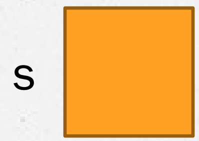
Areas of Triangles and Parallelograms

"There is no substitute for hard work." – Thomas Edison

Concept 1: Postulate 24-26

Postulate 24: Area of a Square Postulate

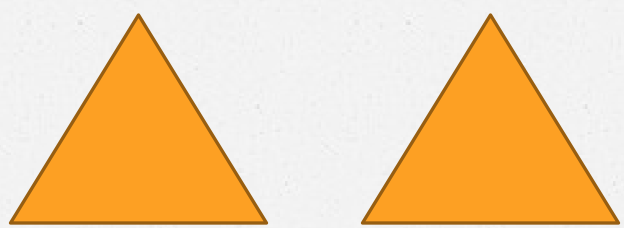
- The area of a square is the square of the length of its side.



$A = s^2$

Postulate 25: Area Congruence Postulate

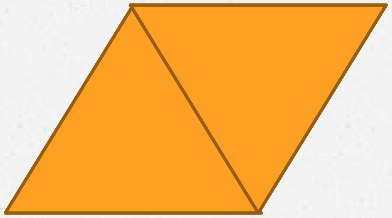
- If two polygons are congruent, then they have the same area.



Concept 1: Postulate 24-26

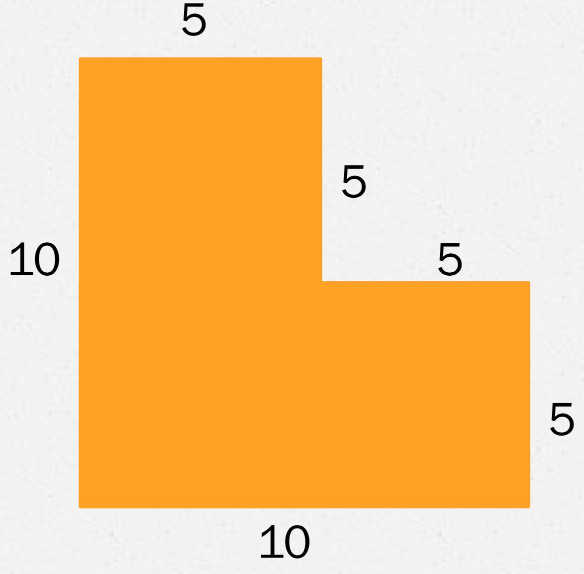
Postulate 26: Area Addition Postulate

- The area of a region is the sum of all the areas of its overlapping parts..



Example 1

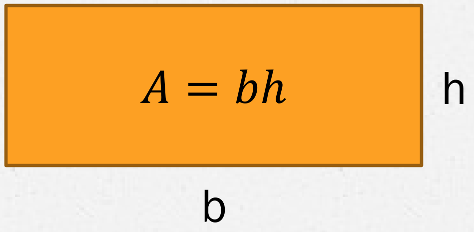
- Find the area of the shape.



Concept 2:Theorem 11.1-3

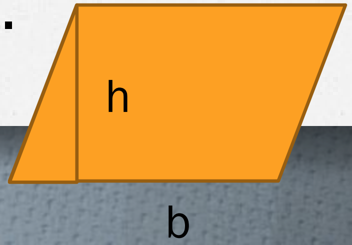
Theorem 11.1: Area of a Rectangle

- The area of a rectangle is the product of its base and height.



Theorem 11.2: Area of a Parallelogram.

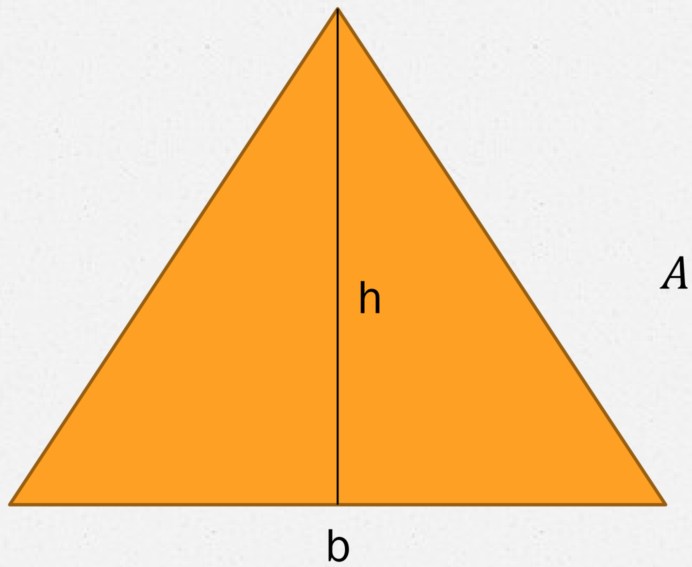
- The area of a parallelogram is the product of a base and its corresponding height.



Concept 2:Theorem 11.1-3

Theorem 11.3: Area of a Triangle

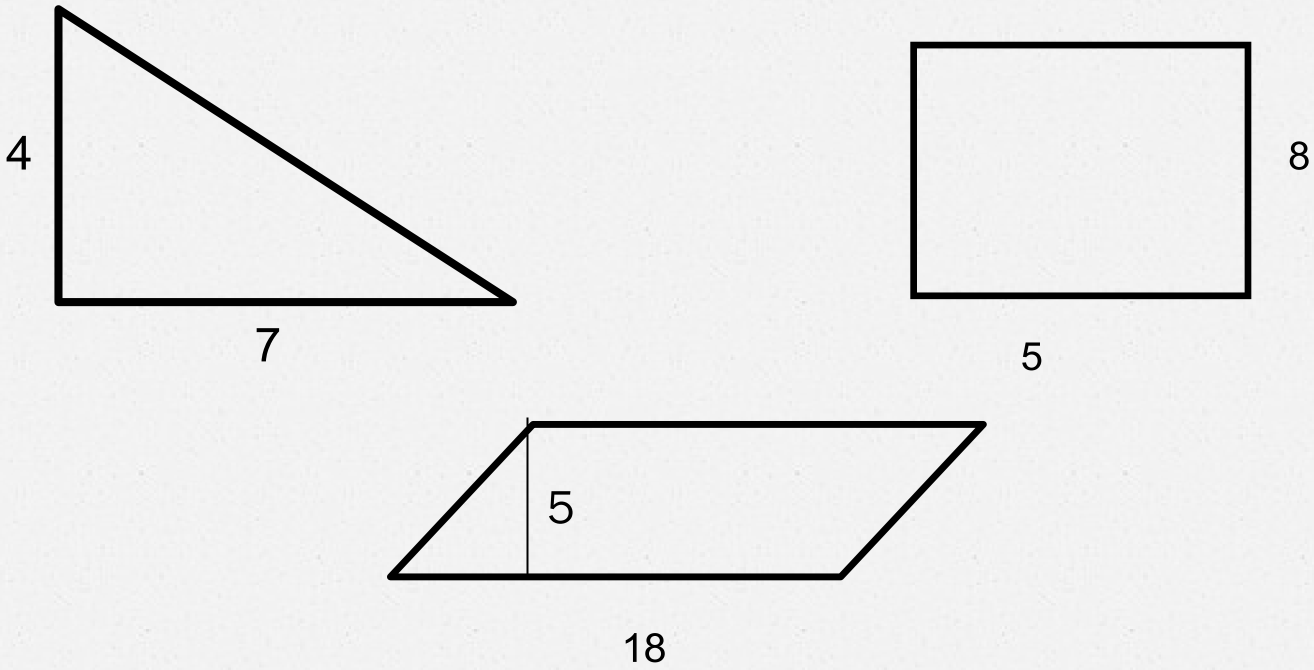
- The area of a triangle is one half the product of a base and its corresponding height.



