## AQA

# AQA Level 2 Certificate FURTHER MATHEMATICS 

Level 2 (8365)
Mark Scheme
Worksheet 10
Factor Theorem

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## Glossary for Mark Schemes

These examinations are marked in such a way as to award positive achievement wherever possible. Thus, for these papers, marks are awarded under various categories.

M Method marks are awarded for a correct method which could lead to a correct answer.

A Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.

B Marks awarded independent of method.
M Dep A method mark dependent on a previous method mark being awarded.

B Dep A mark that can only be awarded if a previous independent mark has been awarded.
ft Follow through marks. Marks awarded following a mistake in an earlier step.

SC Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe Or equivalent. Accept answers that are equivalent.
eg, accept 0.5 as well as $\frac{1}{2}$

## AQA

## 10 Factor Theorem

| Question | Answer | Mark | Comments |
| :--- | :--- | :--- | :--- |
| $\mathbf{1 ( a )}$ | $x\left(x^{2}-5 x-36\right)$ | B1 |  |
| $\mathbf{1}(\mathbf{b})$ | $x=0, x=-4, x=9$ | B2 | B1 For two solutions |


| 2(a) | $f(1)=1+2-5-6=-8$ | B1 |  |
| :--- | :--- | :--- | :--- |
|  | $f(-1)=-1+2+5-6=0$ | B1 |  |
| 2(b) | $f(2)=8+8-10-6=0$ | B1 |  |
| 2(c) | $f(-2)=-8+8+10-6=4$ | B1 |  |
| 2(d) $=27+18-15-6=24$ | B1 |  |  |
|  | $f(-3)=-27+18+15-6=0$ | B1 |  |


| Question | Answer | Mark | Comments |
| :---: | :---: | :---: | :---: |
| 3(a) | $\begin{aligned} & (-5)^{3}+7(-5)^{2}+2(-5)-40 \\ & -125+175-10-40=0 \end{aligned}$ | M1 <br> A1 | oe <br> Clearly shown to $=0$ |
| 3(b) | $\begin{aligned} & x^{3}+7 x^{2}+2 x-40 \\ & \equiv(x+5)\left(x^{2}+k x-8\right) \\ & (x-2) \\ & (x+4) \end{aligned}$ | M1 <br> A1 <br> A1 | Sight of $x^{2}$ and -8 in a quadratic factor |
| $\begin{aligned} & \text { Alt } 1 \\ & \text { 3(b) } \end{aligned}$ | Substitutes another value into the expression and tests for ' $=0$ ' $\begin{aligned} & (x-2) \\ & (x+4) \end{aligned}$ | M1 <br> A1 <br> A1 |  |
| $\begin{aligned} & \text { Alt } 2 \\ & \text { 3(b) } \end{aligned}$ | Long division of polynomials getting as far as $x^{2}+2 x$ $\begin{aligned} & (x-2) \\ & (x+4) \end{aligned}$ | M1 <br> A1 <br> A1 |  |
| 3(c) | $(x=)-5,-4$ and 2 | B1 |  |
| 4 | $\begin{aligned} & (-2)^{3}+5(-2)^{2}+9(-2)+k=0 \\ & -8+20-18+k=0 \\ & k=6 \end{aligned}$ | M1 <br> A1 <br> A1 |  |


| Question | Answer | Mark | Comments |
| :---: | :---: | :---: | :---: |
| 5(a) | $(-3)^{3}+(-3)^{2}+(-3) a-72=0$ | M1 |  |
|  | $-27+9-3 a-72=0$ | A1 |  |
|  | $a=-30$ | A1 |  |
| 5(b) | $\begin{aligned} & x^{3}+x^{2}-30 x-72 \\ & \equiv(x+3)\left(x^{2}+k x-24\right) \end{aligned}$ | M1 | Sight of $x^{2}$ and -24 in a quadratic factor |
|  | $(x+4)$ | A1 |  |
|  | $(x-6)$ | A1 |  |
| 5(b) | Substitutes another value into the expression and tests for ' $=0$ ' | M1 |  |
|  | $(x+4)$ | A1 |  |
|  | $(x-6)$ | A1 |  |
| $\begin{aligned} & \text { Alt } 2 \\ & 5(b) \end{aligned}$ | Long division of polynomials getting as far as $x^{2}-2 x$ | M1 |  |
|  | $(x+4)$ | A1 |  |
|  | $(x-6)$ | A1 |  |


