CAMBRIDGE TECHNOLOGY IN MATHS

Year 12

Arithmetic and geometric sequences for the TI-Nspire

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Published in: ***Essential Further Mathematics 3rd edition TIN/CP Version***

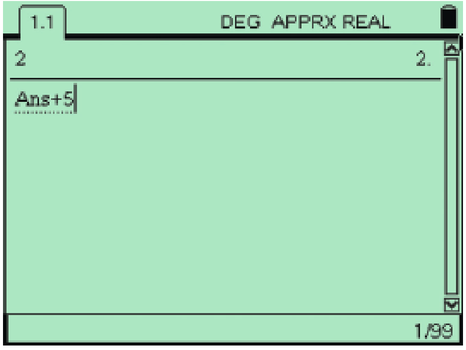
Cambridge University Press

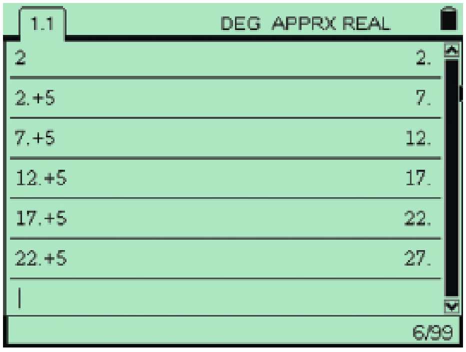
How to generate the terms of an arithmetic sequence using the TI-Nspire

Generate the first five terms of the arithmetic sequence: 2, 7, 12, 17, 22, . . .

Steps

1. Start a new document by pressing ctrl + N.
2. Select **1:Add Calculator**.
   Enter the value of the first term, **2**.
   Press enter.
3. The common difference for the sequence is 5. So, type +5. Press enter.
   The second term in the sequence, **7**, is generated.
4. Pressing enter again generates the next term, **12**.
5. Keep pressing enter until the desired number of terms is generated.





How to generate the terms of a sequence using the TI-Nspire

Generate the terms in an arithmetic sequence with $a = 10$ and $d = 4$.

Steps

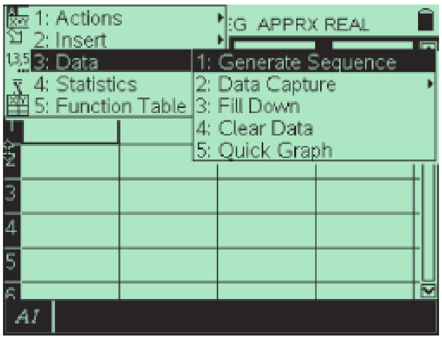
Strategy: Find an expression for the *n*th term of the sequence, as for Example 5. A graphics calculator can then be used to display the sequence in a table.

1 For this sequence, $a = 10$ and $d = 4$. — $a = 10,\ d = 4$

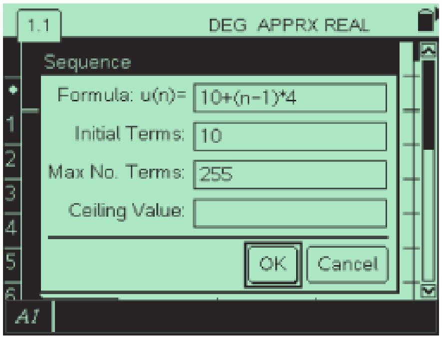
2 Use $t_n = a + (n - 1)d$ to write down an expression for the *n*th term, t_n. Don't simplify. — $t_n = 10 + (n - 1) \times 4$

3 Start a new document by pressing ctrl + N. Select **3:Add Lists & Spreadsheet**.

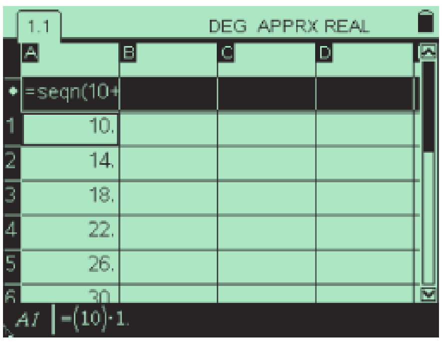
a Place the cursor in any cell in column A and press menu /**3:Data** to generate the screen opposite.



