This document represents an updating of the 2000 IRP. This updating has been undertaken for the purpose of

- clarifying the Prescribed Learning Outcomes
- introducing Suggested Achievement Indicators
- addressing content overload

Resources previously recommended for the 2000 version of the curriculum, where still valid, continue to support this updated IRP. (See the Learning Resources section in this IRP for additional information.)

Applications of Mathematics 10 to 12

Integrated Resource Package 2006

British Columbia
The Best Place on Earth

Ministry of Education

IRP 136

Library and Archives Canada Cataloguing in Publication Data
Main entry under title:
Applications of mathematics 10 to 12 : integrated resource
package. – Rev. ed.

Also available on the Internet.
ISBN 0-7726-5499-9

1. Mathematics – Study and teaching (Secondary).
2. Business mathematics – Study and teaching (Secondary).
3. Finance, Personal – Mathematics – Study and teaching
(Secondary) I. British Columbia. Ministry of Education.

QA39.3.A66 2006 510.71'2 C2006-9600279

Acknowledgments

Preface

Introduction to Applications of Mathematics 10 to 12

Considerations for Program Delivery

Prescribed Learning Outcomes

Student Achievement

Learning Resources

Glossary

Many people contributed their expertise to this document. The Project Co-ordinator was Mr. Richard DeMerchant of the Ministry of Education, working with other ministry personnel and our partners in education. We would like to thank all who participated in this process

Applications of Mathematics 10 to 12 IRP Refinement Team

Ron Coleborn, School District No. 41 (Burnaby)	Preliminary Review
Brad Epp School, District No. 73 (Kamloops/Thompson)	Preliminary Review
Hold Fast Consultants Inc.	IRP writing and editing

This Integrated Resource Package (IRP) provides basic information teachers will require in order to implement Applications of Mathematics 10 to 12. This document supersedes the Applications of Mathematics 10 to 12 portion of the *Mathematics 10 to 12 Integrated Resource Package* (2000).

The information contained in this document is also available on the Internet at www.bced.gov.bc.ca/irp/irp.htm

The following paragraphs provide brief descriptions of the components of the IRP.

INTRODUCTION

The Introduction provides general information about Applications of Mathematics 10 to 12, including special features and requirements.

Included in this section are
- a rationale for teaching Applications of Mathematics 10 to 12 in BC schools
- the curriculum goals
- descriptions of the curriculum organizers – groupings for prescribed learning outcomes that share a common focus
- a suggested timeframe for each curriculum organizer
- a graphic overview of the curriculum content

CONSIDERATIONS FOR PROGRAM DELIVERY

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners.

PRESCRIBED LEARNING OUTCOMES

This section contains the *prescribed learning outcomes*, the legally required content standards for the provincial education system. The learning outcomes define the required knowledge, skills, and attitudes for each subject. They are statements of what students are expected to know and be able to do by the end of the course.

STUDENT ACHIEVEMENT

This section of the IRP contains information about classroom assessment and measuring student achievement, including sets of specific achievement indicators for each prescribed learning outcome. Achievement indicators are statements that describe what students should be able to do in order to demonstrate that they fully meet the expectations set out by the prescribed learning outcomes. Achievement indicators are not mandatory; they are provided to assist in the assessment of how well students achieve the prescribed learning outcomes.

Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of the prescribed learning outcomes.

LEARNING RESOURCES

This section contains general information on learning resources, and provides a link to titles, descriptions, and ordering information for the recommended learning resources in the Applications of Mathematics 10 to 12 Grade Collections.

GLOSSARY

The glossary defines selected terms used in this Integrated Resource Package.

Introduction

Applications of Mathematics 10 to 12

This Integrated Resource Package (IRP) sets out the provincially prescribed curriculum for Applications of Mathematics 10 to 12. The development of this IRP has been guided by the principles of learning:

- Learning requires the active participation of the student.
- People learn in a variety of ways and at different rates.
- Learning is both an individual and a group process.

In addition to these three principles, this document recognizes that British Columbia's schools include young people of varied backgrounds, interests, abilities, and needs. Wherever appropriate for this curriculum, ways to meet these needs and to ensure equity and access for all learners have been integrated as much as possible into the learning outcomes and achievement indicators.

The prescribed learning outcomes in the Applications of Mathematics 10 to 12 IRP are based on *The Common Curriculum Framework for K to 12 Mathematics* (Western and Northern Canadian Protocol for Collaboration in Basic Education, 1996). The Achievement Indicators were developed, in part, using the following documents:

- *Mathematics 10 to 12 Integrated Resource Package* (British Columbia Ministry of Education, 2000);
- *Applications of Mathematics 10 Provincial Examination Specifications* (British Columbia Ministry of Education, 2004);
- *Applications of Mathematics 12 Provincial Examination Specifications* (British Columbia Ministry of Education, 2004);
- *Outcomes with Assessment Standards for Applied Mathematics 10* (Alberta Learning, 2002);
- *Outcomes with Assessment Standards for Applied Mathematics 20* (Alberta Learning, 2002); and,
- *Applied Mathematics 30: Information Bulletin* (Alberta Learning, 2004).

This document represents an updating of the 2000 IRP. This updating has been undertaken for the purpose of

- clarifying the prescribed learning outcomes
- introducing suggested achievement indicators
- addressing content overload

Resources previously recommended for the 2000 version of the curriculum continue to support this updated IRP. (See the Learning Resources section later in this IRP for additional information.)

Applications of Mathematics 10 to 12, in draft form, was available for public review and response from November to December, 2005. Feedback from educators, students, parents, and other educational partners informed the development of this updated IRP.

Rationale

Mathematics is increasingly important in our technological society. Students today require the ability to reason and communicate, to solve problems, and to understand and use mathematics. Development of these skills helps students become numerate.

> Numeracy can be defined as the combination of mathematical knowledge, problem solving and communication skills required by all persons to function successfully within our technological world. Numeracy is more than knowing about numbers and number operations. (British Columbia Association of Mathematics Teachers, 1998)

Becoming numerate involves developing the ability to explore, conjecture, reason logically, and use a variety of mathematical methods to solve problems. It also involves the development of self-confidence and the ability to use quantitative and spatial information in problem solving and decision making. As students develop their numeracy skills and concepts, they generally grow more confident and motivated in their mathematical explorations. This growth occurs as they learn to enjoy and value mathematics, to think analytically, and to understand and appreciate the role of mathematics in everyday life.

The provincial mathematics curriculum emphasizes the development of numeracy skills and concepts and their practical application in higher education and the workplace. The curriculum places emphasis on probability and statistics, reasoning and communication, measurement, and problem solving. To ensure that students are prepared for the demands of both further education and the workplace, the graduate years of the mathematics curriculum (Grades 10 to 12) help students develop a more sophisticated sense of numeracy. At the same time, the curriculum investigates the creative and aesthetic aspects of mathematics by exploring the connections between mathematics, art, and design.

Requirements and Graduation Credits

Applications of Mathematics 10 and 11 *or* 12 are two of the courses available for students to satisfy the Graduation Program mathematics requirement. Applications of Mathematics 10, 11, and 12 are each designated as four-credit courses, and must be reported as such to the Ministry of Education for transcript purposes. Letter grades and percentages must be reported for these courses. It is not possible to obtain partial credit for these courses.

The course codes for Applications of Mathematics 10 to 12 are AMA 10, AMA 11, and AMA 12. These courses are also available in French (Applications des mathématiques 10, Applications des mathématiques 11, Applications des mathématiques 12; course codes AMAF 10, AMAF 11, AMAF 12).

Graduation Program Examination

Applications of Mathematics 10 has a Graduation Program examination, worth 20% of the final course mark. Students are required to take this exam to receive credit for the course. Applications of Mathematics 12 has an optional Graduation Program examination, worth 40% of the final course mark for students who choose to write it. Although students are not required to take this exam to receive credit for the course, they should be advised that some post-secondary institutions require Grade 12 exams to meet entrance requirements, and that writing Grade 12 exams also provides opportunities for provincial scholarships.

> For more information, refer to the Ministry of Education examinations web site: www.bced.gov.bc.ca/exams/

Goals for Applications of Mathematics 10 to 12

The Applications of Mathematics pathway provides a practical, contextual focus that encourages students to develop their mathematical knowledge, skills, and attitudes in the context of their lives and possible careers. The instructional approaches used to develop the required mathematical concepts emphasize concrete activities and modelling, with less emphasis on symbol manipulation. When needed, students should have access to technology that extends their basic skills and knowledge and allows them to repeatedly investigate and model mathematical concepts and issues.

Goals for Applications of Mathematics 10 to 12

The aim of Applications of Mathematics 10 to 12 is to prepare students for non-calculus based post-secondary programs of study such as certificate programs, diploma programs, continuing education programs, trades programs, technical programs, and some university programs.

Students will

- become numerate citizens with the ability to explore, conjecture, reason logically, and use a variety of mathematical methods to solve problems
- develop self-confidence and the ability to use quantitative and spatial information in problem solving and decision making
- learn to enjoy and value mathematics, to think analytically, and to understand and appreciate the role of mathematics in everyday life
- be prepared for the demands of both further education and the workplace and develop a more sophisticated sense of numeracy

Curriculum Organizers

A curriculum organizer consists of a set of prescribed learning outcomes that share a common focus. The prescribed learning outcomes for Applications of Mathematics 10 to 12 progress in age-appropriate ways, and are grouped under the following curriculum organizers and suborganizers.

Note that the ordering of these organizers and suborganizers is not intended to imply an order of instruction.

Curriculum Organizers and Suborganizers

Applications of Mathematics			
Number	**Patterns and Relations** • Patterns • Variables and Equations • Relations and Functions	**Shape and Space** • Measurement • 3-D Objects and 2-D Shapes	**Statistics and Probability** • Data Analysis • Chance and Uncertainty

Number
In this organizer, students further develop their number sense as they develop an intuitive feeling about numbers and their multiple relationships. It is important that students continue to develop computational fluency and the ability to detect arithmetic errors.

The Number organizer describes the knowledge and skills that students need in order to understand and perform calculations proficiently, decide which arithmetic operations can be used to solve problems, and then solve the problems.

Patterns and Relations
Students need to recognize, extend, create, and use patterns as a routine aspect of their lives. This organizer provides opportunities for students to look for relationships and describe these relationships mathematically. These relationships will be described visually, symbolically, orally, and in written form.

The Patterns and Relations organizer includes the following suborganizers:
- Patterns – use patterns to describe the world and solve problems
- Variables and Equations – represent algebraic expressions in multiple ways
- Relations and Functions – use algebraic and graphical models to generalize patterns, make predictions, and solve problems

Shape and Space
It is important that students look for and use similarity, congruence, patterns, transformations, and dilatations in the solution of a range of problems. This organizer provides opportunities for students to study geometric representations of algebraic relations and to develop and refine their reasoning skills.

The Shape and Space organizer includes the following suborganizers:
- Measurement – describe and compare everyday phenomena, using trigonometry
- 3-D Objects and 2-D Shapes – describe the characteristics of geometric objects and shapes and analyse the relationships among them

Statistics and Probability
Students must be able to solve problems involving data sets presented in relevant contexts. Students should also be able to model data graphically and symbolically so that patterns can be identified and used to solve problems.

The language that students use to describe chance becomes more sophisticated and involves the vocabulary of probability theory.

The Statistics and Probability organizer includes the following suborganizers:

- Data Analysis – collect, display, and analyse data to make predications about a population
- Chance and Uncertainty – use experimental or theoretical probability to represent and solve problems involving uncertainty

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems.

The following seven mathematical processes should be integrated within Applications of Mathematics 10 to 12.

Communication

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students need to be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine, 1991, p. 5).

Mental Mathematics and Estimation

Mental mathematics is a combination of cognitive strategies that enhances flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility.

Even more important than performing computational procedures or using calculators is the greater facility that students need – more than ever before – with estimation and mental mathematics (NCTM, May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001).

Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating.

Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type "How would you...?" or "How could you...?" the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

Reasoning

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to
- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels.

Visualization

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with the opportunity to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to decide when to measure, when to estimate and to know several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Applications of Mathematics 10 to 12: At a Glance

Applications of Mathematics 10	Applications of Mathematics 11	Applications of Mathematics 12
Number		
• use basic arithmetic operations on real numbers to solve problems • describe and apply arithmetic operations on tables to solve problems	• solve consumer problems, using arithmetic operations	• describe and apply operations on matrices to solve problems, using technology as required • design or use a spreadsheet to make and justify financial decisions
Patterns and Relations		
Relations and Functions • examine the nature of relations with an emphasis on functions • represent and analyse linear functions	**Variables and Equations** • represent and analyse situations that involve expressions and equations **Relations and Functions** • represent and analyse quadratic and exponential functions	**Patterns** • generate and analyse sinusoidal patterns
Shape and Space		
Measurement • solve problems involving triangles, including those found in 3-D and 2-D applications **3-D Objects and 2-D Shapes** • solve coordinate geometry problems involving lines and line segments	**Measurement** • use measuring devices to make estimates and to perform calculations in solving problems	**Measurement** • analyse objects, shapes, and processes to solve cost and design problems **3-D Objects and 2-D Shapes** • solve problems involving vectors in two and three dimensions
Statistics and Probability		
Data Analysis • implement and analyse sampling procedures, and draw appropriate inferences from collected data	**Data Analysis** • analyse graphs or charts of given situations to derive specific information	**Chance and Uncertainty** • solve problems based on the counting of sets, using the fundamental counting principle

Suggested Timeframe

Provincial curricula are developed in accordance with the amount of instructional time recommended by the Ministry of Education for each subject area. Teachers may choose to combine various curricula to enable students to integrate ideas and make meaningful connections.

In each of Grades 10, 11, and 12, a minimum of 100 hours of instructional time is recommended for the study of Applications of Mathematics. Although a four-credit course is typically equivalent to 120 hours, this timeframe allows for flexibility to address local needs.

The following table shows the number of hours suggested to deliver the prescribed learning outcomes in each curriculum organizer.

These estimations are provided as suggestions only; when delivering the prescribed curriculum teachers should adjust the instructional time as necessary.

Suggested Timeframe for Applications of Mathematics 10 to 12

Curriculum Organizer (Suborganizer)	Suggested Timeframe	Curriculum Organizer (Suborganizer)	Suggested Timeframe	Curriculum Organizer (Suborganizer)	Suggested Timeframe
Applications of Mathematics 10		**Applications of Mathematics 11**		**Applications of Mathematics 12**	
Number	20 - 25 hrs	**Number**	20 - 25 hrs	**Number**	20 - 25 hrs
Patterns and Relations		**Patterns and Relations**		**Patterns and Relations**	
(Relations and Functions)	25 - 30 hrs	(Variables and Equations)	15 - 20 hrs	(Patterns)	20 - 25 hrs
		(Relations and Functions)	20 - 25 hrs		
Shape and Space		**Shape and Space**		**Shape and Space**	
(Measurement)	20 - 25 hrs	(Measurement)	15 - 20 hrs	(Measurement)	15 - 20 hrs
(3-D Objects and 2-D Shapes)	10 - 15 hrs			(3-D Objects and 2-D Shapes)	20 - 25 hrs
Statistics and Probability		**Statistics and Probability**		**Statistics and Probability**	
(Data Analysis)	10 - 15 hrs	(Data Analysis)	15 - 20 hrs	(Chance and Uncertainty)	10 - 15 hrs

Considerations for Program Delivery

Applications of Mathematics 10 to 12

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners. Included in this section is information about

- Alternative Delivery policy
- instructional focus
- fostering the development of positive attitudes
- applying mathematics
- involving parents and guardians
- confidentiality
- inclusion, equity, and accessibility for all learners
- working with the school and community
- working with the Aboriginal community
- information and communications technology
- copyright and responsibility

ALTERNATIVE DELIVERY POLICY

The Alternative Delivery policy does not apply to Applications of Mathematics 10 to 12.

The Alternative Delivery policy outlines how students, and their parents or guardians, in consultation with their local school authority, may choose means other than instruction by a teacher within the regular classroom setting for addressing prescribed learning outcomes contained in the Health curriculum organizer of the following curriculum documents:

- Health and Career Education K to 7, and Personal Planning K to 7 Personal Development curriculum organizer (until September 2008)
- Health and Career Education 8 and 9
- Planning 10

The policy recognizes the family as the primary educator in the development of children's attitudes, standards, and values, but the policy still requires that all prescribed learning outcomes be addressed and assessed in the agreed-upon alternative manner of delivery.

