Ridge Search for Statistical Multi-objective Optimization and Application to IC Geometric Design

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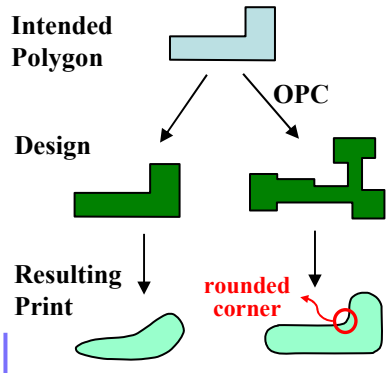
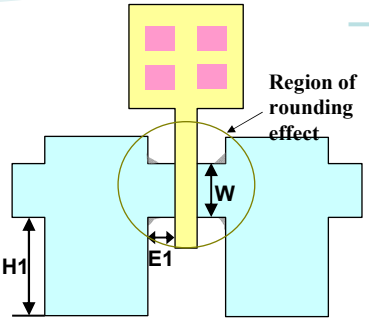
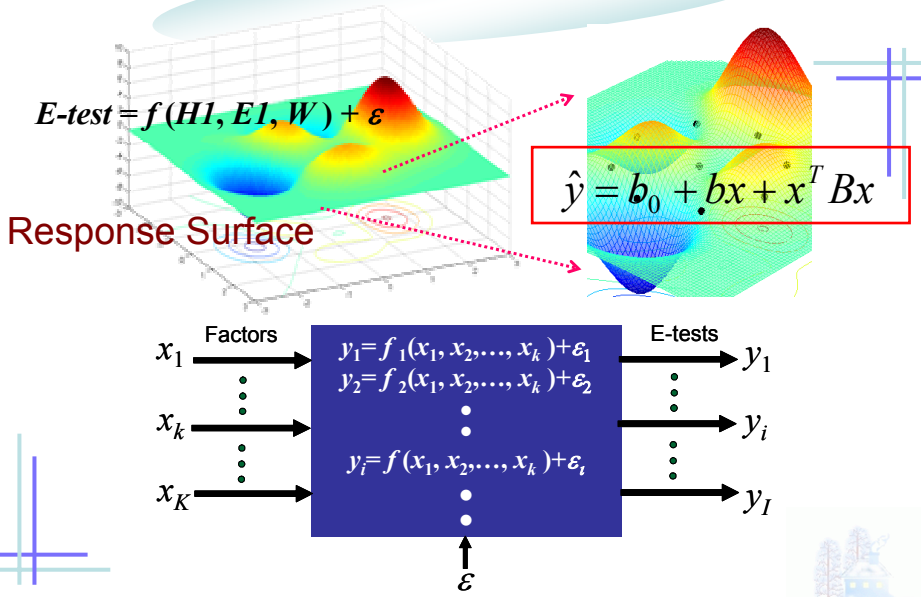
Effects of Rounded Corners on E-Tests





Design Factors	Lower bound	Upper bound
*H*1	0	0.4
*E*1	0.05	0.15
W	0.1	0.3

Empirical Model of Effects on E-Tests



Optimization Formulation and Results

- **Layout factor x_k window:** $L_k \le x_k \le U_k$

- **E-test output with target T_i:**

$$\min_{x_k} \sum_i w_i (y_i - T_i)^2 = \sum_i w_i (b_{i0} + b_i x + x^T B_i x - T_i)^2$$

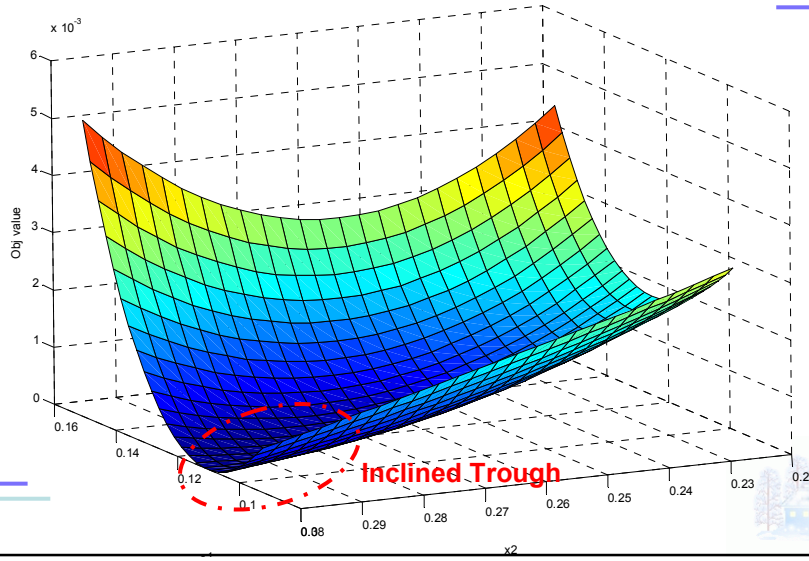
- **Minimizing sensitivity of y_i to x_k changes:**

