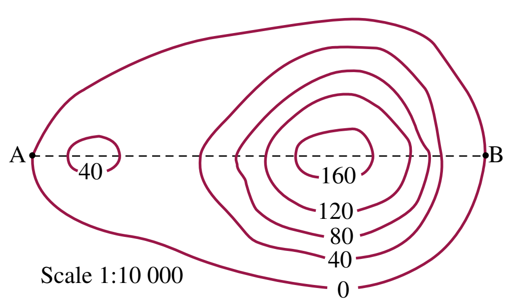
From the given profile, construct an *appropriate* contour map.



THINK / **WRITE**

1. Draw a dashed, horizontal line to represent the cross-section from A to B. It should be the same length as $\overline{AB}$.

2. Transfer points of equal height onto the dashed line $\overline{AB}$. These points are on the same contour line so draw a contour line to connect them and indicate its height. Remember: There is no information on the width and shape of the hill beyond the cross-sectional line, $\overline{AB}$, so the contour that is drawn will be only one possibility.



3. Repeat this process for the next height. Continue until all heights are done. Remember: The shape and exact height of the peak of the hill is unknown. The height is somewhere between the highest altitude given and the next interval.

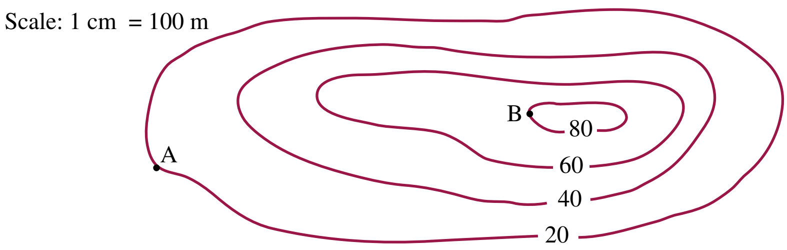
4. Add in the horizontal scale.

WORKED Example 28

For the contour map given, calculate:

a **the direct, straight-line distance between locations A and B (to the nearest metre)**

b **the average slope of the land and the angle of elevation (in degrees and minutes) from the lower point to the upper one.**



THINK / **WRITE**

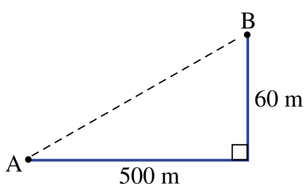
a 1 Identify the difference in height from the contour intervals given.

a Height difference = 80 − 20
= 60 metres

2 Use the scale given to measure and calculate the horizontal distance between the two locations.

Horizontal difference in distance
= 5 cm which represents 500 metres.

3 Draw a simple triangle to represent the profile from A to B.



4 Calculate the direct distance from A to B using Pythagoras' theorem.

$c^2 = a^2 + b^2$

$c^2 = 60^2 + 500^2$

$c^2 = 253\,600$

$c = \sqrt{253\,600}$

$= 503.587\,132\,5$

≈ 504 metres

b 1 Calculate the average slope.

b Average slope $= \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{60}{500}$

$= 0.12$

2 Calculate the angle of elevation using the tangent ratio.

$\tan \theta = 0.12$

$\theta = 6.8428°$

$= 6°51'$

3 Write your answer.

The direct distance between A and B is 504 metres while the angle of elevation of B from A is 6°51′.

remember

1. A contour map is used to represent the shape of the terrain.
2. A contour line is defined as a line that *joins places* that are at the *same height* above sea level or reference point.
3. The closer the contour lines are, the steeper the slope.
4. Intervals are indicated on the line to show the relative difference in height or altitude.
5. Map scales are given as a ratio, for example 1 : 25 000 or 1 cm = 5 km.
6. Average slope between two points is given by:

$$\begin{aligned}\text{Average slope} &= \text{gradient}\\ &= \frac{\text{rise}}{\text{run}}\\ &= \tan\theta\end{aligned}$$

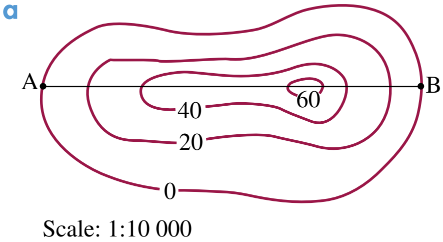
where θ = angle of elevation from lower point to upper one.