$$A = \frac{1}{2}bh$$

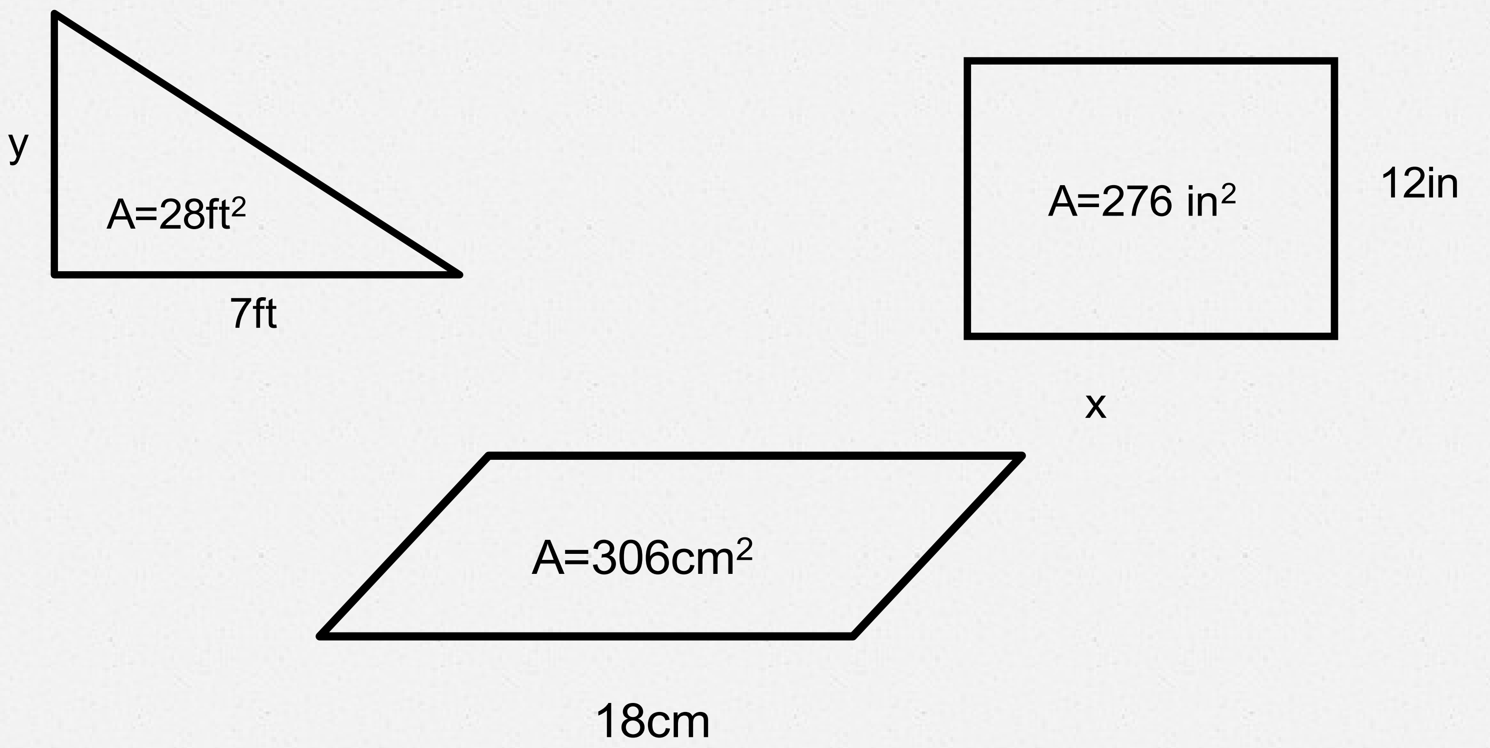
Example 2

- Find the area of the shapes.



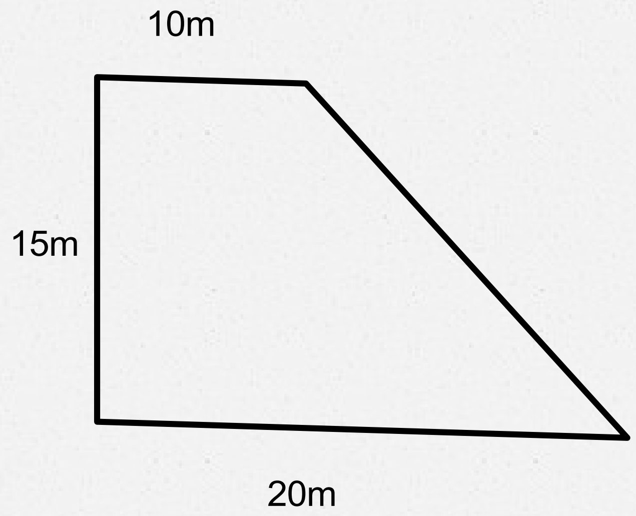
Example 3

- Solve for the unknown in each shape.



Example 4

- Solve for the area of the polygon.



Concept 3: Perimeter/Area

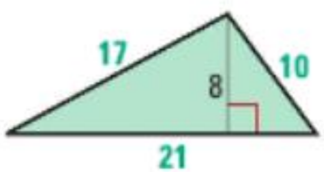
To determine the difference between perimeter and area:

Perimeter	Area
1. It's a measurement of the distance around a polygon.	1. It's a measurement of the space of a polygon.
2. It's units are a length.	2. It's units are length by length or length^2
3. Used to find the amount of fencing needed, the frame of an object, etc.	3. Used to find the amount of paint needed, amount of seed needed for a field, etc.

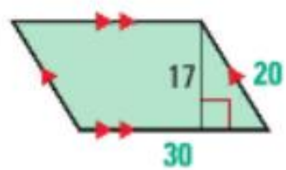
Example 5

Find the perimeter and area of the polygon.

1.



2.



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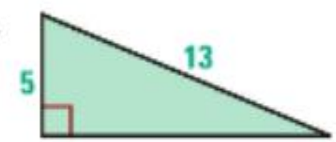


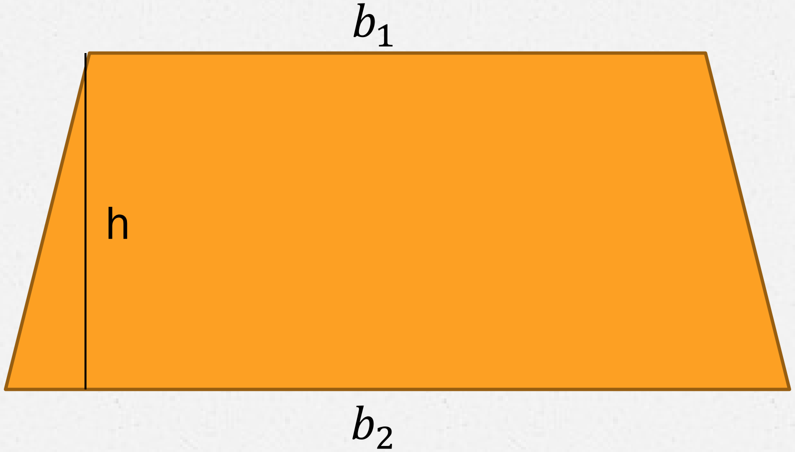
Areas of Trapezoids, Rhombuses, and Kites

"Bad is never good until worse happens."
–Danish Proverb

Concept 4: Area of a Trapezoid

Theorem 11.4: Area of a Trapezoid

- The area of a trapezoid is one half the product of the height and the sum of the lengths of the bases.



$$A = \frac{1}{2}h(b_1 + b_2)$$

Concept 5: Area of Rhombus/Kite

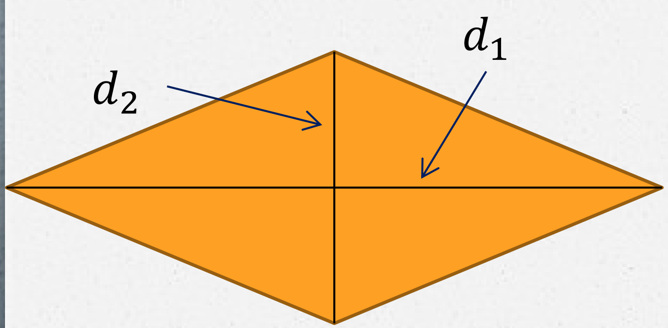
Theorem 11.5: Area of a Rhombus

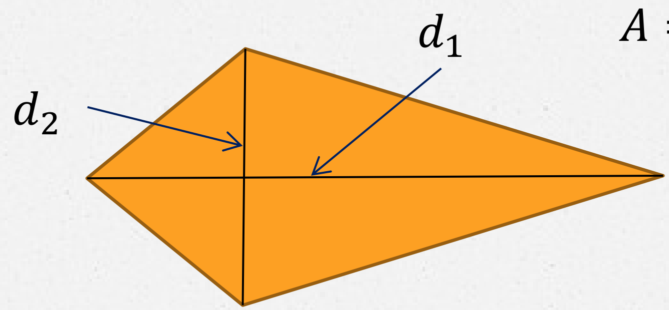
- The area of a rhombus is one half the product of the lengths of its diagonals.

Theorem 11.6: Area of a Kite

- The area of a kite is one half the product of the lengths of its diagonals.

$$A = \frac{1}{2} d_1 d_2$$

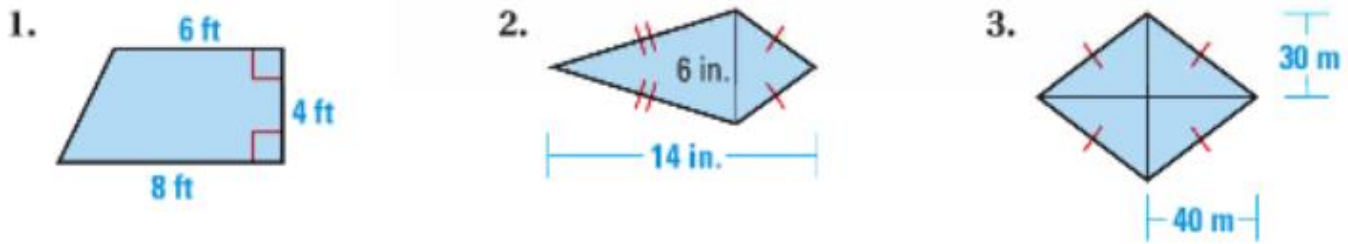




Example 1

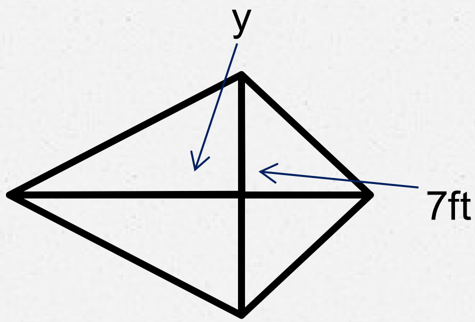
- Find the area of each shape.

