## April 15, Week 13

Today: Chapter 10, Torque

Homework Assignment \#10 - Due April 19.
Mastering Physics: 7 problems from chapter 9
Written Question: 10.86

Help sessions with Jonathan:
M: 1000-1100, RH 111
T: 1000-1100, RH 114
Th: 0900-1000, RH 114

## General Torque

The direction of the force also determines the torque. When $\overrightarrow{\mathbf{F}}$ is not perpendicular to the lever arm ( $\overrightarrow{\mathrm{r}}$ ), only the component of $\vec{F}$ which is perpendicular to $\vec{r}$ causes torque.

## General Torque

The direction of the force also determines the torque. When $\overrightarrow{\mathbf{F}}$ is not perpendicular to the lever arm ( $\overrightarrow{\mathrm{r}}$ ), only the component of $\vec{F}$ which is perpendicular to $\vec{r}$ causes torque.


## General Torque

The direction of the force also determines the torque. When $\overrightarrow{\mathbf{F}}$ is not perpendicular to the lever arm ( $\overrightarrow{\mathrm{r}}$ ), only the component of $\vec{F}$ which is perpendicular to $\vec{r}$ causes torque.


## General Torque

The direction of the force also determines the torque. When $\overrightarrow{\mathbf{F}}$ is not perpendicular to the lever arm ( $\overrightarrow{\mathrm{r}}$ ), only the component of $\vec{F}$ which is perpendicular to $\vec{r}$ causes torque.


## General Torque

The direction of the force also determines the torque. When $\overrightarrow{\mathbf{F}}$ is not perpendicular to the lever arm ( $\overrightarrow{\mathrm{r}}$ ), only the component of $\vec{F}$ which is perpendicular to $\vec{r}$ causes torque.


## General Torque

The direction of the force also determines the torque. When $\overrightarrow{\mathbf{F}}$ is not perpendicular to the lever arm ( $\overrightarrow{\mathrm{r}}$ ), only the component of $\vec{F}$ which is perpendicular to $\vec{r}$ causes torque.


## General Torque

The direction of the force also determines the torque. When $\overrightarrow{\mathbf{F}}$ is not perpendicular to the lever arm ( $\overrightarrow{\mathrm{r}}$ ), only the component of $\vec{F}$ which is perpendicular to $\vec{r}$ causes torque.


## General Torque

The direction of the force also determines the torque. When $\overrightarrow{\mathbf{F}}$ is not perpendicular to the lever arm ( $\overrightarrow{\mathrm{r}}$ ), only the component of $\overrightarrow{\mathrm{F}}$ which is perpendicular to $\overrightarrow{\mathrm{r}}$ causes torque.

$F_{\|}$- component parallel to $\vec{r}$ causes no torque

## General Torque

The direction of the force also determines the torque. When $\overrightarrow{\mathbf{F}}$ is not perpendicular to the lever arm ( $\overrightarrow{\mathrm{r}}$ ), only the component of $\overrightarrow{\mathrm{F}}$ which is perpendicular to $\overrightarrow{\mathrm{r}}$ causes torque.

$F_{\| \mid}$- component parallel to $\overrightarrow{\mathrm{r}}$ causes no torque

## General Torque

The direction of the force also determines the torque. When $\overrightarrow{\mathbf{F}}$ is not perpendicular to the lever arm ( $\overrightarrow{\mathrm{r}}$ ), only the component of $\overrightarrow{\mathrm{F}}$ which is perpendicular to $\overrightarrow{\mathrm{r}}$ causes torque.

$F_{\|}$- component parallel to $\overrightarrow{\mathbf{r}}$ -
causes no torque
$F_{\perp}$ - component perpendicular to $\vec{r}$

- causes torque


## General Torque

The direction of the force also determines the torque. When $\overrightarrow{\mathbf{F}}$ is not perpendicular to the lever arm ( $\overrightarrow{\mathrm{r}}$ ), only the component of $\overrightarrow{\mathrm{F}}$ which is perpendicular to $\overrightarrow{\mathrm{r}}$ causes torque.

