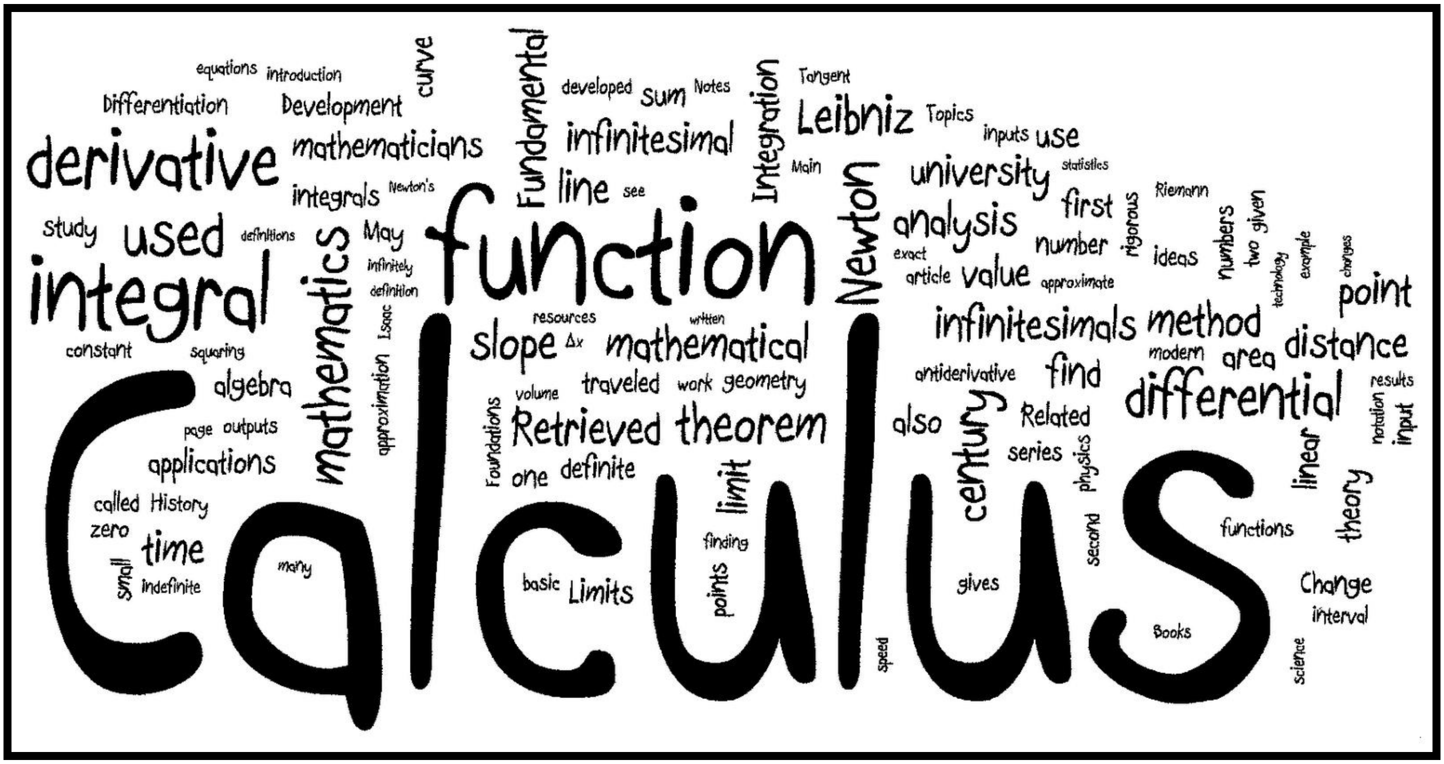


ARE YOU READY FOR CALCULUS?

A. Functions

A **function** is a set of points (*x*, *y*) such that for every *x*, there is one and only one *y*. In short, in a function, the *x*-values cannot repeat while the *y*-values can. In AP Calculus, all of the graphs will come from functions. The notation for functions is either "*y* = " or "*f*(*x*) = ". In the *f*(*x*) notation, you need to take the number or expression inside the parenthesis and plug it in wherever you see *x* in the function.