Find the area of the figure.

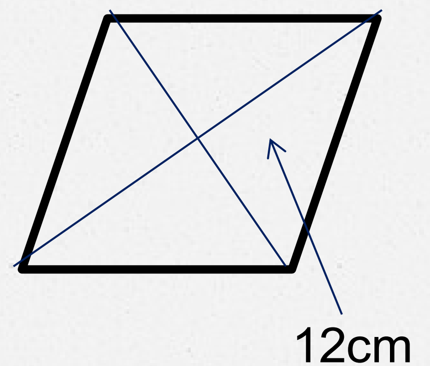


Example 2

- Solve for the variable.

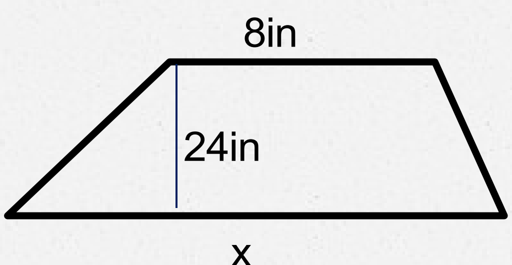


$A=56ft^2$



$A=324cm^2$

$A=264\ in^2$



Example 3

One diagonal of a kite is twice as long as the other diagonal. The area of the kite is 72.25 square inches. What are the lengths of the diagonals?

(A) 6 in., 6 in. (B) 8.5 in., 8.5 in. (C) 8.5 in., 17 in. (D) 6 in., 12 in.

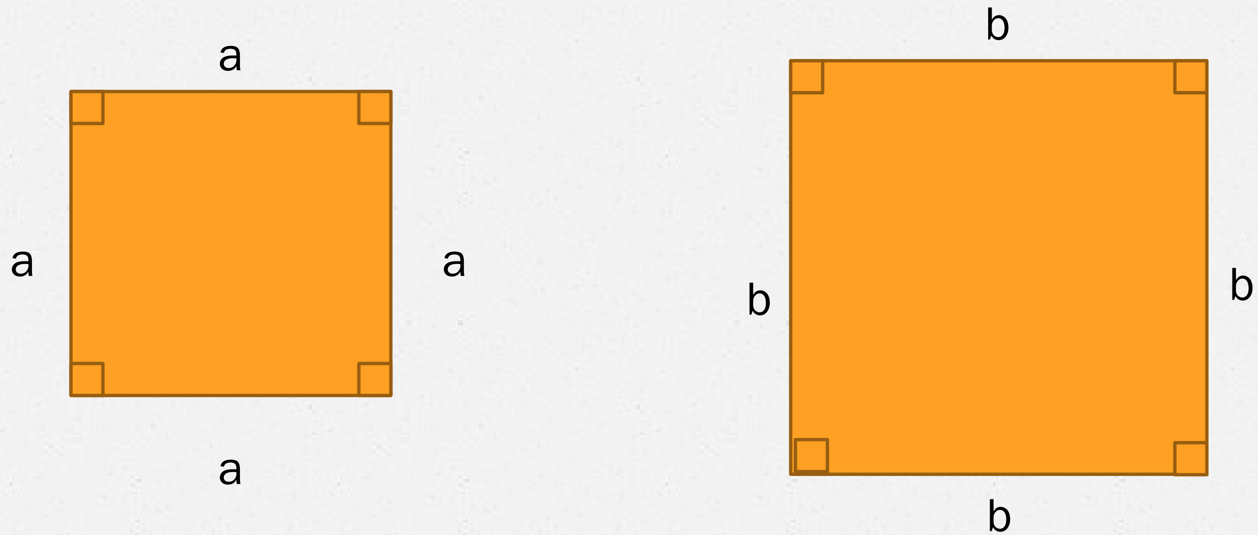
Perimeter and Area of Similar Figures

"Things could be worse. Suppose your errors were counted and published every day, like those of a baseball player." – Anon.

Concept 6: Area of Similar Polygons

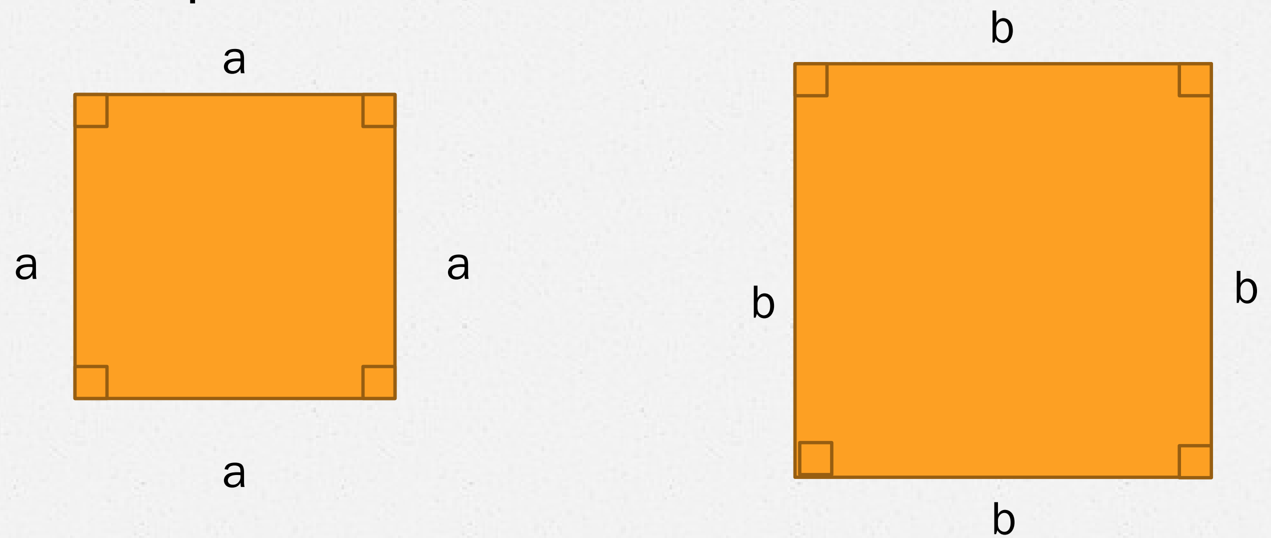
Theorem 11.7: Areas of Similar Polygons

- If two polygons are similar with the lengths of corresponding sides in the ratio of a:b, then the ratio of their areas is $a^2 = b^2$.



Concept 6: Area of Similar Polygons

- Similar-same shape, not necessarily the same size.
- When you encounter this word, you usually have to set up a proportion.
- Ratio of perimeters is the same as the scale factor.



Example 1

- Find the unknown ratios.

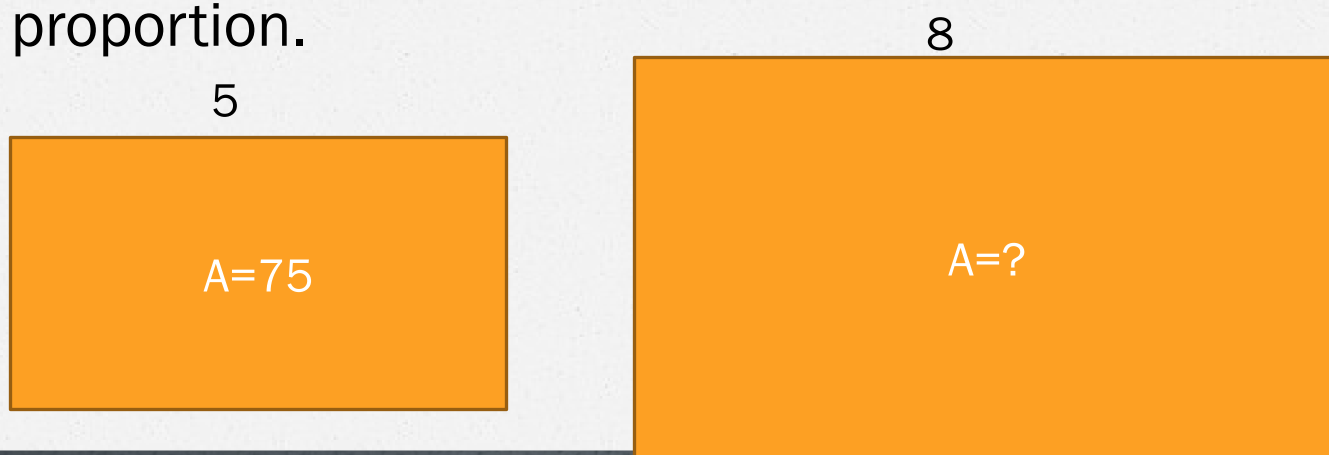
1. Ratio of two corresponding sides is 5:4.
What is the ratio of perimeters and what is the ratio of areas?