$F_{\| \mid}$- component parallel to $\overrightarrow{\mathbf{r}}$ causes no torque
$F_{\perp}$ - component perpendicular to $\vec{r}$

- causes torque
$\tau=r F_{\perp}$


## General Torque

The direction of the force also determines the torque. When $\overrightarrow{\mathbf{F}}$ is not perpendicular to the lever arm ( $\overrightarrow{\mathrm{r}}$ ), only the component of $\overrightarrow{\mathrm{F}}$ which is perpendicular to $\overrightarrow{\mathrm{r}}$ causes torque.

$F_{\| \mid}$- component parallel to $\overrightarrow{\mathbf{r}}$ causes no torque
$F_{\perp}$ - component perpendicular to $\vec{r}$

- causes torque
$\tau=r F_{\perp}$


## General Torque

The direction of the force also determines the torque. When $\overrightarrow{\mathbf{F}}$ is not perpendicular to the lever arm ( $\overrightarrow{\mathrm{r}}$ ), only the component of $\overrightarrow{\mathrm{F}}$ which is perpendicular to $\overrightarrow{\mathrm{r}}$ causes torque.

$F_{\| \mid}$- component parallel to $\overrightarrow{\mathbf{r}}$ -
causes no torque
$F_{\perp}$ - component perpendicular to $\vec{r}$

- causes torque

$$
\tau=r F_{\perp}=r F \sin \phi
$$

## General Torque

The direction of the force also determines the torque. When $\overrightarrow{\mathbf{F}}$ is not perpendicular to the lever arm ( $\overrightarrow{\mathrm{r}}$ ), only the component of $\vec{F}$ which is perpendicular to $\vec{r}$ causes torque.

$F_{\| \|}$- component parallel to $\overrightarrow{\mathrm{r}}$ -
causes no torque
$F_{\perp}$ - component perpendicular to $\vec{r}$

- causes torque
$\phi$ is angle between $\overrightarrow{\mathrm{r}}$ and $\overrightarrow{\mathbf{F}}$

$$
\tau=r F_{\perp}=r F \sin \phi
$$

## General Torque Exercise

In which of the following cases would the torque have the maximum value?

## General Torque Exercise

In which of the following cases would the torque have the maximum value?
(a)


## General Torque Exercise

In which of the following cases would the torque have the maximum value?
(a)

(b)


## General Torque Exercise

In which of the following cases would the torque have the maximum value?
(a)

(b)

(c)


## General Torque Exercise

In which of the following cases would the torque have the maximum value?
(a)

(c)

(d) They each
cause no torque

## General Torque Exercise

In which of the following cases would the torque have the maximum value?
(a)

(c)

(d) They each
cause no torque
(e) They each cause equal torque

## General Torque Exercise

In which of the following cases would the torque have the maximum value?

(c)

$\tau_{3}=(2 m)(1 N) \sin 30^{\circ}=1 N \cdot m$
(d) They each
cause no torque
(e) They each cause equal torque

## The Torque Vector

The direction of the torque vector is given by a cross product.


$$
\tau=r F \sin \phi
$$

## The Torque Vector

The direction of the torque vector is given by a cross product.


$$
\tau=r F \sin \phi
$$

## The Torque Vector

The direction of the torque vector is given by a cross product.


$$
\tau=r F \sin \phi
$$

## The Torque Vector

The direction of the torque vector is given by a cross product.


$$
\tau=r F \sin \phi
$$

## The Torque Vector

The direction of the torque vector is given by a cross product. counter-clockwise rotation

## The Torque Vector

The direction of the torque vector is given by a cross product. counter-clockwise rotation

$$
\vec{\tau}=\odot
$$

$$
\tau=r F \sin \phi
$$

## The Torque Vector

The direction of the torque vector is given by a cross product.


$$
\tau=r F \sin \phi
$$

## Perpendicular Distance

The calculation of torque can be simplified in some cases by the use of the perpendicular distance.