It is important to note the significance of the term "alternative delivery" as it relates to the Alternative Delivery policy. The policy does not permit schools to omit addressing or assessing any of the prescribed learning outcomes within the health and career education curriculum. Neither does it allow students to be excused from meeting any learning outcomes related to health. It is expected that students who arrange for alternative delivery will address the health-related learning outcomes and will be able to demonstrate their understanding of these learning outcomes.

For more information about policy relating to alternative delivery, refer to www.bced.gov.bc.ca/policy/

INSTRUCTIONAL FOCUS

The Applications of Mathematics 10 to 12 courses are arranged into four organizers with problem solving integrated throughout. Decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations allows more time for concept development.

In addition to problem solving, other critical thinking processes – reasoning and making connections – are vital to increasing students' mathematical power and must be integrated throughout the program. A minimum of half the available time within all organizers should be dedicated to activities related to these processes.

Instruction should provide a balance between estimation and mental mathematics, paper and pencil exercises, and the appropriate use of technology, including calculators and computers. (It is assumed that all students have regular access to appropriate technology such as graphing calculators, or computers with graphing software and standard spreadsheet programs.) Concepts should be introduced using manipulatives, and gradually developed from the concrete to the pictorial to the symbolic.

FOSTERING THE DEVELOPMENT OF POSITIVE ATTITUDES

Students should be exposed to experiences that encourage them to enjoy and value mathematics, develop mathematical habits of mind, and understand and appreciate the role of mathematics in human affairs. They should be encouraged to explore, take risks, exhibit curiosity, and make and correct errors, so they gain confidence in their abilities to solve complex problems. The assessment of attitudes is indirect, and based on inferences drawn from students' behaviour. We can see what students do and hear what they say, and from these observations make inferences and draw conclusions about their attitudes.

Applying Mathematics

For students to view mathematics as relevant and useful, they must see how it can be applied to a wide variety of real-world applications. Mathematics helps students understand and interpret their world and solve problems that occur in their daily lives.

Involving Parents and Guardians

The family is essential in the development of students' attitudes and values. The school plays a supportive role by focussing on the prescribed learning outcomes in the Grades 10 to 12 Mathematics curriculum. Parents and guardians are encouraged to support, enrich, and extend the curriculum at home.

It is highly recommended that schools inform parents and guardians about the Applications of Mathematics curriculum. Teachers (along with school and district administrators) may choose to do so by

- informing parents/guardians and students, via a course outline at the beginning of the course, of the prescribed learning outcomes for the course
- responding to parent and guardian requests to discuss course unit plans, learning resources, etc.

Confidentiality

The *Freedom of Information and Protection of Privacy Act* (FOIPPA) applies to students, to school districts, and to all curricula. Teachers, administrators, and district staff should consider the following:

- Be aware of district and school guidelines regarding the provisions of FOIPPA and how it applies to all subjects, including Applications of Mathematics 10 to 12.
- Do not use students' Personal Education Numbers (PEN) on any assignments that students wish to keep confidential.
- Ensure students are aware that if they disclose personal information that indicates they are at risk for harm, then that information cannot be kept confidential.
- Inform students of their rights under FOIPPA, especially the right to have access to their own personal information in their school records. Inform parents of their rights to access their children's school records.
- Minimize the type and amount of personal information collected, and ensure that it is used only for purposes that relate directly to the reason for which it is collected.
- Inform students that they will be the only ones recording personal information about themselves unless they, or their parents, have consented to teachers collecting that information from other people (including parents).
- Provide students and their parents with the reason(s) they are being asked to provide personal information in the context of the Applications of Mathematics 10 to 12 curriculum.
- Inform students and their parents that they can ask the school to correct or annotate any of the personal information held by the school, in accordance with Section 29 of FOIPPA.
- Ensure students are aware that their parents may have access to the schoolwork they create only insofar as it pertains to students' progress.
- Ensure that any information used in assessing students' progress is up-to-date, accurate, and complete.

For more information about confidentiality, refer to www.mser.gov.bc.ca/privacyaccess/

Inclusion, Equity, and Accessibility for All Learners

British Columbia's schools include young people of varied backgrounds, interests, and abilities. The Kindergarten to Grade 12 school system focusses on meeting the needs of all students. When selecting specific topics, activities, and resources to support the implementation of Applications of Mathematics 10 to 12, teachers are encouraged to ensure that these choices support inclusion, equity, and accessibility for all students. In particular, teachers should ensure that classroom instruction, assessment, and resources reflect sensitivity to diversity and incorporate positive role portrayals, relevant issues, and themes such as inclusion, respect, and acceptance.

Government policy supports the principles of integration and inclusion of students for whom English is a second language and of students with special needs. Most of the prescribed learning outcomes and suggested achievement indicators in this IRP can be met by all students, including those with special needs and/or ESL needs. Some strategies may require adaptations to ensure that those with special and/or ESL needs can successfully achieve the learning outcomes. Where necessary, modifications can be made to the prescribed learning outcomes for students with Individual Education Plans.

For more information about resources and support for students with special needs, refer to www.bced.gov.bc.ca/specialed/

For more information about resources and support for ESL students, refer to www.bced.gov.bc.ca/esl/

Working with the School and Community

This curriculum addresses a wide range of skills and understandings that students are developing in other areas of their lives. It is important to recognize that learning related to this curriculum extends beyond the Mathematics classroom.

Community organizations may also support the curriculum with locally developed learning resources, guest speakers, workshops, and field studies. Teachers may wish to draw on the expertise of these community organizations and members.

Working with the Aboriginal Community

The Ministry of Education is dedicated to ensuring that the cultures and contributions of Aboriginal peoples in BC are reflected in all provincial curricula. To address these topics in the classroom in a way that is accurate and that respectfully reflects Aboriginal concepts of teaching and learning, teachers are strongly encouraged to seek the advice and support of local Aboriginal communities. Aboriginal communities are diverse in terms of language, culture, and available resources, and each community will have its own unique protocol to gain support for integration of local knowledge and expertise. To begin discussion of possible instructional and assessment activities, teachers should first contact Aboriginal education co-ordinators, teachers, support workers, and counsellors in their district who will be able to facilitate the identification of local resources and contacts such as elders, chiefs, tribal or band councils, Aboriginal cultural centres, Aboriginal Friendship Centres, and Métis or Inuit organizations.

In addition, teachers may wish to consult the various Ministry of Education publications available, including the “Planning Your Program” section of the resource, *Shared Learnings.* This resource was developed to help all teachers provide students with knowledge of, and opportunities to share experiences with, Aboriginal peoples in BC.

For more information about these documents, consult the Aboriginal Education web site: www.bced.gov.bc.ca/abed/welcome.htm

Information and Communications Technology

The study of information and communications technology is increasingly important in our society. Students need to be able to acquire and analyse information, to reason and communicate, to make informed decisions, and to understand and use information and communications technology for a variety of purposes. Development of these skills is important for students in their education, their future careers, and their everyday lives.

Literacy in the area of information and communications technology can be defined as the ability to obtain and share knowledge through investigation, study, instruction, or transmission of information by means of media technology. Becoming literate in this area involves finding, gathering, assessing, and communicating information using electronic means, as well as developing the knowledge and skills to use and solve problems effectively with the technology. Literacy also involves a critical examination and understanding of the ethical and social issues related to the use of information and communications technology. When planning for instruction and assessment in Applications of Mathematics 10 to 12, teachers should provide opportunities for students to develop literacy in relation to information and communications technology sources, and to reflect critically on the role of these technologies in society.

Copyright and Responsibility

Copyright is the legal protection of literary, dramatic, artistic, and musical works; sound recordings; performances; and communications signals. Copyright provides creators with the legal right to be paid for their work and the right to say how their work is to be used. The law permits certain exceptions for schools (i.e., specific things permitted) but these are very limited, such as copying for private study or research. The copyright law determines how resources can be used in the classroom and by students at home.

In order to respect copyright it is necessary to understand the law. It is unlawful to do the following, unless permission has been given by a copyright owner:

- photocopy copyrighted material to avoid purchasing the original resource for any reason
- photocopy or perform copyrighted material beyond a very small part – in some cases the copyright law considers it "fair" to copy whole works, such as an article in a journal or a photograph, for purposes of research and private study, criticism, and review
- show recorded television or radio programs to students in the classroom unless these are cleared for copyright for educational use (there are exceptions such as for news and news commentary taped within one year of broadcast that by law have record-keeping requirements – see the web site at the end of this section for more details)
- photocopy print music, workbooks, instructional materials, instruction manuals, teacher guides, and commercially available tests and examinations
- show videorecordings at schools that are not cleared for public performance
- perform music or do performances of copyrighted material for entertainment (i.e., for purposes other than a specific educational objective)
- copy work from the Internet without an express message that the work can be copied.

Permission from or on behalf of the copyright owner must be given in writing. Permission may also be given to copy or use all or some portion of copyrighted work through a licence or agreement. Many creators, publishers, and producers have formed groups or "collectives" to negotiate royalty payments and copying conditions for educational institutions. It is important to know what licences are in place and how these affect the activities schools are involved in. Some licences may also require royalty payments that are determined by the quantity of photocopying or the length of performances. In these cases, it is important to assess the educational value and merits of copying or performing certain works to protect the school's financial exposure (i.e., only copy or use that portion that is absolutely necessary to meet an educational objective).

It is important for education professionals, parents, and students to respect the value of original thinking and the importance of not plagiarizing the work of others. The works of others should not be used without their permission.

For more information about copyright, refer to http://cmec.ca/copyright/indexe.stm

Prescribed Learning Outcomes

Applications of Mathematics 10 to 12

Prescribed learning outcomes are content standards for the provincial education system; they are the prescribed curriculum. Clearly stated and expressed in measurable and observable terms, learning outcomes set out the required knowledge, skills, and attitudes – what students are expected to know and be able to do – by the end of the specified course.

Schools have the responsibility to ensure that all prescribed learning outcomes in this curriculum are met; however, schools have flexibility in determining how delivery of the curriculum can best take place.

It is expected that student achievement will vary in relation to the learning outcomes. Evaluation, reporting, and student placement with respect to these outcomes are dependent on the professional judgment and experience of teachers, guided by provincial policy.

Prescribed learning outcomes for Applications of Mathematics 10 to 12 are presented by curriculum organizer and suborganizer, and are coded alphanumerically for ease of reference; however, this arrangement is not intended to imply a required instructional sequence.

Wording of Prescribed Learning Outcomes

All prescribed learning outcomes complete the stem, "It is expected that students will…."

When used in a prescribed learning outcome, the word "**including**" indicates that any ensuing item **must be addressed**. Lists of items introduced by the word "including" represent a set of minimum requirements associated with the general requirement set out by the outcome. The lists are not necessarily exhaustive, however, and teachers may choose to address additional items that also fall under the general requirement set out by the outcome.

Conversely, the abbreviation "**e.g.**" (for example) in a prescribed learning outcome indicates that the ensuing items are provided for illustrative purposes or clarification, and **are not required**. Presented in parentheses, the list of items introduced by "e.g." is neither exhaustive nor prescriptive, nor is it put forward in any special order of importance or priority. Teachers are free to substitute items of their own choosing that they feel best address the intent of the prescribed learning outcome.

Domains of Learning

Prescribed learning outcomes in BC curricula identify required learning in relation to one or more of the three domains of learning: cognitive, psychomotor, and affective. The following definitions of the three domains are based on Bloom's taxonomy.

The **cognitive domain** deals with the recall or recognition of knowledge and the development of intellectual abilities. The cognitive domain can be further specified as including three cognitive levels: knowledge, understanding and application, and higher mental processes. These levels are determined by the verb used in the learning outcome, and illustrate how student learning develops over time.

- *Knowledge* includes those behaviours that emphasize the recognition or recall of ideas, material, or phenomena.
- *Understanding and application* represents a comprehension of the literal message contained in a communication, and the ability to apply an appropriate theory, principle, idea, or method to a new situation.
- *Higher mental processes* include analysis, synthesis, and evaluation. The higher mental processes level subsumes both the knowledge and the understanding and application levels.

The **affective domain** concerns attitudes, beliefs, and the spectrum of values and value systems.

The **psychomotor domain** includes those aspects of learning associated with movement and skill demonstration, and integrates the cognitive and affective consequences with physical performances.

Domains of learning and, particularly, cognitive levels, inform the design and development of the Graduation Program examinations for Applications of Mathematics 10 and 12.

Prescribed Learning Outcomes: Applications of Mathematics 10

It is expected that students will:

NUMBER

A1 perform arithmetic operations on irrational numbers, using appropriate decimal approximations
A2 create and modify tables or spreadsheets from both recursive and non-recursive situations
A3 use and modify a spreadsheet template to model recursive situations
A4 solve problems involving combinations of tables, using
- addition or subtraction of two tables
- multiplication of a table by a real number
- spreadsheet functions and templates

PATTERNS AND RELATIONS

Relations and Functions

B1 plot linear data, using appropriate scales
B2 represent linear data using linear function models, including
- ordered pairs
- word descriptions
- graphs
- equations
- table of values

B3 use function notation to evaluate and represent linear functions
B4 determine the following characteristics of the graph of a linear function, given its equation or graph:
- x- and y-intercepts
- slope
- domain
- range

B5 construct the graph of a linear function given its equation in slope-intercept form ($y = mx + b$):
- manually
- using a graphing calculator

B6 solve problems involving partial variation and arithmetic sequences as applications of linear functions

Prescribed Learning Outcomes: Applications of Mathematics 10

Shape and Space

Measurement

C1 solve problems involving two right triangles using trigonometry and the Pythagorean Theorem
C2 extend the concepts of sine and cosine for angles through to 180°
C3 apply the sine and cosine laws to solve problems (excluding the ambiguous case)
C4 find lengths, areas, volumes, and mass using measurement strategies, appropriate units of measure (SI and Imperial systems), and appropriate instruments, including
- tape measure
- metre/yard stick
- ruler
- trundle wheel
- bathroom scale
- decigram scale
- Vernier calipers
- micrometer

C5 calculate the volume and surface area of a sphere, using formulas that are provided
C6 determine the relationships among linear scale factors, areas, the surface areas, and the volumes of similar figures and objects
C7 solve problems involving length, area, volume, time, mass, and rates derived from these
C8 interpret drawings, and use the information to solve problems

3-D Objects and 2-D Shapes

C9 solve problems involving distances between points in the coordinate plane
C10 solve problems involving midpoints of line segments
C11 solve problems involving rise, run, and slope of line segments
C12 determine the equation of a line, given information that uniquely determines the line

Statistics and Probability

Data Analysis

D1 determine the equation of a line of best fit, using
- estimate of slope and one point
- least squares method with technology

D2 use a calculator to determine the correlation coefficient (r) of a data set
D3 interpret the correlation coefficient (r) and its limitations for varying problem situations, using relevant scatter plots
D4 apply line-fitting and correlation techniques to analyse experimental results

Prescribed Learning Outcomes: Applications of Mathematics 11

It is expected that students will:

NUMBER

A1 solve consumer problems, including
- wages earned in various situations
- property taxation
- exchange rates
- unit prices

A2 reconcile financial statements, including
- chequebooks with bank statements
- cash register tallies with daily receipts

A3 solve budget problems using graphs and tables to communicate solutions

A4 solve investment and credit problems involving simple and compound interest

PATTERNS AND RELATIONS

Variables and Equations

B1 graph linear inequalities, in two variables

B2 solve systems of linear equations, in two variables:
- algebraically (elimination and substitution)
- graphically

B3 solve nonlinear equations (i.e., quadratic and exponential), using a graphing calculator

B4 solve systems of linear inequalities, in two variables, using a graphing calculator

B5 use systems of linear inequalities, in two variables, to model and solve problems

B6 use nonlinear systems of equations (i.e., linear/quadratic and linear/exponential), in two variables, to model and solve problems

B7 apply linear programming to find optimal solutions to decision-making problems

Relations and Functions

B8 determine the following characteristics of the graph of a quadratic function:
- vertex
- domain and range
- axis of symmetry
- intercepts

Prescribed Learning Outcomes: Applications of Mathematics 11

Shape and Space

Measurement

C1 enlarge or reduce a dimensioned object, according to a specified scale
C2 calculate maximum and minimum values, using tolerances, for lengths, areas, and volumes
C3 solve problems involving percentage error when input variables are expressed with percentage errors

Statistics and Probability

Data Analysis

D1 extract information from given graphs of discrete or continuous data, using
- time series
- continuous data
- contour lines