2. Ratio of perimeters is 3:7.
What is the ratio of two corresponding sides and the ratio of areas?

3. Ratio of Areas is 9:4.
What is the ratio of two corresponding sides and the ratio of perimeters?

Concept 7: Setting up proportions.

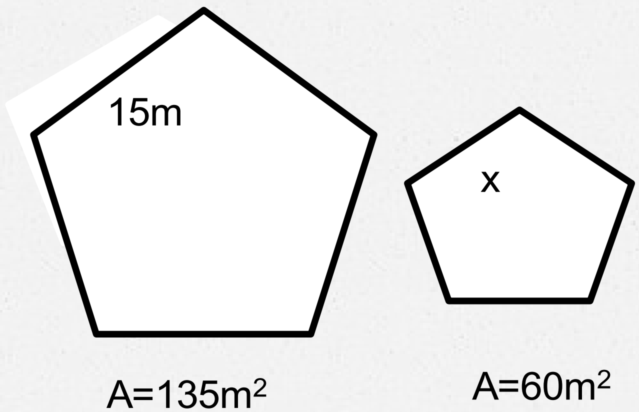
- When setting up proportions, the ratios have to follow the same pattern. Each ratio has to be a ratio of the perimeters or each ratio has to be a ratio of the areas.
- I suggest ratio of areas when setting up the proportion.



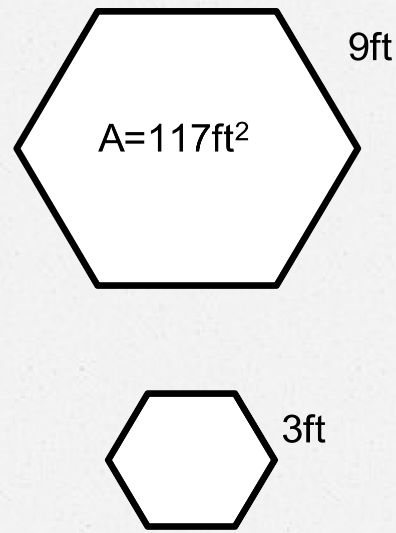
Example 2

Find the unknown area or unknown corresponding side length.

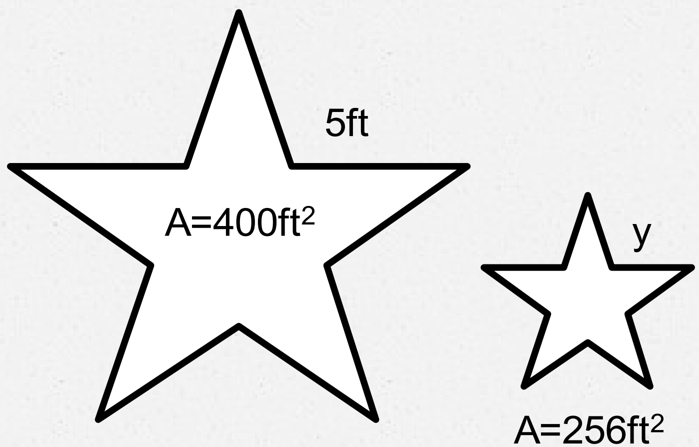
1.



2.



3.



Example 3

- A large rectangular baking pan is 15 inches long and 10 inches wide. A smaller pan is similar to the large pan. The area of the smaller pan is 96 square inches. Find the width of the smaller pan.

Example 4

- On a blueprint the area of a room is 100 in^2 and the length of the north wall is 5 in. The actual length of the north wall is 35 ft. What is the area of the room?

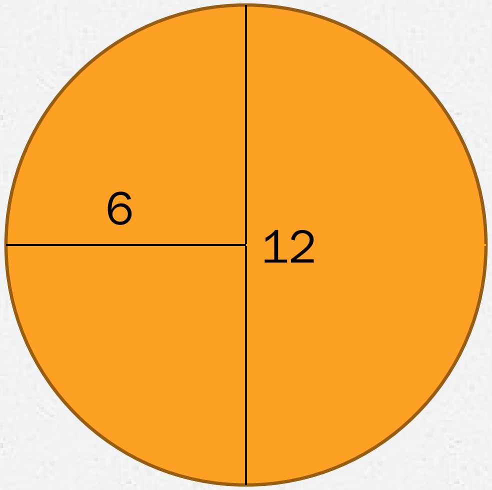
Circumference and Arc Length

"The superior man(/woman) blames himself(/herself), the inferior man(/woman) blames others." –Don Shula

Concept 8: Circumference of a Circle

Theorem 11.8: Circumference of a Circle

- The circumference C of a circle is C=πd or C=2πr, where d is the diameter of the circle and r is the radius of the circle.



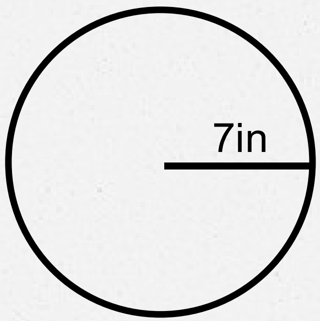
Example 1

- Find the indicated measure.

Circumference:

1. The diameter of the circle is 5cm.

2.



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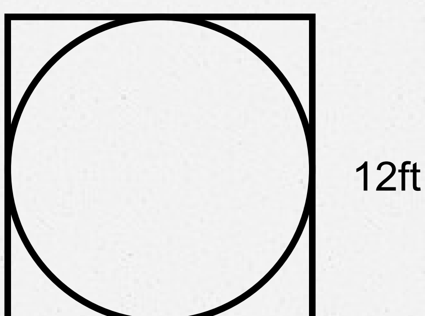
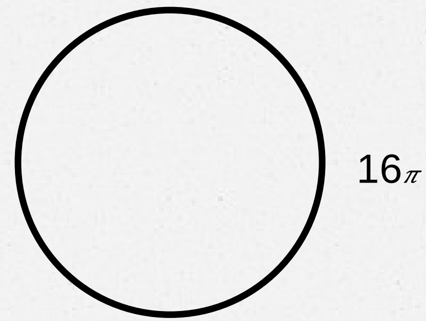


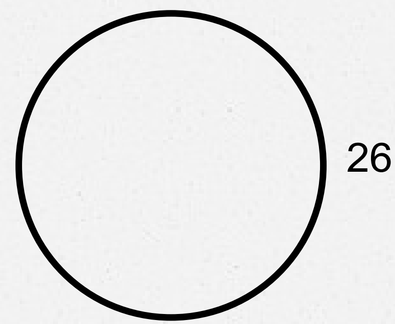
Radius:

4. The circumference is 8π

5.



6.



Example 2

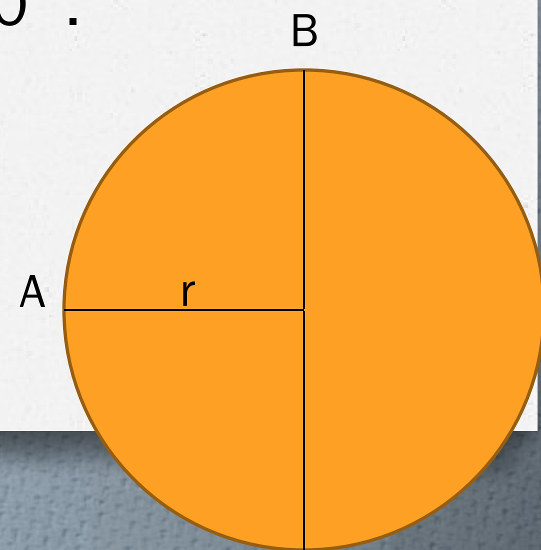
- The radius of a car tire is 21 inches. If the car drives for 5280 feet, how many revolutions does the car tire experience?

Concept 9: Arc Length of a Circle

Arc length is the actual length of a portion of the circle called the arc.

Arc Length Corollary

- In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.



