10-1 Areas of Parallelograms and Triangles

Mathematics Florida Standards

MAFS.912.G-MG.1.1 Use geometric shapes, their measures, and their properties to describe objects.
MAFS.912.G-GPE.2.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles.

MP 3, MP 4, MP 5, MP 6

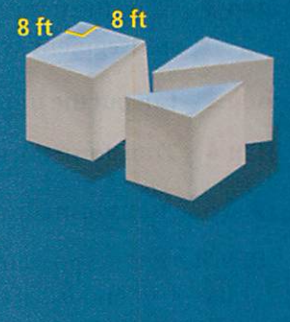
Objective To find the area of parallelograms and triangles

You can combine triangles to make just about any shape!

MATHEMATICAL PRACTICES

SOLVE IT! Getting Ready!

A stage is being set up for a concert at the arena. The stage is made up of blocks with tops that are congruent right triangles. The tops of two of the blocks, when put together, make an 8 ft-by-8 ft square. The band has requested that the stage be arranged to form the shape of an arrow. Draw a diagram that shows how the stage could be laid out in the shape of an arrow with an area of at least 1000 ft^2 but no more than 1400 ft^2.



Essential Understanding You can find the area of a parallelogram or a triangle when you know the length of its base and its height.

A parallelogram with the same base and height as a rectangle has the same area as the rectangle.

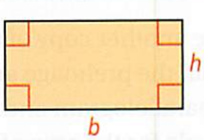
Lesson Vocabulary
- base of a parallelogram
- altitude of a parallelogram
- height of a parallelogram
- base of a triangle
- height of a triangle

take note

Theorem 10-1 Area of a Rectangle

The area of a rectangle is the product of its base and height.

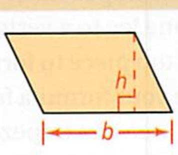
$A = bh$



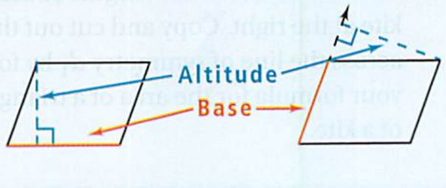
Theorem 10-2 Area of a Parallelogram

The area of a parallelogram is the product of a base and the corresponding height.

$A = bh$



A **base of a parallelogram** can be any one of its sides. The corresponding **altitude** is a segment perpendicular to the line containing that base, drawn from the side opposite the base. The **height** is the length of an altitude.



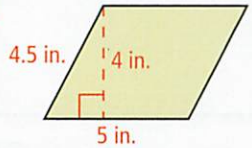
Problem 1 Finding the Area of a Parallelogram

Think

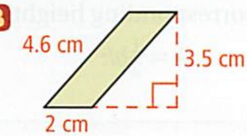
Why aren't the sides of the parallelogram considered altitudes?
Altitudes must be perpendicular to the bases. Unless the parallelogram is also a rectangle, the sides are not perpendicular to the bases.

What is the area of each parallelogram?

A



B



You are given each height. Choose the corresponding side to use as the base.

$A = bh$		$A = bh$
$= 5(4) = 20$	Substitute for b and h.	$= 2(3.5) = 7$
The area is 20 in.^{2}.		The area is 7 cm^2.

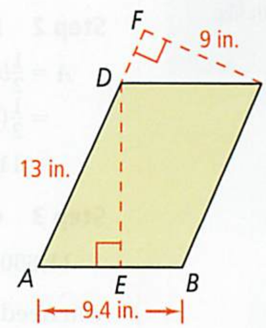
Got It? **1.** What is the area of a parallelogram with base length 12 m and height 9 m?

Problem 2 Finding a Missing Dimension

Think

What does $\overline{CF}$ represent?
$\overline{CF}$ is an altitude of the parallelogram when $\overline{AD}$ and $\overline{BC}$ are used as bases.

For $\square ABCD$, what is DE to the nearest tenth?



First, find the area of $\square ABCD$. Then use the area formula a second time to find DE.

$A = bh$

$= 13(9) = 117$ Use base AD and height CF.

The area of $\square ABCD$ is 117 in.^{2}.

$A = bh$

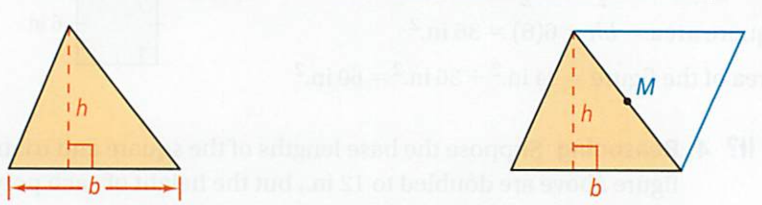
$117 = 9.4(DE)$ Use base AB and height DE.