EXERCISE 10H Contour maps

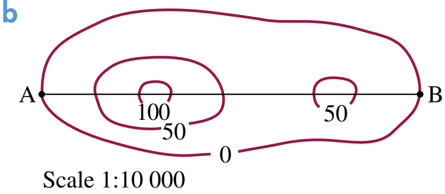
WORKED Example 26

1 For each of the contour maps, give an appropriate profile along the cross-sectional line, $\overline{AB}$.

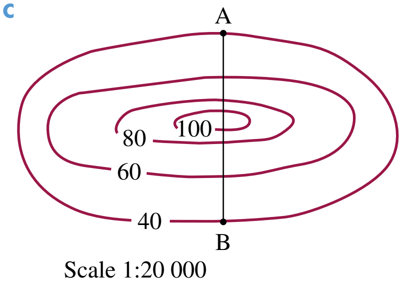
a



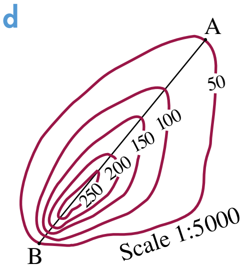
b



c



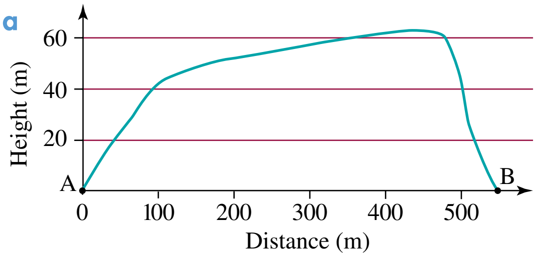
d



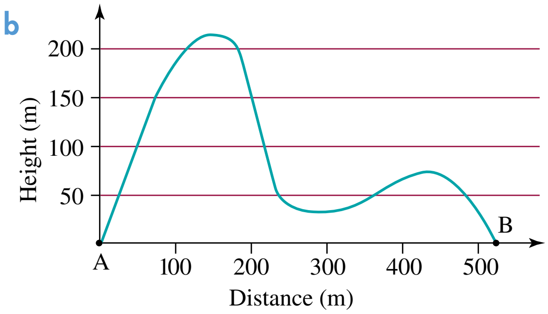
WORKED Example 27

2 For each of the given profiles, construct an appropriate contour map.

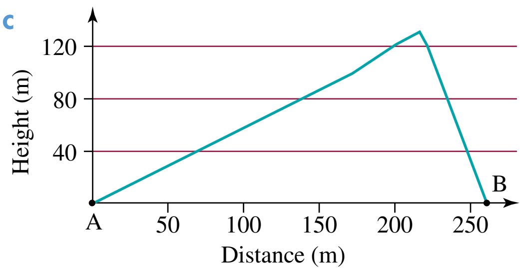
a



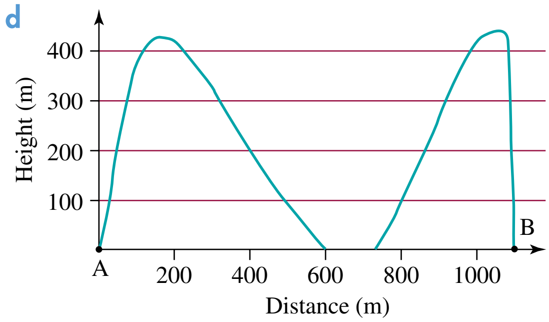
b



c

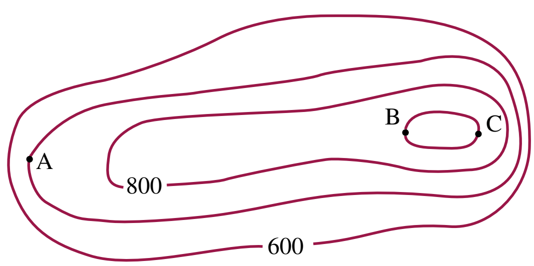


d

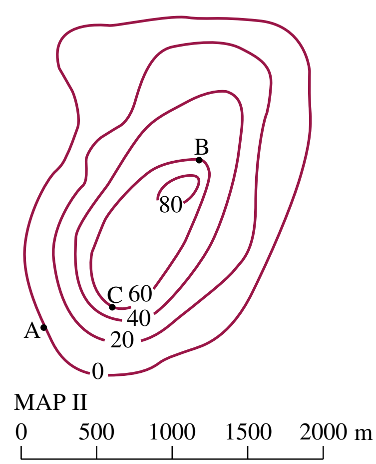


WORKED Example 28

3 For contour maps I and II, calculate:



MAP I Scale 1 cm = 200 m

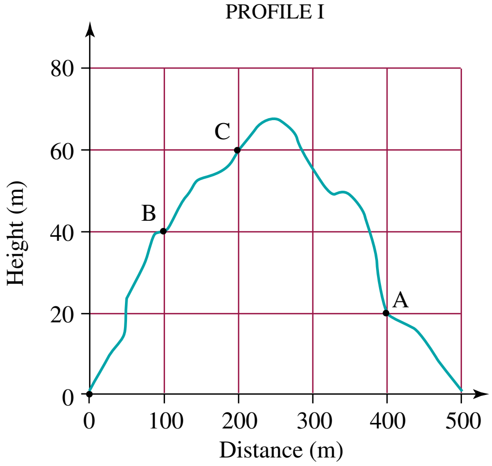


a the direct, straight-line distance (to the nearest metre) between locations:
- **i** A and B
- **ii** A and C
- **iii** B and C

