AQA IGCSE FM "Full Coverage": Trigonometric Equations

This worksheet is designed to cover one question of each type seen in past papers, for each AQA IGCSE Further Maths topic. This worksheet was automatically generated by the DrFrostMaths Homework Platform: students can practice this set of questions interactively by going to www.drfrostmaths.com/homework, logging on, *Practise → Past Papers/Worksheets* (or *Library → Past/Past Papers* for teachers), and using the 'Revision' tab.

Question 1

Categorisation: Simplify algebraic fractions involving trigonometric ratios.

[AQA IGCSE FM June2014-P1 Q15 Edited]

Express

$$\frac{\sin\theta - \sin^3\theta}{\cos^3\theta}$$

as a single trigonometric ratio.

....

Question 2

Categorisation: Simplify an expression involving trigonometric ratios.

[AQA IGCSE FM Jan2013-P2 Q18]

Express $1 - \tan\theta\sin\theta\cos\theta$ in terms of $\cos\theta$.

....

Question 3

Categorisation: Add a fraction with a non-fraction,

e.g. $\tan \theta + 1 \equiv \dfrac{\sin \theta}{\cos \theta} + 1 = \dfrac{\sin \theta + \cos \theta}{\cos \theta}$

[AQA IGCSE FM SAM P1 Q12]

Prove that

$$\tan^2 x - 1 \equiv \frac{1 - 2 \cos^2 x}{\cos^2 x}$$

....

Question 4

Categorisation: Add or subtract algebraic fractions.

[AQA IGCSE FM June2012-P1 Q16]

Prove that

$$\tan \theta + \frac{1}{\tan \theta} \equiv \frac{1}{\sin \theta \cos \theta}$$

....

Question 5

Categorisation: (Further practice of proofs)

[AQA IGCSE FM June2015-P1 Q17]

Prove that

$$2 \tan^2\theta + 1 \equiv \frac{1 + \sin^2\theta}{1 - \sin^2\theta}$$

where $\sin^2\theta \neq 1$.

....

Question 6

Categorisation: Use trigonometric identities in the context of matrices.

[AQA IGCSE FM Practice paper set 4 P2 Q16]

$$\mathbf{P} = \begin{pmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{pmatrix} \qquad \mathbf{Q} = \begin{pmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{pmatrix}$$

Work out **PQ** Give your answer in its simplest form.

....

Question 7

Categorisation: Determine the value of another trigonometric ratio, where θ is specified to be obtuse.

[AQA IGCSE FM June2016-P1 Q16]

Angle θ is obtuse and $\sin\theta = \frac{\sqrt{11}}{6}$
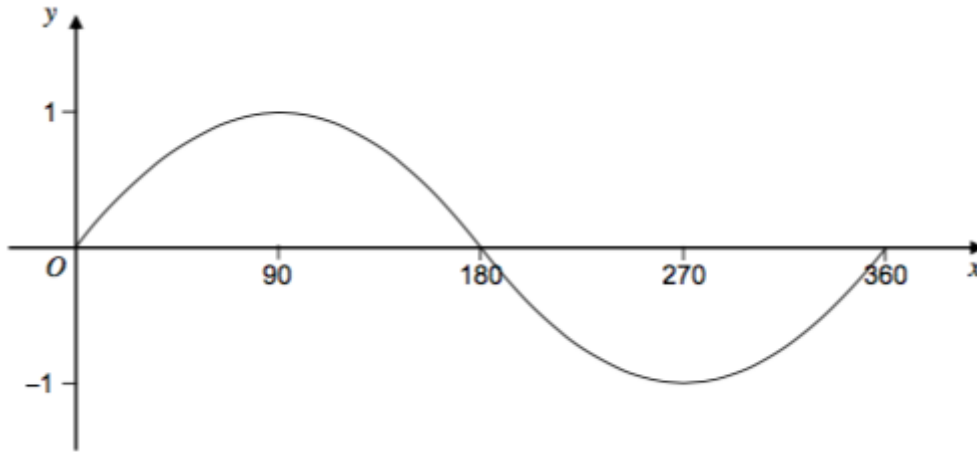
Work out the value of $\cos\theta$

$\cos\theta = $ ....

Question 8

Categorisation: Use a graph to solve a simple trigonometric equation.

