## Applications

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## Applications

1. If you don't brush your teeth regularly, it won't take long for large colonies of bacteria to grow in your mouth. Suppose a single bacterium lands on your tooth and starts multiplying by a factor of 4 every hour.
a. Write an equation that describes the number of bacteria $b$ in the new colony after $n$ hours.
b. How many bacteria will be in the colony after 7 hours?
c. How many bacteria will be in the colony after 8 hours? Explain how you can find this answer by using the answer from part (b) instead of the equation.
d. After how many hours will there be at least $1,000,000$ bacteria in the colony?
e. Suppose that, instead of 1 bacterium, 50 bacteria land in your mouth. Write an equation that describes the number of bacteria $b$ in this colony after $n$ hours.
f. Under the conditions of part (e), there will be $3,276,800$ bacteria in this new colony after 8 hours. How many bacteria will there be after 9 hours and after 10 hours? Explain how you can find these answers without using the equation from part (e).
2. Loon Lake has a "killer plant" problem similar to Ghost Lake in Problem 2.1. Currently, 5,000 square feet of the lake is covered with the plant. The area covered is growing by a factor of 1.5 each year.
a. Copy and complete the table to show the area covered by the plant for the next 5 years.
b. The surface area of the lake is approximately 200,000 square feet. How long will it take before the lake is completely covered?

Growth of Loon Lake Plant

| Year | Area Covered (sq. ft) |
| :---: | :---: |
| 0 | 5,000 |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

3. Leaping Leonora just signed a contract with a women's basketball team. The contract guarantees her $\$ 20,000$ the first year, $\$ 40,000$ the second year, $\$ 80,000$ the third year, $\$ 160,000$ the fourth year, and so on, for 10 years.
a. Make a table showing Leonora's salary each year of this contract.
b. What total amount will Leonora earn over the 10 years?
c. Describe the growth pattern in Leonora's salary.

d. Write an equation for Leonora's salary $s$ for any year $n$ of her contract.
4. As a biology project, Talisha is studying the growth of a beetle population. She starts her experiment with 5 beetles. The next month she counts 15 beetles.
a. Suppose the beetle population is growing linearly. How many

Homework Help nline For: Help with Exercise 4 Web Code: ape-3204 beetles can Talisha expect to find after 2, 3, and 4 months?
b. Suppose the beetle population is growing exponentially. How many beetles can Talisha expect to find after 2,3, and 4 months?
c. Write an equation for the number of beetles $b$ after $m$ months if the beetle population is growing linearly. Explain what information the variables and numbers represent.
d. Write an equation for the number of beetles $b$ after $m$ months if the beetle population is growing exponentially. Explain what information the variables and numbers represent.
e. How long will it take the beetle population to reach 200 if it is growing linearly?
f. How long will it take the beetle population to reach 200 if it is growing exponentially?

5. Fruit flies are often used in genetic experiments because they reproduce very quickly. In 12 days, a pair of fruit flies can mature and produce a new generation. The table below shows the number of fruit flies in three generations of a laboratory colony.
a. What is the growth factor for this fruit-fly population? Explain how you found your answer.

Growth of Fruit-Fly Population

| Generations | 0 | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Number of Fruit Flies | 2 | 120 | 7,200 | 432,000 |

b. Suppose this growth pattern continues. How many fruit flies will be in the fifth generation?
c. Write an equation for the population $p$ of generation $g$.
d. After how many generations will the population exceed one billion?
6. A population of mice has a growth factor of 3 . After 1 month, there are 36 mice. After 2 months, there are 108 mice.
a. How many mice were in the population initially (at 0 months)?
b. Write an equation for the population after any number of months. Explain what information the numbers and variables in your equation represent.
7. Fido did not have fleas when his owners took him to the kennel. The number of fleas on Fido after he returned from the kennel grew according to the equation $f=8\left(3^{n}\right)$, where $f$ is the number of fleas and $n$ is the number of weeks since he returned from the kennel. (Fido left the kennel at week 0.)
a. How many fleas did Fido pick up at the kennel?
b. What is the growth factor for the number of fleas?
c. How many fleas will Fido have after 10 weeks if he is not treated?

8. Consider the equation $y=150\left(2^{x}\right)$.
a. Make a table of $x$ and $y$-values for whole-number $x$-values from 0 to 5 .
b. What do the numbers 150 and 2 in the equation tell you about the relationship?

For Exercises 9-12, find the growth factor and the $y$-intercept of the equation's graph.
9. $y=300\left(3^{x}\right)$
10. $y=300(3)^{x}$
11. $y=6,500(2)^{x}$
12. $y=2(7)^{x}$

For: Multiple-Choice Skills Practice
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13. The following graph represents the population growth of a certain kind of lizard.

14. The following graphs show the population growth for two species.

a. Find the growth factors for the two species. Which species is growing faster? Explain.
b. What are the $y$-intercepts for the graphs of Species X and Species Y?