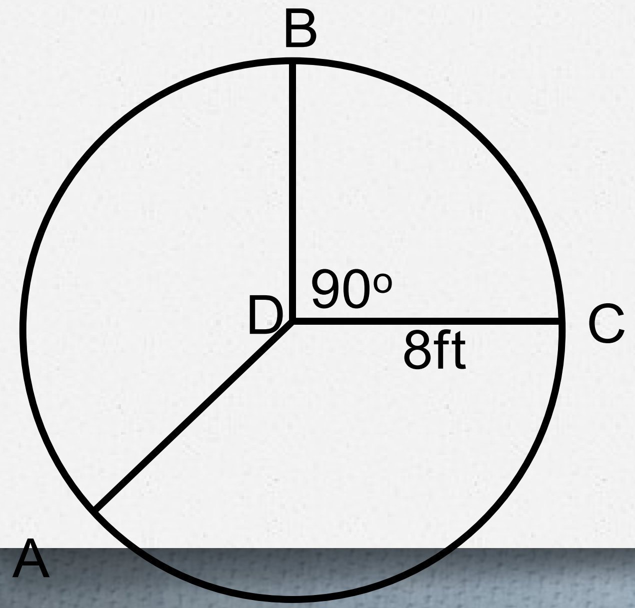
$$\frac{Arc\ length\ of\ \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360°}$$

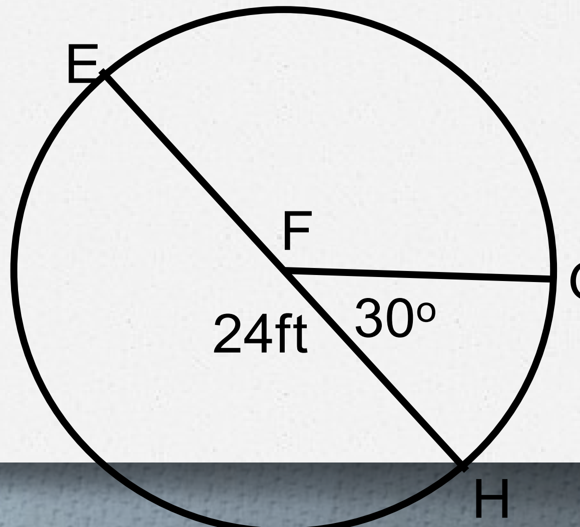
Example 3

Find the indicated measurement.

1. mBAC
2. mBA
3. mEG
4. mGHF

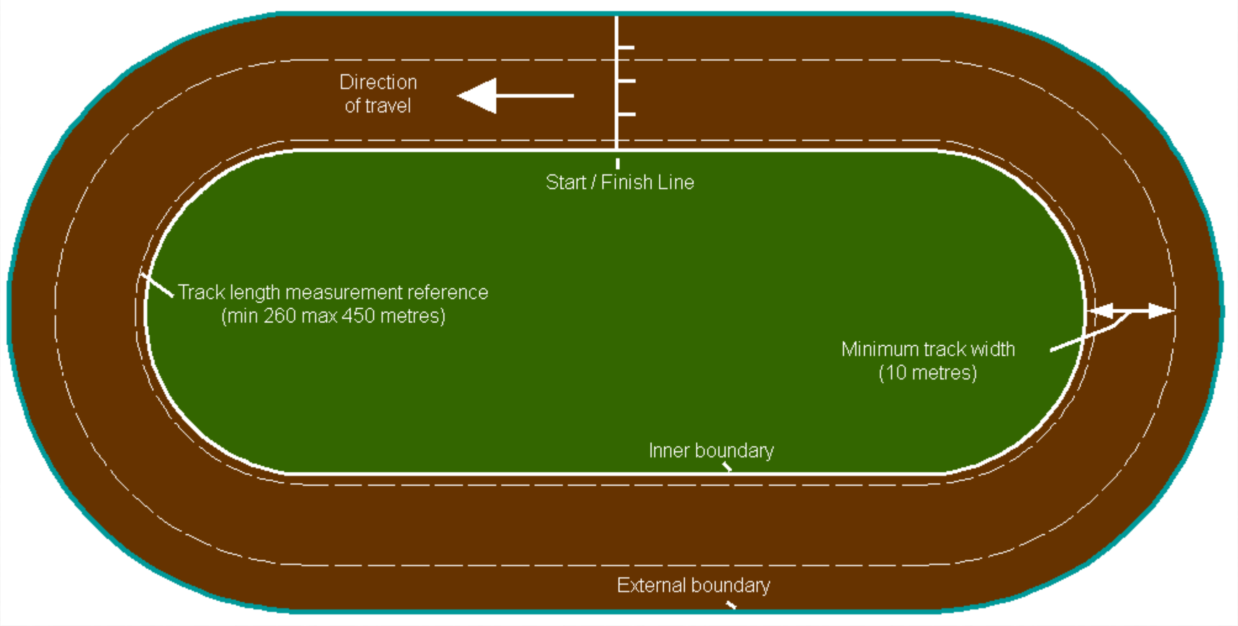
5. Length of BAC
6. Length of GH
7. Length of AC
8. Length of EG





Example 4

- The inside track used for outdoor track and field is 400m long. If the straight parts of the track are 100m what is the radius of the two ends of the track?



Areas of Circles and Sectors

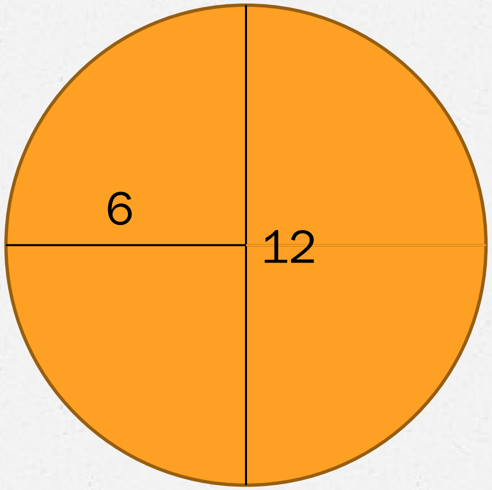
"I think we consider too much the good luck of the early bird, and not enough the bad luck of the early worm."

–Franklin D. Roosevelt

Concept 10: Area of a Circle

- Theorem 11.9: Area of a Circle
- The area of a circle is π times the square of the radius.

$$A = \pi r^2$$



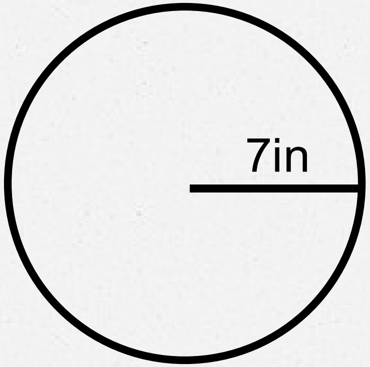
Example 1

- Find the indicated measure.

Area:

1. The diameter of the circle is 5cm.

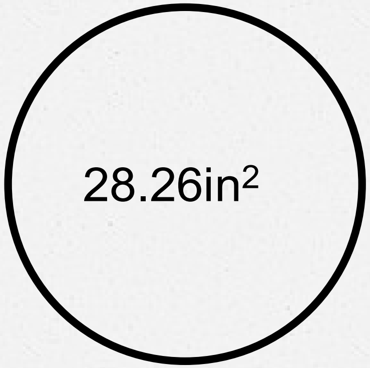
2.



Diameter:

4. The circumference is 10π

5.

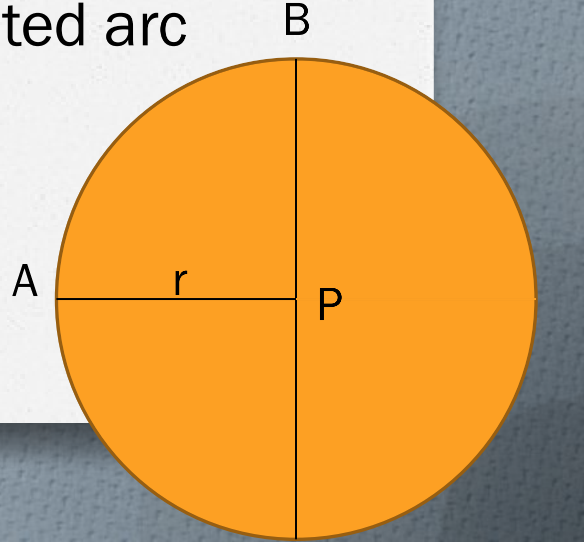


Concept 11: Area of a Sector

- A sector is the part of a circle formed by a central angle and its arc. (basically a slice of pizza/pie)

Theorem 11.10: Area of a Sector

- The ratio of the area of a sector of the circle to the area of the whole circle is equal to the ratio of the measure of the intercepted arc to 360°.