If $f(x)=x^2-5x+8$, find $f(-6)$

$$\begin{aligned} f(-6)&=(-6)^2-5(-6)+8 \\ &=36+30+8=\boxed{74} \end{aligned}$$

If $f(x)=5x+8$, find $f(x+h)$

$$\begin{aligned} f(x+h)&=5(x+h)+8 \\ &=5x-5h+8 \end{aligned}$$

One concept that comes up in very frequently in AP Calculus is **composition of functions** (a function within another function). The format of a composition of functions is: plug a value into one function, determine an answer, and plug that answer into a second function.

If $f(x)=x^2-x+1$ and $g(x)=2x-1$, find a) $f(g(-1))$ b) $g(f(x))$

a)
$$\begin{aligned} &g(-1)=2(-1)-1=-3 \\ &f(g(-1))=f(-3)=9+3+1=\boxed{13} \end{aligned}$$

b)
$$\begin{aligned} g(f(x))&=g(x^2-x+1)=2(x^2-x+1)-1 \\ &=2x^2-2x+1 \end{aligned}$$

Finally, expect to use **piecewise functions**. A piecewise function gives different equations to use, based on the value of *x*.

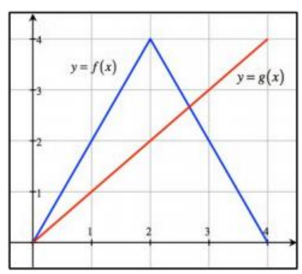
If $f(x)=\begin{cases} x^2-3, & x\ge 0 \\ 2x+1, & x<0 \end{cases}$, find a) $f(5)$ b) $f(f(1))$

a)
$$f(5)=(5)^2-3=22$$

b)
$$\begin{aligned} &f(1)=(1)^2-3=-2 \\ &f(f(1))=f(-2)=2(-2)+1=\boxed{-3} \end{aligned}$$

1) If $f(x)=4x-x^2$, find $\dfrac{f(x+h)-f(x)}{h}$

2)



Find $f(g(3))$

3) If $f(x)=\dfrac{4}{x-1}$ and $g(x)=2x$, then the solution of $f(g(x))=g(f(x))$ is

4) If $f(x)=2x^3+Ax^2+Bx-5$ and if $f(2)=3$ and $f(-2)=-37$, what is the value of $A+B$?

B. Domain and Range

Since questions in AP Calculus usually ask about behavior of functions in intervals, understanding the notation that intervals can be written with is important. One way is with less than (<, ≤) or greater than (>, ≥) symbols and the other way is in interval notation.

Description	Interval notation
$x > a$	(a, ∞)
$x \geq a$	$[a, \infty)$
$x < a$	$(-\infty, a)$

Description	Interval notation
$x \leq a$	$(-\infty, a]$
$a < x < b$	(a, b)
$a \leq x \leq b$	$[a, b]$

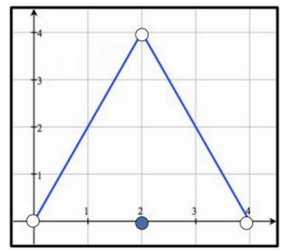
Description	Interval notation
$a \leq x < b$	$[a, b)$
$a < x \leq b$	$(a, b]$
All real numbers	$(-\infty, \infty)$

The **domain** of a function is the set of allowable *x*-values. The domain of a function is (–∞, ∞) except for:

- values of *x* which create a zero in the denominator $y = \dfrac{6}{x-6}$ $\boxed{x \neq 6}$
- an even root of a negative number $y = \sqrt{x+5}$ $\boxed{[-5, \infty)}$
- a logarithm of a non–positive number. $y = \ln(x-10)$ $\boxed{(10, \infty)}$

The **range** of a function is the set of allowable *y*-values.
Finding the range of functions algebraically is not easy (it is really a Calculus problem), but visually it is the lowest possible *y*-value and highest possible *y*-value.

