AQA Qualifications

AQA Level 2 Certificate

FURTHER MATHEMATICS

Level 2 (8365)

Mark Scheme

Worksheet 10

Factor Theorem

Version 2.0

Glossary for Mark Schemes

These examinations are marked in such a way as to award positive achievement wherever possible. Thus, for these papers, marks are awarded under various categories.

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
M Dep	A method mark dependent on a previous method mark being awarded.
B Dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$

10 Factor Theorem

Question	Answer	Mark	Comments
1(a)	$x(x^2 - 5x - 36)$	B1	
1(b)	$x = 0$, $x = -4$, $x = 9$	B2	B1 For two solutions
2(a)	$f(1) = 1 + 2 - 5 - 6 = -8$	B1	
	$f(-1) = -1 + 2 + 5 - 6 = 0$	B1	
2(b)	$f(2) = 8 + 8 - 10 - 6 = 0$	B1	
	$f(-2) = -8 + 8 + 10 - 6 = 4$	B1	
2(c)	$f(3) = 27 + 18 - 15 - 6 = 24$	B1	
	$f(-3) = -27 + 18 + 15 - 6 = 0$	B1	
2(d)	$(x + 1)$, $(x - 2)$ and $(x + 3)$	B1	

Question	Answer	Mark	Comments
3(a)	$(-5)^3 + 7(-5)^2 + 2(-5) - 40$ $-125 + 175 - 10 - 40 = 0$	M1 A1	oe Clearly shown to = 0
3(b)	$x^3 + 7x^2 + 2x - 40$ $\equiv (x + 5)(x^2 + kx - 8)$ $(x - 2)$ $(x + 4)$	M1 A1 A1	Sight of x^2 and -8 in a quadratic factor
Alt 1 3(b)	Substitutes another value into the expression and tests for '= 0' $(x - 2)$ $(x + 4)$	M1 A1 A1	
Alt 2 3(b)	Long division of polynomials getting as far as $x^2 + 2x$ $(x - 2)$ $(x + 4)$	M1 A1 A1	
3(c)	$(x =)$ -5, -4 and 2	B1	

4	$(-2)^3 + 5(-2)^2 + 9(-2) + k = 0$ $-8 + 20 - 18 + k = 0$ $k = 6$	M1 A1 A1	

Question	Answer	Mark	Comments
5(a)	$(-3)^3 + (-3)^2 + (-3)a - 72 = 0$	M1	
	$-27 + 9 - 3a - 72 = 0$	A1	
	$a = -30$	A1	
5(b)	$x^3 + x^2 - 30x - 72$ $\equiv (x + 3)(x^2 + kx - 24)$	M1	Sight of x^2 and -24 in a quadratic factor
	$(x + 4)$	A1	
	$(x - 6)$	A1	
Alt 1 5(b)	Substitutes another value into the expression and tests for '= 0'	M1	
	$(x + 4)$	A1	
	$(x - 6)$	A1	
Alt 2 5(b)	Long division of polynomials getting as far as $x^2 - 2x$	M1	
	$(x + 4)$	A1	
	$(x - 6)$	A1	

Question	Answer	Mark	Comments
6(a)	$(x-3)(x+4)(x+k)$ $\equiv x^3 + ax^2 + bx + 24$	M1	or $-3 \times 4 \times k = 24$
	$(x-2)$	A1	
6(b)	$(x-3)(x+4)(x-2)$	M1	
	$(x-3)(x^2+2x-8)$	M1	oe
	$x^3 - x^2 - 14x + 24$	A1	
	$a = -1$ and $b = -14$	A1 ft	ft Their expansion
Alt 6(b)	Substitutes any two of $x = -4$, $x = 2$ or $x = 3$ into $x^3 + ax^2 + bx + 24$ to create simultaneous equations	M1	
	Any two of $-64 + 16a - 4b + 24 = 0$ or $8 + 4a + 2b + 24 = 0$ or $27 + 9a + 3b + 24 = 0$	M1	
	$a = -1$	A1	
	$b = -14$	A1 ft	ft Their first solution

Question	Answer	Mark	Comments
7(a)	$(5)^3 + k(5)^2 + 9(5) - 20 = 0$	M1	
	$125 + 25k + 45 - 20 = 0$	A1	
	$k = -6$	A1	
7(b)	$x^3 - 6x^2 + 9x - 20$ $\equiv (x - 5)(x^2 + kx + 4)$	M1	Sight of x^2 and 4 in a quadratic factor
	$(x - 5)(x^2 - x + 4)$	A1	
7c	Tests '$b^2 - 4ac$' for the quadratic	M1	ft Their quadratic or attempts to solve their quadratic = 0
	Shows '$b^2 - 4ac$' = -15 (or < 0) and states no more linear factors	A1	States 'no solutions' to their quadratic = 0

Question	Answer	Mark	Comments
8	Substitutes a value of x into the expression and tests for '= 0'	M1	
	Works out first linear factor $(x + 1)$, $(x + 2)$ or $(x - 9)$	A1	
	$x^3 - 6x^2 - 25x - 18$ $\equiv (x + 1)(x^2 + kx - 18)$ or $(x + 2)(x^2 + kx - 9)$ or $(x - 9)(x^2 + kx + 2)$	M1	Attempts to work out the quadratic factor Sight of x^2 and -18 in a quadratic factor or sight of x^2 and -9 in a quadratic factor or sight of x^2 and 2 in a quadratic factor
	2nd and 3rd linear factors	A1	
	-1, -2 and 9	A1	
Alt 1 8	Substitutes a value of x into the expression and tests for '= 0'	M1	
	Works out first linear factor $(x + 1)$, $(x + 2)$ or $(x - 9)$	A1	
	Substitutes another value into the expression and tests for '= 0'	M1	
	2nd and 3rd linear factors	A1	
	-1, -2 and 9	A1	

Question	Answer	Mark	Comments
Alt 2 8	Substitutes a value of x into the expression and tests for '= 0'	M1	
	Works out first linear factor $(x + 1)$, $(x + 2)$ or $(x - 9)$	A1	
	Long division of polynomials getting as far as $x^2 - 7x$ or $x^2 - 8x$ or $x^2 + 3x$	M1	Depending on first linear factor
	2nd and 3rd linear factors	A1	
	-1, -2 and 9	A1	

Question	Answer	Mark	Comments
9(a)	$32 - 32 - 162 + 162$	B1	
9(b)	$(x - 2)(x^4 - 81)$	M1	
	$(x - 2)(x^2 + 9)(x^2 - 9)$	M1	
	$(x - 2)(x^2 + 9)(x + 3)(x - 3)$	M1	
	2, -3 and 3	A1	

Question	Answer	Mark	Comments
10(a)	$3\left(-\frac{2}{3}\right)^3 + 2\left(-\frac{2}{3}\right)^2 - 3\left(-\frac{2}{3}\right) - 2$	M1	Oe
	$-\frac{8}{9} + \frac{8}{9} + 2 - 2 = 0$	A1	Clearly shown to = 0
10(b)	$(3x + 2)(x^2 - 1)$	M1	
	$(3x + 2)(x + 1)(x - 1)$	A1	