D2 draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data
D3 design different ways of presenting data and analysing results, by focussing on the truthful display of data and the clarity of presentation
D4 collect experimental data and use best-fit quadratic and exponential functions, to make predictions and solve problems

Prescribed Learning Outcomes: Applications of Mathematics 12

It is expected that students will:

NUMBER

A1 model and solve problems, including those solved previously, using technology to perform matrix operations, including
- addition
- subtraction
- scalar multiplication

A2 model and solve consumer and network problems using technology to perform matrix multiplication as required

A3 design a financial spreadsheet template to allow users to input their own variables

A4 analyse the costs and benefits of renting or buying an increasing asset, such as land or property, under various circumstances

A5 analyse the costs and benefits of leasing or buying a decreasing asset, such as a vehicle or computer, under various circumstances

A6 analyse an investment portfolio applying such concepts as interest rate, rate of return, and total return

PATTERNS AND RELATIONS

Patterns

B1 describe sinusoidal curves using terms, including
- amplitude
- period
- maximum and minimum values
- vertical and horizontal shift

B2 graph sinusoidal data using technology, and represent the data with a best fit equation of the form $y = a \sin(bx + c) + d$

B3 use best fit sinusoidal equations, and their associated graphs, to make predictions (interpolation, extrapolation)

Prescribed Learning Outcomes: Applications of Mathematics 12

SHAPE AND SPACE

Measurement

C1 use dimensions and unit prices to solve problems involving perimeter, area, and volume
C2 solve problems involving estimation and costing for objects, shapes, or processes when a design is given
C3 design an object, shape, layout, or process within a specified budget
C4 use simplified models to estimate the solutions to complex measurement problems

3-D Objects and 2-D Shapes

C5 use appropriate terminology to describe
- vectors (i.e., direction, magnitude)
- scalar quantities (i.e., magnitude)

C6 assign meaning to the multiplication of a vector by a scalar
C7 determine the magnitude and direction of a resultant vector, using triangle or parallelogram methods
C8 model and solve problems in 2-D and 3-D (with 3-D vectors restricted to those that are mutually orthogonal), using vector diagrams and technology

STATISTICS AND PROBABILITY

Chance and Uncertainty

D1 classify events as independent or dependent
D2 use the fundamental counting principle to determine the number of different ways to perform multi-step operations
D3 construct a sample space for two or three events
D4 solve problems, using the probabilities of mutually exclusive and complementary events

STUDENT ACHIEVEMENT

Applications of Mathematics 10 to 12

This section of the IRP contains information about classroom assessment and student achievement, including specific achievement indicators to assist in the assessment of student achievement in relation to each prescribed learning outcome. Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of prescribed learning outcomes.

Classroom Assessment and Evaluation

Assessment is the systematic gathering of information about what students know, are able to do, and are working toward. Assessment evidence can be collected using a wide variety of methods, such as

- observation
- student self-assessments and peer assessments
- quizzes and tests (written, oral, practical)
- samples of student work
- projects and presentation
- oral and written reports
- journals and learning logs
- performance reviews
- portfolio assessments

Assessment of student performance is based on the information collected through assessment activities. Teachers use their insight, knowledge about learning, and experience with students, along with the specific criteria they establish, to make judgments about student performance in relation to prescribed learning outcomes.

Three major types of assessment can be used in conjunction to support student achievement.

- Assessment **for** learning is assessment for purposes of greater learning achievement.
- Assessment **as** learning is assessment as a process of developing and supporting students' active participation in their own learning.
- Assessment **of** learning is assessment for purposes of providing evidence of achievement for reporting.

Assessment for Learning

Classroom assessment for learning provides ways to engage and encourage students to become involved in their own day-to-day assessment – to acquire the skills of thoughtful self-assessment and to promote their own achievement.

This type of assessment serves to answer the following questions:

- What do students need to learn to be successful?
- What does the evidence of this learning look like?

Assessment for learning is criterion-referenced, in which a student's achievement is compared to established criteria rather than to the performance of other students. Criteria are based on prescribed learning outcomes, as well as on suggested achievement indicators or other learning expectations.

Students benefit most when assessment feedback is provided on a regular, ongoing basis. When assessment is seen as an opportunity to promote learning rather than as a final judgment, it shows students their strengths and suggests how they can develop further. Students can use this information to redirect their efforts, make plans, communicate with others (e.g., peers, teachers, parents) about their growth, and set future learning goals.

Assessment for learning also provides an opportunity for teachers to review what their students are learning and what areas need further attention. This information can be used to inform teaching and create a direct link between assessment and instruction. Using assessment as a way of obtaining feedback on instruction supports student achievement by informing teacher planning and classroom practice.

Assessment as Learning

Assessment as learning actively involves students in their own learning processes. With support and guidance from their teacher, students take responsibility for their own learning, constructing meaning for themselves. Through a process of continuous self-assessment, students develop the ability to take stock of what they have already learned, determine what they have not yet learned, and decide how they can best improve their own achievement.

Although assessment as learning is student-driven, teachers can play a key role in facilitating how this assessment takes place. By providing regular opportunities for reflection and self-assessment, teachers can help students develop, practise, and become comfortable with critical analysis of their own learning.

Assessment of Learning

Assessment of learning can be addressed through summative assessment, including large-scale assessments and teacher assessments. These summative assessments can occur at the end of the year or at periodic stages in the instructional process.

Large-scale assessments, such as Foundation Skills Assessment (FSA) and Graduation Program exams, gather information on student performance throughout the province and provide information for the development and revision of curriculum. These assessments are used to make judgments about students' achievement in relation to provincial and national standards.

Assessment of learning is also used to inform formal reporting of student achievement.

For Ministry of Education reporting policy, refer to www.bced.gov.bc.ca/policy/policies/student_reporting.htm

Assessment *for* Learning	Assessment *as* Learning	Assessment *of* Learning
Formative assessment is *ongoing in the classroom*	**Formative assessment is *ongoing in the classroom***	**Summative assessment occurs *at end of year or at key stages***
• teacher assessment, student self-assessment, and/or student peer assessment • criterion-referenced – criteria based on prescribed learning outcomes identified in the provincial curriculum, reflecting performance in relation to a specific learning task • involves both teacher and student in a process of continual reflection and review about progress • teachers adjust their plans and engage in corrective teaching in response to formative assessment	• self-assessment • provides students with information on their own achievement and prompts them to consider how they can continue to improve their learning • student-determined criteria based on previous learning and personal learning goals • students use assessment information to make adaptations to their learning process and to develop new understandings	• teacher assessment • may be either criterion-referenced (based on prescribed learning outcomes) or norm-referenced (comparing student achievement to that of others) • information on student performance can be shared with parents/guardians, school and district staff, and other education professionals (e.g., for the purposes of curriculum development) • used to make judgments about students' performance in relation to provincial standards

For more information about assessment for, as, and of learning, refer to the following resource developed by the Western and Northern Canadian Protocol (WNCP): *Rethinking Assessment with Purpose in Mind*.

This resource is available online at www.wncp.ca/

Criterion-Referenced Assessment and Evaluation

In criterion-referenced evaluation, a student's performance is compared to established criteria rather than to the performance of other students. Evaluation in relation to prescribed curriculum requires that criteria be established based on the learning outcomes.

Criteria are the basis for evaluating student progress. They identify, in specific terms, the critical aspects of a performance or a product that indicate how well the student is meeting the prescribed learning outcomes. For example, weighted criteria, rating scales, or scoring guides (reference sets) are ways that student performance can be evaluated using criteria.

Wherever possible, students should be involved in setting the assessment criteria. This helps students develop an understanding of what high-quality work or performance looks like.

Criterion-referenced assessment and evaluation may involve these steps:	
Step 1	Identify the prescribed learning outcomes and suggested achievement indicators (as articulated in this IRP) that will be used as the basis for assessment.
Step 2	Establish criteria. When appropriate, involve students in establishing criteria.
Step 3	Plan learning activities that will help students gain the knowledge, skills, and attitudes outlined in the criteria.
Step 4	Prior to the learning activity, inform students of the criteria against which their work will be evaluated.
Step 5	Provide examples of the desired levels of performance.
Step 6	Conduct the learning activities.
Step 7	Use appropriate assessment instruments (e.g., rating scale, checklist, scoring guide) and methods (e.g., observation, collection, self-assessment) based on the particular assignment and student.
Step 8	Review the assessment data and evaluate each student's level of performance or quality of work in relation to criteria.
Step 9	Where appropriate, provide feedback and/or a letter grade to indicate how well the criteria are met.
Step 10	Communicate the results of the assessment and evaluation to students and parents/guardians.

Key Elements

Key elements provide an overview of content in each curriculum organizer. They can be used to determine the expected depth and breadth of the prescribed learning outcomes.

Note that some topics appear at multiple grade levels in order to emphasize their importance and to allow for developmental learning.

Achievement Indicators

To support the assessment of provincially prescribed curricula, this IRP includes sets of achievement indicators in relation to each learning outcome.

Achievement indicators, taken together as a set, define the specific level of knowledge acquired, skills applied, or attitudes demonstrated or by the student in relation to a corresponding prescribed learning outcome. They describe what evidence to look for to determine whether or not a student has fully met the intent of the learning outcome. Since each achievement indicator defines only one aspect of the corresponding learning outcome, the entire set of achievement indicators should be considered when determining whether students have fully met the learning outcome.

In some cases, achievement indicators may also include suggestions as to the type of task that would provide evidence of having met the learning outcome (e.g., problem solving; a constructed response such as a list, comparison, analysis, or chart; a product created and presented such as a report, poster, or model; a particular skill demonstrated).

Achievement indicators support the principles of assessment *for* learning, assessment *as* learning, and assessment *of* learning. They provide teachers and parents with tools that can be used to reflect on what students are learning, as well as provide students with a means of self-assessment and ways of defining how they can improve their own achievement.

Achievement indicators are not mandatory; they are suggestions only, provided to assist in the assessment of how well students achieve the prescribed learning outcomes.

Achievement indicators may be useful to provincial examination development teams and inform the development of exam items. However, examination questions, item formats, exemplars, rubrics, or scoring guides will not necessarily be limited to the achievement indicators as outlined in the Integrated Resource Packages.

> Specifications for provincial examinations are available online at www.bced.gov.bc.ca/exams/specs/

The following pages contain the suggested achievement indicators corresponding to each prescribed learning outcome for the Applications of Mathematics 10 to 12 curriculum. The achievement indicators are arranged by curriculum organizer and suborganizer for each grade; however, this order is not intended to imply a required sequence of instruction and assessment.

STUDENT ACHIEVEMENT

Applications of Mathematics 10

Key Elements: Applications of Mathematics 10

Mathematical Process (Integrated)

The following mathematical processes have been integrated within the prescribed learning outcomes and achievement indicators for the course: communication, problem solving, connections, mental mathematics and estimation, reasoning, technology, and visualization.

NUMBER

- basic arithmetic operations on real numbers, including decimal number approximations (e.g., square roots) and irrational numbers
- spreadsheets to model recursive and non-recursive situations
- arithmetic operations on tables to solve problems

PATTERNS AND RELATIONS

Relations and Functions
- linear data sets and associated graphs
- function notation
- characteristics of linear functions, including
  - slope intercept form
  - partial variation
  - arithmetic sequences
  - interpolation
  - extrapolation

SHAPE AND SPACE

Measurement
- triangle properties (e.g., trigonometric ratios, Pythagorean Theorem)
- sine and cosine law (excluding the ambiguous case)
- measurement instruments and strategies
- linear scale factors, areas, and volume
- diagrams for solving problems involving measurements

3-D Objects and 2-D Shapes
- coordinate geometry problems, including distance between points, midpoints, and line segments
- slopes of lines, including horizontal and vertical lines
- equations of lines

STATISTICS AND PROBABILITY

Data Analysis
- lines of best fit (e.g., scatterplots)
- correlation coefficients

Number

Students demonstrate an understanding of and proficiency with calculations, including making decisions concerning which arithmetic operation or operations to use, to solve a problem and then solve the problem.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
A1 perform arithmetic operations on irrational numbers, using appropriate decimal approximations	❑ express a given irrational number, using appropriate decimal number approximations in a given context ❑ manipulate given equations involving simple operations of multiplying and dividing irrational numbers ❑ manipulate given equations that involve roots, powers, or order of operations ❑ perform arithmetic operations on given irrational numbers such as square roots, cube roots, and π, using a calculator ❑ solve given problems involving at least one arithmetic operation on irrational numbers (e.g., surface area and volume of spheres, perimeter and area of circles)
A2 create and modify tables or spreadsheets from both recursive and non-recursive situations	❑ insert rows and columns in a given table or spreadsheet to accommodate changed contexts ❑ modify an initial value in a given table or spreadsheet and determine the effect on values (recursive and non-recursive) calculated using the initial value (e.g., the effect on total price of grocery items listed in a table or spreadsheet if the sales tax is changed) ❑ calculate the amounts for subsequent cells in a given recursive table ❑ create a table to solve a given problem
A3 use and modify a spreadsheet template to model recursive situations	❑ insert or delete rows and columns in a given spreadsheet as needed to model a recursive situation ❑ modify provided formulas in given spreadsheet templates ❑ state or create new formulas in modifying provided spreadsheets
A4 solve problems involving combinations of tables, using – addition or subtraction of two tables – multiplication of a table by a real number – spreadsheet functions and templates	❑ combine given spreadsheets and tables when provided with formulas (i.e., that have the same dimensions) ❑ combine given spreadsheets using addition, subtraction, multiplication, division, and sum function ❑ combine given spreadsheets and tables when formulas are not given ❑ solve problems that involve combining tables

Patterns and Relations

Students use algebraic and graphical models to generalize patterns, make predictions, and solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Relations and Functions B1 plot linear data, using appropriate scales	❑ plot a given data set using paper and pencil ❑ plot a given data set using a graphing calculator ❑ choose appropriate scales, axis labels, and titles when drawing graphs manually ❑ choose appropriate window settings when using a graphing calculator to graph a given data set
B2 represent linear data using linear function models, including – ordered pairs – word descriptions – graphs – equations – table of values	❑ make a list of values, list ordered pairs, sketch a graph, and write a rule in words for a given linear function ❑ write a rule in words for a linear function whose equation is given ❑ generate sketches of graphs, given real-life relationships between variables ❑ describe real-life matching situations for graphs of linear functions, with the descriptions roughly incorporating the meanings of key points
B3 use function notation to evaluate and represent linear functions	❑ evaluate given functions for rational number inputs ❑ use the features of a given graph or table to interpret notation
B4 determine the following characteristics of the graph of a linear function, given its equation or graph: – x- and y-intercepts – slope – domain – range *Suborganizer 'Relations and Functions' continued on page 37*	❑ identify the slope and y-intercept, given the equation in slope-intercept form ❑ explain the meaning of the slope and y-intercept in a given situation ❑ manipulate a given equation into slope-intercept form ❑ determine the x-intercept and y-intercept from the graph and equation of a given linear function ❑ state the domain and range of given graphs and equations ❑ determine the domain and range from given word problems

Prescribed Learning Outcomes	Suggested Achievement Indicators
Suborganizer 'Relations and Functions' continued from page 36	
B5 construct the graph of a linear function from its equation in slope-intercept form (y = mx + b): – manually – using a graphing calculator	❑ sketch the graph of a function whose equation is given in slope-intercept form ❑ choose appropriate scales, axis labels, and titles when drawing graphs ❑ determine the window parameters that best display the characteristics of a given graph ❑ carry out one-step or two-step manipulations of given linear equations, so the equation is in slope-intercept form that can then be sketched manually ❑ carry out one-step or two-step manipulations of given linear equations, so a form of the equation is produced that can be entered into a graphing calculator
B6 solve problems involving partial variation and arithmetic sequences as applications of linear functions	❑ determine the equation for a given partial variation problem (including direct variation situations) ❑ recognize that direct and partial variation problems are represented by linear functions ❑ create a graph representing a given direct and partial variation problem ❑ determine the numerical value of slope, and interpret slope in a given context, such as speed ❑ interpolate given graphs to make estimates ❑ extrapolate given graphs to make estimates ❑ explain the significance of x- and y-intercepts related to a given linear function or graph of a given linear function