Domain: $(0, 4)$
Range: $[0, 4)$



Find the domain of the following functions using interval notation (see **Section H** to solve inequality in #2, 3, and 5):

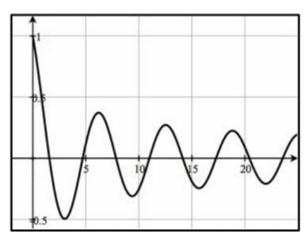
1) $y = \sqrt[3]{x+5}$

2) $y = \sqrt{2x+4}$

3) $f(x) = \ln(x^2 - 4)$

4) $y = \dfrac{2x}{x^2 - 2x - 3}$

5) $f(x) = \dfrac{\sqrt{x^2 - 4}}{x - 3}$

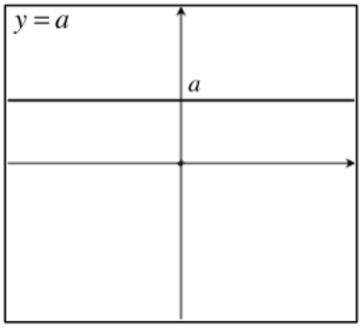
6) 

Domain:

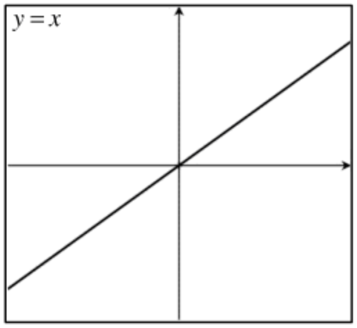
Range:

C. Graphs of Common Functions

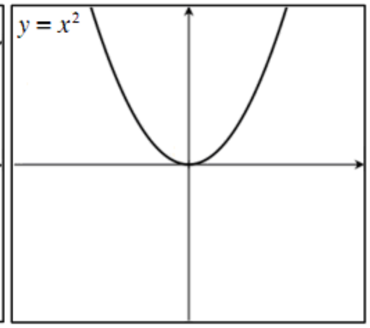
There are certain graphs that occur all the time in Calculus and students should know the general shape of them, where they cross the *x*-axis (zeros) and *y*-axis (*y*-intercept), as well as the domain and range. There are no practice problems for this section other than students knowing the shape of all of these functions.



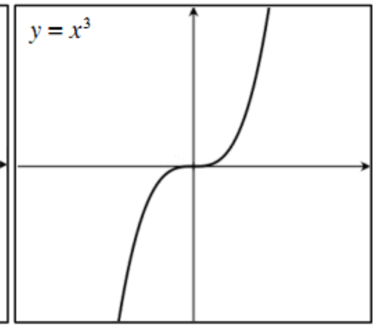
x-intercept: none
y-intercept: a
Domain: $(-\infty,\infty)$
Range: $[a,a]$



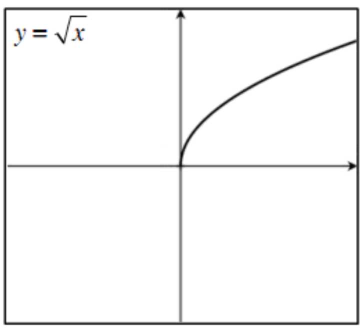
x-intercept: 0
y-intercept: 0
Domain: $(-\infty,\infty)$
Range: $(-\infty,\infty)$



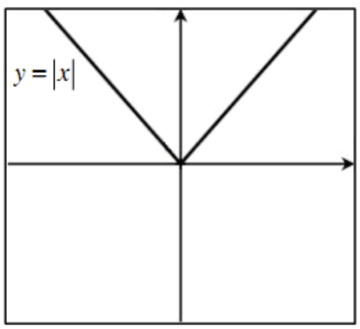
x-intercept: 0
y-intercept: 0
Domain: $(-\infty,\infty)$
Range: $[0,\infty)$



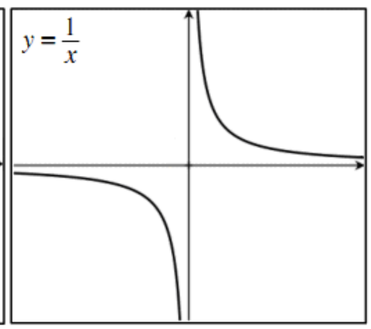
x-intercept: 0
y-intercept: 0
Domain: $(-\infty,\infty)$
Range: $(-\infty,\infty)$