$$\min_{x_k} \sum_{(i,k)} \left(\frac{\partial y_i}{\partial x_k} - 0\right)^2$$

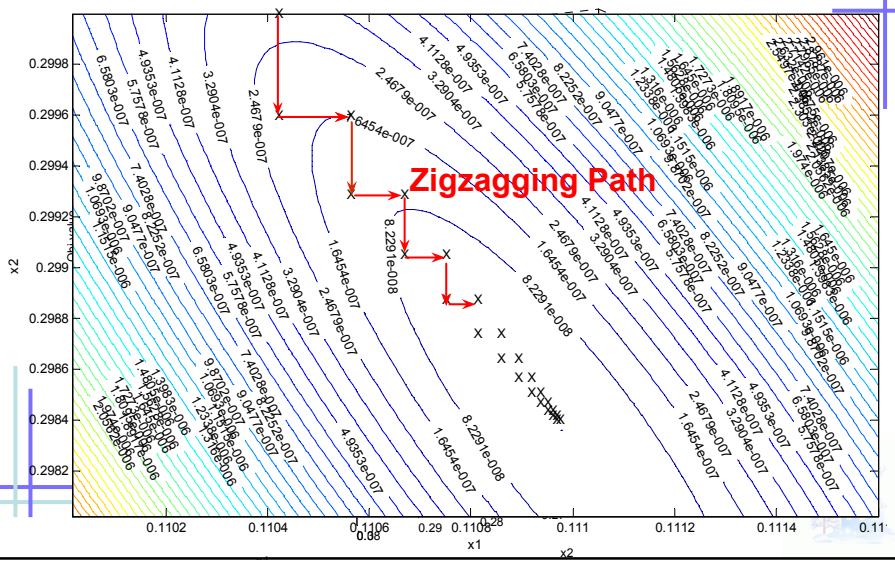
- **E-test output with spec. windows :**

$$L_j \le y_j \le U_j \Rightarrow L_j \le b_{j0} + b_j x + x^T B_j x \le U_j$$

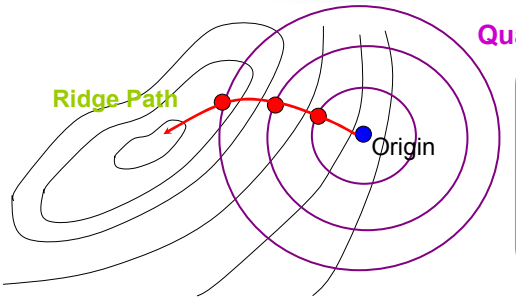
Problem of Gradient Search Method (Ex. GRG)



Zigzag of GRG Method



Ridge Analysis

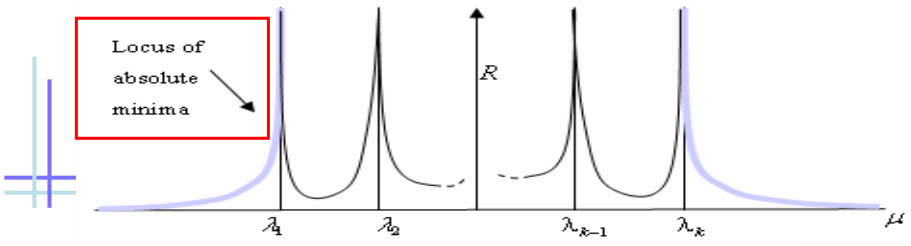


Quadratic Response Surface

Minimize $\hat{y} = b_0 + \mathbf{b}^T\mathbf{x} + \mathbf{x}^T\mathbf{B}\mathbf{x}$

subject to $\mathbf{x}^T\mathbf{x} = R^2$

$$\Rightarrow \hat{\mathbf{x}}_s = -\frac{1}{2}(\mathbf{B} - \mu\mathbf{I})^{-1}\mathbf{b}$$



