Arithmetic and Geometric Sequences

A sequence is a list of numbers or objects, called terms, in a certain order. In an **arithmetic sequence**, the difference between one term and the next is always the same. This difference is called a **common difference**. The common difference is added to each term to get the next term.

2, 5, 8, 11, 14, …

This is an increasing arithmetic sequence with a common difference of 3.

32, 26, 20, 14, 8, …

This is a decreasing arithmetic sequence with a common difference of –6.

Example: What are the next three terms in the sequence?

1, 5, 9, 13, … I can see that this is an arithmetic sequence with a common difference of 4. To get the next three terms, add 4 to 13 which equals 17, the next term in the sequence. Then add 4 to 17 to get the next term to get 21, etc. So the next three terms are 17, 21, and 25.

Use the following formula to find any term of an arithmetic sequence.

$$a_n = a_1 + (n-1)d$$

a_n = the term in the sequence you are trying to find (n represents the desired term number)
a_1 = the first term in the sequence
d = the common difference

Example: What is the 10^{th} term of the following sequence?

1, 5, 9, 13, …

$$a_{10} = 1 + (10-1)4 = 1 + 9 \cdot 4 = 1 + 36 = 37$$

So the 10^{th} term of this sequence is 37.

Example: What is the 12^{th} term of the following sequence?

34, 31, 28, 25, 22, …

$$a_{12} = 34 + (12-1)(-3) = 34 + 11(-3) = 34 + (-33) = 1$$

The 12^{th} term of this sequence is 1.

Practice:

1. Find the next three terms: 3, 10, 17, 24, 31, _____, _____, _____
2. Find the 25th term: 53, 50, 47, 44, 41, … _____
3. Find the 20th term: 25, 40, 55, 70, 85, … _____
4. Find the 75th term: 88, 81, 74, 67, 60, … _____

A **geometric sequence** is a sequence of numbers where the ratio of consecutive terms is constant. This ratio is called the **common ratio** (r). Sometimes the terms of a geometric sequence get so large that you may need to express the terms in scientific notation rounded to the nearest tenth.

2, 6, 18, 54, … This is an increasing geometric sequence with a common ratio of 3.

1,000, 200, 40, 8, … This is a decreasing geometric sequence with a common ratio or 0.2 or $\frac{1}{5}$.

Example: What are the next three terms of the following sequence?

4, 20, 100, 500, …

$500 \cdot 5 = 2500$

$2500 \cdot 5 = 12{,}500$ The next three terms are 2,500, 12,500, and 62,500.

$12{,}500 \cdot 5 = 62{,}500$

Explicit sequences also have a formula for finding any term in a sequence.

$$a_n = a_1 r^{(n-1)}$$

a_n = the term in the sequence you are trying to find (n represents the desired term number)
a_1 = the first term in the sequence
r = the common ratio

Example: Find the 7th term in the following sequence: 6, 18, 54, 162, …

Finding the common ratio can be harder than finding the common difference. One way to find it is the divide each term by the term before it.

$18 \div 6 = 3$, $54 \div 18 = 3$, $162 \div 54 = 3$ So the common ratio is 3.

$a_7 = 6 \cdot 3^{(7-1)} = 6 \cdot 3^6 = 6 \cdot 729 = 4{,}374$ So the 7th term of the sequence is 4, 374.

Example: Find the 8th term in the following sequence: 96, 48, 24, 12, 6, …

To find the common ratio, divide each term by the one before it.

$48 \div 96 = \frac{1}{2}$, $24 \div 48 = \frac{1}{2}$, $12 \div 24 = \frac{1}{2}$ The common ratio is $\frac{1}{2}$.

$a_8 = 96 \cdot \frac{1}{2}^{(8-1)} = 96 \cdot \frac{1}{2}^{7} = 96 \cdot \frac{1}{128} = 0.75$ The 8th term of the sequence is 0.75.

Practice:

1. Find the next three terms: 128, 64, 32, 16, 8, _____, _____, _____ .
2. Find the 9th term: 0.01, 0.1, 1, 10, 100, … _____
3. Find the 7th term: 1, 6, 36, 216, 1,296, … _____
4. Find the 11th term: 1, –2, 4, –8, 16, … _____

Arithmetic and Geometric Sequences

Determine if the sequence is arithmetic or geometric, and then find the next three terms.

1. -2, -4, -8, -16, _____, _____, _____

2. 65, 60, 55, 50, 45, _____, _____, _____

3. 8, 13, 18, 23, 28, _____, _____, _____

4. 1, 1.5, 2.25, 3.375, _____, _____, _____

Determine if the sequence is arithmetic or geometric, and then find the given term.