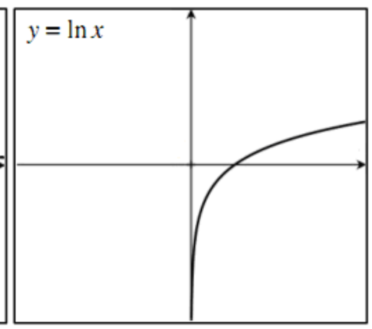
x-intercept: 0
y-intercept: 0
Domain: $[0,\infty)$
Range: $[0,\infty)$



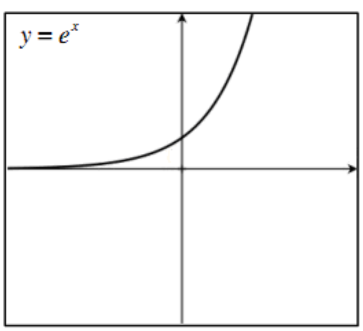
x-intercept: 0
y-intercept: 0
Domain: $(-\infty,\infty)$
Range: $[0,\infty)$



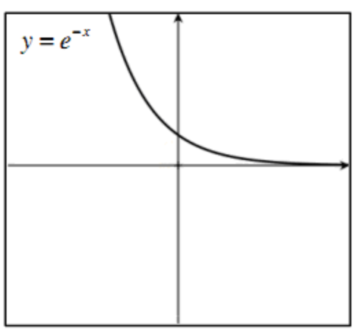
x-intercept: none
y-intercept: none
Domain: $x \neq 0$
Range: $y \neq 0$



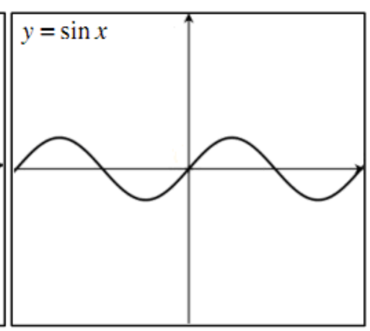
x-intercept: 1
y-intercept: none
Domain: $(0,\infty)$
Range: $(-\infty,\infty)$



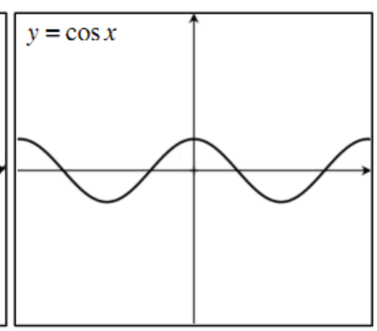
x-intercept: none
y-intercept: 1
Domain: $(-\infty,\infty)$
Range: $(0,\infty)$



x-intercept: none
y-intercept: 1
Domain: $(-\infty,\infty)$
Range: $(0,\infty)$



x-intercept: $\ldots,-\pi,0,\pi,\ldots$
y-intercept: 0
Domain: $(-\infty,\infty)$
Range: $[-1,1]$



x-intercept: $\ldots,-\frac{\pi}{2},\frac{\pi}{2},\ldots$
y-intercept: 1
Domain: $(-\infty,\infty)$
Range: $[-1,1]$

D. Linear Functions

Probably **the most important concept** from prior math classes that is required for AP Calculus is that of linear functions. The concepts and procedures to solve problems you need to know backwards and forwards are:

Slope (rate of change): given two points (x_1, y_1) and (x_2, y_2), the slope of the line passing through the points can be written as:

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Point−slope form: the equation of a line passing through the points (x_1, y_1) and slope m is given by

$$y - y_1 = m(x - x_1)$$

While you might have preferred slope-intercept form (~~$y = mx + b$~~) from in your prior math classes, the form $\boldsymbol{y - y_1 = m(x - x_1)}$ is far more useful in AP Calculus.

Parallel lines: two distinct lines are parallel if they have the same slope: $m_1 = m_2$.

Normal lines: two lines are normal if they are perpendicular (slopes are negative reciprocals): $m = a/b \rightarrow m_{\perp} = -b/a$

Horizontal lines have slope zero. **Vertical lines** have no slope (slope is undefined).