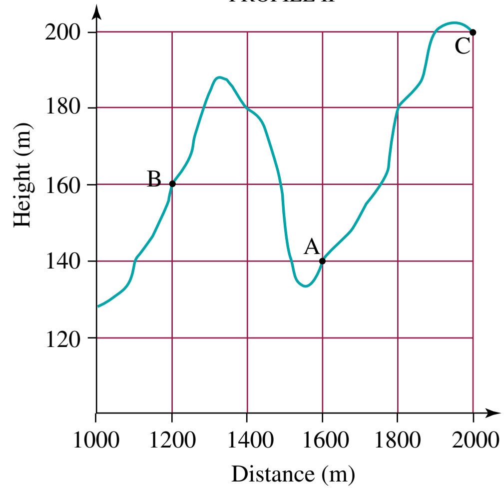
b the average slope of the land and the angle of elevation (in degrees and minutes) from:
- **i** A to B
- **ii** A to C.

4 For each of the given profiles, calculate:

PROFILE I



PROFILE II

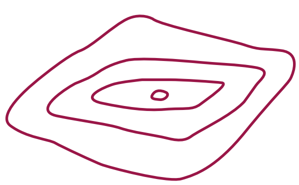


a the direct, straight-line distance (to the nearest metre) between locations:
  i A and B
  ii A and C
  iii B and C

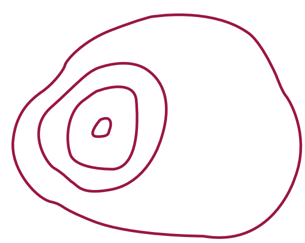
b the average slope of the land and the angle of elevation (in degrees and minutes) from:
  i A to B
  ii A to C.

5 Match up each of the contour maps below with an appropriate profile.

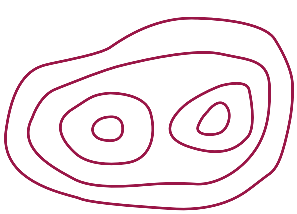
a



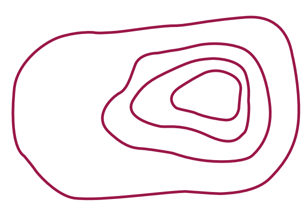
b



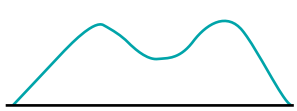
c



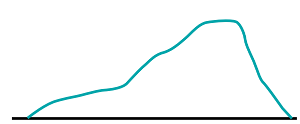
d



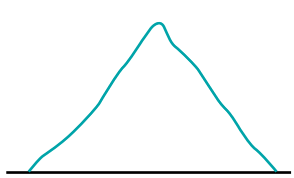
i



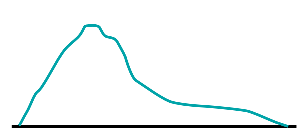
ii



iii

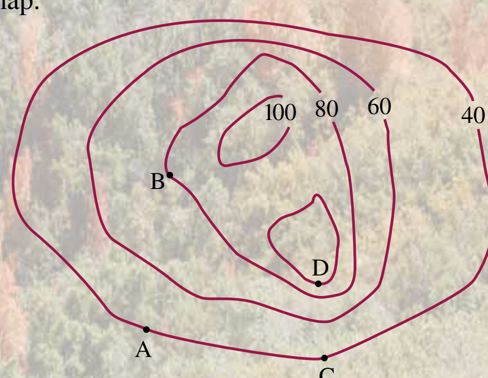


iv



6 **multiple choice**

Examine the contour map.



Scale 1:10 000

a The direct, straight-line distance from A to B above is closest to:

A 108 m **B** 201 m **C** 204 m **D** 209 m **E** 215 m

b The angle of depression of C from D is closest to:

A 16°42′ **B** 21°48′ **C** 30°58′ **D** 45° **E** 59°2′

A day at the beach

A young family is enjoying a day at the beach. They bring along beach towels and many spades and sand buckets. One of the children observes a sailing boat passing a buoy out at sea and inquires what is the distance to the yacht and the buoy.