5. 11th term: 5, 3, 1, –1, … _____
6. 23rd term: 0.1, 0.15, 0.2, 0.25, … _____
7. 6th term: 25, 75, 225, 675, … _____
8. 22nd term: –2, –5, –8, –11, –14, … _____
9. 10th term: $a_1 = 320$, $r = 0.5$
10. 50th term: –9, 2, 13, 24, 35, … _____
11. Mariano received a bonus of $50 for working the day after Thanksgiving, plus his regular wage of $9.45 an hour. If his total wages for the day were $135.05, how many hours did he work?
12. Heather makes $6.50 per hour. Every three months, she is eligible for a 2% raise. How much will she make after 2 years if she gets a raise every time she is eligible?

Date: _____

Arithmetic and Geometric Sequences
Worksheet

Arithmetic Sequence - is a sequence of terms that have a common _____ between them.

General Term:

$a_n =$

Geometric Sequence - is a sequence of terms that have a common _____ between them.

General Term:

$a_n =$

1. Are the following sequences arithmetic, geometric, or neither? If they are arithmetic, state the value of d. If they are geometric, state r.

   a) 6, 12, 18, 24, ... _____

   b) 6, 11, 17, ... _____

   c) 2, 14, 98, 686, ... _____

   d) 160, 80, 40, 20, ... _____

   e) -40, -25, -10, 5, ... _____

   f) 7, -21, 63, -189, ... _____

2. For the following arithmetic sequences, find a and d and state the formula for the general term. Don't forget to simplify!

a) - 10, - 4, 2, 8, 14, ...

b) 10, 8, 6, 4, ...

c) 36, 31, 25, 21, ...

3. Use your formula from question 2c to find the values of a_7 and a_{20}.

4. For the following geometric sequences, find a and r and state the formula for the general term.

a) 1, 3, 9, 27, ... b) 12, 6, 3, 1.5, ... c) 9, -3, 1, ...

5. Use your formula from question 4c) to find the values of the a_4 and a_{12}

6. Find the number of terms in the following arithmetic sequences. Hint: you will need to find the formula for t_n first!

a) 2, 5, 8, ... , 299 b) 9, 5, 1, ... - 251.

Answers:

1a) arithmetic d = 6 b) neither c) geometric r = 7 d) geometric r = 0.5 or r = ½ e) arithmetic d = 15 f) geometric r = -3 2a) a = -10; d=6; t_n =6n-16 b) a = 10; d=-2; t_n=-2n+12 c) a = 36; d=-5; t_n= - 5n+41 3. t_7=6; t_{20} = -59 4. a) a = 1; r = 3; $t_n = 1(3)^{n-1}$ b) a = 12; r = $\frac{1}{2}$; $t_n = 12\left(\frac{1}{2}\right)^{n-1}$ c) a = 9; r = - 3; $t_n = 9(-3)^{n-1}$ 5. t_4 = -243 t_{12} = -177147 6. a) t_n = 3n-1; n = 100 b) t_n = -4n+13; n=66

Arithmetic and Geometric Series – Worksheet

General formula for an arithmetic series:

General formula for a geometric series:

1) Find the designated sum of the arithmetic series

a) S_{14} of $3+7+11+15+\cdots$

b) S_{11} of $-13-11-9-7-\cdots$

c) S_9 of $22+20+18+16+\cdots$

d) S_{35} of $-2-5-8-11-\cdots$

2) Determine the sum of each arithmetic series

a) $6+13+20+\cdots+69$

b) $4+15+26+\cdots+213$

c) $5-8-21-\cdots-190$

d) $100+90+80+\cdots-100$

3) Find the designated sum of the geometric series

a) S_7 of $4+8+16+32+\cdots$

b) S_{13} of $1-6+36-216+\cdots$

c) S_{17} of $486+162+54+18+\cdots$

d) S_6 of $3+15+75+375+\cdots$

4) Determine S_n for each geometric series

a) $a=6,\ r=2,\ n=9$

b) $f(1)=2,\ r=-2,\ n=12$

c) $f(1)=729,\ r=-3,\ n=15$

d) $f(1)=2700,\ r=10,\ n=8$

5) If the first term of an arithmetic series is 2, the last term is 20, and the increase constant is +2 …

a) Determine the number of terms in the series

b) Determine the sum of all the terms in the series

6) A geometric series has a sum of 1365. Each term increases by a factor of 4. If there are 6 terms, find the value of the first term.

Answers

1) a) 406 **b)** -33 **c)** 126 **d)** -1855

2) a) 375 **b)** 2170 **c)** -1480 **d)** 0

3) a) 508 **b)** 1 865 **c)** 3 172 **d)** 11

4) a) 3066 **b)** -2730 **c)** 2 615 088 483 **d)** $2.999\,999\,97 \times 10^{!"}$

5) a) $n = 10$ **b)** $S_{!"} = 110$

6) $t_{!} = 1$