b With the cursor on **1:Generate Sequence**, press enter to display the pop-up screen shown opposite.
Type in the entries as shown. Use tab to move between entry boxes. Leave the **Max No. Terms** at 255.
Note: The calculator uses u(*n*) for the *n*th term.



c Press enter to close the pop-up screen and display the sequence of terms.
The term number can be read directly from the row number (left-hand side) of the spreadsheet. For example, the 5th term would be 26.
Use the ▼ arrow to move down through the sequence to see further terms.



Original location: Chapter 9 Examples 7-8 (p.269-273)

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Example: Finding when a term in a sequence first exceeds a given value

How many terms would we have to write down in the arithmetic sequence 10, 14, 18, 22, . . . before we found a term greater than 51?

Solution

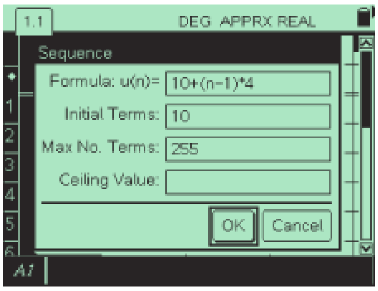
Strategy: Find an expression for the *n*th term of the sequence, as for Example 5. A graphics calculator can then be used to display the sequence in a table. The first term that exceeds 51 can then be found.

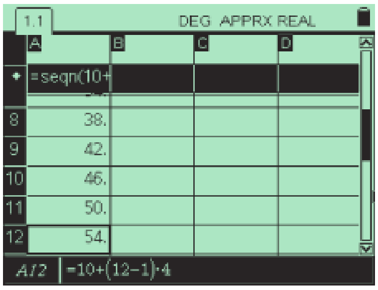
1 For this sequence, $a = 10$ and $d = 4$

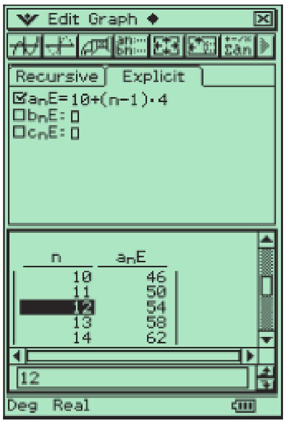
$a = 10, d = 4$

$t_n = 10 + (n - 1) \times 4$

2 Use your calculator to generate the sequence of terms and move down through this sequence of terms until you find the first term that exceeds 51; in this case, the 12th term.







3 Write down key values in the sequence (to show how you solved the problem) and your answer.

n	1	2	. . .	10	11	12	. . .
t_n	10	14	. . .	46	50	54	. . .

The first term to exceed 51 is t_{12}.

Example: Application of the *n*th term of an arithmetic sequence

Before starting on a weight-loss program a man weighs 124 kg. He plans to lose weight at a rate of 1.5 kg a week until he reaches his recommended weight of 94 kg.

a Write down a rule for the man's weight, W_n, at the start of week n.

b If he keeps to his plan, how many weeks will it take the man to reach his target weight of 94 kg?

Solution

Strategy: You need to recognise that by losing a constant amount of weight each week, the man's weekly weight follows an arithmetic sequence. Using this information, you can write down an expression for his weight in the nth week. You can then use this expression to display the sequence of weights in a table and hence determine when the target weight is reached.

1 Arithmetic sequence with $a = 124$ and $d = -1.5$. Use the rule $W_n = a + (n - 1)d$ to write down an expression for W_n.