Your task is to prepare a set of instructions and simple calculations that both the father and mother could follow, so as to determine distances to any objects out to sea, such as the yacht or the buoy. The equipment available to the parents are a towel (assume the length of a towel is known) and the buckets and spades.

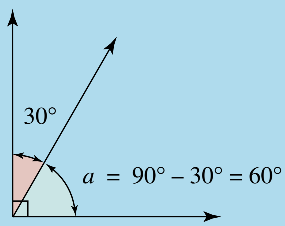
summary

Angles

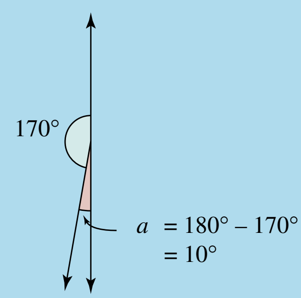
- Angles and bearings are measured in degrees and minutes.
- 60 minutes = 1 degree

Angle laws

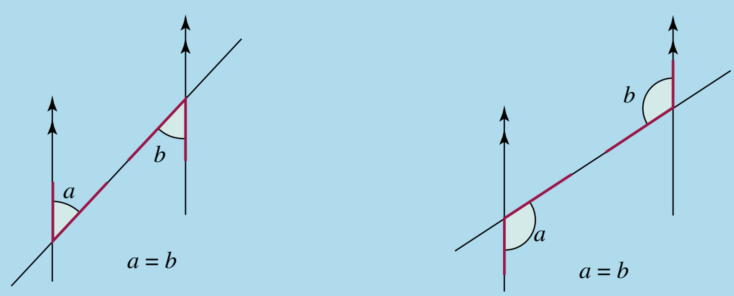
- Complementary angles add up to 90°.



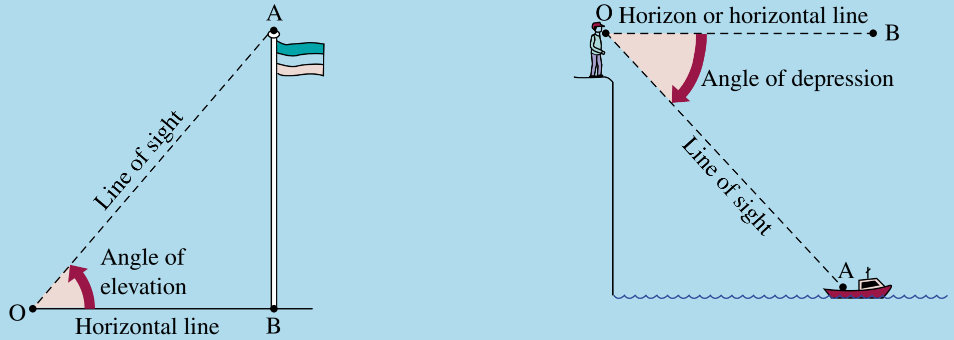
- Supplementary angles add up to 180°.



- Alternate angles are equal.

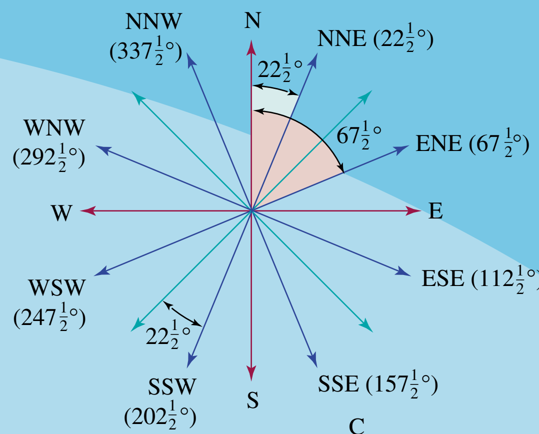


Angles of elevation and depression



Bearings

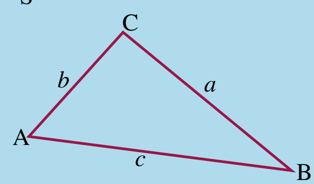
- Standard compass bearings
  Common angles between directions are 22.5°, 45°, 67.5°, 90°, 135°, 180°, 225°, 270°, 315°.
- Other compass bearings
  Start at north or south, then turn through an angle towards east or west, for example N20°W, S80°E.
- True bearings
  Start at north and then turn through an angle in a clockwise direction, for example 157°T, 030°T, 287°T.



