Applications of the Smoothed Particle Hydrodynamics method: The Need for Supercomputing

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We shortly describe the numerical method *Smoothed Particle Hydrodynamics* (SPH) and report on our parallel implementation of the code. One major application of our code is the simulation of astrophysical problems. We present some recent results of simulations of accretion disks in close symbiotic binary stars.

1. Smoothed Particle Hydrodynamics

Smoothed Particle Hydrodynamics (SPH; [11], [5], [13]) is a meshless Lagrangian particle method for solving a system of hydrodynamic equations for compressible fluids. SPH is especially suited for problems with free boundaries, a commonplace situation in astrophysics. Rather than solving the equations on a grid, the equations are solved at the positions of the so-called particles, each representing a mass packet with a certain density, velocity, temperature etc. and moving with the flow.

The principle of the SPH method is to transform a system of coupled partial differential equations into a system of coupled ordinary differential equations which can be solved by a standard integration scheme. This is achieved by a convolution of all variables with an appropriate smoothing kernel W and an approximation of the integral by a sum over particle quantities:

$$f(\mathbf{x}) \longrightarrow \int d^3x' \; f(\mathbf{x}')W(\mathbf{x}-\mathbf{x}') \longrightarrow \sum_i V_i f_i W(\mathbf{x}-\mathbf{x}_i)$$

Then all spatial derivatives can be computed as derivatives of the analytically known kernel function. Thus only the derivatives in time are left in the equations.

The main advantage of SPH is that it is a Lagrangian formulation where no advection terms are present. Furthermore conservation of mass comes for free, and the particles can

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be almost arbitrarily distributed which removes the need for a computational grid. By varying the kernel function W in space or time one can adapt the resolution, if necessary.

The Euler equation, for example, in its SPH form reads

$$\frac{d\mathbf{v}_i}{dt} = -\sum_j m_j \frac{p_j + p_i}{\varrho_i \varrho_j} \nabla W(\mathbf{x}_i - \mathbf{x}_j)$$

which has been derived as outlined above and then antisymmetrized. In order to efficiently evaluate this equation it is important to use a kernel function W that has compact support, thus reducing the number of non-zero contributions to the sum which runs over all particles. Finding interacting particles is an important part of every SPH implementation; this is done using well-known grid- or tree structures.

In contrast to many other flavors of SPH used in astrophysics, in our approach the viscous stress tensor is not a rather arbitrary artificial viscosity. Instead it is implemented according to the Navier-Stokes equation to describe the physical viscosity correctly [4].

2. The parallel implementation

The usual approach of using high level languages (such as High Performance Fortran, HPF) for a parallelization of the code proved not feasible. The irregular particle distributions create irregular data structures, and nowaday's compilers unfortunately cannot create efficient code in this situation. We instead decided to use the low level MPI library as it is available for all common platforms.

2.1. Straightforward Domain Decomposition

The main principle we settled for was using a domain decomposition where a modified serial version of the code runs on every node. The communication across domain boundaries is taken care of by a special kind of boundary condition, akin to periodic boundaries. This way the communication routines are separated from the routines implementing the physics. We hope that this will make future additions to the physics easier, because people adding to the physics will need only a basic knowledge of the way communication is handled.

This inter-domain boundary condition takes care of (almost) all necessary communication and sets up ghost particles for the SPH routines. The same approach had already successfully been implemented for periodic boundaries, only that now the ghost particles come from other nodes. Of course particle interactions that cross domain boundaries are calculated on only one node.

The disadvantage of this method is that a low number of particles cannot efficiently be distributed onto many nodes. The ghost particle domain of each node has the size of the interaction range, and for increasing numbers of nodes the ghost particles eventually outnumber the real particles. Although the numerical workload stays the same, managing the particles becomes more expensive. The common remedy is to increase the number of particles proportionally to the number of nodes.

2.2. Not Wasting Memory

An SPH code needs at least three passes over all particles, computing the density, the viscous stress tensor, and the acceleration, respectively. If those passes are run one after the other, then the interaction information for all particles has to be kept in memory. Given that there are about 100 interactions per particle this information requires by far (a factor of 10) the largest amount of memory of the overall simulation. This severely limits the total number of particles that fit into a given computer system.

In order to save on memory we run these passes in parallel, where each particle begins the next pass as soon as it and all its neighbours have finished the previous one. This can be realized with only negligible overhead by calculating the interactions by sweeping through the simulation domain combining all three passes. The interactions of a particle are determined on the fly when the particle is first encountered and are dropped from the interaction list as soon as the particles has finished the third pass.