Shape and Space

Students describe and compare everyday phenomena, using either direct or indirect measurement, describe the characteristics of 3-D objects and 2-D shapes, and analyse the relationships among them.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Measurement C1 solve problems involving two right triangles using trigonometry and the Pythagorean Theorem	❑ solve given problems involving two right triangles, where the shared side is a given value ❑ solve two-dimensional problems, if the diagram is given ❑ solve given problems involving two right triangles, where the shared side is determined from given information and is needed to solve the second triangle
C2 extend the concepts of sine and cosine for angles through to 180°	❑ use a calculator to find the values of the sine and cosine of given angles from 0° to 180° ❑ use a calculator to find the angle between 0° and 180°, given the value for the cosine of the angle ❑ use a calculator to find the angle between 0° and 180°, given the value for the sine of the angle ❑ illustrate the use of sine and cosine ratios in a given triangle with one angle between 90° and 180°
C3 apply the sine and cosine laws to solve problems (excluding the ambiguous case) *Suborganizer 'Measurement' continued on page 39*	❑ determine any side of a triangle, given the diagram and using either the sine or cosine law provided ❑ determine any acute angle of a triangle, given the diagram and using the sine law provided ❑ determine any angle of a triangle, given the diagram and using the cosine law provided ❑ determine any side or angle of a triangle, using the provided sine or cosine law, whether or not a diagram is given ❑ use sine and cosine laws to solve given problems involving more than one triangle

Prescribed Learning Outcomes	Suggested Achievement Indicators
Suborganizer 'Measurement' continued from page 38	
C4 find lengths, areas, volumes, and mass using measurement strategies, appropriate units of measure (SI and Imperial systems), and appropriate instruments, including – tape measure – metre/yard stick – ruler – trundle wheel – bathroom scale – decigram scale – Vernier calipers – micrometer	❑ select and justify a measuring device to determine given linear measurements ❑ determine a given linear measurement and explain the strategy used ❑ select and justify a measuring device to determine the area of a given object ❑ determine the area of a given shape or the surface area of a given object and explain the strategy used ❑ select and justify a measuring device to determine the volume of a given object ❑ determine the volume of a given object and explain the strategy used ❑ determine the mass of a given object and explain the strategy used
C5 calculate the volume and surface area of a sphere, using formulas that are provided	❑ use given formulas and dimensions to calculate the volume and surface area of a sphere ❑ determine the volume of a given sphere in mL or cm^3 ❑ determine the radius or diameter of a given sphere, using an appropriate formula
C6 determine the relationships among linear scale factors, areas, the surface areas, and the volumes of similar figures and objects	❑ determine linear scale factors from given length measures ❑ find area and volume scale factors from given linear factors ❑ calculate unknown lengths from given scale factors ❑ find areas and volumes, given the linear scale factors
C7 solve problems involving length, area, volume, time, mass, and rates derived from these	❑ solve problems involving simple areas and volumes, including drawing relevant diagrams ❑ solve problems involving composite areas and volumes, if relevant diagrams are given ❑ substitute input data into given formulas describing areas and volumes ❑ solve problems involving rates and combinations of rates
C8 interpret drawings, and use the information to solve problems	❑ read input data in SI or Imperial units from given drawings ❑ solve problems that require the use of a formula using the information provided in given drawings

Prescribed Learning Outcomes	Suggested Achievement Indicators
3-D Objects and 2-D Shapes	
C9 solve problems involving distances between points in the coordinate plane	❑ calculate the distance between two given points on a plane using the following formula for distance: $\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ❑ plot given points on a coordinate plan, and use the Pythagorean Theorem to determine the distance between the points ❑ solve given distance problems using a variety of strategies, when intermediate steps are not provided
C10 solve problems involving midpoints of line segments	❑ express midpoints as ordered (coordinate) pairs ❑ solve given midpoint problems involving finding a second endpoint, when a diagram is provided ❑ solve midpoint problems given one endpoint and the midpoint
C11 solve problems involving rise, run, and slope of line segments	❑ find the slope of a line, given the graph of the line ❑ find the slope of a line, given any two points on the line, using the following slope formula: $m(\overline{AB}) = \dfrac{y_2 - y_1}{x_2 - x_1}$ ❑ distinguish between negative and positive slopes ❑ graph a line, given a point and the slope of the line ❑ identify that vertical lines have an undefined slope and horizontal lines have a slope equal to zero ❑ explain why vertical lines have an undefined slope and horizontal lines have a slope equal to zero ❑ interpret slope in terms of rate or steepness in contexts that are provided ❑ interpret the units for slope in given contexts
C12 determine the equation of a line, given information that uniquely determines the line	❑ identify the slope and y-intercept from a given equation in the form y = mx + b ❑ write an equation in the form y = mx + b, given the slope and y-intercept ❑ differentiate between the slope-intercept form and other forms of linear equations ❑ manipulate a given linear equation into any “y =” form that can be graphed ❑ recognize the equations of horizontal and vertical lines ❑ determine the equation of a line, in slope-intercept form, given a point and the slope ❑ determine the equation of a line given a graph of the line with a labelled point on the line ❑ plot given points, determine the slope of the line segment joining the points, and determine the equation of the line segment in the form y = mx + b ❑ manipulate a given linear equation into the slope-intercept form, which clearly identifies the slope and y-intercept ❑ solve given real-world problems related to linear equations

Statistics and Probability

Students collect, display, and analyse data to make predictions about a population.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Data Analysis D1 determine the equation of a line of best fit, using – estimate of slope and one point – least squares method with technology	❑ create a scatterplot from a collected or given data set using paper and pencil ❑ create a scatterplot from a collected or given data set using technology ❑ use technology to draw a line of best fit, using a given data set and the least squares method ❑ determine the equation of the line of best fit using the slope-intercept method ❑ determine the equation of the line of best fit using the least squares method with technology ❑ evaluate the validity of a given line of best fit with respect to the real-life data used to generate the line ❑ compare the lines of best fit produced by various methods
D2 use a calculator to determine the correlation coefficient (r) of a data set	❑ calculate r for a given data set, using a calculator ❑ state one or more generalizations regarding the value of r ❑ explain the significance of the sign of r
D3 interpret the correlation coefficient (r) and its limitations for varying problem situations, using relevant scatter plots	❑ sketch scatterplots from given data sets for different values of r ❑ make and justify inferences about the correlation coefficient r with respect to a given scatterplot, or vice versa ❑ suggest real-life situations where negative r values occur
D4 apply line-fitting and correlation techniques to analyse experimental results	❑ make and justify inferences from given data and the equation of the line of best fit generated from the data set ❑ interpolate and extrapolate points from a given line of best fit when the view screen does not require any manipulation ❑ relate the slope and y-intercept to the real-life context of a given investigation

STUDENT ACHIEVEMENT

Applications of Mathematics 11

Key Elements: Applications of Mathematics 11

Mathematical Process (Integrated)
The following mathematical processes have been integrated within the prescribed learning outcomes and achievement indicators for the course: communication, problem solving, connections, mental mathematics and estimation, reasoning, technology, and visualization.

Number

- consumer problems, including wages, property taxes, exchange rates, unit prices
- financial mathematics, including chequebooks, credit charges, and balances
- budgets
- simple and compound interest

Patterns and Relations

Variable and Equations
- linear inequalities in two variables, including algebraic and graphic solution methods
- non-linear equations, including quadratic and exponential equations
- linear and non-linear systems of equations in two variables
- optimization problems involving linear programming

Relations and Functions
- characteristics of quadratic functions, including vertex, domain and range, axis of symmetry, intercepts

Shape and Space

Measurement
- scale drawings and scale factors, including reductions and enlargements
- tolerances
- measurement errors
- percentage errors

Statistics and Probability

Data Analysis
- discrete and continuous data
- interpolations and extrapolations from data sets and their graphs
- graphical representations of data
- quadratic and exponential regression equations and models

NUMBER

Students demonstrate an understanding of and proficiency with calculations, including making decisions concerning which arithmetic operation or operations to use to solve a problem and then solve the problem.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
A1 solve consumer problems, including – wages earned in various situations – property taxation – exchange rates – unit prices	❑ calculate unit prices for a given context ❑ calculate wages, including minimum wage rates, regular and overtime pay, and combined salary and commission ❑ calculate wages involving gratuities and piecework ❑ solve single-step exchange rate problems
A2 reconcile financial statements, including – chequebooks with bank statements – cash register tallies with daily receipts	❑ reconcile a given financial statement, and explain the processes involved ❑ explain the meaning of entries on a given financial statement ❑ calculate credit charges and balances due on given credit card statements
A3 solve budget problems using graphs and tables to communicate solutions	❑ create a budget for a given context ❑ draw and interpret graphs that display given budget data – limited to circle graphs, bar graphs, pictographs ❑ make budget predictions based on given data ❑ create a revised budget to reflect a single change in income or expenses
A4 solve investment and credit problems involving simple and compound interest	❑ calculate the amount of accumulated interest, using a formula, graphing calculator, table of values, or a spreadsheet, in a given context ❑ calculate the appropriate value of the interest rate in the simple interest formula, in a given context

PATTERNS AND RELATIONS

Students represent algebraic expressions in multiple ways and use algebraic and graphical models to generalize patterns, make predictions, and solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Variables and Equations B1 graph linear inequalities, in two variables	❑ graph the boundary line between two half planes ❑ use a test point, usually (0,0), to determine the solution region that satisfies the inequality, given a boundary line ❑ graph a given linear inequality expressed in the form $y = mx + b$, using <, >, ≤, ≥ ❑ rewrite any given inequality expressed in the $Ax + By = C$ form in the $y = mx + b$ form, where A, B, C are integral and $B > 0$ ❑ determine when solid or broken lines should be used to display the solution for a given inequality ❑ explain why the shaded half plane represents the solution region of a given inequality
B2 solve systems of linear equations, in two variables: – algebraically (elimination and substitution) – graphically	❑ determine the possible solution of a given linear system of equations by direct substitution ❑ graphically solve given linear systems of equations with integral coefficients with manipulation into a "$y =$" form ❑ determine the point of intersection for a given system of linear equations ❑ explain the meaning of the point of intersection for a given linear system ❑ translate given word problems into a system of equations ❑ solve given linear systems of equations algebraically by substitution, where at least one variable has a coefficient of one ❑ solve given linear systems of equations with integral coefficients by elimination ❑ determine the most efficient method (substitution or elimination) to solve a given system of linear equations and justify
B3 solve nonlinear equations (i.e., quadratic and exponential), using a graphing calculator *Suborganizer 'Variables and Equations' continued on page 47*	❑ solve a given quadratic equation graphically, using more than one method (e.g., for $y = 3x^3 - 2x - 7$ use the graphing calculator to find the x-intercepts; then use it to find the intersection of $y = 3x^3 - 2x$ and $y = 7$) ❑ solve a given exponential equation graphically, using more than one method (e.g., for $y = 3^x - 5$ use the graphing calculator to find the x-intercept; then use it to find the intersection of $y = 3^x$ and $y = 5$) ❑ explain the difference between determining the x-intercepts of the graph of a function and the determining the intersection of the two graphs

Prescribed Learning Outcomes	Suggested Achievement Indicators
Suborganizer 'Variables and Equations' continued from page 46	
B4 solve systems of linear inequalities, in two variables, using a graphing calculator	❑ graph the boundary line between two half planes ❑ determine the solution region that satisfies a given inequality using a test point when given a boundary line ❑ graph a linear inequality expressed in the form $y = \mathrm{m}x + \mathrm{b}$, using $<, >, \le, \ge$ ❑ rewrite an inequality expressed in the $Ax + By = C$ form in the $y = \mathrm{m}x + \mathrm{b}$ form, where A, B, C are integral and $B > 0$ ❑ determine the solution region for a system of linear inequalities and verify the solution
B5 use systems of linear inequalities, in two variables, to model and solve problems	❑ use a test point to determine the solution region that satisfies a given inequality ❑ distinguish between the use of solid and broken lines in given solution regions ❑ graph any given linear inequality in two variables ❑ explain why the shaded half plane represents the solution region of a given inequality
B6 use nonlinear systems of equations (i.e., linear/quadratic and linear/exponential), in two variables, to model and solve problems	❑ use a test point to determine the solution region that satisfies a given inequality ❑ distinguish between the use of solid and broken lines in given solution regions ❑ graph any given linear/quadratic and linear/exponential inequality in two variables ❑ explain why the shaded half plane represents the solution region of a given inequality
B7 apply linear programming to find optimal solutions to decision-making problems	❑ write the system of inequalities corresponding to a problem ❑ graph the inequalities ❑ find vertices ❑ write an expression for the objective function ❑ substitute vertices into an expression for the objective function to find the optimal solution
Relations and Functions	
B8 determine the following characteristics of the graph of a quadratic function: – vertex – domain and range – axis of symmetry – intercepts	❑ graph a given quadratic function on a graphing calculator ❑ determine the following properties of a given quadratic function, using a graphing calculator: – vertex – domain – range – equation of the axis of symmetry – x- and y-intercepts ❑ solve problems that require the calculation of the domain, range, intercepts, or vertex of a quadratic function

Shape and Space

Students describe and compare everyday phenomena, using either direct or indirect measurement.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Measurement C1 enlarge or reduce a dimensioned object, according to a specified scale	❑ read and interpret given scale drawings to solve problems ❑ enlarge or reduce given 1-D or 2-D drawings by a given scale factor ❑ enlarge or reduce given simple shapes in 3-D, using orthographic drawings
C2 calculate maximum and minimum values, using tolerances, for lengths, areas, and volumes	❑ calculate maximum and minimum values, using measured tolerances in routine/familiar problems ❑ calculate the sum or difference of two given measurements, including measurement errors
C3 solve problems involving percentage error when input variables are expressed with percentage errors	❑ convert between given tolerances and percentage error ❑ solve simple problems involving tolerance and percentage error

Statistics and Probability

Students collect, display, and analyse data to make predictions about a population.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Data Analysis	
D1 extract information from given graphs of discrete or continuous data, using – time series – continuous data – contour lines	❑ extract information from a given graphical presentation to solve a problem ❑ extract information from a given pictograph to solve a problem ❑ use the information presented in a given graph to perform mathematical calculations and to make interpretations and provide explanations for the interpretations
D2 draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data	❑ analyse and interpret trends in given data presented in graphical and tabular form ❑ interpolate or extrapolate, given a set of data in graphical or tabular form, to solve problems
D3 design different ways of presenting data and analysing results, by focussing on the truthful display of data and the clarity of presentation	❑ determine if a given graphical presentation is misleading and explain why ❑ discuss the advantages and disadvantages associated with different given graphical presentations ❑ provide alternative graphical presentations for given data to stress a particular point of view
D4 collect experimental data and use best-fit quadratic and exponential functions, to make predictions and solve problems	❑ use a graphing calculator to create a scatterplot of collected data to determine the quadratic or exponential regression equation ❑ choose the appropriate regression equation for a given or student collected data set, based on the context and the correlation coefficient ❑ explain the choice of regression model used in a particular context ❑ use technology to interpolate and extrapolate information from a given or calculated regression equation

STUDENT ACHIEVEMENT

Applications of Mathematics 12

Key Elements: Applications of Mathematics 12

Mathematical Process (Integrated)

The following mathematical processes have been integrated within the prescribed learning outcomes and achievement indicators for the course: communication, problem solving, connections, mental mathematics and estimation, reasoning, technology, and visualization.