Find the equation of the line in slope-intercept form passing through (4, 5) and (−2, −1).

$$\boxed{m = \frac{5+1}{4+2} = 1} \qquad \boxed{y - 5 = 1(x-4) \Longrightarrow y = x + 1}$$

Write the equation of the line through (4, 7) and **a)** parallel and **b)** normal to $4x - 2y = 1$.

$$\boxed{y = 2x - \frac{1}{2} \Longrightarrow m = 2} \qquad \boxed{\begin{array}{l} \text{a) } y - 7 = 2(x-4) \\ \text{b) } y - 7 = \frac{-1}{2}(x-4) \end{array}}$$

1) Find the equation of the line in slope-intercept form passing through (−7, 1) and (3, −4).

2) Let f be a linear function where $f(2) = -5$ and $f(-3) = 1$. Find $f(x)$.

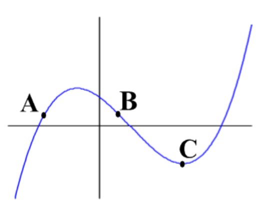
3) Find k if the lines $3x - 5y = 9$ and $2x + ky = 11$ are **a)** parallel and **b)** perpendicular.

4) Write the equation of the line through (−6, 2) and **a)** parallel and **b)** normal to $5x + 2y = 7$.

5) Use the table to calculate the average rate of change from $t = 1$ to $t = 4$.

t	0	1	2	3	4
$x(t)$	8	7	5	1	2

6) Order the points **A**, **B**, and **C**, from least to greatest, by their rates of change.



E. Negative and Fractional Exponents

In AP Calculus, you will be required to perform algebraic manipulations with **negative exponents** and **fractional exponents**. These manipulations are imperative to master as AP Calculus problems will need to be altered prior to applying a certain calculus skill.

- **Negative exponent**: $x^{-n} = \dfrac{1}{x^n}$ (Note: negative powers don't make expressions negative; they create fractions)
- **Fractional exponent**: $x^{1/2} = \sqrt{x}$ $x^{a/b} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$
- **When we multiply, we add exponents**: $\left(x^a\right)\left(x^b\right) = x^{a+b}$
- **When we divide, we subtract exponents**: $\dfrac{x^a}{x^b} = x^{a-b}$
- **When we raise powers, we multiply exponents**: $\left(x^a\right)^b = x^{ab}$

Simplify using only positive exponents.

1) $(-5x^3)^{-2}$

2) $(36x^{10})^{1/2}$

3) $-5\left(\frac{3}{2}\right)(4-9x)^{-1/2}(-9)$

4) $2\left(\dfrac{2}{2-x}\right)\left[\dfrac{-2}{(2-x)^2}\right]$

5) $-\frac{2}{3}(8x)^{-5/3}(8)$

6) $(4x^{-1})^{-1}$

7) $(x^3-1)^{-2}$

8) $(4x^2-12x+9)^{-1/2}$

9) $\left(\dfrac{-3}{x^4}\right)^{-2}$

10) $\dfrac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

F. Eliminating Complex Fractions

AP Calculus frequently uses **complex fractions**, which are fractions within fractions. Answers are never left with complex fractions and they must be simplified. To simply complex fractions use the **"Copy Dot Flip"** method.

When the problem is in the form of $\dfrac{\frac{a}{b}}{\frac{c}{d}}$ we can multiply by the reciprocal of the denominator and write $\dfrac{a}{b}\cdot\dfrac{d}{c}=\dfrac{ad}{bc}$

However, this does not work when the numerator and denominator are not single fractions. In those cases the numerator and denominator first need to be simplified into a single fraction.

Copy Dot Flip

$$\dfrac{\frac{2}{3}}{\frac{5}{6}} = \frac{2}{3} \bullet \frac{6}{5} = \frac{12}{15} = \boxed{\frac{4}{5}}$$

Simplify the following.