This sweeping happens (almost) independently on all nodes. In between the passes information about the particles may have to be exchanged between nodes, which is taken care of by the boundary condition module.

2.3. Simple Load Balancing

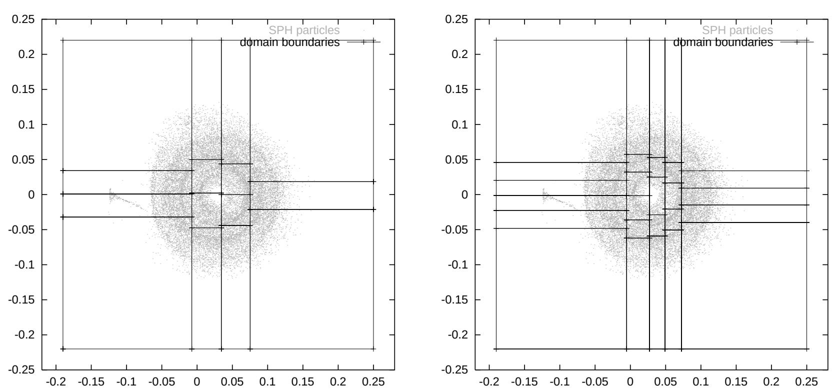

Fig. 1. Two examples of domain decompositions. On the left for 15 nodes, on the right for 32 nodes.

Load balancing, although of vital importance, has only been implemented in an ad hoc fashion. The domains are cuboid and of different sizes. They do not form a grid but rather a tree structure. The initial domains are chosen by distributing the particles evenly; this distribution is refined during the simulation by monitoring computing time and resizing the domains after every time step. In order to keep the domain shapes as cubic as possible, nodes may be transferred to a different subtree, thus reorganizing the overall domain structure. Two examples of domain decompositions are shown in Figure 1.

Thanks to MPI our code runs on many different platforms. It has been tested on our workstation cluster, a Beowulf cluster, the IBM SP/2, and the Cray T3E. It performs reasonably well on all those platforms; the load balancing takes only a negligible amount of time. The typical overall time spent waiting is less than about 12 %. Typical runs have 300 000 particles on about 50 nodes, where one evaluation of the right hand side takes about one second.

3. Accretion Disks in Symbiotic Binary Stars

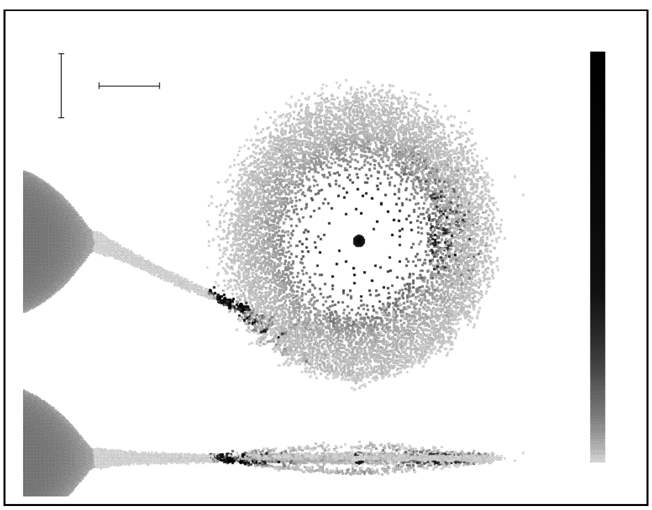

Fig. 2. Pole-on and edge-on view of a simulation of the accretion disk of the Dwarf Nova OY Car. Mass of the White Dwarf: 0.696 $M_\odot$, mass of the donor star: 0.069 $M_\odot$. The scales indicate 0.1 solar radii. The donor star is on the left. Greycoded is the dissipated energy.

Symbiotic binary stars are so close to each other that their evolution and appearance changes dramatically compared to single stars. Dwarf Novae are a class of variable symbiotic binary stars where mass transfer from one star to the other occurs. The donor is a light main sequence star, the accretor a more massive, but much smaller White Dwarf (WD). Due to its intrinsic angular momentum the overflowing gas cannot be accreted by the WD right away, instead a thin gaseous disk around the WD forms and the subsequent accretion is governed by viscous processes in the disk, [18]. The physics of these accretion disks is far from being well understood. Existing models of long term outburst behaviour are essentially 1D and neglect the tidal influence of the donor star [12]. Observationally, the disks show variability on timescales from minutes to decades, occasionally increasing in brightness up to 5 magnitudes. Numerical simulations , especially in 3D, require enormous amounts of grid points — or particles in our case — to achieve the necessary resolution. Since the problem size is so large, and the integration time so long, parallel programs on supercomputers are the only possible way to go.