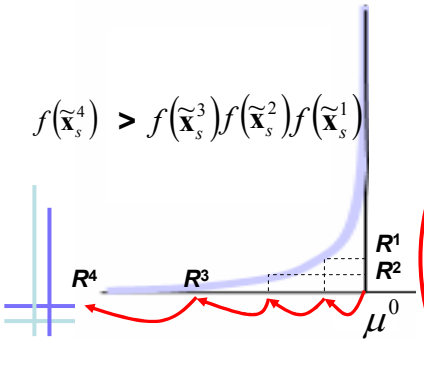
Generalized Ridge Search Method

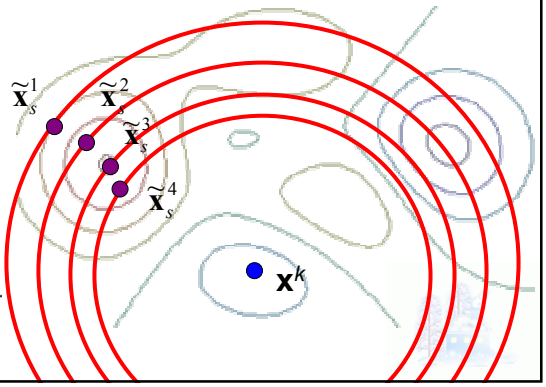
Hessian Matrix for Quadratic Approximation

$$\tilde{\mathbf{x}}_s = \mathbf{x}^k - \frac{(\frac{\mathbf{G}^k}{2} - \mu\mathbf{I})^{-1}\boldsymbol{\beta}^k}{2}$$

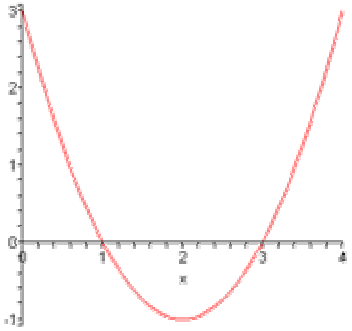
$$\mu^{(\gamma+1)} = \mu^{(\gamma)} - \Delta \times \alpha^{\gamma}$$

$$f(\tilde{\mathbf{x}}_s^4) > f(\tilde{\mathbf{x}}_s^3) f(\tilde{\mathbf{x}}_s^2) f(\tilde{\mathbf{x}}_s^1)$$