Sine and cosine rules

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $c^2 = a^2 + b^2 - 2ab \times \text{Cos}\, C$

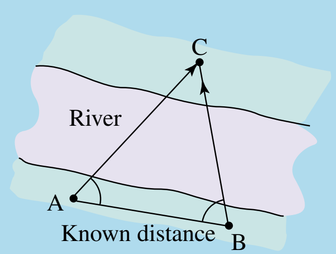


Navigation and specification of locations

- Bearings are directions in the horizontal plane, not just angles.
- When solving navigation problems, in most cases the angle laws will need to be used.
- When determining a bearing, be clear on where the direction is taken *from* and *to* the starting and finishing points.
- There is a 180° difference between the bearing of A from B compared to the bearing of the return, that is, of B from A.

Triangulation

- Triangulation involves finding dimensions in inaccessible regions.
- Sine and cosine rules may be used if:
  (a) the distance between two locations is known and
  (b) the direction from the two locations to a third is known.
- Alternatively, we may use *similarity* when two similar triangles are given.



Traverse surveying

- Offsets are at 90° to the traverse line.
- Use Pythagoras' theorem to calculate lengths.
- Use the tangent ratio to calculate angles.

Contour maps

- A contour map represents the shape of the terrain.
- Contour lines join locations that are at the same height (or altitude) above sea level or a reference point.
- Contour lines that are close together indicate steep terrain.
- Contour lines that are far apart indicate gentle slopes.
- The vertical distance between two locations can be found from the difference in the values of the two contour lines.
- Average slope $= \text{gradient}$

$$= \frac{\text{rise}}{\text{run}}$$

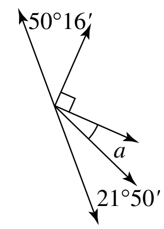
$= \tan\theta$, where θ = angle of elevation from lower point to upper point.

CHAPTER review

Multiple choice

10A

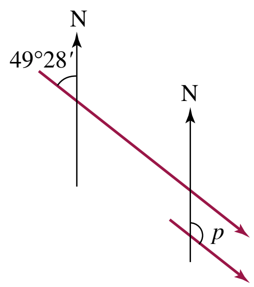
1 The value of a at right is:



A 17°54′
B 18°54′
C 19°
D 21°50′
E 28°20′

10A

2 The value of p at right is:



A 40°32′
B 49°
C 49°28′
D 120°
E 130°32′

10B

3 A fly is hovering above a frog sitting on the ground, at an angle of elevation of 42°20′ and 12 cm directly above the ground. The minimum length (to 1 decimal place) that the frog's tongue needs to be to touch the fly is:

A 8.1 cm **B** 12.0 cm **C** 13.2 cm **D** 16.2 cm **E** 17.8 cm

10B

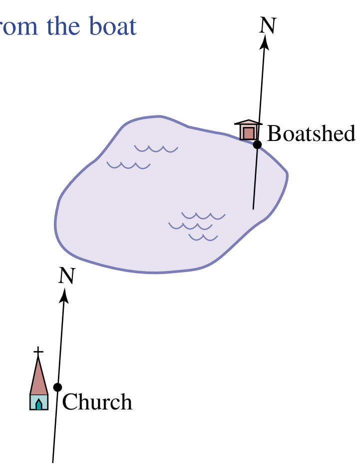
4 A helicopter has its 8-metre rescue ladder hanging vertically from the doorway of the craft. The ladder's free end is just touching the surface of the floodwater below. From the top of the ladder the angle of depression of a stranded person in the water, clinging to a pole, is 54°. The distance that the helicopter must fly horizontally toward the person to rescue him is:

A 4.7 m **B** 5.8 m **C** 6.5 m **D** 9.9 m **E** 11.0 m

10C

5 Using a protractor, the bearing of the church from the boat shed in the diagram at right is:

A S35°W
B N35°E
C SW
D 055°T
E S55°W



10C

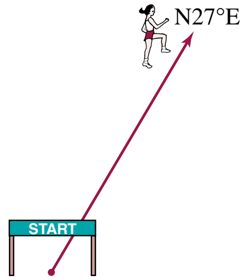
6 There is a fork in a road. The road heading eastwards is on a bearing of S27°E. If the angle between the two roads is 135°, the most likely direction of the other westward-bound road is:

A S27°E **B** S72°W **C** N72°W **D** SE **E** SW

10D

7 The bearing required to return to the starting line in the situation at right is:

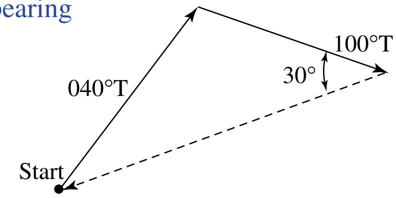
A N27°S
B N27°E
C S27°W
D N27°W
E S27°E



10D

8 For the final leg of the journey shown, the bearing to return to the start is:

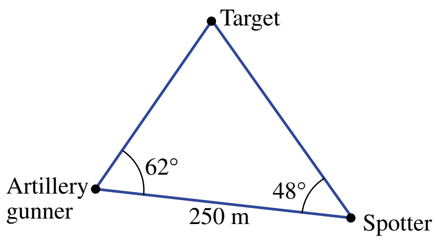
A 070°T
B 130°T
C 250°T
D 110°T
E 250°T



10E

9 The horizontal distance (to the nearest metre) that the artillery gunner shown at right needs to fire to reach the target is closest to:

A 198 m
B 210 m
C 235 m
D 250 m
E 266 m



10E

10 Two boats, P and Q, are to rendezvous at port R. At the moment boat P is 15 km due west of Q. For boat P, port R has a bearing of N49°E and for boat Q the bearing of the port is 334°T. The distance (to the nearest kilometre) that Q has to travel is:

A 10 km B 12 km C 13 km D 15 km E 18 km

10F

11 To find the distance across a large excavation, the measurements as shown in the diagram were found. The distance, AB, across the excavation in metres is:

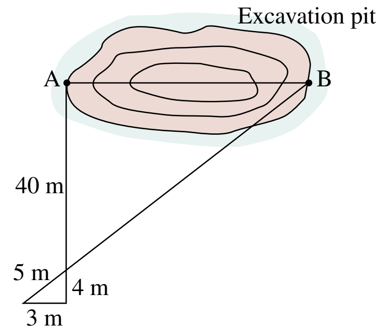
A $\frac{40}{4} \times 3$

B $\frac{40}{4} \times 5$

C $\frac{40}{3} \times 4$

D $\frac{4}{40} \times 3$

E $\frac{40}{5} \times 3$



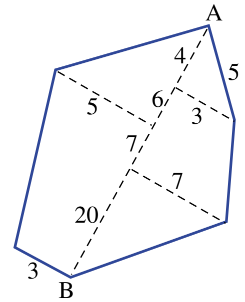
10F

12 Jennifer is standing 2 metres directly in front of her bedroom window, which is 1.5 metres wide. The width (to the nearest metre) of her view of a mountain 3 kilometres from her window is:

A 4003 metres B 4000 metres C 3000 metres D 2250 metres E 2252 metres

10G

13 For the irregular block shown, the appropriate surveyor's notes are:



A

	A	
	0	
	4	3
5	13	
	20	7
3	40	
	B	

B

	A	
	0	
	4	5
5	10	
	17	7
3	37	
	B	

C

	B	
	0	
	4	3
5	10	
	17	7
3	37	
	A	

D

	A	
	0	
3	4	
	10	5
7	17	
	37	3
	B	

E

	A	
	0	
	4	3
5	10	
	17	7
3	37	
	B	

10G

14 From the surveyor's notes at right the length (to the nearest metre) of the line joining the two vertices A and E is:

A 25 m
B 38 m
C 43 m
D 57 m
E 60 m

	G	
	0	
A20	20	
B25	30	
	45	10F
	55	5E
C10	80	
	100	
	D	

10H

15 For the contour map given, the average slope from A to B can be stated as:

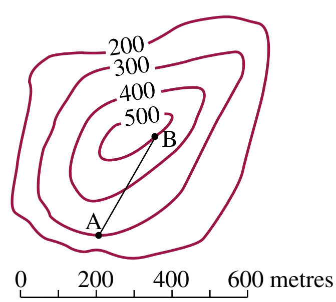
A $\tan\theta = \frac{500}{300}$

B $\sin\theta = \frac{300}{3}$

C $\tan\theta = \frac{300}{200}$

D $\tan\theta = \frac{200}{300}$

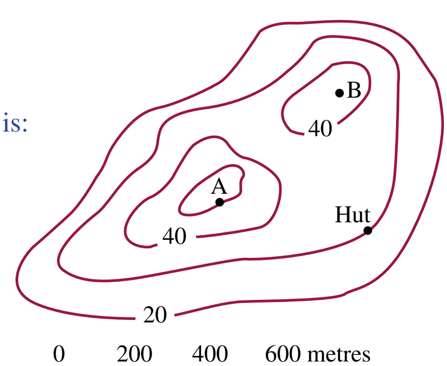
E $\cos\theta = \frac{200}{300}$



10H

16 The gradient from the hut to peak A is:

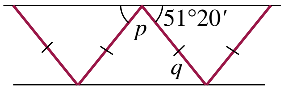
A 0.10
B 0.05
C $\frac{20}{300}$
D $\tan 20°$
E 20



Short answer

10A

1 A steel truss, which is to be used to reinforce the roof in a building, is designed as shown in the diagram at right. Find the values of p and q.

