4-1 Classifying Triangles
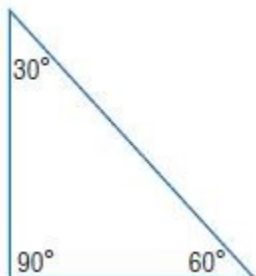
ARCHITECTURE Classify each triangle as *acute, equiangular, obtuse,* or *right*.
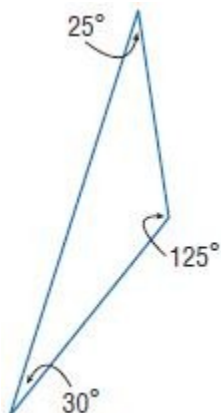
1. **Refer to the figure on page 240.**

ANSWER:
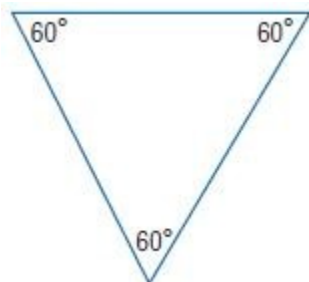right

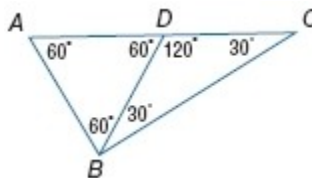2. **Refer to the figure on page 240.**

ANSWER:
obtuse

3. **Refer to the figure on page 240.**

ANSWER:
equiangular

Classify each triangle as *acute, equiangular, obtuse,* or *right*. Explain your reasoning.

4. △*ABD*

ANSWER:
equiangular; all three angles are 60°

5. △*BDC*

ANSWER:
obtuse; ∆ *BDC* > 90°

6. △*ABC*

ANSWER:
right; ∆ *ABC* = 90°

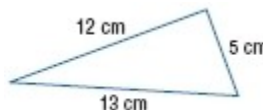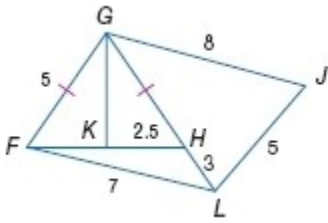**APPLY MATH Classify each triangle as *equilateral, isosceles,* or *scalene*.**

7.

ANSWER:
isosceles

8.

ANSWER:
scalene

If point *K* is the midpoint of $\overline{FH}$, classify each triangle in the figure as *equilateral, isosceles,* or *scalene*.

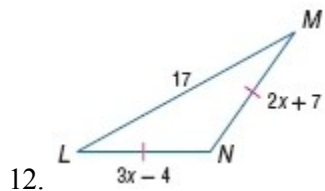9. $\triangle FGH$

   ANSWER:
   equilateral

10. $\triangle GJL$

    ANSWER:
    isosceles

11. $\triangle FHL$

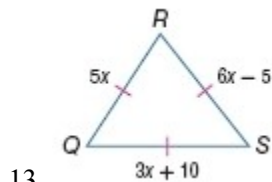   *ANSWER:*
    scalene

ALGEBRA Find *x* and the measures of the unknown sides of each triangle.

12.

   ANSWER:
   $x = 11$, $LN = 29$, and $MN = 29$

13.

   ANSWER:
 $x = 5$, $QR = RS = QS = 25$

14. **JEWELRY** Suppose you are bending stainless steel wire to make the earring shown. The triangular portion of the earring is an isosceles triangle. If 1.5 centimeters are needed to make the hook portion of the earring, how many earrings can be made from 45 centimeters of wire? Explain your reasoning.

   ANSWER:
   4; The total amount of wire needed, including the hook, is $2.1 + 3.2 + 3.2 + 1.5$ or 10 cm. 45 cm ÷ 10 cm/earring = 4.5 earrings. There is not enough wire to make 5 earrings, only 4 can be made from 45 cm of wire.
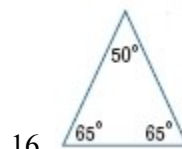
Classify each triangle as *acute, equiangular, obtuse,* or *right*.
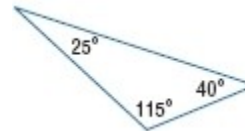
15.

   ANSWER:
   obtuse

16.

   ANSWER:
   acute

17.

   ANSWER:
   right

18.

ANSWER:
equiangular

19.

ANSWER:
acute

20.