Number

- matrices to model tables and problems
- matrix operations, including addition, subtraction, and scalar multiplication
- financial spreadsheets, including recursive and non-recursive situations, asset management, leasing, and purchasing
- investment portfolios

Patterns and Relations

Patterns

- properties of sinusoidal curves, including amplitude, period, maximum and minimum values, and translations
- graphs of sinusoidal data
- sinusoidal regression

Shape and Space

Measurement

- design problems involving perimeter, area, and volume
- estimate and cost materials
- design objects or processes within a specified budget
- complex measurement problems

3-D Objects and 2-D Shapes

- scalar and vector quantities
- directional and bearing notation
- vector multiplication by negative and positive scalars
- magnitude and direction of resultant vectors
- 2-D and 3-D applications of vectors

Statistics and Probability

Chance and Uncertainty

- independent and dependent events
- fundamental counting principle
- sample spaces
- mutually exclusive and complementary events

Number

Students identify the types and functions of insurance, the preparation of a personal financial plan, the types of taxes imposed by different levels of government, and the costs associated with foreign purchasing and currency exchange.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
A1 model and solve problems, including those solved previously, using technology to perform matrix operations, including – addition – subtraction – scalar multiplication A2 model and solve consumer and network problems using technology to perform matrix multiplication as required	❑ create a matrix from a given table ❑ produce a row matrix or a column matrix from a given problem context ❑ describe the dimensions of a given matrix ❑ identify conditions required to perform operations on given matrices ❑ identify the required matrix operation for a given context ❑ perform operations of addition, subtraction, matrix multiplication, and scalar multiplication on given matrices ❑ make modifications to one or two components of a given matrix in response to new scenarios ❑ model a simple problem with a matrix ❑ solve problems in which the matrix is given or developed
A3 design a financial spreadsheet template to allow users to input their own variables *Organizer 'Number' continued on page 54*	❑ distinguish between entered values, fixed values, and computed values in a given spreadsheet ❑ identify appropriate algebraic functions used in the solution of given spreadsheet problems ❑ interpret a given written problem and develop an appropriate spreadsheet ❑ solve non-recursive or recursive problems by working within the parameters of an existing spreadsheet ❑ design a simple spreadsheet by using given formulas and functions to solve non-recursive problems, such as billing and design calculations, or recursive problems such as loan calculations ❑ determine the algebraic formulas used to construct a given spreadsheet

Prescribed Learning Outcomes	Suggested Achievement Indicators
Organizer 'Number' continued from page 53	
A4 analyse the costs and benefits of renting or buying an increasing asset, such as land or property, under various circumstances	❑ use correct terminology to discuss given payment options ❑ modify an existing spreadsheet to accommodate changing needs, and analyse differing factors in each model ❑ identify the more appropriate payment model, given two schedules, and justify the choice in terms of cost
A5 analyse the costs and benefits of leasing or buying a decreasing asset, such as a vehicle or computer, under various circumstances	❑ prepare a spreadsheet to create a cost-and-benefit analysis that includes – payments – interest rates – amount paid on loan – amount paid to interest – amount paid to principal ❑ determine the pros and cons with respect to a chosen payment model and explain why the model is preferred ❑ determine whether renting or buying would be preferred given a specific set of circumstances ❑ determine if the value of a given asset is either increasing or decreasing ❑ represent the value of a given increasing asset by the regression model $y = a\left(b^x\right)$, where $b > 1$ ❑ represent the value of a given decreasing asset by the regression model $y = a\left(b^x\right)$, where $0 < b < 1$
A6 analyse an investment portfolio applying such concepts as interest rate, rate of return, and total return	❑ describe a given investment portfolio using investment terminology ❑ compare two given similar portfolios and determine the strengths and weaknesses of each with regard to interest rate, rate of return, and total return ❑ calculate the total return and the average annual rate of return of a given investment

PATTERNS AND RELATIONS

Students use patterns to describe the world and to solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Patterns B1 describe sinusoidal curves using terms, including – amplitude – period – maximum and minimum values – vertical and horizontal shift	❑ describe a sinusoidal event, using correct terminology, given a diagram, a context, an equation, or a graph ❑ determine the amplitude, period, maximum or minimum value, vertical and horizontal shift for a given sinusoidal curve ❑ describe the transformational effect(s) when parameters such as amplitude, period, horizontal shift, and vertical shift are changed for a given periodic event
B2 graph sinusoidal data using technology, and represent the data with a best fit equation of the form $y = a \sin (bx + c) + d$	❑ use technology to graph given data that represents a sinusoidal curve ❑ determine and state appropriate graphing calculator window settings as ordered triples in the form x: $[x_{min,}\ x_{max,}\ x_{scale}]$ and y: $[y_{min,}\ y_{max,}\ y_{scale}]$ ❑ determine a best-fit equation for a given data set using the regression model $y = a \sin (bx + c) + d$
B3 use best fit sinusoidal equations, and their associated graphs, to make predictions (interpolation, extrapolation)	❑ use the TRACE or CALC function on a graphing calculator to make predictions in a given problem situation ❑ make predictions from the sinusoidal regression equation derived from a given data set ❑ sketch transformations of a given graph ❑ relate values of a, b, and d of $y = a \sin (bx + c) + d$ to given window settings ❑ determine the values of a, b, and d in the equation $y = a \sin (bx + c) + d$ or in a given graph ❑ recognize that when $c \neq 0$ a horizontal transformation occurs and when $d \neq 0$ a vertical transformation occurs ❑ choose appropriate window settings for a graphing calculator, based on given values for amplitude, period, and vertical shift

Shape and Space

Students describe and compare everyday phenomena, using either direct or indirect measurement, describe the characteristics of 3-D objects and 2-D shapes, and analyse the relationships among them.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Measurement C1 use dimensions and unit prices to solve problems involving perimeter, area, and volume	❑ draw sketches and scale diagrams in 2-D from various views when a written description of a design is given ❑ draw sketches and scale diagrams in 3-D from various views when a written description of a design is given ❑ calculate perimeter, surface area, and volume, and determine the cost of composite designs when calculations (e.g., Pythagorean Theorem, trigonometry, solving for any variable in given formulas) are required to find values of necessary dimensions
C2 solve problems involving estimation and costing for objects, shapes, or processes when a design is given	❑ solve given cost and estimation problems related to perimeter, surface area, and volume by creating a diagram and performing the necessary calculations ❑ justify the solution and procedures to solve a problem involving the costing of a design
C3 design an object, shape, layout, or process within a specified budget	❑ design objects based on given criteria within a specified budget ❑ determine the effects on the budget for a design or process when one or more of the parameters is changed
C4 use simplified models to estimate the solutions to complex measurement problems	❑ estimate the solution for a given problem and explain the method used to determine the estimate ❑ determine the most efficient process to yield a desired result for a given problem and justify the method
3-D Objects and 2-D Shapes C5 use appropriate terminology to describe – vectors (i.e., direction, magnitude) – scalar quantities (i.e., magnitude)	❑ describe given vectors and scalar quantities using appropriate terminology to state the magnitude and direction ❑ describe a given angle using directional notation (e.g., heading – W35°S or 35° south of west) ❑ describe a given angle using bearing notation (e.g., clockwise from north – bearing of 125°) ❑ distinguish between a scalar quantity and a vector quantity
C6 assign meaning to the multiplication of a vector by a scalar *Suborganizer '3-D Objects and 2-D Shapes' continued on page 57*	❑ explain the effect of multiplying a given vector by a positive scalar ❑ explain the effect of multiplying a given vector by a negative scalar ❑ solve given problems involving scalar multiplication

Prescribed Learning Outcomes	Suggested Achievement Indicators
Suborganizer '3-D Objects and 2-D Shapes' continued from page 56	
C7 determine the magnitude and direction of a resultant vector, using triangle or parallelogram methods	❑ identify properties of a given parallelogram and use the properties to draw appropriate vector diagrams ❑ construct an appropriate vector diagram from given information ❑ determine relative magnitude of given vectors by comparing lengths
C8 model and solve problems in 2-D and 3-D (with 3-D vectors restricted to those that are mutually orthogonal), using vector diagrams and technology	❑ solve given 2-D problems involving right triangles ❑ solve 2-D problems involving oblique triangles in which a vector diagram is given ❑ solve 2-D problems involving oblique triangles without being given a parallelogram or head-to-tail diagram ❑ model and solve mutually orthogonal 3-D vector problems ❑ solve 3-D problems that can be represented by two 2-D diagrams (one in a horizontal plane and one in a vertical plane)

STATISTICS AND PROBABILITY

Students use experimental and theoretical probability to represent and solve problems involving uncertainty.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
Chance and Uncertainty D1 classify events as independent or dependent	❑ use examples to explain the difference between independent and dependent events
D2 use the fundamental counting principle to determine the number of different ways to perform multi-step operations	❑ determine $P(A$ and $B)$ for given independent events ❑ determine $P(A$ and $B)$ for given dependent events where order is specified
D3 construct a sample space for two or three events	❑ determine sample spaces for given problems involving two or three components ❑ generate a sample space and use it to draw conclusions about the outcomes of given routine and non-routine problems
D4 solve problems, using the probabilities of mutually exclusive and complementary events	❑ identify if given events are complementary ❑ determine the complement to a given event to solve problems ❑ given the probability of an event, determine the probability of the complement ❑ given the probability of one event and the probability of combined events, determine the probability of the other event ❑ determine $P(A$ and $B)$ for given single events that are mutually exclusive to solve problems

Learning Resources

Application of Mathematics 10 to 12

This section contains general information on learning resources, and provides a link to the titles, descriptions, and ordering information for the recommended learning resources in the Applications of Mathematics 10 to 12 Grade Collections.

What Are Recommended Learning Resources?
Recommended learning resources are resources that have undergone a provincial evaluation process using teacher evaluators and have Minister's Order granting them provincial recommended status. These resources may include print, video, software and CD-ROMs, games and manipulatives, and other multimedia formats. They are generally materials suitable for student use, but may also include information aimed primarily at teachers.

Information about the recommended resources is organized in the format of a Grade Collection. A Grade Collection can be regarded as a "starter set" of basic resources to deliver the curriculum. In many cases, the Grade Collection provides a choice of more than one resource to support curriculum organizers, enabling teachers to select resources that best suit different teaching and learning styles. Teachers may also wish to supplement Grade Collection resources with locally approved materials.

How Can Teachers Choose Learning Resources to Meet Their Classroom Needs?
Teachers must use either:
- provincially recommended resources OR
- resources that have been evaluated through a local, board-approved process

Prior to selecting and purchasing new learning resources, an inventory of resources that are already available should be established through consultation with the school and district resource centres. The ministry also works with school districts to negotiate cost-effective access to various learning resources.

What Are the Criteria Used to Evaluate Learning Resources?
The Ministry of Education facilitates the evaluation of learning resources that support BC curricula, and that will be used by teachers and/or students for instructional and assessment purposes. Evaluation criteria focus on content, instructional design, technical considerations, and social considerations.

> Additional information concerning the review and selection of learning resources is available from the ministry publication, *Evaluating, Selecting and Managing Learning Resources: A Guide* (Revised 2002)
> www.bced.gov.bc.ca/irp/resdocs/esm_guide.pdf

What Funding is Available for Purchasing Learning Resources?
As part of the selection process, teachers should be aware of school and district funding policies and procedures to determine how much money is available for their needs. Funding for various purposes, including the purchase of learning resources, is provided to school districts. Learning resource selection should be viewed as an ongoing process that requires a determination of needs, as well as long-term planning to co-ordinate individual goals and local priorities.

What Kinds of Resources Are Found in a Grade Collection?
The Grade Collection charts list the recommended learning resources by media format, showing links to the curriculum organizers and suborganizers. Each chart is followed by an annotated bibliography. Teachers should check with suppliers for complete and up-to-date ordering information. Most suppliers maintain web sites that are easy to access.

APPLICATIONS OF MATHEMATICS 10 TO 12 GRADE COLLECTIONS

The Grade Collections for Applications of Mathematics 10 to 12 list the recommended learning resources for these courses. Resources previously recommended for the 2000 version of the curriculum, where still valid, continue to support this updated IRP. The ministry updates the Grade Collections on a regular basis as new resources are developed and evaluated.

> Please check the following ministry web site for the most current list of recommended learning resources in the Applications of Mathematics 10 to 12 Grade Collections: www.bced.gov.bc.ca/irp_resources/lr/resource/gradcoll.htm

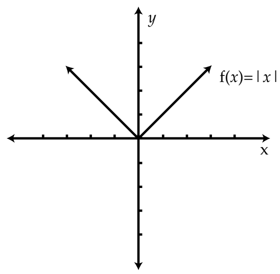
GLOSSARY

Applications of Mathematics 10 to 12

This appendix provides an illustrated glossary of terms used in this Integrated Resource Package. The terms and definitions are intended to be used by readers unfamiliar with mathematical terminology. For a more complete definition of each term, refer to a mathematical dictionary such as the *Nelson Canadian School Mathematics Dictionary* (ISBN 17-604800-6).

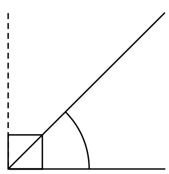
A

absolute value of a number
How far the number is from 0. Example: the absolute values of -4.2 and of 4.2 are each 4.2.



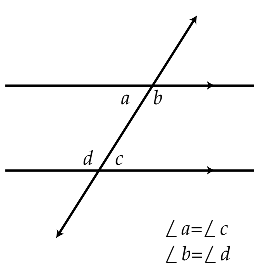
absolute value function
The function f defined by $f(x)=|x|$, where $|x|$ denotes the absolute value of x.

accuracy
A measure of how far an estimate is from the true value.

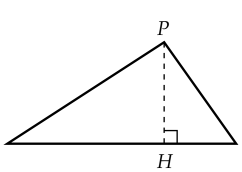


acute angle
An angle whose measure is between 0° and 90°.

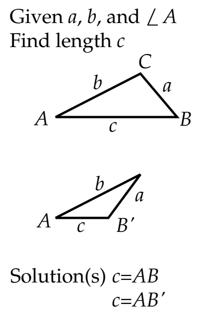
algorithm
A mechanical method for solving a certain type of problem, often a method in which one kind of step is repeated a number of times.



alternate interior angles
In the diagram to the left, the angles labelled a and c are alternate interior angles, as are the angles b and d.



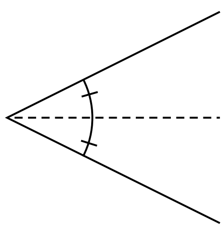
altitude of a triangle
A line segment PH, where P is a vertex of the triangle, H lies on the line through the other two vertices, and PH is perpendicular to that line.



ambiguous case
Two sides of a triangle and the angle opposite one of them are specified, and we want to calculate the remaining angles or side. There may be no solution, exactly one, or exactly two.

amplitude (of a periodic curve)
The maximum displacement from a reference level in either a positive or negative direction. That reference level is often chosen halfway between the biggest and smallest values taken on by the curve.

analytic geometry (coordinate geometry)
An approach to geometry in which position is indicated by using coordinates, lines, and curves, and other objects are represented by equations, and algebraic techniques are used to solve geometric problems.



angle bisector
A line that divides an angle into two equal parts.

antiderivative
If $f(x)$ is the derivative of $F(x)$, then $F(x)$ is an *antiderivative* of $f(x)$. *Indefinite integral* means the same thing.

antidifferentiation
The process of finding antiderivatives.

arc
A connected segment of a circle or curve.

arc sine (of x)
The angle (in radians) between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is x.
Notation: $\sin^{-1} x$ or $\arcsin x$.

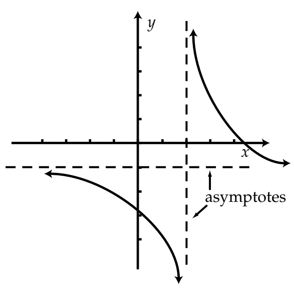
arc tangent
The angle (in radians) between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose tangent is x.
Notation: $\tan^{-1} x$ or $\arctan x$.

arithmetic operation
Addition, subtraction, multiplication, and division.

arithmetic sequence (arithmetic progression)
A sequence in which any term is obtained from the preceding term by adding a fixed amount, the *common difference*.

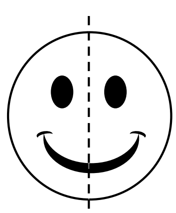
If a is the first term and d the common difference, then the sequence is $a, a + d, a + 2d, a + 3d, \ldots$. The n-th term t_n is given by the formula $t_n = a + (n - 1)d$.

arithmetic series
The sum S_n of the first n terms of an arithmetic sequence. If the sequence has first term a and common difference d, then
$S_n = \frac{1}{2}n\left[2a + \left(n-1\right)d\right] = \frac{1}{2}n\left(a + l\right)$ where l is $a + (n - 1)d$, the "last" term.



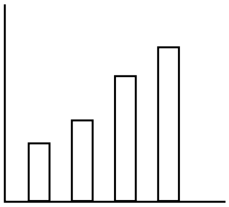
asymptote (to a curve)
A line l such that the distance from points P on the curve to l approaches zero as the distance of P from an origin increases without bound as P travels on a certain path along the curve.

average velocity
The net change in position of a moving object divided by the elapsed time.



axis of symmetry (of a geometric figure)
A line such that for any point P of the figure, the mirror image of P in the line is also in the figure.

B



bar graph
A graph using parallel bars (vertical or horizontal) that are proportional in length to the data they represent.

base
In the expression s^t, the number or expression s is called the base, and t is the exponent. In the expression $\log_a u$, the base is a.

binomial
The sum of two monomials.

binomial distribution
The probabilities associated with the number of successes when an experiment is repeated independently a fixed number of times. For example, the number of times a six is obtained when a fair die is tossed 100 times has a binomial distribution.

Binomial Theorem
A rule for expanding expressions of the form $(x + y)^n$.

bisect
To divide into two equal parts.

broken-line graph
A graph using line segments to join the plotted points to represent data.