Explain what these $y$-intercepts tell you about the populations.
c. Write an equation that describes the growth of Species X.
d. Write an equation that describes the growth of Species Y.
e. For which equation is $(5,1215)$ a solution?

## Connections

15. Multiple Choice Choose the answer that best approximates $3^{20}$ in scientific notation.
A. $3.5 \times 10^{-9}$
B. $8 \times 10^{3}$
C. $3 \times 10^{9}$
D. $3.5 \times 10^{9}$
16. Multiple Choice Choose the answer that is closest to $2.575 \times 10^{6}$.
F. $2^{18}$
G. $12^{6}$
H. $6^{12}$
J. $11^{9}$
17. Approximate $5^{11}$ in scientific notation.

For Exercises 18-20, decide whether each number is less than or greater than one million without using a calculator. Explain your reasoning.
18. $3^{6}$
19. $9^{5}$
20. $12^{6}$
21. The prime factorization of 54 is $3 \times 3 \times 3 \times 2$. This can be written using exponents as $3^{3} \times 2$. Write the prime factorization of each number using exponents.
a. 45
b. 144
c. 2,024
22. Consider these equations.

Equation 1: $y=10-5 x \quad$ Equation 2: $y=(10) 5^{x}$
a. What is the $y$-intercept of each equation?
b. For each equation, explain how you could use a table to find how the $y$-values change as the $x$-values increase. Describe the change.
c. Explain how you could use the equations to find how the $y$-values change as the $x$-values increase.
d. For each equation, explain how you could use a graph to find how the $y$-values change as the $x$-values increase.
23. Maria enlarges a $2-\mathrm{cm}-\mathrm{by}-3-\mathrm{cm}$ rectangle by a factor of 2 to get a $4-\mathrm{cm}-\mathrm{by}-6-\mathrm{cm}$ rectangle. She then enlarges the $4-\mathrm{cm}-$ by $-6-\mathrm{cm}$ rectangle by a factor of 2 . She continues this process, enlarging each new rectangle by a factor of 2 .

a. Copy and complete the table to show the dimensions, perimeter, and area of the rectangle after each enlargement.

Rectangle Changes

| Enlargement | Dimensions (cm) | Perimeter (cm) | Area (cm ${ }^{2}$ ) |
| :---: | :---: | :---: | :---: |
| 0 (original) | 2 by 3 |  | $\square$ |
| 1 | 4 by 6 |  | $\square$ |
| 2 | $\square$ |  | $\square$ |
| 3 | $\square$ |  | $\square$ |
| 4 | $\square$ | $\square$ | $\square$ |
| 5 | $\square$ | $\square$ | $\square$ |

b. Is the pattern of growth for the perimeter linear, exponential, or neither? Explain.
c. Is the pattern of growth for the area linear, exponential, or neither? Explain.
d. Write an equation for the perimeter $P$ after $n$ enlargements.
e. Write an equation for the area $A$ after $n$ enlargements.
f. How would your answers to parts (a)-(e) change if the copier were set to enlarge by a factor of 3 ?

Write an equation for each line. Identify the slope and $\boldsymbol{y}$-intercept.
24.

26.

25.

27.


Kele enlarged the figure below by a scale factor of 2 . Ahmad enlarged the figure $\mathbf{2 5 0} \%$. Use this information for Exercises 28 and 29.

28. Who made the larger image?
29. Multiple Choice Which scale factor would give an image whose size is between those of Ahmad's image and Kele's image?
A. $\frac{2}{5}$
B. $\frac{3}{5}$
C. $\frac{9}{4}$
D. $\frac{10}{4}$
30. Companies sometimes describe part-time jobs by comparing them to full-time jobs. For example, a job that requires working half the number of hours of a full-time job is described as a $\frac{1}{2}$-time job or a $50 \%$-time job.
ACME, Inc. has three part-time job openings:

- A $\frac{5}{6}$-time job as a gadget inspector
- A $75 \%$-time job as a widget designer
- A 0.875-time job as a gizmo seller

Order these jobs from the one requiring the most time to the one requiring the least time.

## Extensions

31. a. Make a table and a graph for the exponential equation $y=1^{x}$.
b. How are the patterns in the table and the graph of $y=1^{x}$ similar to patterns you have observed for other exponential relationships? How are they different?
32. You can find the equation for an exponential relationship if you know two points on its graph. Find the equation of the exponential relationship whose graph passes through each pair of points. Explain.
a. $(1,6)$ and $(2,12)$
b. $(2,90)$ and $(4,810)$
33. Leaping Leonora from Exercise 3 also considered an offer from another team. They promised her $\$ 1$ million a year for the next 25 years. The same team offered Dribbling Dawn $\$ 1$ the first year, $\$ 2$ the second year, $\$ 4$ the third year, $\$ 8$ the fourth year, and so on for 25 years.
a. Suppose Leonora and Dawn had both accepted the offers and played for 20 years. At the end of 20 years, who would have received more money?
b. Tell which player would have received more after 21 years, 22 years, 23 years, and 25 years.