3.1. 3D-SPH Simulation of the Stream-Disk Interaction in a Dwarf Nova

One aspect of Dwarf Nova disks is the impact of the overflowing gas stream onto the rim of the accretion disk. Both flows are highly supersonic and two shock regions form [10]. The shocked gas becomes very hot, a bright spot develops, which sometimes can be brighter than the rest of the disk. The relative heights of the stream and the rim of the disk are unclear. If the stream is thicker than the disk, a substantial portion of the infalling gas could stream over and under the disk and impact at much smaller radii [1], [8].

Figure 2 shows a snapshot of the simulation of the accretion disk of the Dwarf Nova OY Carinae. Grey-coded is the energy release due to viscous dissipation. One can clearly see the bright spot where the stream hits the disk rim. Furthermore, on the far side of the donor star, a secondary bright spot is visible where overflowing stream material finally impacts onto the disk. In this simulation, about 10 to 20 % of the stream material can flow over and under the disk.

3.2. Superhumps in AM CVn

AM Canem Venaticorum stars are thought to be the helium counterparts to dwarf novae. AM CVn stars are believed to consist of two helium white dwarfs, a rather massive primary and a very light, Roche-lobe filling secondary. Roche-lobe overflow feeds an accretion disk around the primary. Tsugawa & Osaki [16] showed that such helium disks undergo thermal instabilities similar to the hydrogen disks in Dwarf Novae. In three AM CVn stars, Dwarf Nova-like outbursts indeed have been observed.

In order to investigate whether AM CVn exhibit superhumps we performed 3D-SPH simulations of the accretion disk. Initially, there was no disk around the primary. Particles were inserted at the inner Lagrangian point according to the mass transfer rate. Already after about 30 orbital periods the disk grew to a point where it was subject to the 3:1 inner Lindblad resonance [19]. Subsequently, the disk became more and more tidally distorted and started to precess rapidly in the frame of reference corotating with the stars (see Figure 3), which translates to a slow prograde precession in the observers' frame. Every time the bulk of the disk passes the secondary, the tidal stresses and hence the viscous heating are strongest, giving rise to modulations in the photometric lightcurve,

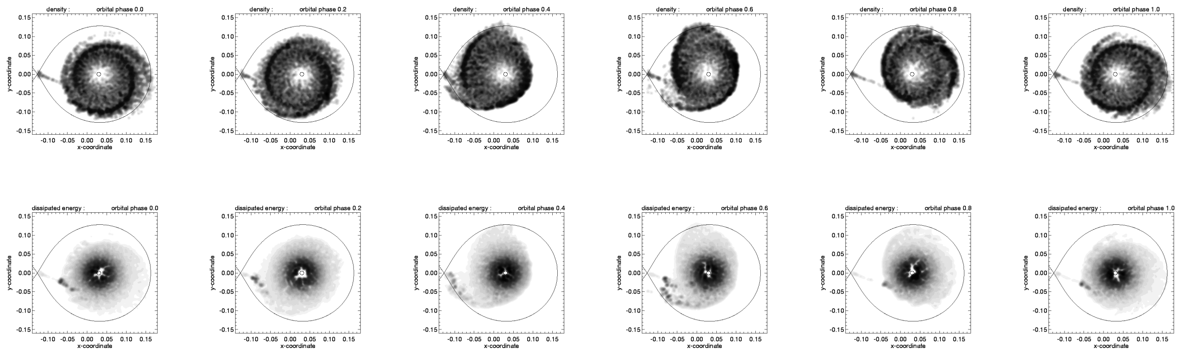

Fig. 3. A series of snapshots of the disk of Am CVn, 0.2 orbital phases apart. Upper panel: density distribution; lower panel: dissipated energy. The parameters used are: $M_1 = 1\ M_\odot$, $M_2 = 0.15\ M_\odot$, mass transfer rate $10^{-10}\ M_\odot$/yr. A ploytropic equation of state with $\gamma = 1.01$ was used. One can see how the precession of the tidally distorted disk leads periodically to higher dissipation, resulting in superhumps in the lightcurve; see Figure 4.

the superhumps. A Fourier transform of the obtained lightcurve reveals a superhump period excess of 4.4 %. This is in good agreement with the periods given by Warner [17], which differ by 3.8 %. A former study of the superhump phenomenon by Kunze et al. [9] showed that the period excess is a function of the mass transfer rate, the mass ratio of the stars, and the kinematic viscosity of the disk. These parameters are not well known for AM CVn.