ANSWER:
right

APPLY MATH Classify each triangle as *acute, equiangular, obtuse,* or *right*.
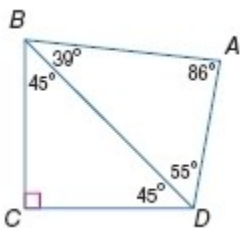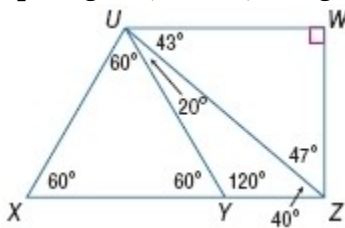
21. △*UYZ*

ANSWER:
obtuse

22. △*BCD*

ANSWER:
right

23. △*ADB*

ANSWER:
acute

24. △*UXZ*

ANSWER:
acute

25. △*UWZ*

ANSWER:
right

26. △*UXY*

ANSWER:
equiangular

Classify each triangle as *equilateral*, *isosceles*, or *scalene*.

27. **Refer to the figure on page 241.**
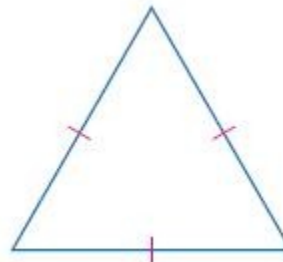
ANSWER:
equilateral

28. **Refer to the figure on page 241.**
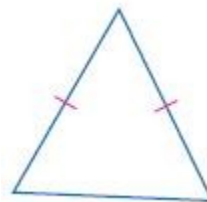
ANSWER:
isosceles

29. **Refer to the figure on page 241.**

ANSWER:
scalene

If point C is the midpoint of $\overline{BD}$ and point E is the midpoint of $\overline{DF}$, classify each triangle as *equilateral*, *isosceles*, or *scalene*.
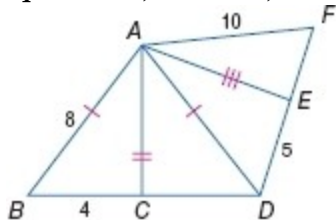
30. $\triangle ABC$

ANSWER:
scalene

31. $\triangle AEF$

ANSWER:
scalene

32. $\triangle ADF$

ANSWER:
isosceles

33. $\triangle ACD$

ANSWER:
scalene

34. $\triangle AED$

ANSWER:
scalene

35. $\triangle ABD$

ANSWER:
equilateral

36. **ALGEBRA** Find x and the length of each side if $\triangle ABC$ is an isosceles triangle with $\overline{AB} \cong \overline{BC}$.
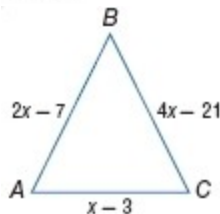
ANSWER:
$x = 7; AB = 7, BC = 7, CA = 4$

37. **ALGEBRA** Find x and the length of each side if $\triangle FGH$ is an equilateral triangle.
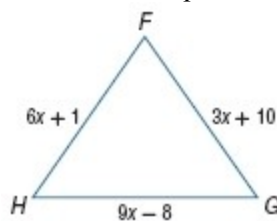
ANSWER:
$x = 3; FG = GH = HF = 19$

38. **GRAPHIC ART** Classify each numbered triangle in Kat by its angles and by its sides. Use the corner of a sheet of notebook paper to classify angle measures and a ruler to measure sides.
Refer to the figure on page 242.

ANSWER:
$\triangle$ 1: right scalene, $\triangle$ 2: right scalene, $\triangle$ 3: obtuse scalene, $\triangle$ 4: acute isosceles, $\triangle$ 5: right scalene, $\triangle$ 6: obtuse scalene

39. **KALEIDOSCOPE** Josh is building a kaleidoscope using PVC pipe, cardboard, bits of colored paper, and a 12-inch square mirror tile. The mirror tile is to be cut into strips and arranged to form an open prism with a base like that of an equilateral triangle. Make a sketch of the prism, giving its dimensions. Explain your reasoning.

ANSWER:

Because the base of the prism formed is an equilateral triangle, the mirror tile must be cut into three strips of congruent width. Since the original tile is a 12-inch square, each strip will be 12 inches long by 12 ÷ 3 or 4 inches wide.