| Question | Answer | Mark | Comments |
| :---: | :---: | :---: | :---: |
| 6(a) | $\begin{aligned} & (x-3)(x+4)(x+k) \\ & \equiv x^{3}+a x^{2}+b x+24 \\ & (x-2) \end{aligned}$ | M1 <br> A1 | or $-3 \times 4 \times k=24$ |
| 6(b) | $\begin{aligned} & (x-3)(x+4)(x-2) \\ & (x-3)\left(x^{2}+2 x-8\right) \\ & x^{3}-x^{2}-14 x+24 \\ & a=-1 \text { and } b=-14 \end{aligned}$ | M1 <br> M1 <br> A1 <br> A1 ft | oe <br> ft Their expansion |
| Alt 6(b) | Substitutes any two of $x=-4, x=2$ or $x=3$ into $x^{3}+a x^{2}+b x+24$ to create simultaneous equations <br> Any two of $-64+16 a-4 b+24=0$ <br> or $8+4 a+2 b+24=0$ <br> or $\begin{aligned} & 27+9 a+3 b+24=0 \\ & a=-1 \\ & b=-14 \end{aligned}$ | M1 <br> M1 <br> A1 <br> A1 ft | ft Their first solution |


| Question | Answer | Mark | Comments |
| :---: | :---: | :---: | :---: |
| 7(a) | $\begin{aligned} & (5)^{3}+k(5)^{2}+9(5)-20=0 \\ & 125+25 k+45-20=0 \\ & k=-6 \end{aligned}$ | M1 <br> A1 <br> A1 |  |
| 7(b) | $\begin{aligned} & x^{3}-6 x^{2}+9 x-20 \\ & \equiv(x-5)\left(x^{2}+k x+4\right) \\ & (x-5)\left(x^{2}-x+4\right) \end{aligned}$ | M1 <br> A1 | Sight of $x^{2}$ and 4 in a quadratic factor |
| 7c | Tests ' $b^{2}-4 a c$ ' for the quadratic <br> Shows ' $b^{2}-4 a c$ ' $=-15(\mathrm{or}<0)$ and states no more linear factors | M1 <br> A1 | ft Their quadratic or attempts to solve their quadratic $=0$ <br> States 'no solutions' to their quadratic $=0$ |
| 8 | Substitutes a value of $x$ into the expression and tests for ' $=0$ ' <br> Works out first linear factor $(x+1),(x+2)$ or $(x-9)$ $x^{3}-6 x^{2}-25 x-18$ $\equiv(x+1)\left(x^{2}+k x-18\right)$ <br> or $(x+2)\left(x^{2}+k x-9\right)$ <br> or $(x-9)\left(x^{2}+k x+2\right)$ <br> 2nd and 3rd linear factors <br> $-1,-2$ and 9 | M1 <br> A1 <br> M1 <br> A1 <br> A1 | Attempts to work out the quadratic factor Sight of $x^{2}$ and -18 in a quadratic factor or sight of $x^{2}$ and -9 in a quadratic factor or sight of $x^{2}$ and 2 in a quadratic factor |
| Alt 1 <br> 8 | Substitutes a value of $x$ into the expression and tests for ' $=0$ ' <br> Works out first linear factor $(x+1),(x+2)$ or $(x-9)$ <br> Substitutes another value into the expression and tests for ' $=0$ ' <br> 2nd and 3rd linear factors $-1,-2 \text { and } 9$ | M1 <br> A1 <br> M1 <br> A1 <br> A1 |  |


| Question | Answer | Mark | Comments |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Alt } 2 \\ & 8 \end{aligned}$ | Substitutes a value of $x$ into the expression and tests for ' $=0$ ' <br> Works out first linear factor $(x+1),(x+2)$ or $(x-9)$ <br> Long division of polynomials getting as far as $x^{2}-7 x$ <br> or $x^{2}-8 x$ <br> or $x^{2}+3 x$ <br> 2nd and 3rd linear factors <br> $-1,-2$ and 9 | M1 <br> A1 <br> M1 <br> A1 <br> A1 | Depending on first linear factor |
| $\begin{aligned} & \text { 9(a) } \\ & \text { 9(b) } \end{aligned}$ | $\begin{aligned} & 32-32-162+162 \\ & (x-2)\left(x^{4}-81\right) \\ & (x-2)\left(x^{2}+9\right)\left(x^{2}-9\right) \\ & (x-2)\left(x^{2}+9\right)(x+3)(x-3) \end{aligned}$ <br> $2,-3$ and 3 | B1 <br> M1 <br> M1 <br> M1 <br> A1 |  |
| 10(a) <br> 10(b) | $\begin{aligned} & 3\left(-\frac{2}{3}\right)^{3}+2\left(-\frac{2}{3}\right)^{2}-3\left(-\frac{2}{3}\right)-2 \\ & -\frac{8}{9}+\frac{8}{9}+2-2=0 \\ & (3 x+2)\left(x^{2}-1\right) \\ & (3 x+2)(x+1)(x-1) \end{aligned}$ | M1 <br> A1 <br> M1 <br> A1 | Oe <br> Clearly shown to $=0$ |