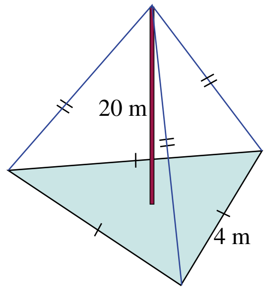


10B

2 Three guy-wires are used to support a 20-metre tall pole (as shown). Find:

a the length of a guy-wire (to 1 decimal place)

b the angle of elevation of the guy-wires.



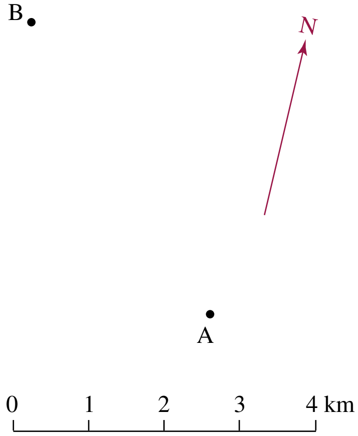
10C

3 a Use your protractor to find the compass bearing of:

i B from A **ii** A from B.

b What is the actual distance between A and B?

c Redraw the diagram and add a point C so that it lies at 050°T from B and 345°T from A.

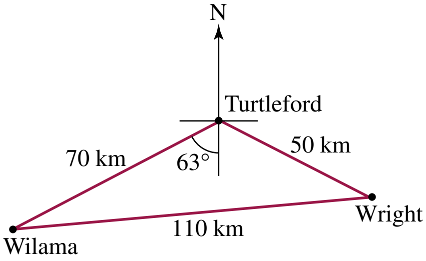


10D

4 A country valley is bounded by three major roads as shown.

a What is the direction from Turtleford to Wright if Turtleford to Wilama is S63°W?

b What is the bearing of Wilama from Wright?



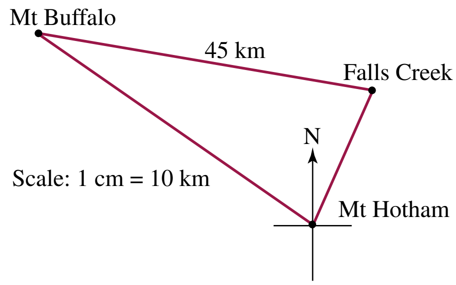
10D

5 Three communication towers are located as shown.

a Find the distance from the tower at Mt Hotham to:

i Falls Creek

ii Mt Buffalo.



b Find the direction from Mt Hotham to Falls Creek, given the direction from Mt Hotham to Mt Buffalo is N30°W and the distance from Falls Creek to Mt Buffalo is 45 km.

10E

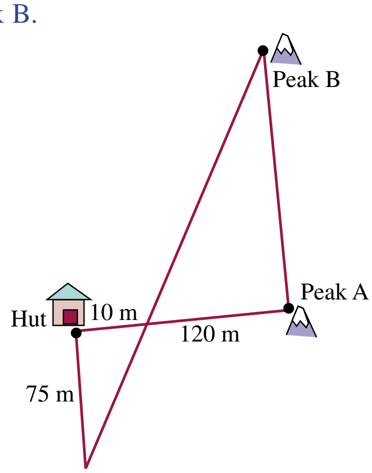
6 A communications station has received a distress signal from a yacht on a bearing of 206°T at a distance of 38 km. A rescue ship is 21 km from the station in a direction of S17°E.

a Find the distance in kilometres (to 1 decimal place) that the ship must travel to reach the stricken yacht.

b On what bearing must the ship sail to reach the yacht?

10F

7 A bushwalker has taken some measurements and drawn the diagram at right. Find the distance from peak A to peak B.



10G

8 Given the surveyor's notes shown at right, for an irregular shaped region, find:

a the bearing of F from A, given that D lies north of A

b the distance (to the nearest metre) from B to C.

	D	
	110	
	85	45E
C50	75	
	35	40F
B60	30	
	10	20G
	0	
	A	

10H

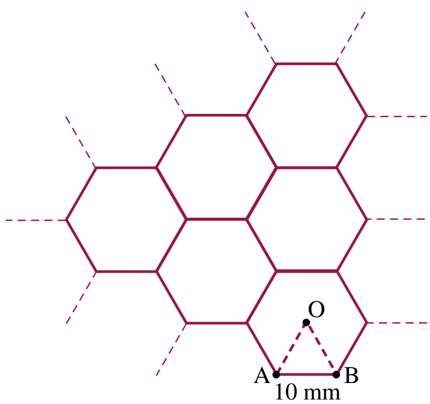
9 a Draw an appropriate contour map of an Egyptian pyramid.
b If the slope of the pyramid is $\frac{1}{2}$, find the angle of elevation (in degrees and minutes) of the top from the base edge.