40. **MULTI-STEP** You are given the equations of three lines: $4x - 3y = -5, y = 3, 2x + y = 15$.
a. Without graphing the lines, determine whether the three lines form a triangle. If so, find the vertices and classify the triangle as equilateral, isosceles, or scalene.
b. Describe your solution process.
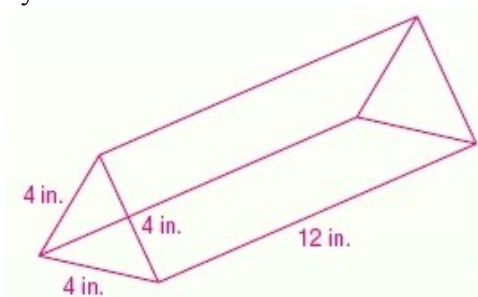
ANSWER:

a. yes; (1, 3), (6, 3), (4, 7); isosceles
b. Sample answer: In order to determine whether the lines formed a triangle, I needed to see if each pair of lines intersected at exactly one point. I used substitution to determine that the first and second lines intersected at (1, 3). Next, I used substitution to find that the second and third lines intersected at (6, 3). Then, I used elimination by multiplication to determine that the first and third lines intersected at (4, 7). The three points of intersection are the three vertices of the triangle. I then used the Distance Formula to determine that the distance between (1, 3) and (6, 3) is 5 units, the distance between (1, 3) and (4, 7) is 5 units, and the distance between (6, 3) and (4, 7) is $\sqrt{20}$ units. So the triangle is isosceles.

COORDINATE GEOMETRY Find the measures of the sides of $\triangle XYZ$ and classify each triangle by its sides.

41. $X(-5, 9), Y(2, 1), Z(-8, 3)$

ANSWER:

scalene; $XZ = 3\sqrt{5}, XY = \sqrt{113}, YZ = 2\sqrt{26}$

42. $X(7, 6), Y(5, 1), Z(9, 1)$

ANSWER:

isosceles; $XZ = \sqrt{29}, XY = \sqrt{29}, YZ = 4$

43. $X(3, -2), Y(1, -4), Z(3, -4)$

ANSWER:

isosceles; $XZ = 2, XY = 2\sqrt{2}, YZ = 2$

44. $X(-4, -2), Y(-3, 7), Z(4, -2)$

ANSWER:

scalene; $XZ = 8, XY = \sqrt{82}, YZ = \sqrt{130}$

45. **PROOF** Write a paragraph proof to prove that $\triangle DBC$ is an acute triangle if $m\angle ADC = 120$ and $\triangle ABC$ is acute.
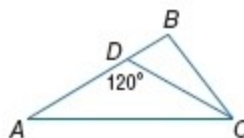
ANSWER:

Given: $m\angle ADC = 120$
Prove: $\triangle DBC$ is acute.
Proof: $\angle ADC$ and $\angle BDC$ form a linear pair. $\angle ADC$ and $\angle BDC$ are supplementary because if two angles form a linear pair, then they are supplementary. So, $m\angle ADC + m\angle BDC = 180$. We know $m\angle ADC = 120$, so by substitution, $120 + m\angle BDC = 180$. Subtract to find that $m\angle BDC = 60$. We already know that $\angle B$ is acute because $\triangle ABC$ is acute. $\angle BCD$ must also be acute because $\angle C$ is acute and $m\angle C = m\angle ACD + m\angle BCD$. $\triangle DBC$ is acute by definition.

46. **PROOF** Write a two-column proof to prove that $\triangle BCD$ is equiangular if $\triangle ACE$ is equiangular and $\overline{BD} \parallel \overline{AE}$.
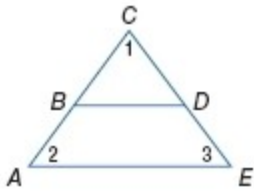
ANSWER:

Given: $\triangle ACE$ is equiangular and $\overline{BD} \parallel \overline{AE}$.
Prove: $\triangle BCD$ is equiangular.
Proof:
Statements (Reasons)
1. $\triangle ACE$ is equiangular and $\overline{BD} \parallel \overline{AE}$. (Given)
2. $\angle 1 \cong \angle 2 \cong \angle 3$ (Def. of equiangular $\triangle$)
3. $\angle 2 \cong \angle CBD$ and $\angle 3 \cong \angle CDB$ (Corr. $\angle s$ Post.)
4. $\angle 1 \cong \angle CBD \cong \angle CDB$ (Substitution)
5. $\triangle BCD$ is equiangular. (Def. of equiangular $\triangle$)

ALGEBRA For each triangle, find x and the measure of each side.

47. $\triangle FGH$ is an equilateral triangle with $FG = 3x - 10$, $GH = 2x + 5$, and $HF = x + 20$.

ANSWER:

$x = 15$; $FG = 35$, $GH = 35$, $HF = 35$

48. $\triangle JKL$ is isosceles with $\overline{JK} \cong \overline{KL}$, $JK = 4x - 1$, $KL = 2x + 5$, and $LJ = 2x - 1$.