$DE = \frac{117}{9.4} \approx 12.4$

DE is about 12.4 in.

Got It? **2.** A parallelogram has sides 15 cm and 18 cm. The height corresponding to a 15-cm base is 9 cm. What is the height corresponding to an 18-cm base?

You can rotate a triangle about the midpoint of a side to form a parallelogram.



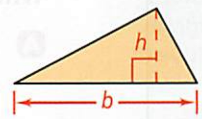
The area of the triangle is half the area of the parallelogram.

take note

Theorem 10-3 Area of a Triangle

The area of a triangle is half the product of a base and the corresponding height.

$A = \frac{1}{2}bh$



A **base of a triangle** can be any of its sides. The corresponding **height** is the length of the altitude to the line containing that base.

Problem 3 Finding the Area of a Triangle

Plan

Why do you need to convert the base and the height into inches?
You must convert them both because you can only multiply measurements with like units.

Sailing **You want to make a triangular sail like the one at the right. How many square feet of material do you need?**

Step 1 Convert the dimensions of the sail to inches.

$(12 \text{ ft} \cdot \frac{12 \text{ in.}}{1 \text{ ft}}) + 2 \text{ in.} = 146 \text{ in.}$ Use a conversion factor.

$(13 \text{ ft} \cdot \frac{12 \text{ in.}}{1 \text{ ft}}) + 4 \text{ in.} = 160 \text{ in.}$

Step 2 Find the area of the triangle.

$A = \frac{1}{2}bh$

$= \frac{1}{2}(160)(146)$ Substitute 160 for b and 146 for h.

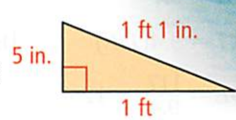
$= 11{,}680$ Simplify.

Step 3 Convert 11,680 in.^{2} to square feet.

$11{,}680 \text{ in.}^2 \cdot \frac{1 \text{ ft}}{12 \text{ in.}} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} = 81\frac{1}{9} \text{ ft}^2$

You need $81\frac{1}{9}$ ft^2 of material.

Got It? **3.** What is the area of the triangle?

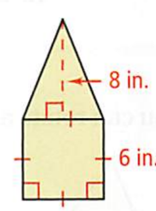


Problem 4 Finding the Area of an Irregular Figure

Plan

How do you know the length of the base of the triangle?
The lower part of the figure is a square. The base length of the triangle is the same as the base length of the square.

What is the area of the figure at the right?



Find the area of each part of the figure.

triangle area $= \frac{1}{2}bh = \frac{1}{2}(6)8 = 24$ in.^{2}

square area $= bh = 6(6) = 36$ in.^{2}

area of the figure $= 24$ in.$^2 + 36$ in.$^2 = 60$ in.^{2}

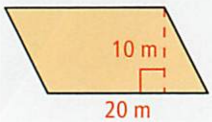
Got It? **4. Reasoning** Suppose the base lengths of the square and triangle in the figure above are doubled to 12 in., but the height of each polygon remains the same. How is the area of the figure affected?

Lesson Check

Do you know HOW?

Find the area of each parallelogram.

1.



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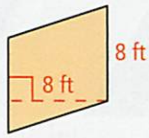
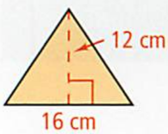


Find the area of each triangle.

3.



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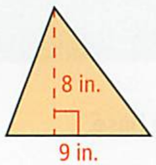
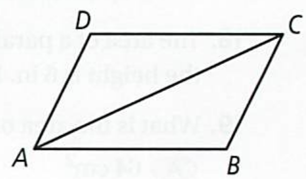


Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Vocabulary** Does an altitude of a triangle have to lie inside the triangle? Explain.

6. **Writing** How can you show that a parallelogram and a rectangle with the same bases and heights have equal areas?

7. $\square ABCD$ is divided into two triangles along diagonal $\overline{AC}$. If you know the area of the parallelogram, how do you find the area of $\triangle ABC$?

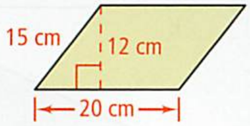


Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Find the area of each parallelogram. See Problem 1.

8.



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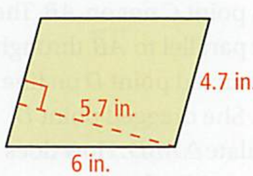

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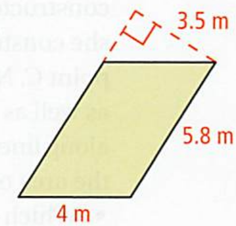
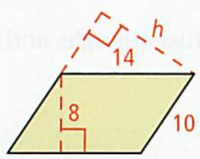


Find the value of h for each parallelogram. See Problem 2.

11.