10H

10 A ski run rises 120 metres from the baseline and the run is 1500 metres long.
a What is the angle of elevation of the slope (in degrees and minutes)?
b What is the gradient of the slope (expressed as a scale ratio, for example 1:20)?

Analysis

1 The laws of geometry are evident in the life of bees. In a cross-sectional view of a beehive with its regular hexagonal compartments:



a i find the height of $\triangle AOB$ (to 3 decimal places)
ii find the length of $\overline{AO}$
iii find the area of $\triangle AOB$ and thus the area (to 1 decimal place) of one hexagonal compartment.

In a beehive slab there are 500 compartments. Each compartment is 12 mm deep.

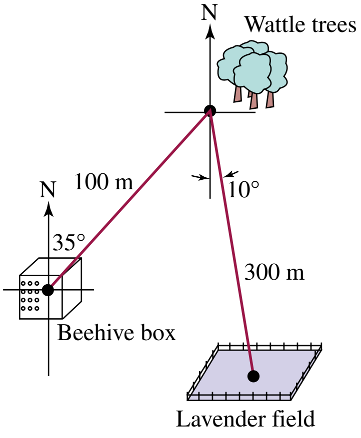
b i Find the volume of honey (to 1 decimal place) that would be stored in a single compartment.
ii Find the volume of honey collected from one slab of beehive. Express your answer to the nearest cubic centimetre.

The beekeeper decides to trial a synthetic beehive where the *linear dimensions* of the hexagonal compartment are *doubled*.

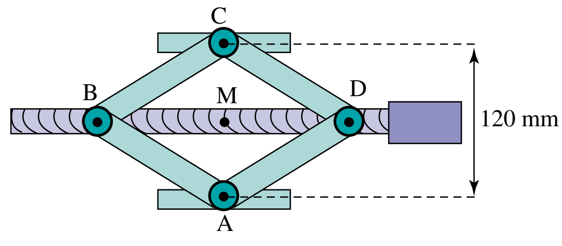
c i What is the scale factor for the new artificial hive compartment compared to the original?
ii What is the area (to 1 decimal place) of the new artificial hexagon?
iii What is the volume of honey that would be stored in a single artificial hexagonal compartment? (Answer in cubic centimetres to 1 decimal place.)

The beekeeper wishes the bees to collect the nectar from two prime locations — a field of lavender and group of wattle trees. A map of their respective locations is shown.

d i Find the distance (to the nearest metre) the bees have to travel to the lavender field from the hive box.
ii If the direction from the hive to the wattle trees is 035°T, what is the direction (to the nearest degree) of the lavender field from the hive box?
iii How far north (to the nearest metre) must the beekeeper move the hive box so that it is directly west from the wattle trees?



2 Every car should carry a jack . One type of jack used to raise a car is a scissor-jack. A simple diagram of a scissor-jack is given at right.



AB = BC = CD = AD = 200 mm

The threaded rod is rotated to increase or decrease the length of the line segment BD.

a **i** In the triangle BCD, M is the midpoint of BD. What is the length of CM?

ii If ∠BCD = 160°, what is the length of BD (to the nearest millimetre)?

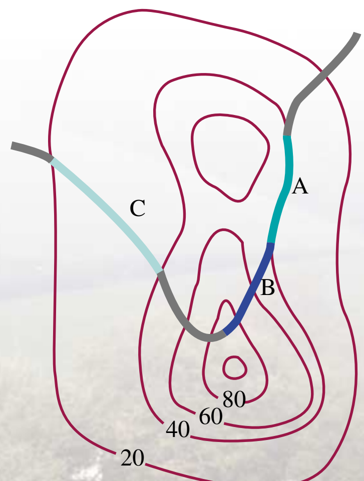
iii What is the size of angle MBC?

The jack is raised by reducing the length of the line segment BD.

b **i** If the height of the jack, AC, is raised to 250 mm, what is the length of BD (to the nearest mm)?

ii If angle MBC is 70°, what is the length of BD and what is the height of the jack?

3 **a** The scenic route, taken on a car trip through the hills, is shown on the contour map at right.



State the difference between sections A and B of the trip. Explain how this is indicated on the contour map.

b The average slope along section C of the route is 0.005.

i What is the horizontal distance covered by the car along this section of road?

ii What is the direct distance between the start and finish of this section of the road?

iii Find the map scale (in its simplest form) for the contour map

____ cm = ____ metres.