C

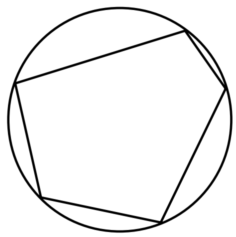
Cartesian (rectangular) coordinate system
A coordinate system in which the position of a point is specified by using its signed distances from two perpendicular reference lines (axes).

central angle
An angle determined by two radii of a given circle; equivalently, an angle whose vertex is at the centre of the circle.

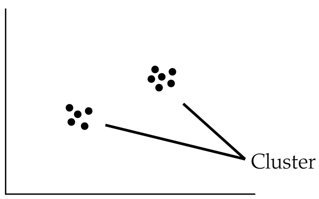
chain rule
A rule for differentiating composite functions.
If $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$.

chord
The line segment that joins two points on a curve, usually a circle.

circumference
The boundary of a closed curve, such as a circle; also, the measure (length) of that boundary. Please see *perimeter*.



circumscribed
The polygon P is *circumscribed* about the circle C if P is inside C and the edges of P are tangent to C. The circle C is *circumscribed* about the polygon Q if Q is inside C and the vertices of Q are on the boundary of C. The notion can be extended to other figures, and to three dimensions.



cluster
A collection of closely grouped data points.

coefficient
A numerical or constant multiplier in an algebraic expression. The coefficient of x^2 in $4x^2 - 2axy$ is 4, and the coefficient of xy is $-2a$.

collinear
Lying on the same line.

combination
A set of objects chosen from another set, with no attention paid to the order in which the objects are listed (see also *permutation*). The number of possible combinations of r objects selected from a set of n distinct objects is ${}_nC_r$, pronounced "*n* choose *r*."

common factor (CF)
A number that is a factor of two or more numbers. For example, 3 is a common factor of 6 and 12. *Common divisor* means the same thing. The term is also used with polynomials. For example, $x - 1$ is a common factor of $x^2 - x$ and $x^2 - 2x + 1$.

common fraction
A number written as $\frac{a}{b}$, where the numerator a and the denominator b are integers, and b is not zero. Examples: $\frac{4}{5}, \frac{-13}{6}, \frac{3}{1}$.

compass
An instrument for drawing circles or arcs of circles.

complementary angles
Two angles that add up to a right angle.

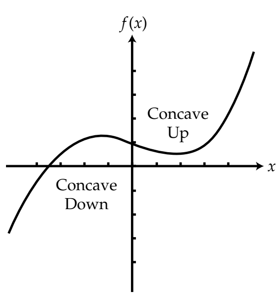
completing the square
Rewriting the quadratic polynomial $ax^2 + bx + c$ in the form $a(x - p)^2 + q$, perhaps to solve the equation $ax^2 + bx + c = 0$.

complex fraction
A fraction in which the numerator or the denominator, or both, contain fractions.

composite function
A function $h(x)$ obtained from two functions f and g by using the rule $h(x) = f(g(x))$ (first do g to x, then do f to the result).

composite number
An integer greater than 1 that is not prime, such as 9 or 14.

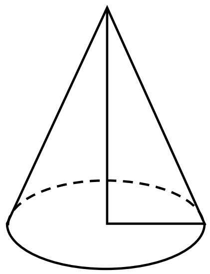
compound interest
The interest that accumulates over a given period when each successive interest payment is added to the principal in order to calculate the next interest payment.



concave down (or downward)
The function $f(x)$ is concave down on an interval if the graph of $y = f(x)$ lies below its tangent lines on that interval.

concave up (or upward)
The function $f(x)$ is concave up on an interval if the graph of $y = f(x)$ lies above its tangent lines on that interval.

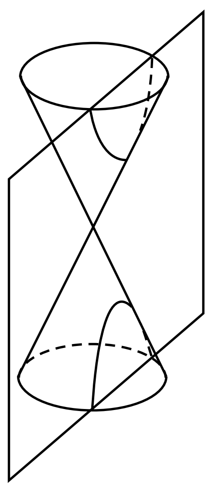
conditional probability
The probability of an event given that another event has occurred. The (conditional) probability that someone earns more than $200,000 a year, given that the person plays in the NHL, is different from the probability that a randomly chosen person earns more than $200,000.



cone (right circular)
The three-dimensional object generated by rotating a right triangle about one of its legs.

confidence interval(s)
An interval that is believed, with a preassigned degree of confidence, to include the particular value of some parameter being estimated.

congruent
Having identical shape and size.



conic section
A curve formed by intersecting a plane and the surface of a double cone. Apart from degenerate cases, the conic sections are the ellipses, the parabola, and the hyperbolas.

conjecture
A mathematical assertion that is believed, at least by some, to be true, but has not been proved.

constant
A fixed quantity or numerical value.

continuous data
Data that can, in principle, take on any real value in some interval. For example, the exact height of a randomly chosen individual, or the exact length of life of a U-235 atom can be modelled by a continuous distribution.

continuous function
Informally, a function $f(x)$ is *continuous* at a if $f(x)$ does not make a sharp jump at a. More formally, $f(x)$ is continuous at a if $f(x)$ approaches $f(a)$ as x approaches a.

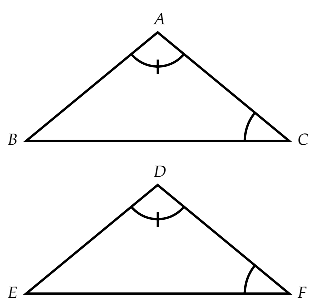
contrapositive
The contrapositive of "Whenever A is true, B must be true" is "Whenever B is false, then A must be false." Any assertion is logically equivalent to its contrapositive, so one strategy for proving an assertion is to prove its contrapositive.

converse (of a theorem)
The assertion obtained by interchanging the premise and the conclusion. If the theorem is "Whenever A happens, B must happen," then its converse is "Whenever B happens, A must happen." The converse of a theorem need not be true.

coordinate geometry
Please see *analytic* geometry.

coordinates
Numbers that uniquely identify the position of a point relative to a coordinate system.

correlation coefficient
A number (between -1 and 1) that measures the degree to which a collection of data points lies on a line.



corresponding angles and corresponding sides
Angles or sides that have the same relative position in geometric figures.

cosecant (of x)
This is $\frac{1}{\sin x}$. Notation: csc x.

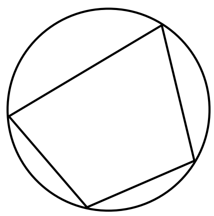
cosine law (law of cosines)
A formula used for solving triangles in plane geometry.
$c^2 = a^2 + b^2 - 2ab\cos C$

cosine (function)
See *primary trigonometric functions*.

cotangent (of x)
This is $\frac{1}{\tan x}$. Notation: cot x.

coterminal angles
Angles that are rotations between the same two lines, termed the initial and terminal arms. For example: 20º, –340º, 380º are coterminal angles.

critical number (of a function)
A number where the function is defined, and where the derivative of the function is equal to 0 or doesn't exist.



cyclic (inscribed) quadrilateral
A quadrilateral whose vertices all lie on a circle.

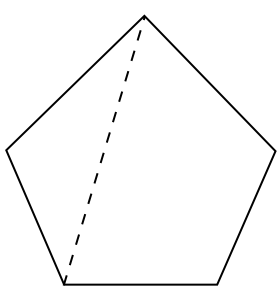
D

decimal fraction
In principle, a fraction $\frac{a}{b}$ where a is an integer and b is a power of 10.
For example, $\frac{1}{4} = \frac{25}{100}$, so $\frac{1}{4}$ can be expressed as a decimal fraction, usually written as 0.25.

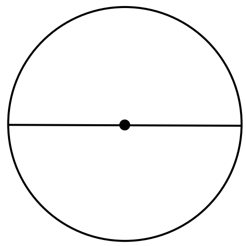
decreasing function
The function $f(x)$ is decreasing on an interval for any numbers s and t in that interval, if t is greater than s then $f(t)$ is less than $f(s)$.

deductive reasoning
A process by which a conclusion is reached from certain assumptions by the use of logic alone.

degree
The highest power or sum of powers that occurs in any term of a given polynomial or polynomial equation. For example, $6x + 17$ has degree 1, and $2 + x^3 + 7x$ has degree 3, as does $2 + 6x + 7y + xy^2$.



diagonal
A line segment that joins two non-adjacent vertices in a polygon or polyhedron.



diameter
A line segment that joins two points on a circle or sphere and passes through the centre. All diameters of a circle or sphere have the same length. That common length is called the diameter.

difference of squares
An expression of the form $A^2 - B^2$, where A and B are numbers, polynomials, or perhaps other mathematical expressions. We can factor $A^2 - B^2$ as $(A+B)(A-B)$.

differentiable
A function is *differentiable* at $x = a$ if under extremely high magnification, the graph of the function looks almost like a straight line near a. Most familiar functions are differentiable everywhere that they are defined.

differential equation
An equation that involves only two variable quantities, say x and y, and the first derivative, or higher derivatives, of y with respect to x.
Example: $3y^2 \frac{dy}{dx} = e^x$.

differentiate; differentiation
To find the derivative; the process of finding derivatives.

direct variation
The quantity Q varies directly with x if $Q = ax$ for some constant a. This can be contrasted with *inverse* variation, in which $Q = \frac{a}{x}$ for some a.

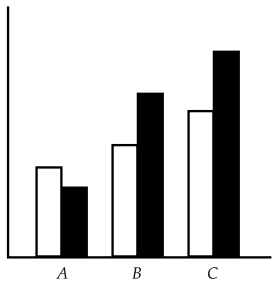
discrete data
Data arising from situations in which the possible outcomes lie in a finite or infinite sequence.

discriminant
The *discriminant* of the quadratic polynomial $ax^2 + bx + c$ (or of the equation $ax^2 + bx + c = 0$) is $b^2 - 4ac$.

displacement
Position, as measured from some reference point.

distance formula
The formula used in coordinate geometry to find the distance between two points. If A has coordinates (x_1,y_1) and B has coordinates (x_2,y_2), then the distance from A to B is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

domain (of a function)
The set of numbers where the function is defined. For example, if $f(x) = \frac{\sqrt{x-2}}{x-5}$, then the domain of $f(x)$ consists of all real numbers greater than or equal to 2, except for the number 5.



double bar graph
A bar graph that uses bars to represent two sets of data visually.

E

edge
The straight line segment that is formed where two faces of a polyhedron meet.

ellipse
A closed curve obtained by intersecting the surface of a cone with a plane. Please see *conic section*.

equation
A statement that two mathematical expressions are equal, such as $3x + y = 7$.

equidistant
Having equal distances from some specified object, point, or line.

estimate
v. To approximate a quantity, perhaps only roughly.
n. The result of estimating. Also, an approximation, based on sampling, to some number associated with a population, such as the average age.

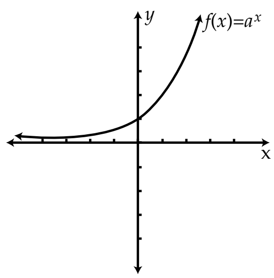
Euclidean geometry
Geometry based on the definitions and axioms set out in Euclid's Elements.

event
A subset of the sample space of all possible outcomes of an experiment.

experimental probability
An estimate of the probability of an event obtained by repeating an experiment many times. If the event occurred in k of the n experiments, it has experimental probability k / n.

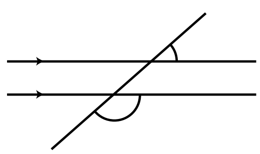
exponent
The number that indicates the power to which the base is raised. For example: 3^4: exponent is 4.

exponential decay
A quantity undergoes exponential decay if its rate of decrease at any time is proportional to its size at the time. Exponential decay models well the decay of radioactive substances.



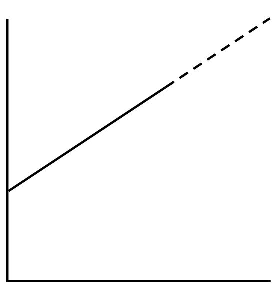
exponential function
An exponential function is a function of the form $f(x) = a^x$, where $a > 0$ and the variable x occurs as the exponent. The exponential function is the function $f(x) = e^x$, where e is a mathematical constant roughly equal to 2.7182818284.

exponential growth
A quantity undergoes exponential growth if its rate of increase at any time is proportional to its size at the time. Exponential growth models well the growth of a population of bacteria under ideal conditions.



exterior angles on the same side of the transversal
A transversal of two parallel lines forms two supplementary exterior angles.

extraneous root
A spurious root obtained by manipulating an equation. For example, if we square both sides of $1 - x + \sqrt{x - 1}$ and simplify, we obtain $(x - 1)(x - 2) = 0$, that is, $x = 1$ or $x = 2$. Since 2 is not a root of the original equation, it is sometimes called an extraneous root.



extrapolate
Estimate the value of a function at a point from values at places on one side of the point only.

extreme values
The highest and lowest numbers in a set.

F

face
One of the plane surfaces of a polyhedron.

factor
n. A *factor* of a number n is a number (usually taken to be positive) that divides n exactly. For example, the factors of 18 are 1, 2, 3, 6, 9, and 18. Similarly, a factor of a polynomial $P(x)$ is a polynomial that divides $P(x)$ exactly. Thus x and $x - 1$ are two of the factors of $x^3 - x$.
v. To *factor* a number or polynomial is to express it as a product of basic terms. For example, $x^3 - x$ factors as $x(x - 1)(x + 1)$.

Factor Theorem
If $P(x)$ is a polynomial, and a is a root of the equation $P(x) = 0$, then $x - a$ is a factor of $P(x)$.

factor(s)
Numbers multiplied to produce a specific product. For example: $2 \times 3 \times 3 = 18$: factors are 2 and 3; $(x - 2)$ and $(x + 1)$ are factors of $x^2 + x - 2$.

first-hand data
Data collected by an individual directly from observations or measurements.

flip
Another word for *reflection*.

frequency diagram
A diagram used to record the number of times various events occurred.

function
A rule that produces, for any element x of a certain set A, an object $f(x)$. The set A is the *domain* of the function; the set of values taken on by $f(x)$ is the *range* of the function.

More formally, a function is a collection of ordered pairs (x, y) such that the second entry y is completely determined by the first entry x.

function notation
If a quantity y is completely determined by a quantity x, then y is called a function of x. For example, the area of a circle of general radius x might be denoted by $A(x)$ (pronounced "A of x.) In this case, $A(x) = \pi x^2$.

fundamental counting principle
If an event can happen in x different ways, and for each of these ways a second event can happen in y different ways, then the two events can happen in $x \times y$ different ways.

G

general polynomial equation
An equation of the form $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n = 0$.

general term (of a sequence)
If n is unspecified, a_n is called the *general term* of the sequence $a_1, a_2, a_3, \ldots$ Sometimes there is an explicit formula for a_n in terms of n.

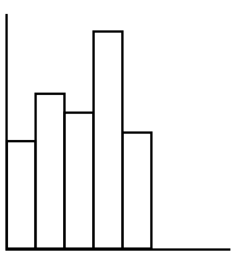
geometric sequence (progression)
A sequence in which each term except the first is a fixed multiple of the preceding term. If the first term is a, and each term is r times the previous one (r is the *common ratio*), then the general term t_n is given by $t_n = ar^{n-1}$.

geometric series
The sum S_n of the first n terms of a geometric sequence. If a is the first term, r the common ratio, where $r \neq 1$, then $S_n = \dfrac{a\left(1-r^n\right)}{1-r}$.
See also *infinite geometric series*.

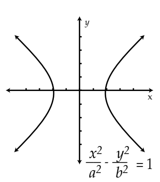
greatest common factor (GCF)
The largest positive integer that divides two or more given numbers. For example, the GCF of 12 and 18 is 6. The GCF is also called the greatest common divisor (GCD).

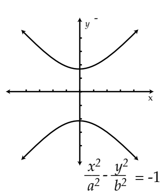
H

higher derivatives
The derivative of the derivative of $f(x)$, the derivative of the derivative of the derivative of $f(x)$, and so on.



histogram
A bar graph showing the frequency in each class using class intervals of the same length.





hyperbola
A curve with two branches where a plane and a circular conical surface meet. Please see *conic section*.

hypotenuse
The side opposite the right angle in a right triangle.

hypothesis
A statement or condition from which consequences are derived.

I

identity
A statement that two mathematical expressions are equal for all values of their variables.

if–then proposition
A mathematical statement that asserts that *if* certain conditions hold, *then* certain other conditions hold.