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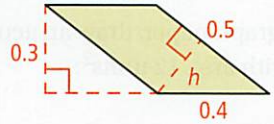

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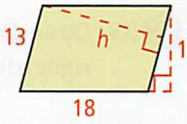
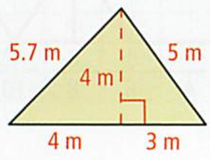


Find the area of each triangle. See Problem 3.

14.



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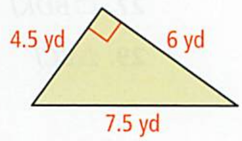

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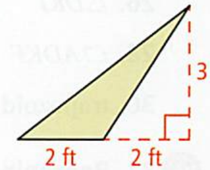
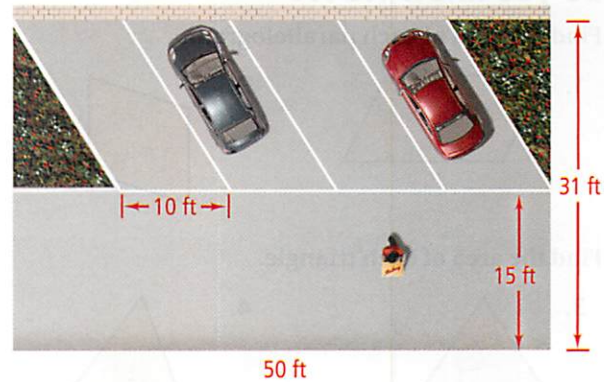


17. **Urban Design** A bakery has a 50 ft-by-31 ft parking lot. The four parking spaces are congruent parallelograms, the driving region is a rectangle, and the two areas for flowers are congruent triangles.

See Problem 4.



a. Find the area of the paved surface by adding the areas of the driving region and the four parking spaces.

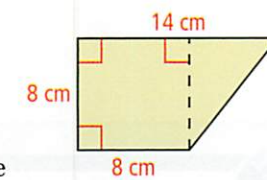
b. Describe another method for finding the area of the paved surface.

c. Use your method from part (b) to find the area. Then compare answers from parts (a) and (b) to check your work.

B Apply

18. The area of a parallelogram is 24 in.^{2} and the height is 6 in. Find the length of the corresponding base.

19. What is the area of the figure at the right?

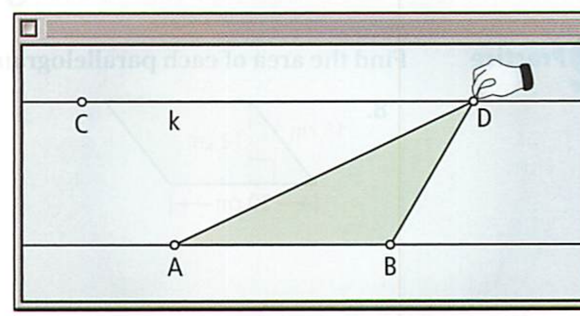


(A) 64 cm^2 (B) 88 cm^2 (C) 96 cm^2 (D) 112 cm^2

20. A right isosceles triangle has area 98 cm^2. Find the length of each leg.

21. **Algebra** The area of a triangle is 108 in.^{2}. A base and corresponding height are in the ratio 3 : 2. Find the length of the base and the corresponding height.

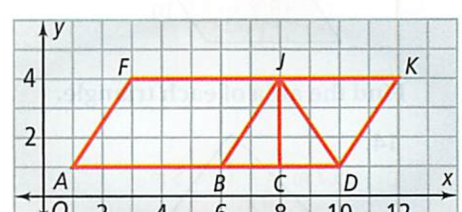
22. **Think About a Plan** Ki used geometry software to create the figure at the right. She constructed $\overleftrightarrow{AB}$ and a point C not on $\overleftrightarrow{AB}$. Then she constructed line k parallel to $\overleftrightarrow{AB}$ through point C. Next, Ki constructed point D on line k as well as $\overline{AD}$ and $\overline{BD}$. She dragged point D along line k to manipulate $\triangle ABD$. How does the area of $\triangle ABD$ change? Explain.



- Which dimensions of the triangle change when Ki drags point D?
- Do the lengths of AD and BD matter when calculating area?

23. **Open-Ended** Using graph paper, draw an acute triangle, an obtuse triangle, and a right triangle, each with area 12 units^2.

Find the area of each figure.



24. $\square ABJF$

25. $\triangle BDJ$

26. $\triangle DKJ$

27. $\square BDKJ$

28. $\square ADKF$

29. $\triangle BCJ$

30. trapezoid $ADJF$

31. **Reasoning** Suppose the height of a triangle is tripled. How does this affect the area of the triangle? Explain.

For Exercises 32–35, (a) graph the lines and (b) find the area of the triangle enclosed by the lines.