## Perpendicular Distance

The calculation of torque can be simplified in some cases by the use of the perpendicular distance.

Perpendicular Distance, $d$ - The distance from the axis of rotation to the force's line of action that is perpendicular to the line of action.

## Perpendicular Distance

The calculation of torque can be simplified in some cases by the use of the perpendicular distance.

Perpendicular Distance, $d$ - The distance from the axis of rotation to the force's line of action that is perpendicular to the line of action.


## Perpendicular Distance

The calculation of torque can be simplified in some cases by the use of the perpendicular distance.

Perpendicular Distance, $d$ - The distance from the axis of rotation to the force's line of action that is perpendicular to the line of action.


## Perpendicular Distance

The calculation of torque can be simplified in some cases by the use of the perpendicular distance.

Perpendicular Distance, $d$ - The distance from the axis of rotation to the force's line of action that is perpendicular to the line of action.


## Perpendicular Distance

The calculation of torque can be simplified in some cases by the use of the perpendicular distance.

Perpendicular Distance, $d$ - The distance from the axis of rotation to the force's line of action that is perpendicular to the line of action.


## Perpendicular Distance

The calculation of torque can be simplified in some cases by the use of the perpendicular distance.

Perpendicular Distance, $d$ - The distance from the axis of rotation to the force's line of action that is perpendicular to the line of action.


## Perpendicular Distance

The calculation of torque can be simplified in some cases by the use of the perpendicular distance.

Perpendicular Distance, $d$ - The distance from the axis of rotation to the force's line of action that is perpendicular to the line of action.


## Perpendicular Distance

The calculation of torque can be simplified in some cases by the use of the perpendicular distance.

Perpendicular Distance, $d$ - The distance from the axis of rotation to the force's line of action that is perpendicular to the line of action.


## Perpendicular Distance

The calculation of torque can be simplified in some cases by the use of the perpendicular distance.

Perpendicular Distance, $d$ - The distance from the axis of rotation to the force's line of action that is perpendicular to the line of action.


## Perpendicular Distance

The calculation of torque can be simplified in some cases by the use of the perpendicular distance.

Perpendicular Distance, $d$ - The distance from the axis of rotation to the force's line of action that is perpendicular to the line of action.


## Perpendicular Distance

The calculation of torque can be simplified in some cases by the use of the perpendicular distance.

Perpendicular Distance, $d$ - The distance from the axis of rotation to the force's line of action that is perpendicular to the line of action.


## Perpendicular Distance II

The perpendicular distance is particularly useful in finding the torque exerted by vertical forces.

## Perpendicular Distance II

The perpendicular distance is particularly useful in finding the torque exerted by vertical forces.


## Perpendicular Distance II

The perpendicular distance is particularly useful in finding the torque exerted by vertical forces.


## Perpendicular Distance II

The perpendicular distance is particularly useful in finding the torque exerted by vertical forces.


## Perpendicular Distance II

The perpendicular distance is particularly useful in finding the torque exerted by vertical forces.


## Perpendicular Distance II

The perpendicular distance is particularly useful in finding the torque exerted by vertical forces.


## Perpendicular Distance II

The perpendicular distance is particularly useful in finding the torque exerted by vertical forces.


## Perpendicular Distance II

The perpendicular distance is particularly useful in finding the torque exerted by vertical forces.


For vertical forces:

$$
\tau=x F
$$

## Center of Gravity

Real objects consist of many particles. When experiencing a gravitational torque, each individual particle experiences a torque.

## Center of Gravity

Real objects consist of many particles. When experiencing a gravitational torque, each individual particle experiences a torque.

Center of Gravity - The position at which the sum of the torques on the individual particles equals the single torque exerted by the total weight of the object.

## Center of Gravity

Real objects consist of many particles. When experiencing a gravitational torque, each individual particle experiences a torque.

Center of Gravity - The position at which the sum of the torques on the individual particles equals the single torque exerted by the total weight of the object.