[AQA IGCSE FM SAM P1 Q9]

The sketch shows $y = \sin x$ for $0° \leq x \leq 360°$

The value of $\sin 73° = 0.956$ to 3 significant figures.

Use the sketch to find **two** angles between $0°$ and $360°$ for which $\sin x = -0.956$

$x = $ .... $^{\circ}$

$x = $ .... $^{\circ}$

Question 9

Categorisation: Solve a simple trigonometric equation without the aid of a graph (but with the aid of a calculator).

[AQA IGCSE FM Practice paper set 4 P2 Q14]

Solve $2 \sin \theta = -1.36$ for $0° \leq \theta \leq 360°$

$\theta = $ .... $^{\circ}$

$\theta = $ .... $^{\circ}$

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Question 10

Categorisation: Solve an equation of the form $sin^2 x = k$ or $cos^2 x = k$.

[AQA IGCSE FM SAM P2 Q18]

Solve the equation $cos^2 x = 0.8$ for $0° \leq x \leq 360°$

Input note: give your solutions correct to 1 decimal place

$x = $ .... °

$x = $ .... °

$x = $ .... °

$x = $ .... °

Question 11

Categorisation: Solve an equation of the form $sin\ x = k\ cos\ x$.

[AQA IGCSE FM Practice paper set 1 P2 Q16b]

Solve $2\ sin\ x = -3\ cos\ x$ for $0° \leq x \leq 360°$

$x = $ .... °

$x = $ .... °

Question 12

Categorisation: Solve a trigonometric equation involving factorisation.

[AQA IGCSE FM Practice paper set 3 P2 Q24b]

Solve $5 \sin^2 x - 2 \sin x = 0$ for $0° \leq x \leq 360°$

$x = $ .... $°$

$x = $ .... $°$

$x = $ .... $°$

$x = $ .... $°$

$x = $ .... $°$

Question 13

Categorisation: As above.

[AQA IGCSE FM June2013-P2 Q22]
Solve

$$tan^2 \theta + 3 \tan \theta = 0$$

for $0° \leq \theta \leq 360°$

....

Question 14

Categorisation: Solve a trigonometric equation involving factorisation into two brackets.

[AQA IGCSE FM Practice paper set 2 P2 Q18b Edited]

It can be shown that $(2s + 1)(s - 1) = 2s^2 - s - 1$

Hence, or otherwise, solve $2\sin^2\theta - \sin\theta - 1 = 0$ for $0° \le \theta \le 360°$

$\theta = $ .... $°$

$\theta = $ .... $°$

$\theta = $ .... $°$

Answers

Question 1

$tan\,\theta$

Question 2

$cos^2\theta$

Question 3

$$= \frac{\sin^2 x}{\cos^2 x} - 1 = \frac{\sin^2 x - \cos^2 x}{\cos^2 x}$$
$$= \frac{1 - \cos^2 x - \cos^2 x}{\cos^2 x} = \frac{1 - 2\cos^2 x}{\cos^2 x}$$

Question 4

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{1}{\sin\theta\cos\theta}$$

Question 5

$$= \frac{2\sin^2\theta}{\cos^2\theta} + 1 = \frac{2\sin^2\theta + \cos^2\theta}{\cos^2\theta}$$
$$= \frac{2\sin^2\theta + 1 - \sin^2\theta}{1 - \sin^2\theta} = \frac{1 + \sin^2\theta}{1 - \sin^2\theta}$$

Question 6

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} or\ \mathbf{I}$

Question 7

$cos\,\theta = -\frac{5}{6}$

Question 8

$x = 253\,^{\circ}$ and $x = 287\,^{\circ}$

Question 9

$\theta = 317.2\,^{\circ}$ and $\theta = 222.8\,^{\circ}$

Question 10

$x = 26.6\,^{\circ}$ and $x = 153.4\,^{\circ}$ and $x = 206.6\,^{\circ}$ and $x = 333.4\,^{\circ}$

Question 11

$x = 123.7\,^{\circ}$ and $x = 303.7\,^{\circ}$

Question 12

$x = 0\,^{\circ}$ and $x = 180\,^{\circ}$ and $x = 360\,^{\circ}$ and $x = 23.6\,^{\circ}$ and $x = 156.4\,^{\circ}$

Question 13

$\theta = 108$ or $\theta = 288$

Question 14

$\theta = 90\,^{\circ}$ and $\theta = 210\,^{\circ}$ and $\theta = 330\,^{\circ}$