Arithmetic sequence

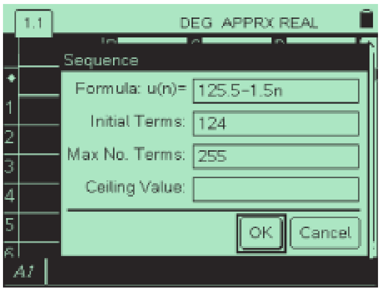
$a = 124,\ d = -1.5$

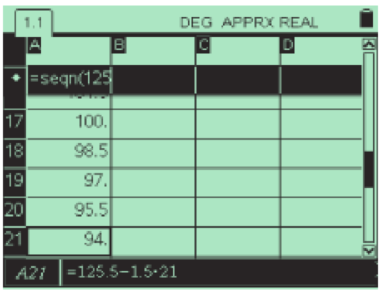
$W_n = 124 + (n - 1) \times (-1.5)$

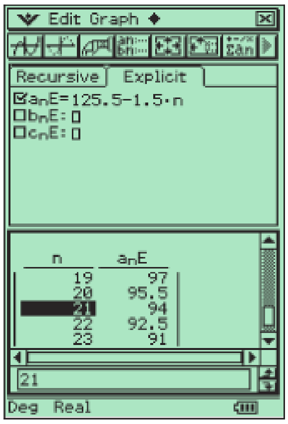
$= 124 - 1.5n + 1.5$

$\therefore W_n = 125.5 - 1.5n$

2 Use your calculator to generate the sequence of terms and move down through this sequence of terms until you find the first term that is 94 or less; in this case, the 21st term.







3 Write down key values in the sequence (to show how you solved the problem) and your answer.

n	1	2	…	19	20	21	…
W_n	124	122.5	…	97	95.5	94	…

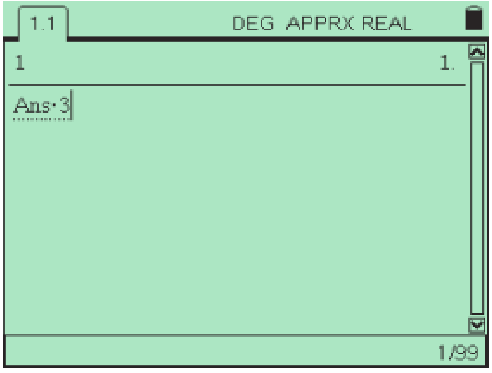
If the man keeps to his plan, he will reach his target weight by the start of week 21, or after 20 weeks of being on the program.

How to generate the terms of a geometric sequence using the TI-Nspire

Generate the first five terms of the geometric sequence 1, 3, 9, 27, . . .

Steps

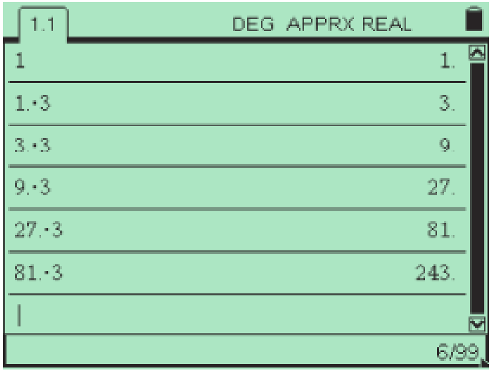
1 Start a new document by pressing ctrl + N.



2 Select **1:Add Calculator**.
Enter the value of the first term, **1**.
Press enter.

3 The common ratio for the sequence is 3. So, type × **3**. Press enter. The second term in the sequence, 3, is generated.

4 Pressing enter again generates the next term, 9.



5 Keep pressing enter until the desired number of terms is generated.

How to graph the terms of a sequence using the TI-Nspire

Plot the terms of the following sequences on the same graph:

- sequence 1: arithmetic with $a = 2$ and $d = 2$
- sequence 2: geometric with $a = 2$ and $r = 2$

for $n = 1, 2, \ldots 6$. These are the sequences plotted previously.