1) $\dfrac{x}{x-\frac{1}{2}}$

2) $\dfrac{\frac{1}{x}+4}{\frac{1}{x}-2}$

3) $\dfrac{x-\frac{1}{x}}{x+\frac{1}{x}}$

4) $\dfrac{\frac{x^2-y^2}{xy}}{\frac{x+y}{y}}$

5) $\dfrac{x^{-3}-x}{x^{-2}-1}$

G. Solving Fractional Equations

To solve **fractional equations** both sides of the equation must have only one fraction and then cross multiplication can be performed to solve for *x*. Algebra has taught you that in order to add or subtract fractions, you need to find an LCD and multiply each fraction by one … in such a way that you obtain the LCD in each fraction. The easiest way to achieve this is to take the denominator of the 1st fraction and multiply it over itself to the 2nd fraction and vice versa.

Solve $\frac{12}{x+2} - \frac{4}{x} = 1$

$$\frac{12}{x+2}\left(\frac{x}{x}\right) - \frac{4}{x}\left(\frac{x+2}{x+2}\right) = 1$$

$$\frac{12x}{x(x+2)} - \frac{4x+8}{x(x+2)} = 1$$

$$\frac{12x-4x-8}{x(x+2)} = 1$$

$$\frac{8x-8}{x(x+2)} = 1$$

$$8x-8 = x^2+2x$$

$$x^2-6x+8=0$$

$$(x-2)(x-4)=0$$

$$\boxed{x=2, \quad x=4}$$

Solve each equation.

1) $\frac{2}{3} - \frac{5}{6} = \frac{1}{x}$

2) $\frac{x-5}{x+1} = \frac{3}{5}$

3) $\frac{x+1}{3} - \frac{x-1}{2} = 1$

4) $x + \frac{6}{x} = 5$

5) $\frac{2}{x+5} + \frac{1}{x-5} = \frac{16}{x^2-25}$

H. **Solving Inequalities**

You may think that **solving inequalities** are just a matter of replacing the equal sign with an inequality sign. In reality, they can be more difficult and are fraught with dangers.

Solving inequalities are a simple matter if they are based on linear equations. They are solved exactly like linear equations, remembering that if you multiply or divide both sides by a negative number, the direction of the inequality sign must be reversed.

Solve the following inequalities. a) $2x-8 \le 6x+2$ b) $-5 \le 6x-1 < 11$

a) $-4x \le 10$, $x \ge -\dfrac{5}{2}$

b) $-4 \le 6x \le 12$, $-\dfrac{2}{3} \le x \le 2$

If the inequality involves an absolute value, create two equations, replacing the absolute value with positive parentheses and negative parentheses and solve each.

Solve the following inequalities. a) $|x| < 2$ b) $|x-1| > 3$

a) $x < 2$ | $-(x) < 2$, $x > -2$; $\boxed{-2 < x < 2}$

b) $x-1 > 3$, $\boxed{x > 4}$ | $-(x-1) > 3$, $x-1 < -3$, $\boxed{x < -2}$

If the inequality is quadratic, it is advised to bring all terms to one side and pretend for a moment it is an equation and solve. Then create a sign chart in which values are chosen to the left and right of the solutions and checked in the inequality to be true or false. The intervals that produce true inequalities are the correct intervals.

Solve the following inequalities.

1) $5(x-3) \le 8(x+5)$

2) $\dfrac{3}{4} > x+1 > \dfrac{1}{2}$

3) $|2x-1| \le x+4$

4) $x^2 - 3x - 18 > 0$

I. Exponential and Natural Log Functions

AP Calculus spends a great deal of time on **exponential functions** in the form of $y = e^x$. Students should know that the value of e = 2.71828…

In AP Calculus, we primarily use logs with base ***e***, which are called **natural logs** (ln).

$$\log_a x = y \Longleftrightarrow x = a^y$$

$$\ln x = y \Longleftrightarrow x = e^y$$

The inverse of a logarithmic function is an exponential function. $e^{\ln x} = x$

There are several rules that students must keep in mind that will simplify problems involving natural logs.

a. $\ln 1 = 0$	*d.* $\ln(a)^n = n(\ln a)$
b. $\ln e = 1$	*e.* $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
c. $\ln(ab) = \ln a + \ln b$	

Common Mistakes

1. $\ln(x+3) \neq \ln x + \ln 3$
2. $(\ln x)^3 \neq 3\ln x$

To solve an equation that has a variable inside a natural log, you must **exponentiate** (raise both sides of equation to a power with base *e*) both sides of the equation. To solve an equation that has a variable in a power with base *e*, the natural log of both sides must be taken.