ANSWER:

$x = 3$; $JK = 11$, $KL = 11$, $LJ = 5$

49. $\triangle MNP$ is isosceles with $\overline{MN} \cong \overline{NP}$. MN is two less than five times x, NP is seven more than two times x, and PM is two more than three times x.
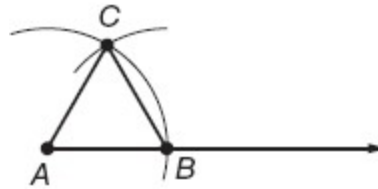
ANSWER:

$x = 3$; $MN = 13$, $NP = 13$, $PM = 11$

50. $\triangle RST$ is equilateral. RS is three more than four times x, ST is seven more than two times x, and TR is one more than five times x.

ANSWER:

$x = 2$; $RS = ST = TR = 11$

51. **CONSTRUCTION** Construct an equilateral triangle. Verify your construction using measurement and justify it using mathematics. (*Hint*: Use the construction for copying a segment.)

ANSWER:

Sample answer: In $\triangle ABC$, $AB = BC = AC = 1.3$ cm. Since all sides have the same length, they are all congruent. Therefore the triangle is equilateral. $\triangle ABC$ was constructed using AB as the length of each side. Since the arc for each segment is the same, the triangle is equilateral.

52. **STOCKS** Technical analysts use charts to identify patterns that can suggest future activity in stock prices. Symmetrical triangle charts are most useful when the fluctuation in the price of a stock is decreasing over time.
a. Classify by its sides and angles the triangle formed if a vertical line is drawn at any point on the graph.
b. How would the price have to fluctuate in order for the data to form an obtuse triangle? Draw an example to support your reasoning.
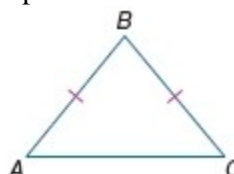

ANSWER:

a. isosceles; acute
b. Sample answer: The fluctuation would have to be high and decrease quickly in order to form an obtuse triangle.
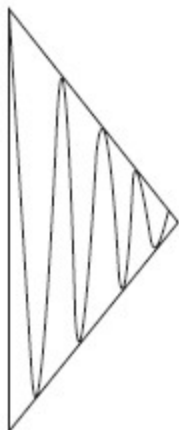
53. **MULTIPLE REPRESENTATIONS** In the diagram, the vertex *opposite* side $\overline{BC}$ is $\angle A$.
a. **GEOMETRIC** Draw four isosceles triangles, including one acute, one right, and one obtuse isosceles triangle. Label the vertices opposite the congruent sides as A and C. Label the remaining vertex B. Then measure the angles of each triangle and label each angle with its measure.
b. **TABULAR** Measure all the angles of each triangle. Organize the measures for each triangle into a table. Include a column in your table to record the sum of these measures.
c. **VERBAL** Make a conjecture about the measures

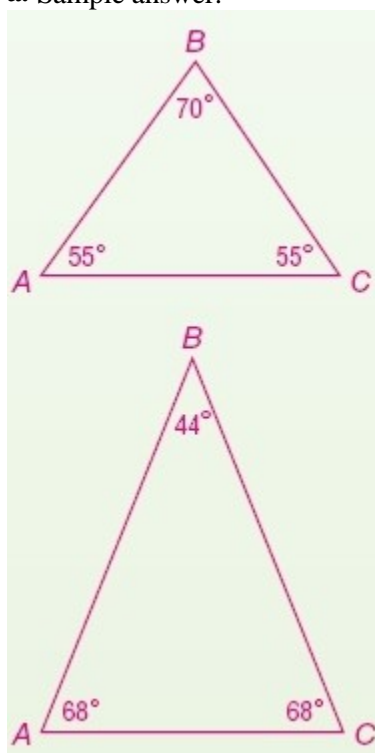of the angles that are opposite the congruent sides of an isosceles triangle. Then make a conjecture about the sum of the measures of the angles of an isosceles triangle.
d. **ALGEBRAIC** If x is the measure of one of the angles opposite one of the congruent sides in an isosceles triangle, write expressions for the measures of each of the other two angles in the triangle. Explain.
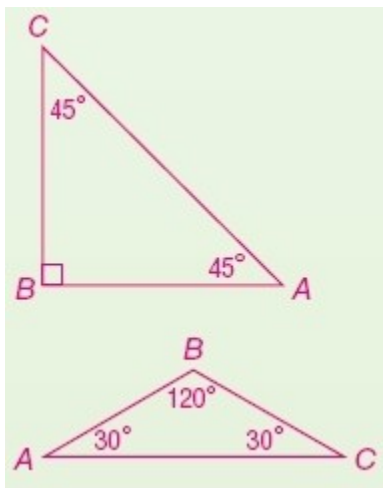