4. Conclusions

The SPH method is very well suited for solving astrophysical problems with compressible flow and free boundaries. An efficient parallel implementation requires some effort but allows three-dimensional long-term simulations. This is especially helpful for exploring and validating theoretical models where the underlying parameters are not well known. Global properties of the system can be reproduced quite accurately.

References

P. J. Armitage and M. Livio, Astrophys. J., 470 (1996) 1024

W. Benz, in: Numerical Modelling of Stellar Puslations: Problems and prospects, J. R. Buchler (ed.), Kluwer Academic Press, Dordrecht 1990

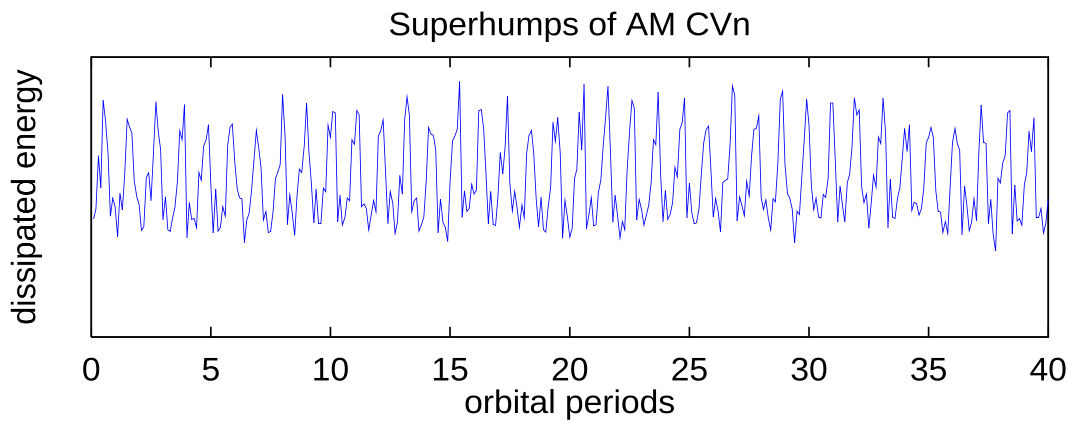

Fig. 4. Shown is the total dissipated energy of the disk over a time span of 40 orbital periods. A Fourier transform of the dissipated energy reveals a superhump period excess of 4.4%.

T. Bubeck, M. Hipp, S. Hüttemann, S. Kunze, M. Ritt, W. Rosenstiel, H.Ruder and R. Speith, in: High Performance Computing in Science and Engineering '98, E. Krause and W. Jäger (eds.), Springer, Berlin, 1999

O. Flebbe, S. Münzel, H. Herold, H. Riffert and H. Ruder, Astrophys. J., 431 (1994) 754

R. A. Gingold and J. J. Monaghan, Mon. Not. R. Astr. Soc., 181 (1977) 375

L. Greengard, V. Rokhlin, A Fast Algorithm for Particle Simulations, J. Comp. Phys., 73 (1987) 325

L. Hernquist, Some Cautionary Remarks About Smoothed Particle Hydrodynamics, Astrophys. J., 404 (1993) 717

F. V. Hessman, Astrophys. J., 510 (1999) 867

S. Kunze, R. Speith and H. Riffert, Mon. Not. R. Astr. Soc., 289 (1997) 889

S. H. Lubow and F.H. Shu, Astrophys. J., 198 (1975) 383

L. B. Lucy, Astron. J., 82 (1977) 1013

F. Meyer and E. Meyer-Hofmeister, Astron. Astrophys., 132 (1983) 143

J. J. Monaghan, Ann. Rev. Astron. Astrophys., 30 (1992) 543

H. Riffert, H. Herold, O. Flebbe, H. Ruder, in: CPC Topical Issue: Numerical Methods in Astrophysics, W. J. Duschl and W. M. Tscharnuter (eds.), in press

M. Steinmetz, E. Müller, Astron. Astrophsy., 268 (1993) 391

M. Tsugawa, Y. Osaki, Publ. Astron. Soc. Japan, 49 (1995) 75

B. Warner, Astron. & Space Sci., 255 (1995) 249

B. Warner, Cataclysmic Variable Stars, Cambridge University Press, Cambridge, 1995

R. Whitehurst, Mon. Not. R. Astr. Soc., 232 (1988) 35