Steps

1 Write an expression for the nth term of the two sequences using the rules.

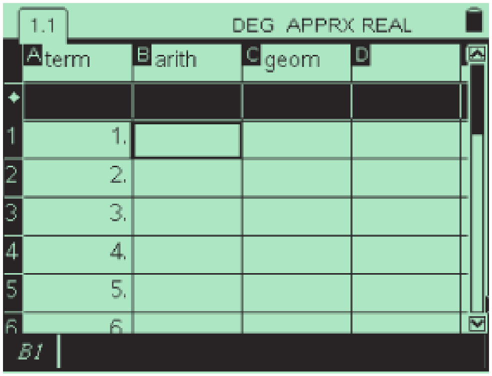
arithmetic: $t_n = 2 + (n - 1) \times 2$
geometric: $t_n = 2 \times 2^{(n-1)}$

2 Start a new document by pressing (ctrl) + (N).

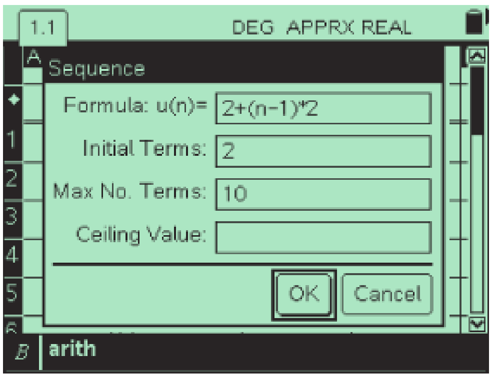
3 Select **3:Add Lists & Spreadsheet**.

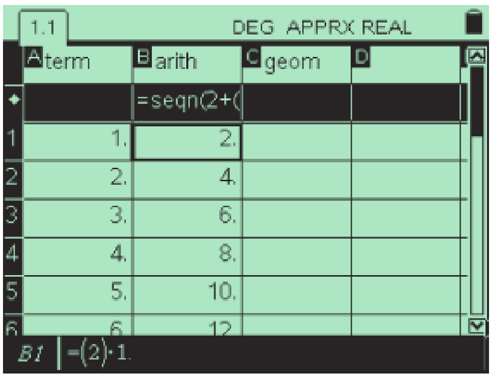
a Enter the numbers **1** to **6** into a list named *term*, as shown.
Note: You can also use the sequence command to do this.

b Name column B *arith* and column C *geom*.

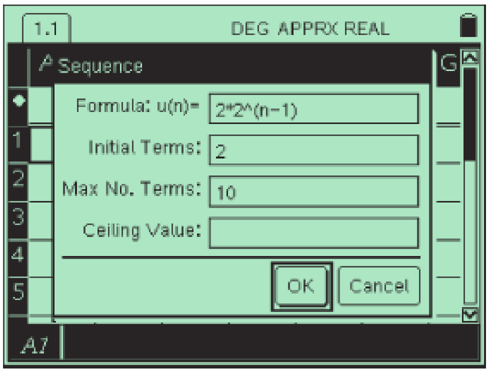


4 a Place the cursor in any cell in column B. Press (menu) **/3:Data/1:Generate Sequence** and type in the entries as shown (below left). Use (tab) to move between entry boxes. Press (enter) to close the pop-up screen and display the values of the first six terms in the arithmetic sequence in column B (below right).





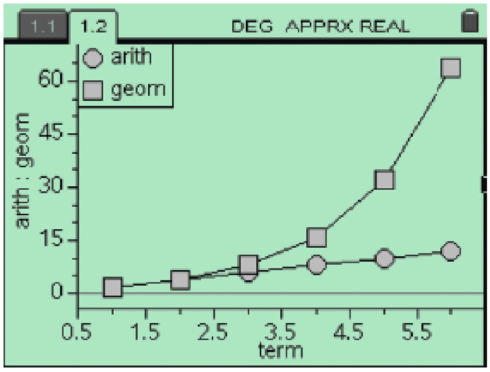
b Place the cursor in any cell in column C. Press menu /**3:Data/1:Generate Sequence** and type in the entries shown (below left). Press enter to close the pop-up screen and display the values of the first six terms in the geometric sequence in column C (below right).