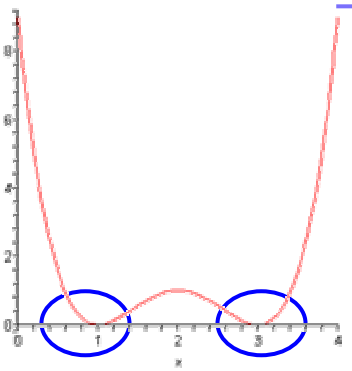




Quartic Function

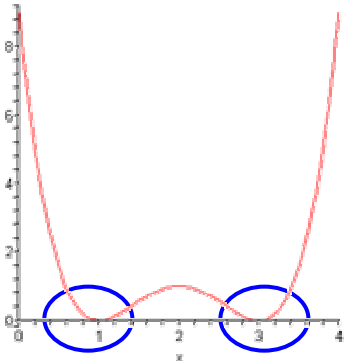


Quadratic function



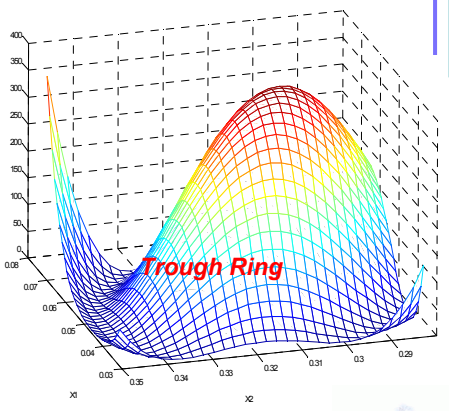
Quadratic of quadratic

Trough in Quartic Function

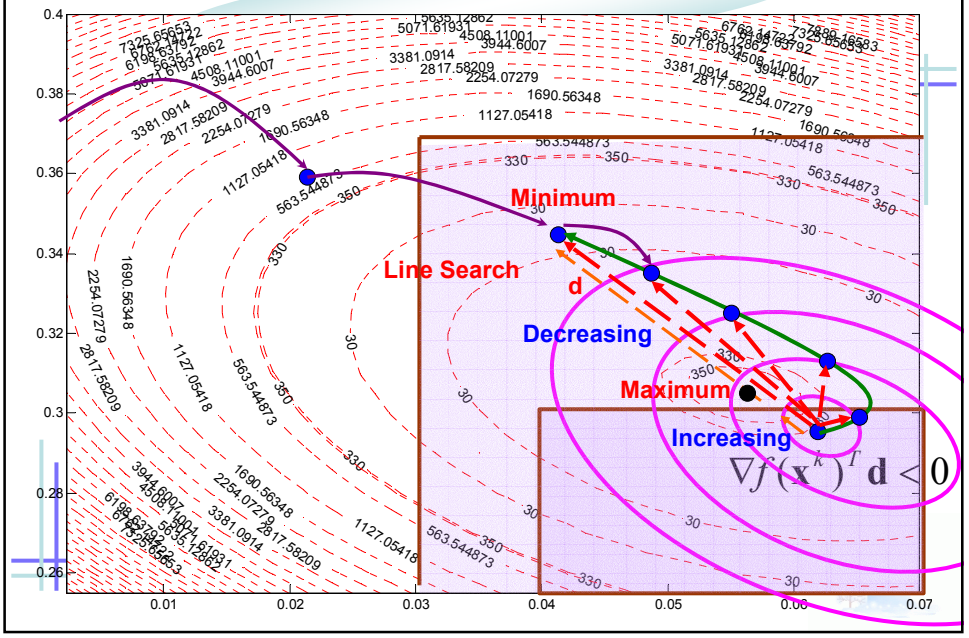


Trough

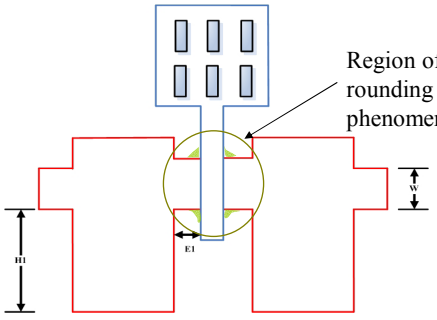
Quadratic of quadratic



Constrained Ridge Search Method



Design for Manufacturability (DFM) Case (i)



Design factors on geometric layout

Factor Name	Lower bound	Upper bound
*H*1	0	0.4
*E*1	0.05	0.15
W	0.1	0.3

Response	Desired Target T_i	Specification window	
		L_i	U_i
A1	0.5	0.4345	0.5311
A2	0.45	0.40598	0.4961
A3	-0.3	-0.3375	-0.2761
A4	-0.25	-0.2724	-0.2228
A5	0.5	0.4381	0.5354
A6	-0.31	-0.3423	-0.2801
A7	0.5	0.4425	0.5408
A8	-0.3	-0.3408	-0.2789
B1	0.47	0.4212	0.5148
C1	240	217.3421	265.6403

- *H*1 →0.4 ; *E*1 →0.05
- changes in E1 and H1 should not have much influence on the responses

Design for Manufacturability (DFM) Case (ii)

Stopping Criterion: bias = 10^{-6} or 700 iterations

3 Variables 26 constraints

2^3 initial solutions —*GRR + Zoutendijk*→ 8 feasible initial solutions

Local Search

	Best Objective Value	*Average Objective Value*	*Average Step*	*Average Computing Time*
GRG + Zoutendijk	1.559253×10^7	2.129230×10^7	619.13	2.44 seconds
GRR + Zoutendijk	1.559223×10^7	1.559223×10^7	34.63	0.51 seconds
Lingo	1.559223×10^7	1.559223×10^7	12	Less than 1 second

Global Search

	Objective Value	*Step*	*Computing Time*
GRG + Zoutendijk	1.559253×10^7	4953	19.69 seconds
GRR + Zoutendijk	1.559223×10^7	277	4.30 seconds
Lingo	1.559223×10^7	8492	2 seconds