Imperial measure
The system of units (foot, pound, and so on) for measuring length, mass, and so on that was once the legal standard in Great Britain.

implicit function
A function y of x defined by a formula of shape $H(x, y) = 0$.
For example, $y^3 - x^2 + 1 = 0$ defines y implicitly as a function *of* x.
In this case, y is given *explicitly* by $y = \left(x^2 - 1\right)^{\frac{1}{3}}$. But often (example: $H(x, y) = y^7 + \left(x^2 + 1\right)y - 1$), it is not possible to give an explicit formula for y.

improper fraction
A *proper fraction* is a fraction whose numerator is less in absolute value than its denominator. An *improper fraction* is a fraction that is not a proper fraction.

increasing function
The function $f(x)$ is increasing on an interval for any numbers s and t in that interval, if t is greater than s then $f(t)$ is greater than $f(s)$.

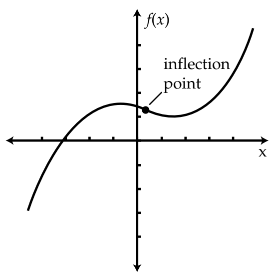
indefinite integral
Another word for *antiderivative.*

independent events
Two events are *independent* if whether or not one of them occurs has no effect on the probability that the other occurs.

inductive reasoning
A form of reasoning in which the truth of an assertion in some particular cases is used to leap to the (tentative) conclusion that the assertion is true in general.

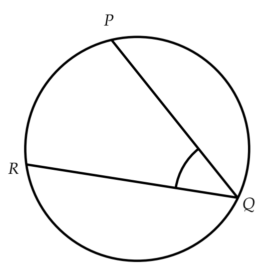
inequality
A mathematical statement that one quantity is greater than or less than the other. The statement $s > t$ means that s is greater than t, while $s < t$ means that s is less than t.

infinite geometric series
The sum $a + ar + ar^2 + \quad + ar^{n-1} + \quad$ of all of the terms of a geometric sequence. If $|r| < 1$, then this sum is equal to $\frac{a}{1-r}$.



inflection point
A point on a curve that separates a part of the curve that is concave up from one that is concave down.

initial value problem
A function is described by specifying a differential equation that it satisfies, together with the value of the function at some "initial" point; the problem is to find the function.

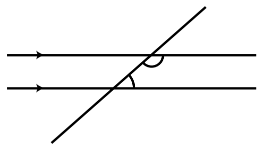


inscribed angle
The angle PQR, where P, Q, and R are three points on a curve, in most cases a circle.

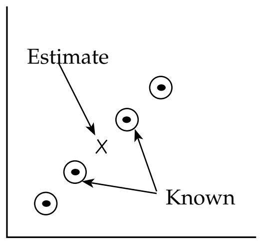
instantaneous velocity (at a particular time)
The exact rate at which the position is changing at that time.

integer
One of 0, 1, -1, 2, -2, 3, -3, 4, -4, and so on.

integration
In part, the process of finding antiderivatives.



interior angles on the same side of the transversal
The transversal of two parallel lines forms interior supplementary angles.



interpolate
Estimate the value of a function at a point from values of the function at places on both sides of the point.

intersection
The point or points where two curves meet.

interval
The set of all real numbers between two given numbers, which may or may not be included. The set of all real numbers from a given point on, or up to a given point, is also an interval, as is the set of all real numbers.

inverse (of a function)
The function $g(x)$ is the inverse of the function $f(x)$ if $f(g(x)) = x$ and $g(f(x)) = x$ for all x, or more informally if each function undoes what the other did.

inverse operations
Operations that counteract each other. For example, addition and subtraction are inverse operations.

inverse trigonometric functions
Inverses of the six basic trigonometric functions. For the two most commonly used, please see *arc sine* and *arc tan*.

irrational number
A number that cannot be expressed as a quotient of two integers. For example, $\sqrt{2}$, π, and e are irrational numbers.

irregular
Lacking in symmetry or pattern.

isosceles triangle
A triangle that has two or more equal sides. Occasionally defined as a triangle that has *exactly* two equal sides.

L

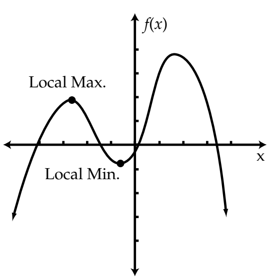
least squares
A criterion used to find the line of best fit, namely that the sum of the squares of the differences between "predicted values" and actual values should be as small as possible.

limit
The *limit* of $f(x)$ as x approaches a (notation: $\lim_{x\to a} f(x)$) is the number that $f(x)$ tends to as x moves closer and closer to a. There may not be such a number. For example, if x is measured in radians, $\lim_{x\to 0}\frac{\sin x}{x} = 1$, but $\lim_{x\to 0}\frac{1}{x}$ does not exist.

line of best fit
For a collection of points in the plane obtained from an experiment, a line that comes in some sense closest to the points. Please see *least squares*.

linear function
A function f given by a formula of the type $f(x) = ax + b$, where a and b are specific numbers.

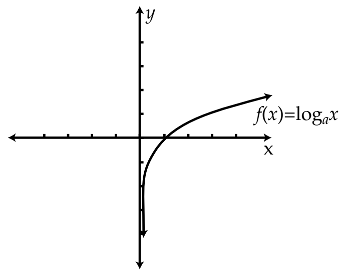
linear programming
Finding the largest or smallest value taken on by a given function $a_1x_1 + a_2x_2 + \ldots + a_nx_n$ (the *objective function*) given that $x_1, x_2, \ldots, x_n$ satisfy certain *linear constraints*. The constraints are inequalities of the form $b_1x_1 + b_2x_2 + \ldots + b_n \geq c$. Many applied problems, such as designing the cheapest animal feed that meets given nutritional goals, can be formulated as linear programming problems.



local maximum
The function $f(x)$ is said to reach a local maximum at $x = a$ if there is a neighbourhood of a such that $f(x) \leq f(a)$ for any x in the neighbourhood; informally, $(a, f(a))$ is at the top of a hill.

local minimum
The function $f(x)$ is said to reach a local minimum at $x = a$ if there is a neighbourhood of a such that $f(x) \geq f(a)$ for any x in the neighbourhood; informally, $(a, f(a))$ is at the bottom of a valley.

logarithmic differentiation
The process of differentiating a product/quotient of functions by finding the logarithm and then differentiating. For example, let $y = \frac{(1+x)^2}{(1+3x)}$. Then $\ln y = 2\ln(1+x) = \ln(1+3x)$ and $\frac{1}{y}\frac{dy}{dx} = \frac{2}{1+x} - \frac{3}{1+3x}$.



logarithmic function
Let a be positive and not equal to 1. The logarithm of x to the base a is the number u such that $a^u = x$, and is denoted by $\log_a x$. Any function of the form $f(x) = \log_a x$ is called a logarithmic function.

lowest common multiple (LCM)
The smallest positive integer that is a multiple of two or more given positive integers. For example, the LCM of 3, 4, and 6 is 12. The LCM is often called the *least* common multiple.

M

matrix
A rectangular array of numbers. For example:

$$\begin{pmatrix} 3 & 4 \\ -2 & 5 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$$

2 × 2 matrix   3 × 1 matrix

maximum point (or value)
The greatest value of a function.

mean (of a sequence of numerical data)
A measure of the average value, obtained by adding up the terms of the sequence and dividing by the number of items.

median (of a sequence of numerical data)
The "middle value" when the data are arranged in order of size. If there is an even number of data, then the average of the two middle values. For example, the median of 5, 3, 7.4, 5, 8, is 5, while the median of 5, 7.4, 5, and 8 is 6.2.

median (of a triangle)
The line segment that joins a vertex of the triangle to the midpoint of the opposite side.

minimum point (or value)
The lowest value of a function.

mixed number
A number that is expressed as the sum of a whole number and a fraction.
For example: $3\frac{2}{5}$

mode
The value that occurs most often in a sequence of data.

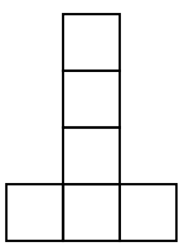
monomial
An algebraic expression that is a product of variables and constants.
Examples: $6x^2$, 1, $\frac{3}{4}x^2y$

multiple (of an integer)
The result obtained when the given integer is multiplied by some integer. Equivalently, an integer that has the given integer as a factor. (Often negative integers are not allowed.)

N

natural logarithm
Logarithm to the base e, where e is a fundamental mathematical constant roughly equal to 2.7182818284. The natural logarithm of x is usually written $\ln x$.

natural number (counting number)
One of the numbers 1, 2, 3, 4, . . . *Positive integer* means the same thing.

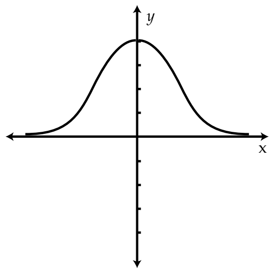


net
A flat diagram consisting of plane faces arranged so that it may be folded to form a solid.

Newton's Law of Cooling
The assertion that if a warm object is placed in a cool room, its temperature decreases at a rate proportional to the difference in temperature between the object and its surroundings.

Newton's Method
An often highly efficient iterative method for approximating the roots of $f(x) = 0$. If r_n is the current estimate, then the next estimate is the x-intercept of the tangent line to $y = f(x)$ at $x = r_n$.

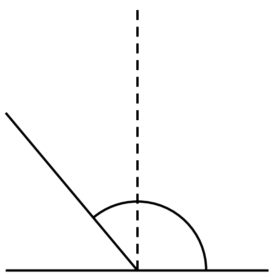
non-differentiable (function)
A function $f(x)$ is non-differentiable at $x = a$ if it does not have a derivative there. Example: if $f(x) = |x|$ then $f(x)$ is not differentiable at $x = 0$, basically because the curve $y = |x|$ has a sharp kink there.



normal distribution curve
The standard normal distribution curve has equation $y = \dfrac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$.

The general normal is obtained by shifting the standard normal to the left or right, and/or rescaling. These curves are sometimes called bell-shaped curves. They figure importantly in probability, statistics, and signal processing.

O



obtuse angle
An angle whose measure is between 90° and 180°.

one-sided limit
Sometimes $f(x)$ exhibits different behaviour depending on whether x approaches a from the right (through values of x greater than a) or from the left. For example, let $f(x) = \dfrac{1}{\left(1 + 2^{\frac{1}{x}}\right)}$. As x approaches 0 from the right, $f(x)$ approaches 0 (notation: $\lim\limits_{x \to 0^+} f(x) = 0$) while $f(x)$ approaches 1 as x approaches 0 from the left.

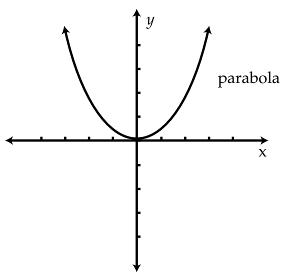
optimization problem
A problem, often of an applied nature, in which we need to find the largest or smallest possible value of a quantity; also called a *max/min problem*.

ordered pair
A sequence of length 2. Ordered pairs (x, y) of real numbers are used to indicate the x and y coordinates of a point in the plane.

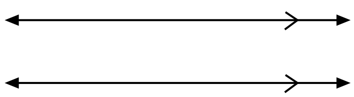
ordinal number
A number designating the place occupied by an item in an ordered sequence (e.g., first, second, and third).

origin
The point in a coordinate system at the intersection of the axes.

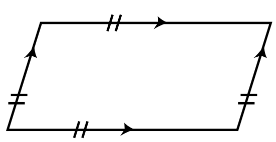
P



parabola
The intersection of a conical surface and a plane parallel to a line on the surface.



parallel lines
Two lines in the *plane* are *parallel* if they do not meet. In three-dimensional space, two lines are parallel if they do not meet *and* there is a plane that contains them both. Alternately, in the plane or in space, two lines are parallel if they stay a constant distance apart.



parallelogram
A quadrilateral such that pairs of opposite sides are parallel.

percentage
In a problem such as "Find 15% of 400," the number 400 is sometimes called the *base*, 15% or 0.15 is called the *rate*, and the answer 60 is sometimes called the *percentage*.

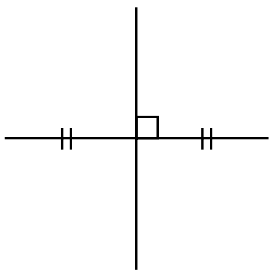
percent error
The relative error expressed in parts per hundred. Let A be an estimate of a quantity whose true value is T. Then $A - T$ is the *error*, and $(A - T)/T$ is the *relative error*.

percentile
The k-th *percentile* of a sequence of numerical data is the number x such that k percent of the data points are less than or equal to x. (Often x is not precisely determined, particularly if the data set is not large.)

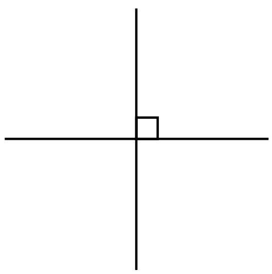
perimeter
The length of the boundary of a closed figure.

period
The interval taken to make one complete oscillation or cycle.

permutation
An ordered arrangement of objects. The number of ways of producing a permutation of r (distinct) objects from a collection of n objects is ${}_{n}P_{r}$, where ${}_{n}P_{r} = n(n - 1)(n\ \ 2)\ldots(n - r + 1)$.

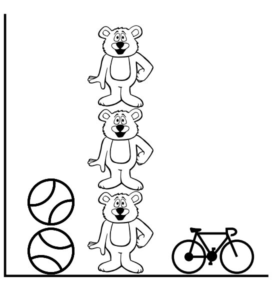


perpendicular bisector
A line that intersects a line segment at a right angle and divides the line segment into two equal parts.

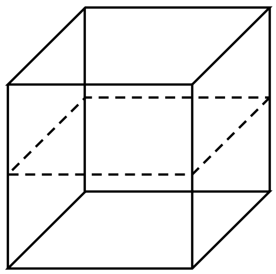


perpendicular line
Two lines that intersect at a right angle.

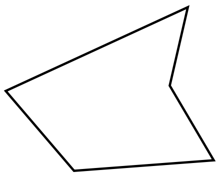
phase shift
A horizontal translation of a periodic function. For example, the function $\cos 2\left(x - \frac{\pi}{3}\right)$ is $\cos 2x$ with a phase shift of $\frac{\pi}{3}$.

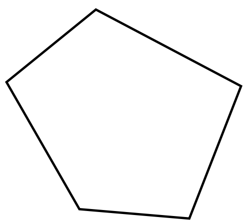


pictograph
A graph that uses pictures or symbols to represent similar data.



plane of symmetry
A 2-D flat surface that cuts through a 3-D object, forming two parts that are mirror images.





polygon
A closed curve formed by line segments that do not intersect other than at the vertices.

polygonal region
A part of a plane that has a polygon as a boundary.

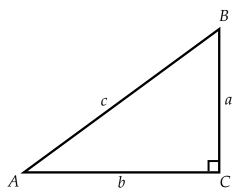
polyhedron
A solid bounded by plane polygonal regions.

polynomial
A mathematical expression that is a sum of monomials. Examples:
$4x^3 - 3x - \frac{1}{2}, \pi x^2 + 2\pi xy, xyz$

population
The items, actual or theoretical, from which a sample is drawn.

power
A power of q is any term of the form q^k. Often but not always, k is taken to be a positive integer.

precision
A measure of the estimated degree of repeatability of a measurement, often described by a phrase such as correct to two decimal places.



Sine A = $\frac{a}{c}$

primary trigonometric functions

$$\sin A = \frac{a}{c} = \frac{opposite}{hypotenuse}$$

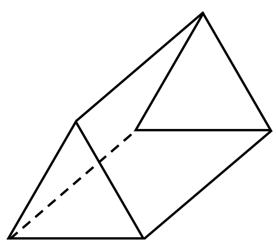
$$\cos A = \frac{b}{c} = \frac{adjacent}{hypotenuse}$$

$$\tan A = \frac{a}{b} = \frac{opposite}{adjacent}$$

Functions of angles defined, for an acute angle, as ratios of sides in a right triangle.

prime
A positive integer that is divisible by exactly two positive integers, namely 1 and itself. The first few primes are 2, 3, 5, 7, 11, and 13.

prime factorization (of a positive integer)
The given integer expressed as a product of primes. For example, $2 \times 5 \times 3 \times 2$ is a prime factorization of 60. Usually the primes are listed in increasing order. The standard prime factorization of 60 is $2^2 \times 3 \times 5$.