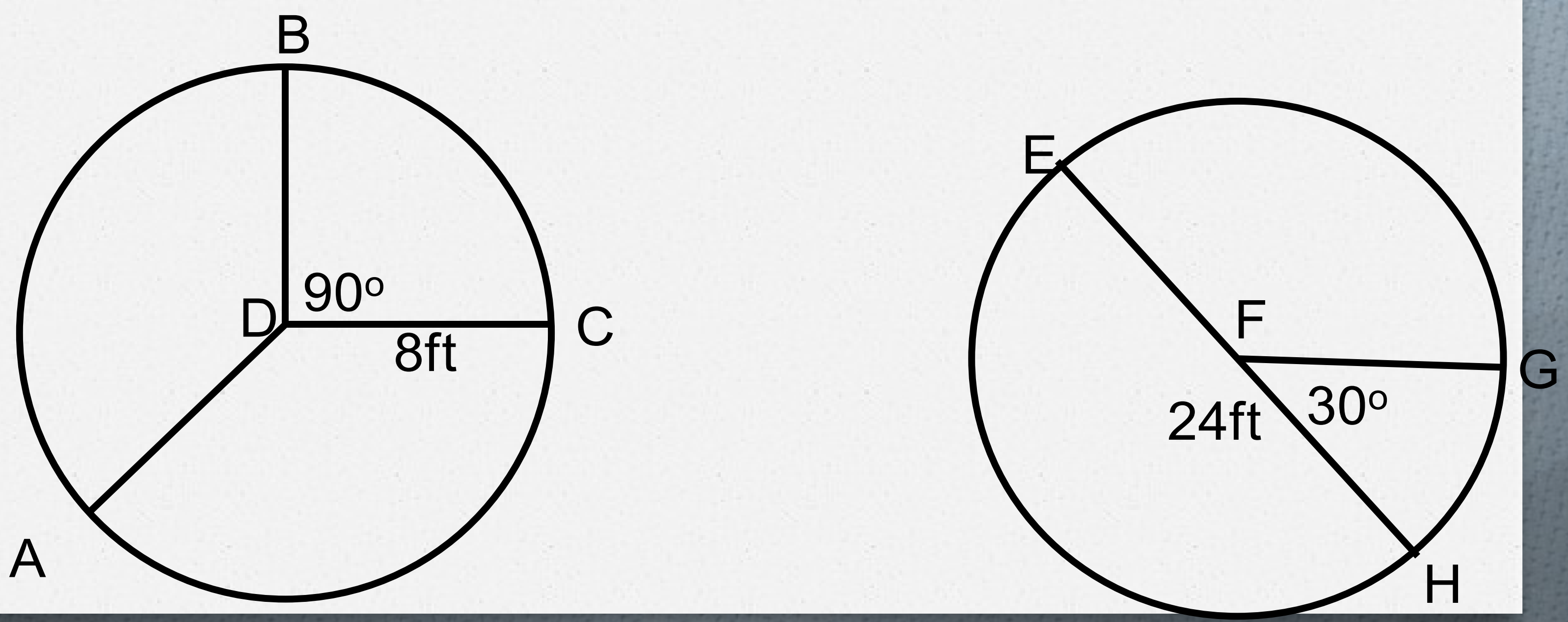


$$\frac{Area\ of\ sector\ APB}{\pi r^2} = \frac{m\widehat{AB}}{360°}$$

Example 2

- Find the area of the given sector.

1. Sector BDC
2. Sector ADC
3. Sector EFG
4. Sector GFH



Example 3

- For a local competition you are required to paint windows. The shape of your window is like the one pictured below. The radius of the semicircle is 20 inches and the full height of the window is 54 inches. What is the area that you have to paint?

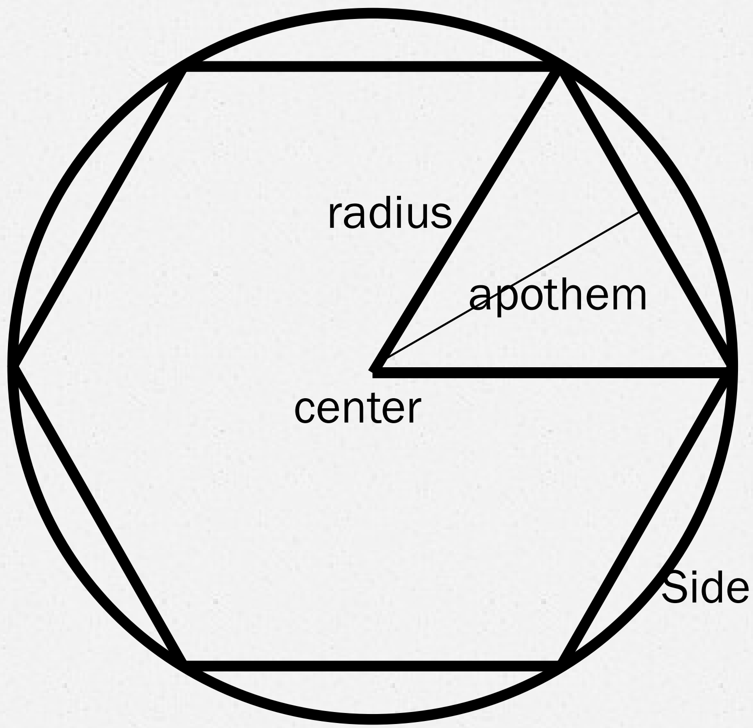
Areas of Regular Polygons

"Doing what's right isn't the problem. It's knowing what's right." –Lyndon B. Johnson

Concept 12: Sides, Apothems, and Radii

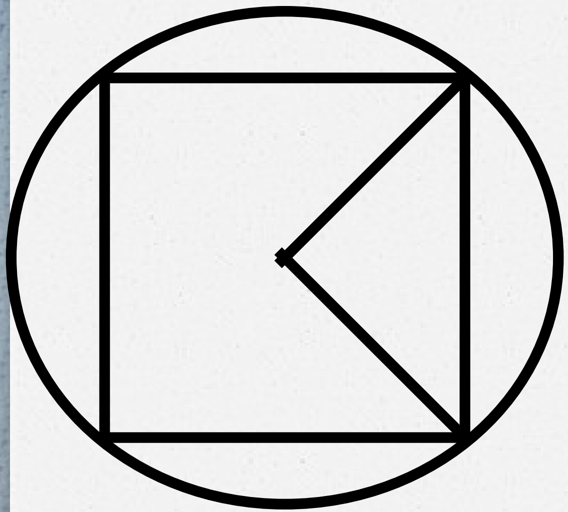
- For regular polygons we can construct these three shapes. In order to be able to create these three we need to think of the circumscribed circle.
- Center of a regular polygon- the center of the circumscribed circle.
- Radius of a regular polygon-the radius of the circumscribed circle.
- Side-the side of the polygon.
- Apothem- the segment perpendicular to a side of the polygon that passes through the center.
- Central angle-the angle formed by two radii of the polygon.