ANSWER:

a. Sample answer:

b.

| m∠A	m∠C	m∠B	Sum of Angle Measures
55	55	70	180
68	68	44	180
45	45	90	180
30	30	120	180

c. Sample answer: In an isosceles triangle, the angles opposite the congruent sides have the same measure. The sum of the measures of the angles of an isosceles triangle is 180.

d. x and $180 - 2x$; If the measures of the angles opposite the congruent sides of an isosceles triangle have the same measure, then if one angle measures x, the other angle also measures x. The sum of the measures of the angles of an isosceles triangle is 180, thus the measure of the third angle is $180 - 2x$.

54. **ERROR ANALYSIS** Elaina says that $\triangle DFG$ is obtuse. Ines disagrees, explaining that the triangle has more acute angles than obtuse angles so it must be acute. Is either of them correct? Explain your reasoning.
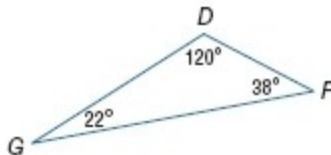
ANSWER:
Sample answer: Elaina; all triangles have at least two acute angles, so using Ines' reasoning all triangles would be classified as acute. Instead, triangles are classified by their third angle. If the third angle is also acute, then the triangle is acute. If the third angle is obtuse, as in the triangle shown, the triangle is classified as obtuse.

ANALYZE RELATIONSHIPS Classify each triangle in the figure by its angles and sides.
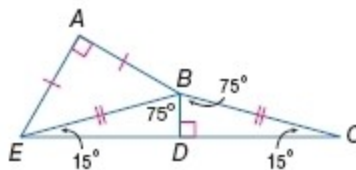
55. $\triangle ABE$

ANSWER:
isosceles right

56. $\triangle EBC$

ANSWER:
isosceles obtuse

57. $\triangle BDC$

ANSWER:
scalene right

58. **JUSTIFY ARGUMENTS** An equilateral triangle has sides that measure $5x + 3$ units and $7x - 5$ units. What is the perimeter of the triangle? Explain.
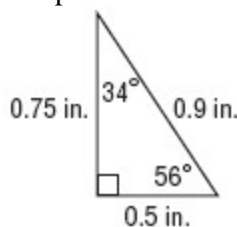
    ANSWER:

    Sample answer: Since the triangle is equilateral, the sides are equal. Setting $5x + 3$ equal to $7x - 5$ and solving, x is 4. The length of one side is $5(4) + 3$ or 23 units. The perimeter of an equilateral triangle is the sum of the three sides or three times one side. The perimeter is $3(23)$ or 69 units.

    ORGANIZE IDEAS Draw an example of each type of triangle below using a protractor and a ruler. Label the sides and angles of each triangle with their measures. If not possible, explain why not.
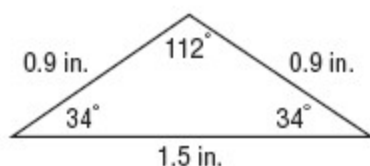
59. scalene right

    ANSWER:
    Sample answer:

60. isosceles obtuse

    ANSWER:
    Sample answer:


61. equilateral obtuse

    ANSWER:
    Not possible; all equilateral triangles have three acute angles.

62. **WRITING IN MATH** Explain why classifying an equiangular triangle as an *acute* equiangular triangle is unnecessary.

    ANSWER:

    Sample answer: An acute triangle has three acute angles and an equiangular triangle has three angles that measure 60°. Since an angle that measures 60° is an acute angle, all equiangular triangles are acute. Therefore, acute equiangular is redundant.

63. Triangle *DEG* is an equilateral triangle.
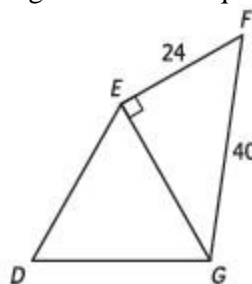
    What is the perimeter of quadrilateral *DEFG*?

    A 96 units
    B 128 units
    C 158 units
    D 160 units

    ANSWER:
    B

64. Refer to the statements below. Which statement is always true?
    F Equilateral triangles are equiangular.
    G Isosceles triangles are scalene.
    H Scalene triangles are acute.
    J Right triangles are isosceles.

    ANSWER:
    F

65. **ACT/SAT** Which of the following best describes the two triangles formed by drawing the diagonal of a square?
    A Scalene right
    B Isosceles acute
    C Equiangular acute
    D Isosceles right
    E Equilateral

    ANSWER:
    D