5 Press home and select **5:Data & Statistics**.

a Construct a scatterplot using *term* as the independent variable and *arith* as the dependent variable.

b Press menu /**2:Plot Properties/6:Add Y Variable**. Select the variable *geom*. Press enter. This will show the terms of the geometric sequence on the same plot.



Note: The arithmetic sequence increases in a linear manner, whereas the geometric sequence increases in an exponential manner.

Questions on graphing the terms of a sequence using the TI-Nspire

Using a graphics calculator, on the same axes plot the first five terms of the following pairs of sequences. In each case, comment on the differences that you notice in the two plots.

Note: You will need to readjust the window settings as you move through the exercises.

1 a arithmetic sequence: $a = 32$ and $d = 0.5$ geometric sequence: $a = 32$ and $r = 0.5$

b arithmetic sequence: $a = 1$ and $d = 2$ geometric sequence: $a = 1$ and $r = 2$

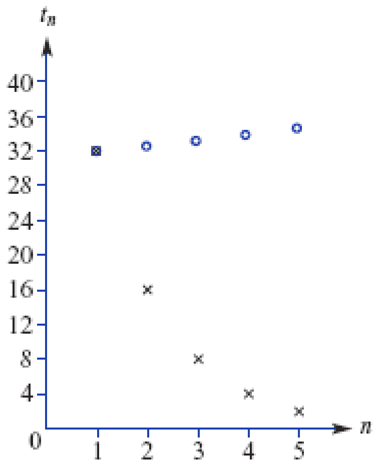
c arithmetic sequence: $a = 100$ and $d = -5$ geometric sequence: $a = 100$ and $r = 0.5$

d arithmetic sequence: $a = 100$ and $d = 5$ geometric sequence: $a = 1$ and $r = 5$

Answers

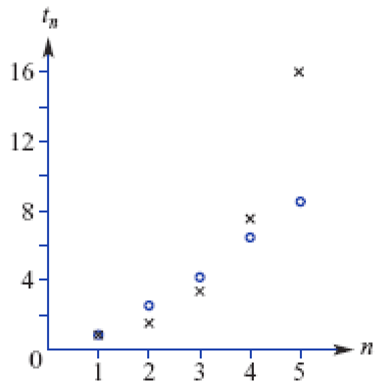
Graphing questions

1 a



The arithmetic sequence is linear and increasing.
The geometric sequence is exponential and decreasing rapidly.

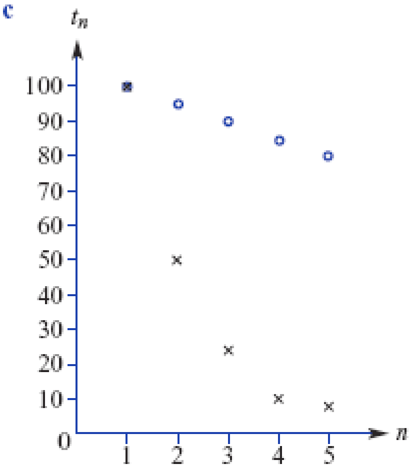
b



The arithmetic sequence is linear and increasing.
The geometric sequence is exponential and increasing.

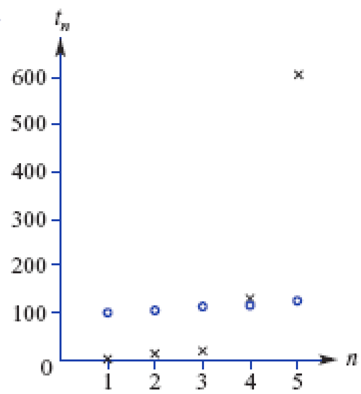
The geometric sequence falls behind the arithmetic sequence but quickly passes it.

c



The arithmetic sequence is linear and decreasing.
The geometric sequence is exponential and decreasing rapidly.

d



The arithmetic sequence is linear and increasing.
The geometric sequence is exponential and increasing.
The terms in the geometric sequence are initially smaller than those of the arithmetic sequence, but soon exceed them in size.