## Center of Gravity

Real objects consist of many particles. When experiencing a gravitational torque, each individual particle experiences a torque.

Center of Gravity - The position at which the sum of the torques on the individual particles equals the single torque exerted by the total weight of the object.


## Center of Gravity

Real objects consist of many particles. When experiencing a gravitational torque, each individual particle experiences a torque.

Center of Gravity - The position at which the sum of the torques on the individual particles equals the single torque exerted by the total weight of the object.


## Center of Gravity

Real objects consist of many particles. When experiencing a gravitational torque, each individual particle experiences a torque.

Center of Gravity - The position at which the sum of the torques on the individual particles equals the single torque exerted by the total weight of the object.


## Center of Gravity

Real objects consist of many particles. When experiencing a gravitational torque, each individual particle experiences a torque.

Center of Gravity - The position at which the sum of the torques on the individual particles equals the single torque exerted by the total weight of the object.


Each particle has
$\tau_{i}=x_{i} w_{i}$

## Center of Gravity

Real objects consist of many particles. When experiencing a gravitational torque, each individual particle experiences a torque.

Center of Gravity - The position at which the sum of the torques on the individual particles equals the single torque exerted by the total weight of the object.


Each particle has
$\tau_{i}=x_{i} w_{i}$
Center of gravity is at $x_{c g}$ where
$\sum_{i} \tau_{i}=x_{c g} w$

## Center of Gravity

Real objects consist of many particles. When experiencing a gravitational torque, each individual particle experiences a torque.

Center of Gravity - The position at which the sum of the torques on the individual particles equals the single torque exerted by the total weight of the object.


## Center of Gravity

Real objects consist of many particles. When experiencing a gravitational torque, each individual particle experiences a torque.

Center of Gravity - The position at which the sum of the torques on the individual particles equals the single torque exerted by the total weight of the object.


$$
\tau_{g}=x_{c g} w
$$

## Center of Gravity Exercise

A $300-N$ uniform bar is leaning against a wall as shown. What is the gravitational torque magnitude about the point $O$ ?


## Center of Gravity Exercise

A $300-N$ uniform bar is leaning against a wall as shown. What is the gravitational torque magnitude about the point $O$ ?
(a) $300 \mathrm{~N} \cdot \mathrm{~m}$


## Center of Gravity Exercise

A $300-N$ uniform bar is leaning against a wall as shown. What is the gravitational torque magnitude about the point $O$ ?
(a) $300 \mathrm{~N} \cdot \mathrm{~m}$

(b) $450 \mathrm{~N} \cdot \mathrm{~m}$

## Center of Gravity Exercise

A $300-N$ uniform bar is leaning against a wall as shown. What is the gravitational torque magnitude about the point $O$ ?
(a) $300 \mathrm{~N} \cdot \mathrm{~m}$

(b) $450 \mathrm{~N} \cdot \mathrm{~m}$
(c) $600 \mathrm{~N} \cdot \mathrm{~m}$

## Center of Gravity Exercise

A $300-N$ uniform bar is leaning against a wall as shown. What is the gravitational torque magnitude about the point $O$ ?
(a) $300 \mathrm{~N} \cdot \mathrm{~m}$


## Center of Gravity Exercise

A $300-N$ uniform bar is leaning against a wall as shown. What is the gravitational torque magnitude about the point $O$ ?
(a) $300 \mathrm{~N} \cdot \mathrm{~m}$

(b) $450 \mathrm{~N} \cdot \mathrm{~m}$
(c) $600 \mathrm{~N} \cdot \mathrm{~m}$
(d) $900 \mathrm{~N} \cdot \mathrm{~m}$
(e) $1200 \mathrm{~N} \cdot \mathrm{~m}$

## Center of Gravity Exercise

A $300-N$ uniform bar is leaning against a wall as shown. What is the gravitational torque magnitude about the point $O$ ?
(a) $300 \mathrm{~N} \cdot \mathrm{~m}$