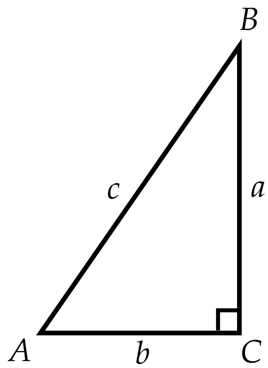
prism
A solid with two parallel and congruent bases in the shape of polygons; the other faces are parallelograms.

probability (of an event)
A number between 0 and 1 that measures the likelihood that the event will occur. P(A) often denotes the probability of the event A.

product
The *product* of two or more objects (numbers, functions, etc.) is the result of multiplying these objects together.

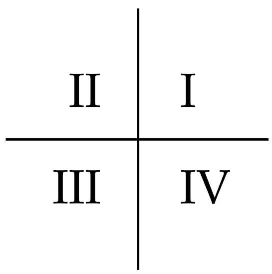
product rule
The rule for finding the derivative of a product of two functions.
If $p(x) = f(x)g(x)$ then $p'(x) = f(x)g'(x) + g(x)f'(x)$.

pyramid
A polyhedron one of whose faces is an arbitrary polygon (called the *base*) and whose remaining faces are triangles with a common vertex called the *apex*).



Pythagorean theorem
In a right-angled triangle, the sum of the squares of the lengths of the sides containing the right angle is equal to the square of the hypotenuse ($a^2 + b^2 = c^2$).

Q

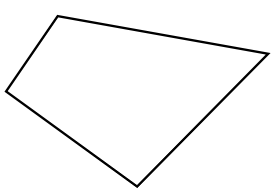


quadrant
One of the four regions that the plane is divided into by two perpendicular lines. When these lines are the usual coordinate axes, the quadrants are called the first quadrant, the second quadrant, and so on as in the diagram.

quadratic formula
A formula used to determine the roots of a quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

quadratic function
A function of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. The graph of such a function is a parabola.



quadrilateral
A polygon with four sides.

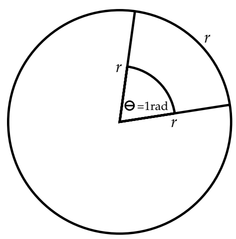
quartile
The 25th percentile is the first quartile, the 50th percentile is the second quartile (or median), and the 75th percentile is the third quartile. Please see *percentile*.

quotient
The result of dividing one object (number, function) by another.

quotient rule
The rule for differentiating the quotient of two functions.

If $q(x) = \dfrac{f(x)}{g(x)}$ then $q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{\left(g(x)\right)^2}$.

R



radian
Equal to the central angle subtended by an arc of unit length at the centre of a circle of unit radius.

radical
The square root, or cube root, and so on of a quantity. For example, the cube root of the quantity Q is the quantity R such that R^3 (the cube of R) is equal to Q. The square root of Q is written $\sqrt{Q}$ ($\sqrt{}$ is the *radical sign*). The cube root of Q is written $\sqrt[3]{Q}$.

radius
A line segment that joins the centre of a circle or sphere to a point on the boundary. All radii of a circle or sphere have the same length. That common length is called the *radius*.

range
A measure of variability of a sequence of data, defined to be the difference between the extremes in the sequence. For example, if the data are 27, 22, 27, 20, 35, and 34, then the range is 15.

range (of a function)
The set of values taken on by a function. Please see *function*.

rank ordering
Ordering (of a sample) according to the value of some statistical characteristic.

rate
A comparison of two measurements with different units. For example, the speed of an object measured in kilometres per hour.

rate of change (of a function at a point)
How fast the function is changing. If $f(x)$ is the function, its rate of change with respect to x at $x = a$ is the derivative of $f(x)$ at $x = a$.

ratio
Another word for quotient. Also, an indication of the relative size of two quantities. We say that P and Q are in the ratio a:b if the size of A divided by the size of B is a / b.

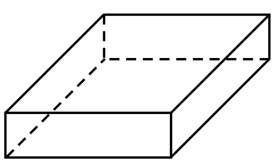
rational expression
The quotient of two polynomials.

rational number
A number that can be expressed as a / b, where a and b are integers.

rationalize the denominator
Transform a quotient P / Q where the denominator Q involves radicals into an equivalent expression with the denominator free of radicals.
For example: $\frac{4}{4-\sqrt{7}} = \frac{(4)\left(4+\sqrt{7}\right)}{\left(4-\sqrt{7}\right)\left(4+\sqrt{7}\right)} = \frac{(4)\left(4+\sqrt{7}\right)}{9}$.

real number
An indicator of location on a line with respect to an origin; a quantity represented by an arbitrary decimal expansion.

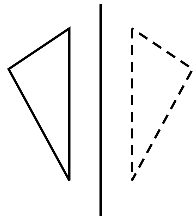
reciprocal
The number or expression produced by dividing 1 by a given number or expression.



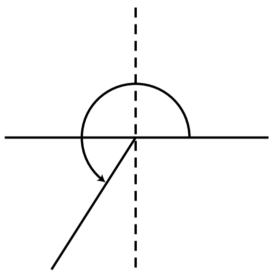
rectangular prism
A prism whose bases are congruent rectangles.

recursive definition (of a sequence)
A way of defining a sequence by possibly specifying some terms directly, and giving an algorithm by which any term can be obtained from its predecessors. For example, the Fibonacci sequence is defined recursively by the rules $F_0 = F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$.

reference angle
The acute angle between the ray line and the x-axis. For example, the reference angles of a 165° and of a 195° angle are each 15° angles.



reflection (in a line)
The transformation that takes any 2-dimensional object to the object that is symmetrical to it with respect to the line, that is, to its mirror image in the line. *Flip* means the same thing. In three-dimensions, we can define analogously *reflection in a plane*.



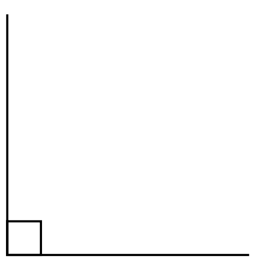
reflex angle
An angle greater than 180° and less than 360°.

relative maximum, minimum
Please see *local maximum*, *local minimum*.

Remainder Theorem
If we divide the polynomial $P(x)$ by $x - a$, the remainder is equal to $P(a)$.

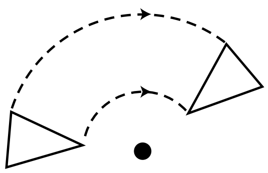
repeating decimal
A decimal expansion that has a block of digits that ultimately cycles forever. For example, $\frac{23}{22}$ has the decimal expansion 1.0454545 . . ., with the block 45 ultimately cycling forever. A terminating decimal like 0.25 is usually viewed as being a repeating decimal, indeed in two ways: as 0.25000 . . . and 0.24999 . . .

resultant
The sum of two or more vectors.



right angle
An angle whose measure is 90°.

root of an equation (in one variable)
If the equation has the form $F(x) = G(x)$, a root of the equation is a number a such that $F(a) = G(a)$.



rotation (in the plane)
A transformation in which an object is turned through some angle about a point. An analogous notion can be defined in three dimensions; there the turn is about a line.

rounding
A process to follow when making an approximation to a given number by using fewer significant figures.

S

sample
A selection from a population.

sample space
The set of all possible outcomes of an experiment.

scalar
A number. Usually used in contexts where there are also vectors around, or functions. Examples of usage: "the length of a vector is a scalar"; "-3 sin x is a scalar multiple of sin x."

scatter plot
If each item in a sample yields two measurements, such as the height x and weight y of the individual chosen, the point with coordinates (x,y) is plotted. If this is repeated for all members of the sample, the resulting collection of points is a scatter plot.

secant (of x)
This is $\frac{1}{\cos x}$. Notation: sec x.

secant line
A line that passes through two points on a curve.

second derivative
The second derivative of $f(x)$ is the derivative of the derivative of $f(x)$. Two common notations: $f''(x)$ and $\frac{d^2 f}{dx^2}$.

second derivative test
Suppose that $f(a) = 0$. The second derivative test gives a way of checking whether at $x = a$ the function $f(x)$ reaches a local minimum or a local maximum.

second-hand data
Data not collected directly by the researcher. For example: encyclopedia.

semicircle
A half-circle; any diameter cuts a circle into two semicircles.

sequence
A finite ordered list $t_1, t_2, \ldots, t_n$ of terms (finite sequence) or a list $t_1, t_2, \ldots, t_n, \ldots$ that goes on forever (an infinite sequence).

series
Any sum of $t_1 + t_2 + \ldots + t_n$, the first n terms of a sequence. The sum $t_1 + t_2 + \ldots + t_n + \ldots$ of all the terms of an infinite sequence is an *infinite series*. The concept of limit is required to define the sum of infinitely many terms.

SI measure
Abbreviation for Système International d'Unités – International System of Units – kilogram, second, ampere, kelvin, candela, mole, radian, and so on.

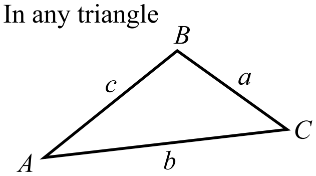
side (of a polygon)
Any of the line segments that make up the boundary of the polygon.

sigma notation
The use of the sign (Greek capital sigma) to denote sum.
For example: $\sum_{i=1}^{5} a_i = a_1 + a_2 + a_3 + a_4 + a_5$.

simple interest
Interest computed only on the original principal of a loan or bank deposit.

sine (function)
Please see *primary trigonometric functions*.

In any triangle

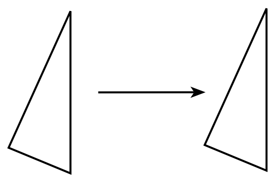


$$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$$

sine law (law of sines)
A formula used for solving triangles in plane trigonometry.

skeleton
A representation of the edges of a polyhedron.

skip counting
Counting by multiples of a number. For example: 2, 4, 6, 8.

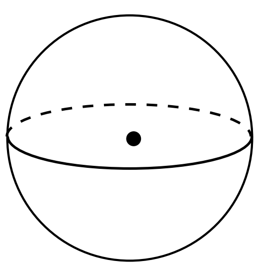


slide
A transformation of a figure by moving it up/down and/or left/right without any rotation. The word is a synonym for the more standard mathematical term translation.

slope
The slope of a (non-vertical) line is a measure of how fast the line is climbing. It can be defined as the change in the y-coordinate of a point on the line when the x-coordinate is increased by 1. If a curve has a (non-vertical) tangent line at the point, the slope of the curve at the point is defined to be the slope of that tangent line.

slope-intercept form (of the equation of a line)
An equation of the form $y = mx + b$. All lines in the plane except for vertical lines can be written in this form. The number m is the slope and b is the y-intercept.

solution (of a differential equation)
A function that satisfies the differential equation. For example, for any constant C, the function given $y = \left(x^2 + C\right)^{\frac{1}{3}}$ is a solution of the differential equation $3y^2 \dfrac{dy}{dx} = 2x$.

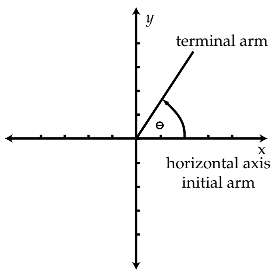


sphere
A solid whose surface is all points equidistant from a centre point.

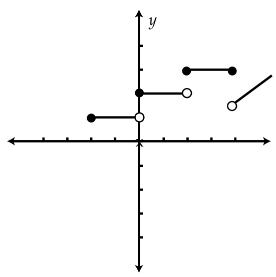
square root
The square root of x is the non-negative number that when multiplied by itself produces x. For example, 5 is the square root of 25. In general the square root of x^2 is $|x|$, the absolute value of x.

standard deviation
Sample standard deviation is the square root of the sample variance. Population standard deviation is the square root of the population variance.

standard form
The usual form of an equation. For example, the standard form of the equation of a circle is $(x-a)^2+(y-b)^2=r^2$, because it reveals geometrically important features, the centre and the radius.



standard position (angle in)
The initial arm of the angle is the positive horizontal axis (x-axis.) Counterclockwise rotation gives a positive angle.



step function
A function whose graph is flat except at a finite number of points, where it takes a sudden jump.

supplementary angles
Two angles whose sum is 180°.

symmetrical (has symmetry)
A geometrical figure is symmetrical if there is a rotation reflection, or combination of these that takes the figure to itself but moves some points. For example, a square has symmetry because it is taken to itself by a rotation about its centre through 90°.

system of equations
A set of equations. A *solution* of the system is an assignment of values to the variables such that all of the equations are (simultaneously) satisfied. For example, $x = 1$, $y = 2$, $z = -3$ is a solution of the system $x + y + z = 0$, $x - y - 4x = 11$.

T

tangent (function)
Please see *primary trigonometric functions.*

tangent (to a curve)
A line is tangent to a curve at the point P if under very high magnification, the line is nearly indistinguishable from the curve at points close to P. A tangent line to a circle can be thought of as a line that meets the circle at only one point.

tangent line approximation
If P is a point on a curve, then close to P the curve can be approximated by the tangent line at P. In symbols, if x is close to a, then $f(x)$ is very closely approximated by $f(a) + (x - a)f'(a)$.

tangram
A square cut into seven shapes: two large triangles, one medium triangle, two small triangles, one square, and one parallelogram.

term
Part of an algebraic expression. For example, x^3 and $5x$ are terms of the polynomial $x^3 + 3x^2 + 5x - 1$.

terminating decimal
A decimal expansion that (ultimately) ends. Example: 3.73.

tesselation
A covering of a surface (usually the entire plane) without overlap or bare spots, by copies of a given geometric figure or of a finite number of given geometric figures. The word comes from *tessala*, the Latin word for a small tile.

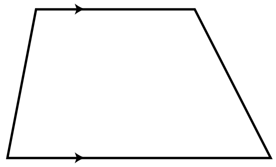
theoretical probability (of an event)
A numerical measure of the likelihood that the event will occur, based on a probability model. If, as can happen with dice or coins, an experiment has only a finite number n of possible outcomes, all equally likely, and in k of these the event occurs, then the theoretical probability of the event is k / n.

tolerance (interval)
The set of numbers that are considered acceptable as the dimension of an item. Example: a manufacturer's tolerance interval for the weight of a "400 gram" box of cereal might be from 395 grams to 420 grams.

transformation
A change in the position of an object, and/or a change in size, and related changes. Also, a change in the form of a mathematical expression.

translation
Please see *slide*.

transversal
A line that intersects two or more lines at different points.



trapezoid
A quadrilateral that has two parallel sides. Some definitions require that the remaining two sides not be parallel.

tree diagram
A pictorial way of representing the outcomes of an experiment that involves more than one step.

trigonometry
The branch of mathematics concerned with the properties and applications of the trigonometric functions, in particular their use in "solving" triangles, in surveying, in the study of periodic phenomena, and so on.

trinomial
A polynomial that has three terms. For example: $ax^2 + bx + c$.

turn
Please see *rotation*.

U

unbiased
A sampling procedure for estimating a population parameter (like the proportion of BC teenagers who smoke) is unbiased if on average it should yield the correct value. At a more informal level, a polling procedure is unbiased if proper randomization procedures are used to select the sample, the wording of the questions is neutral, and so on.

unit circle
A circle of radius 1.

unit vector
A vector of length 1.

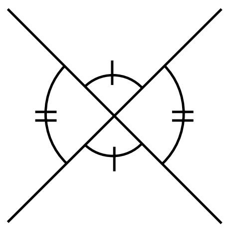
V

variable
A mathematical entity that can stand for any of the members of a given set.

variance
Sample variance is a measure of the variability of a sample, based on the sum of the squared deviations of the data values about the mean. *Population variance* is a theoretical measure of the variability of a population.

vector
A directed line segment (arrow) used to describe a quantity that has direction as well as magnitude.

vertex (pl. vertices)
In a polygon, a point of intersection of two sides. In a polyhedron, a vertex of a face.



vertically opposite angles
Opposite (and equal) angles resulting from the intersection of two lines.

W

whole number
One of the counting numbers 0, 1, 2, 3, 4, and so on; a non-negative integer.

X

***x*-intercept(s)**
The point(s) at which a curve meets the *x*-axis (horizontal axis).

Y

***y*-intercept(s)**
The point(s) at which a curve meets the *y*-axis (vertical axis).

Z

***z*-score**

If x is the numerical value of one observation in a sample, the z-score of x is $\frac{(x - \bar{x})}{s}$, where $\bar{x}$ is the sample mean and s is the sample standard deviation. The z-score measures how far x is from the mean.

zero (root) of $f(x)$
Any number a such that $f(a)=0$.