Simplify the following expressions using properties.

1) $\ln\sqrt{e}$

$\ln e^{1/2}$

$\frac{1}{2}(\ln e)$

$\frac{1}{2}(1) = \boxed{\frac{1}{2}}$

2) $\ln\left(\frac{1}{1-x}\right)$

3) $x e^{\ln x^2}$

4) $e^{\ln 12}$

5) $\ln\sqrt[5]{e^3}$

6) $\ln\frac{1}{\sqrt[3]{e^2}}$

Solve the following equations.

7) $e^{-2x} = 5$

$\ln e^{-2x} = \ln 5$

$-2x(\ln e) = \ln 5$

$-2x = \ln 5 \Rightarrow \boxed{x = \frac{-\ln 5}{2}}$

8) $\ln x - \ln(x-1) = 1$

9) $\ln x^3 - \ln x^2 = \frac{1}{2}$

10) The number of elk after *t* years in a state park is modeled by the function $P(t) = \frac{1216}{1+75e^{-0.03t}}$.

a) What was the initial population?
b) When will the number of elk be 750?

J. Trigonometric Identities

Trigonometric identities are equalities involving trigonometric functions that are true for all values of the occurring angles. While you are not asked to memorize these identities specifically in AP Calculus, knowing them can make problems easier. The following chart gives the major trigonometric identities that you should know. To prove trigonometric identities, you usually start with the more complicated expression and use algebraic rules and the fundamental trigonometric identities. A useful technique is to change all trigonometric functions to sines and cosines.

Fundamental Trig Identities

$$\csc x = \frac{1}{\sin x},\quad \sec x = \frac{1}{\cos x},\quad \cot x = \frac{1}{\tan x},\quad \tan x = \frac{\sin x}{\cos x},\quad \cot x = \frac{\cos x}{\sin x}$$

$$\sin^2 x + \cos^2 x = 1,\quad 1 + \tan^2 x = \sec^2 x,\quad 1 + \cot^2 x = \csc^2 x$$

Sum Identities

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

Double Angle Identities

$$\sin(2x) = 2\sin x\cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

Verify the following identities:

1) $\left(\tan^2 x + 1\right)\left(\cos^2 - 1\right) = -\tan^2 x$

$$\left(\sec^2 x\right)\left(-\sin^2 x\right) =$$

$$\left(\frac{1}{\cos^2 x}\right)\left(-\sin^2 x\right) =$$

$$-\left(\frac{\sin^2 x}{\cos^2 x}\right) =$$

$$-\tan^2 x = -\tan^2 x$$

2) $\left(1 + \sin x\right)\left(1 - \sin x\right) = \cos^2 x$

3) $\sec x - \cos x = \sin x \tan x$

4) $\dfrac{1 - \sec x}{1 - \cos x} = -\sec x$

5) $\sec^2 x + 3 = \tan^2 x + 4$

K. Solving Trigonometric Equations

Trigonometric equations are equations involving trigonometric functions. Typically they have many (or infinite) number of solutions so usually they are solved within a specific domain. These type of equations usually appear on non-calculator questions so a thorough knowledge of the unit circle and the sine/cosine curves is invaluable.

Solve each equation on the interval $[0, 2\pi)$. Give exact values (ex: $\frac{\pi}{3}$).

1) $x\cos x = 3\cos x$

$$x\cos x - 3\cos x = 0$$
$$\cos x(x-3) = 0$$
$$\cos x = 0 \qquad x - 3 = 0$$
$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}} \qquad \boxed{x = 3}$$

2) $2\cos x + \sqrt{3} = 0$

3) $\cos^2 x = \cos x$

4) $2\sin^2 x + \sin x = 1$

5) $2\sin x\cos x + \sin x = 0$

6) $\sin^2 x - \cos^2 x = 0$