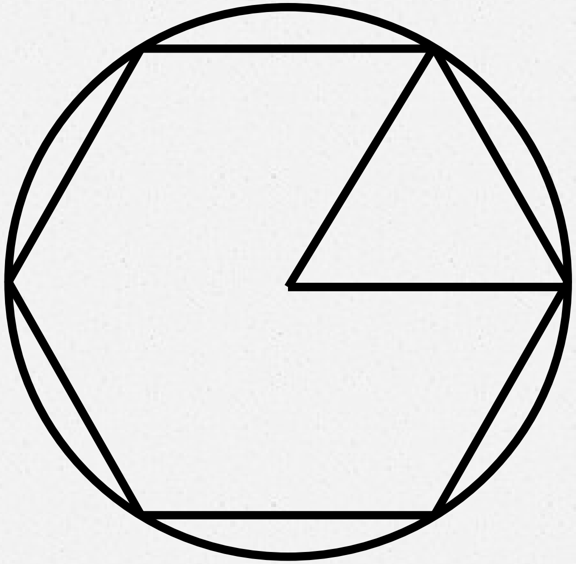
Concept 12: Sides, Apothems, and Radii

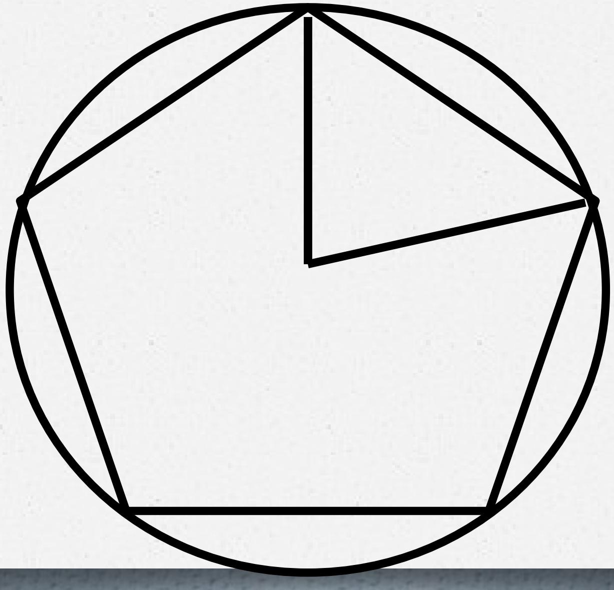


Example 1

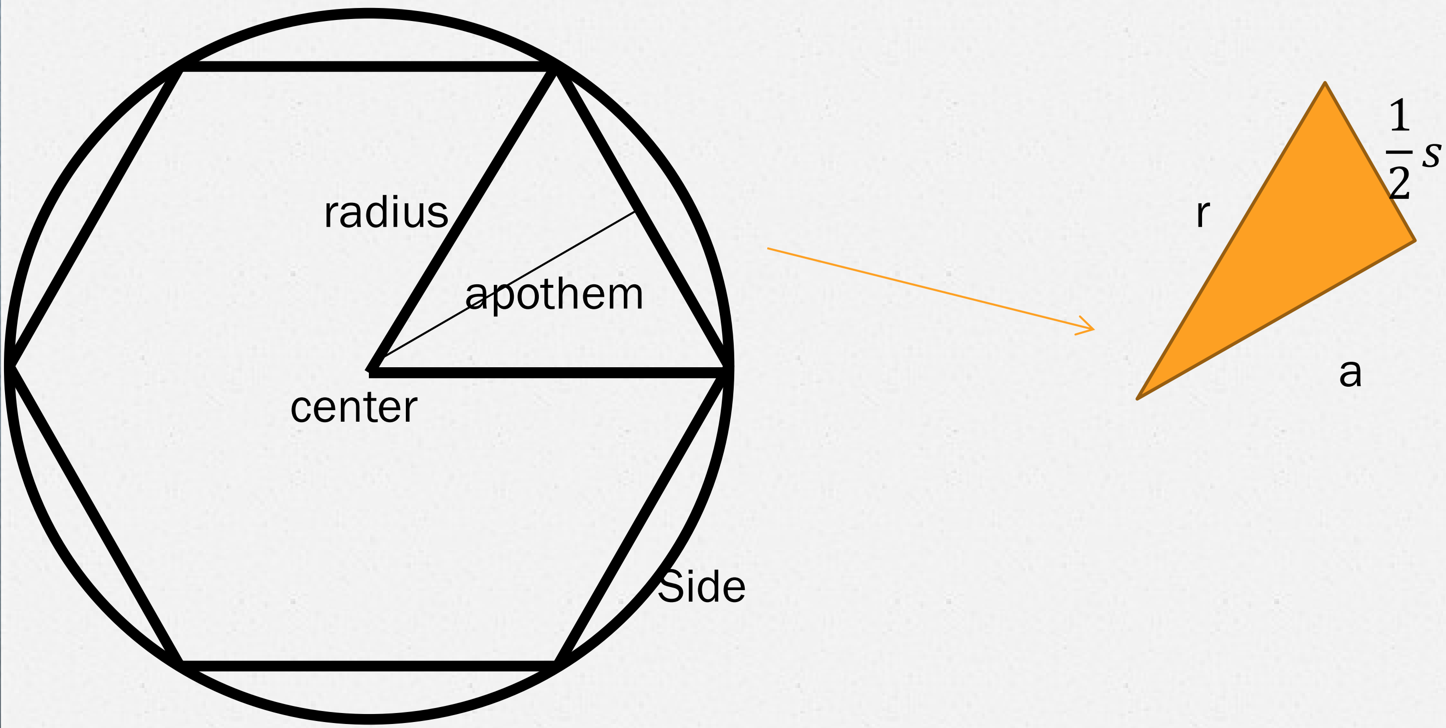
- Find the central angle for each regular polygon.





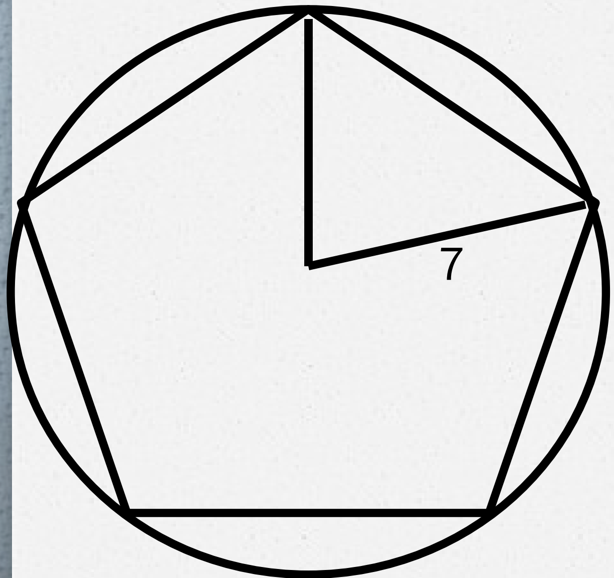


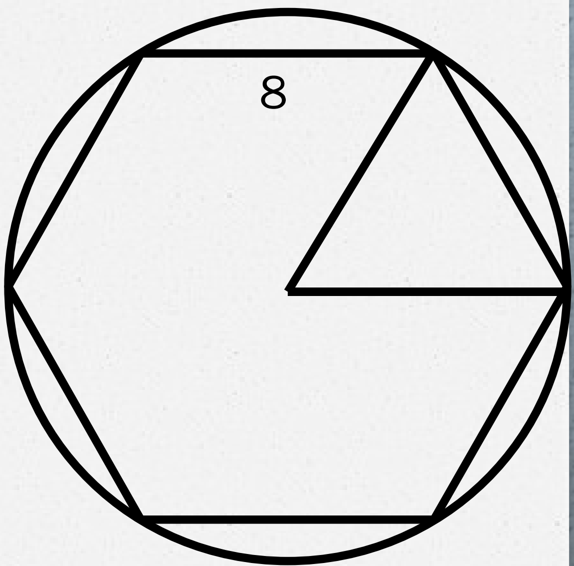
Concept 12: Sides, Apothems, and Radii



Example 2

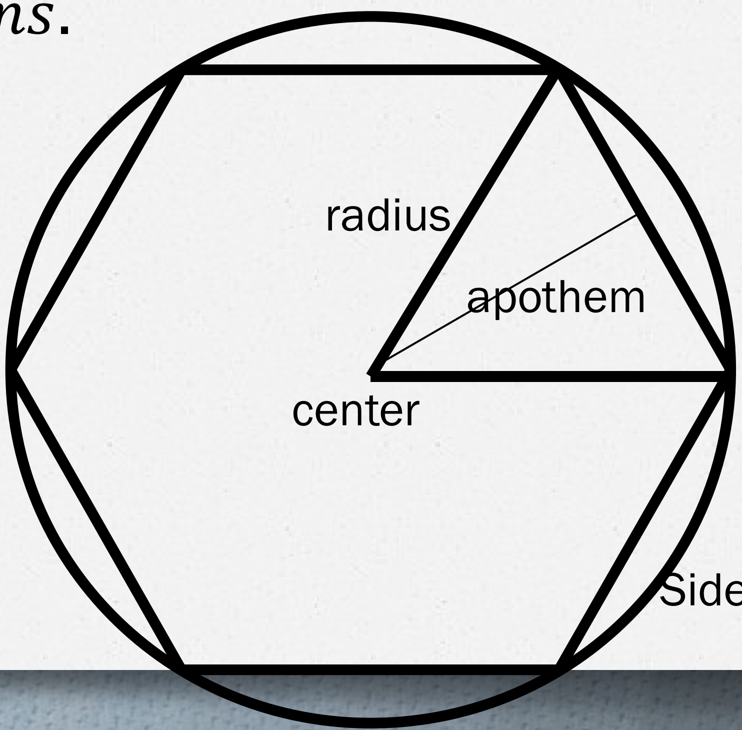
- Find the unknown radius, apothem and/or side lengths.





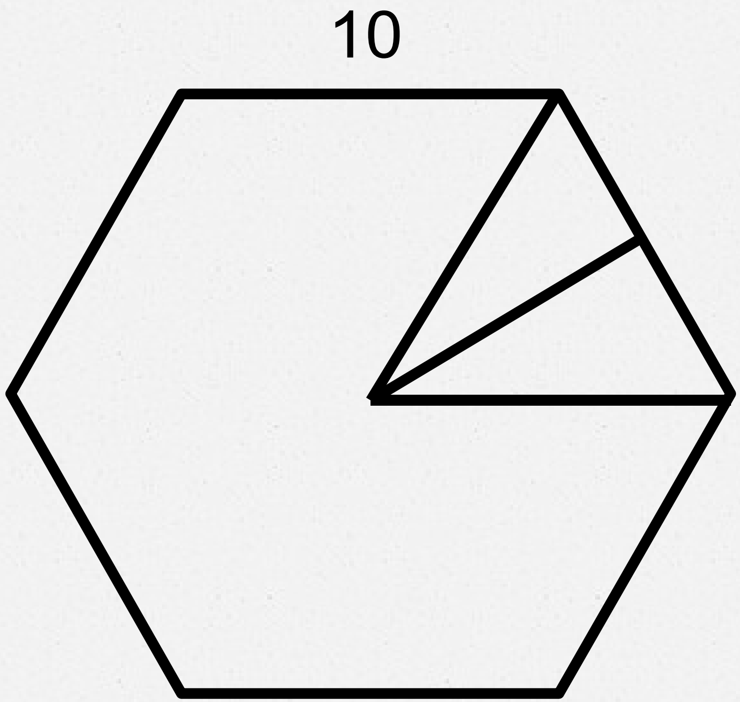
Concept 13: Area of a Regular Polygon

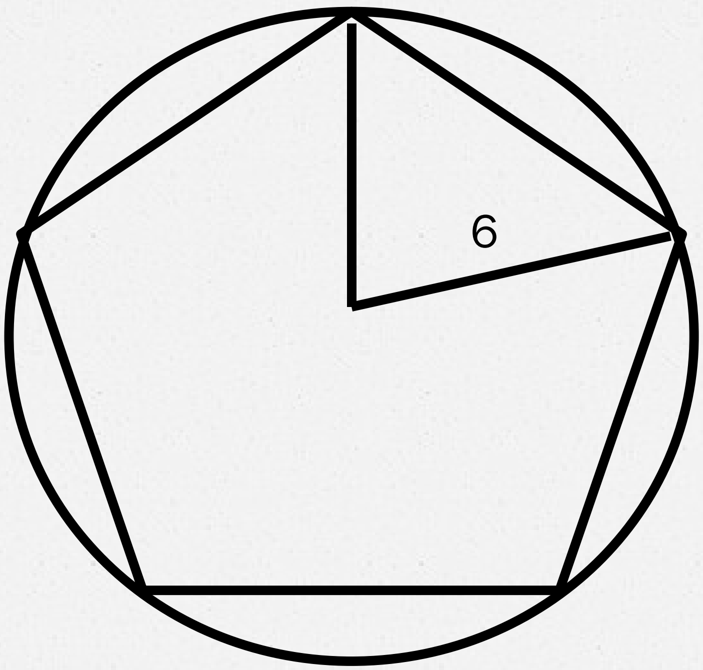
- The area of a regular n-gon with side length s is one half the product of the apothem a and the perimeter P, so $A = \frac{1}{2}aP$, or $A = \frac{1}{2}ans$.