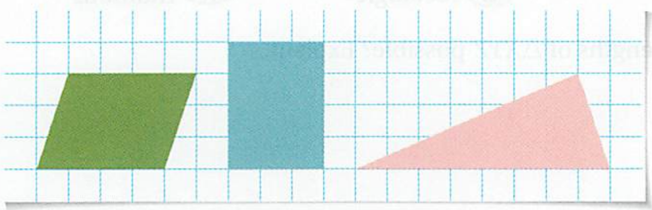
32. $y = x, x = 0, y = 7$

33. $y = x + 2, y = 2, x = 6$

34. $y = -\frac{1}{2}x + 3, y = 0, x = -2$

35. $y = \frac{3}{4}x - 2, y = -2, x = 4$

36. Probability Your friend drew these three figures on a grid. A fly lands at random at a point on the grid.



a. **Writing** Is the fly more likely to land on one of the figures or on the blank grid? Explain.

b. Suppose you know the fly lands on one of the figures. Is the fly more likely to land on one figure than on another? Explain.

Coordinate Geometry Find the area of a polygon with the given vertices.

37. $A(3, 9), B(8, 9), C(2, -3), D(-3, -3)$

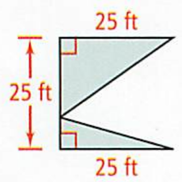
38. $E(1, 1), F(4, 5), G(11, 5), H(8, 1)$

39. $D(0, 0), E(2, 4), F(6, 4), G(6, 0)$

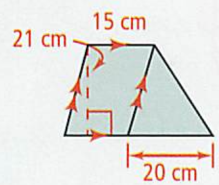
40. $K(-7, -2), L(-7, 6), M(1, 6), N(7, -2)$

Find the area of each figure.

41.



42.



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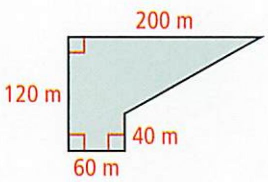


Challenge

History The Greek mathematician Heron is most famous for this formula for the area of a triangle in terms of the lengths of its sides *a*, *b*, and *c*.

$$A = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{1}{2}(a+b+c)$$

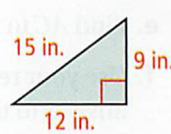
Use Heron's Formula and a calculator to find the area of each triangle. Round your answer to the nearest whole number.

44. $a = 8$ in., $b = 9$ in., $c = 10$ in.

45. $a = 15$ m, $b = 17$ m, $c = 21$ m

46. a. Use Heron's Formula to find the area of this triangle.

b. Verify your answer to part (a) by using the formula $A = \frac{1}{2}bh$.



Standardized Test Prep

SAT/ACT

47. The lengths of the sides of a right triangle are 10 in., 24 in., and 26 in. What is the area of the triangle?

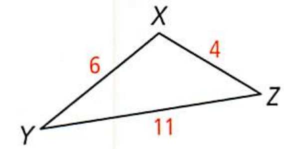
Ⓐ 116 in.^2 Ⓑ 120 in.^2 Ⓒ 130 in.^2 Ⓓ 156 in.^2

48. In quadrilateral *ABCD*, $AB \cong BC \cong CD \cong DA$. Which type of quadrilateral could *ABCD* never be classified as?

Ⓕ square Ⓖ rectangle Ⓗ rhombus Ⓘ kite

Short Response

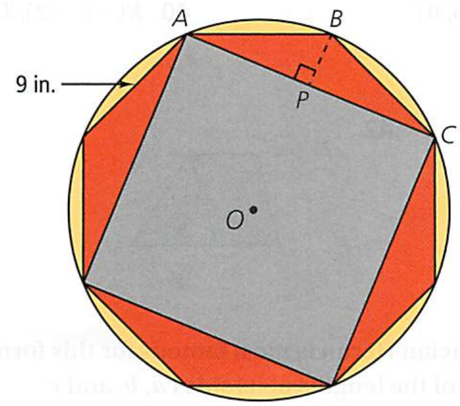
49. Are the side lengths of $\triangle XYZ$ possible? Explain.



Apply What You've Learned

MATHEMATICAL PRACTICES
MP 5

Look back at the information given about the target on page 613. The diagram of the target is shown again below, with three vertices of the regular octagon labeled *A*, *B*, and *C*. $\overline{BP}$ is drawn perpendicular to $\overline{AC}$.



a. What is the measure of $\angle ABC$? Justify your answer.

b. Are the four red triangles congruent? Justify your answer.

c. What are the measures of the angles of $\triangle ABP$?

d. Use a trigonometric ratio to find *BP* to the nearest hundredth of an inch.

e. Find *AC* to the nearest hundredth of an inch.

f. Use your results from parts (d) and (e) to find the area of $\triangle ABC$. Round your answer to the nearest tenth of a square inch.