(b) $450 \mathrm{~N} \cdot \mathrm{~m}$
(c) $600 \mathrm{~N} \cdot \mathrm{~m}$
(d) $900 \mathrm{~N} \cdot \mathrm{~m}$
(e) $1200 \mathrm{~N} \cdot \mathrm{~m}$

## Center of Gravity Exercise

A $300-N$ uniform bar is leaning against a wall as shown. What is the gravitational torque magnitude about the point $O$ ?

$$
\begin{aligned}
& \text { Uniform } \Rightarrow \text { center of gravity at } \\
& \text { the center } \Rightarrow \tau_{g}=(2 \mathrm{~m})(300 \mathrm{~N})
\end{aligned}
$$

$$
\text { (c) } 600 \mathrm{~N} \cdot \mathrm{~m}
$$



## Perpendicular Distance Exercise

What normal force must the wall exert on the bar at point $A$ to keep it from rotating? Ignore any friction between the bar and the wall.


## Perpendicular Distance Exercise

What normal force must the wall exert on the bar at point $A$ to keep it from rotating? Ignore any friction between the bar and the wall.
(a) 150 N


## Perpendicular Distance Exercise

What normal force must the wall exert on the bar at point $A$ to keep it from rotating? Ignore any friction between the bar and the wall.
(a) 150 N

(b) 200 N

## Perpendicular Distance Exercise

What normal force must the wall exert on the bar at point $A$ to keep it from rotating? Ignore any friction between the bar and the wall.
(a) 150 N

(b) 200 N
(c) 300 N

## Perpendicular Distance Exercise

What normal force must the wall exert on the bar at point $A$ to keep it from rotating? Ignore any friction between the bar and the wall.
(a) 150 N

(b) 200 N
(c) 300 N
(d) 400 N

## Perpendicular Distance Exercise

What normal force must the wall exert on the bar at point $A$ to keep it from rotating? Ignore any friction between the bar and the wall.
(a) 150 N


(b) 200 N
(c) 300 N
(d) 400 N
(e) 900 N

## Perpendicular Distance Exercise

What normal force must the wall exert on the bar at point $A$ to keep it from rotating? Ignore any friction between the bar and the wall.
(a) 150 N

(b) 200 N
(c) 300 N
(d) 400 N
(e) 900 N

## Perpendicular Distance Exercise

What normal force must the wall exert on the bar at point $A$ to keep it from rotating? Ignore any friction between the bar and the wall.
(a) 150 N

$$
\begin{aligned}
& \text { No rotation } \Rightarrow \tau_{\text {net }}=0 \\
& \Rightarrow \tau_{N}-\tau_{g}=0 \\
& \Rightarrow \tau_{N}=\tau_{g}=600 N \cdot m \\
& \tau_{N}=(3 m)\left(N_{A}\right)
\end{aligned}
$$



(b) 200 N
(c) 300 N
(d) 400 N
(e) 900 N

## Newton's Second Law for Rotation

Newton's Second law can be modified for rotation.

## Newton's Second Law for Rotation

Newton's Second law can be modified for rotation.

Original Version: $\sum \overrightarrow{\mathbf{F}}=M \overrightarrow{\mathbf{a}}$

## Newton's Second Law for Rotation

Newton's Second law can be modified for rotation.

Original Version: $\sum \overrightarrow{\mathbf{F}}=M \overrightarrow{\mathbf{a}}$
Rotational Version: $\sum \overrightarrow{\boldsymbol{\tau}}=$ ?

## Newton's Second Law for Rotation

Newton's Second law can be modified for rotation.


Rotational Version: $\sum \overrightarrow{\boldsymbol{\tau}}=$ ?

## Newton's Second Law for Rotation

Newton's Second law can be modified for rotation.


## Newton's Second Law for Rotation

Newton's Second law can be modified for rotation.


## Newton's Second Law for Rotation

Newton's Second law can be modified for rotation.


Newton's Second Law for Rotation: $\sum \vec{\tau}=I \vec{\alpha}$