Example 3

- Find the perimeter and area of each regular polygon.

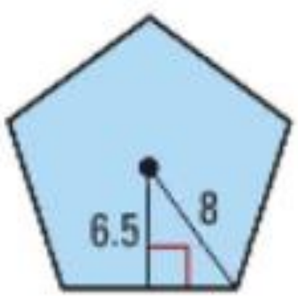




Example 4

Find the perimeter and the area of the regular polygon.

3.



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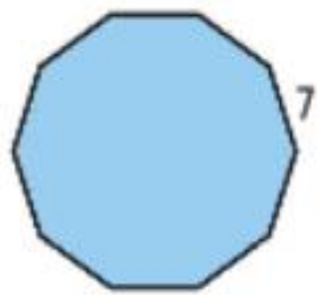

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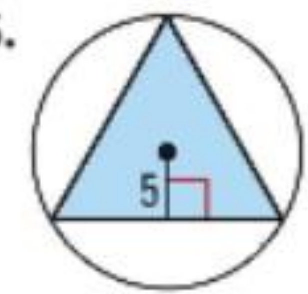


6. Which of Exercises 3–5 above can be solved using special right triangles?

CONCEPT SUMMARY *For Your Notebook*

Finding Lengths in a Regular *n*-gon

To find the area of a regular *n*-gon with radius *r*, you may need to first find the apothem ***a*** or the side length ***s***.

You can use ...	... when you know *n* and ...	... as in ...
Pythagorean Theorem: $\left(\frac{1}{2}s\right)^2 + a^2 = r^2$	Two measures: *r* and ***a***, or *r* and ***s***	Example 2 and Guided Practice Ex. 3.
Special Right Triangles	Any one measure: *r* or ***a*** or ***s*** **And** the value of *n* is 3, 4, or 6	Guided Practice Ex. 5.
Trigonometry	Any one measure: *r* or ***a*** or ***s***	Example 3 and Guided Practice Exs. 4 and 5.

Use Geometric Probability

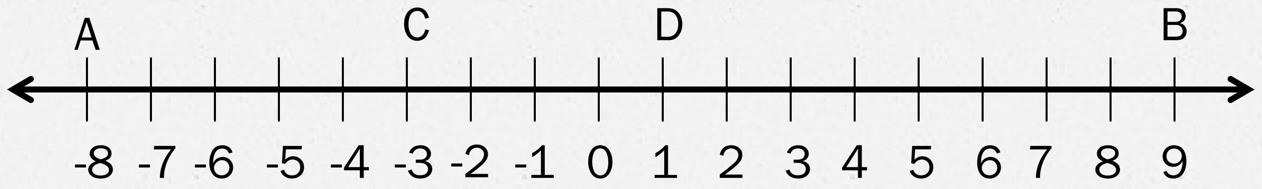
Concept 14: Probability

- The probability of an event is a measure of the likelihood that the event will occur. It is between 0 and 1. 0 represents an impossible event and 1 represents a guaranteed event.

- Geometric probability is a ratio that involves a geometric measure.

Concept 14: Probability and Length

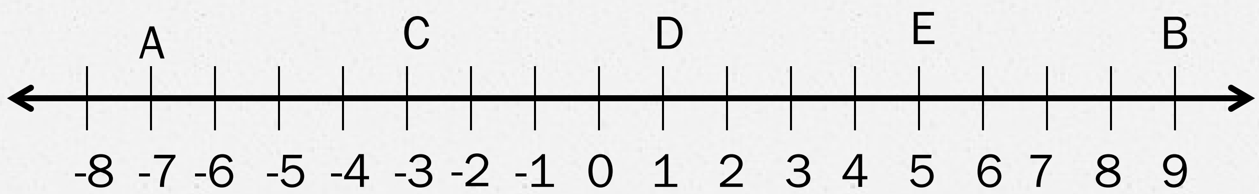
- Let $\overline{AB}$ be a segment that contains the segment $\overline{CD}$. If a point K on $\overline{AB}$ is chosen at random, then the probability that it is on $\overline{CD}$ is the ratio of the length of $\overline{CD}$ to the length of $\overline{AB}$.



Example 1

- Find the probability that a point chosen at random on _______ is on $\overline{AB}$.

1) $\overline{AC}$
2) $\overline{CE}$
3) $\overline{CB}$

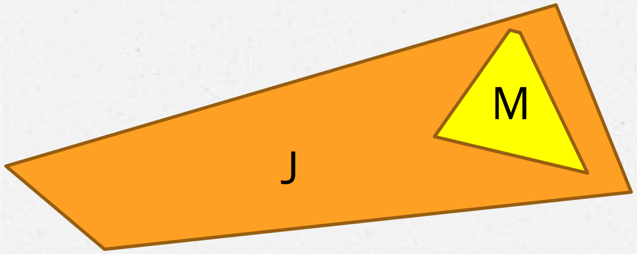


Example 2

- A monorail runs every 12 minutes. The ride from the station near your home to the station near your work takes 9 minutes. One morning, you arrive at the station near your home at 8:46. You want to get to the station near your work near your work by 8:58. What is the probability you will get there by 8:58?

Concept 15: Probability and Area

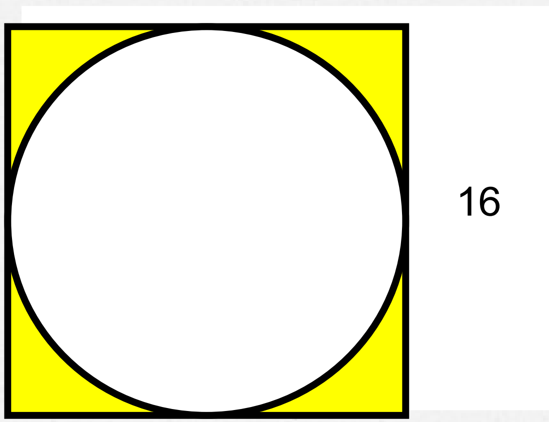
- Let J be a region that contains region M. If a point K in J is chosen at random, then the probability that it is in region M is the ratio of the area of M to the area of J.



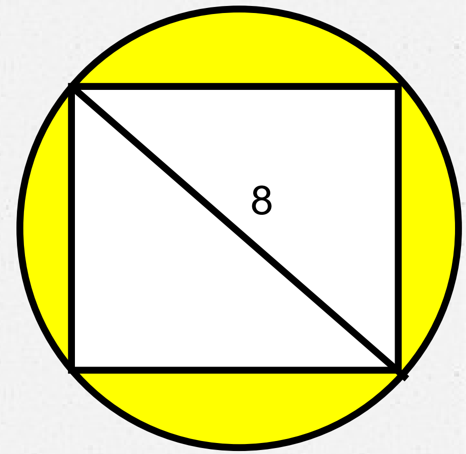
Example 3

- Find the probability that a randomly chosen point is in the yellow shaded region.

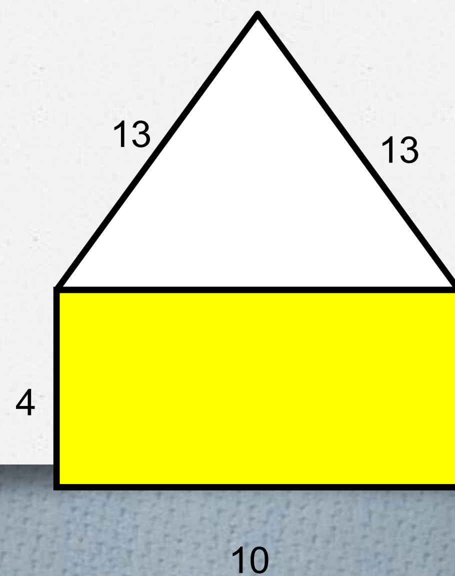
1.



2.



3.



Example 4

- The diameter of a target is 80cm. If you shot an arrow at the target, what is the likelihood that you would hit the red circle in the middle that has a diameter of 10cm?