1. Determine if each sequence below is *arithmetic*, *geometric* or *neither*. If the sequence is arithmetic, state the common difference. If the sequence is geometric, state the common ratio.

a) $\frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \ldots$

Geometric

$r = \frac{1}{3}$

b) -4, -16, -52, …

Neither

c) -19.2, -13.9, -8.6, …

Arithmetic

d = 5.3

2. Write an explicit formula to find the *n*th term.

Arithmetic Explicit Formula: $a_n = a_1 + d(n - 1)$
Geometric Explicit Formula: $a_n = a_1 \bullet r^{n-1}$

a) 5, 8, 11, 14, …

Arithmetic $a_1 = 5$ $d = 3$

$a_n = 5 + 3(n - 1)$

or

$a_n = 2 + 3n$

Simplified Explicit

$5 + 3(n - 1)$

$5 + 3n - 3$

$2 + 3n$

b) $a_3 = 54$ and $r = 3$

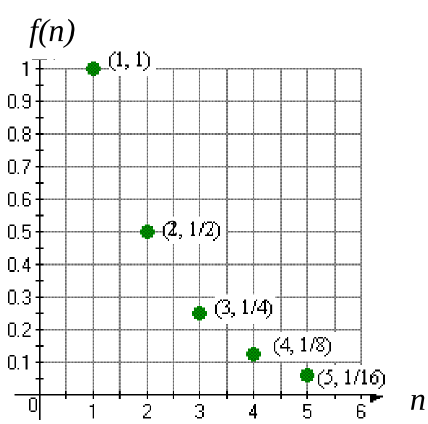
Geometric $a_1 = 6$ $r = 3$

Work backwards by dividing by 3 to find the first term of the sequence

$a_2 = 54 \div 3 = 18$ $a_1 = 18 \div 3 = 6$

$a_n = 6(3)^{n-1}$

c)

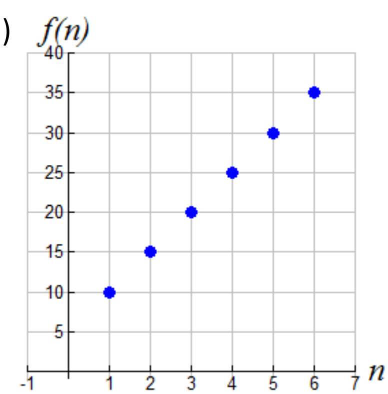


n	f(n)
1	1
2	$\frac{1}{2}$
3	$\frac{1}{4}$
4	$\frac{1}{8}$
5	$\frac{1}{16}$

Geometric
$f(1) = 1$ $r = ½$

$f(n) = 1(½)^{n-1}$

d)



n	f(n)
1	10
2	15
3	20
4	25
5	30
6	35

Arithmetic
$f(1) = 10$ $d = 5$

$f(n) = 10 + 5(n - 1)$

or

$f(n) = 5 + 5n$

3. Write an explicit rule for an arithmetic sequence if $a_1 = 7$ and $a_{14} = 85$. Using your rule, find the 25th term.

n	a_n
1	7
14	85

Arithmetic sequences are linear functions. The common difference is the same value as the constant rate of change (slope).

(1, 7) (14, 85)

$$\frac{\Delta y}{\Delta x} = \frac{85-7}{14-1} = \frac{78}{13} = 6$$

$a_1 = 7 \quad d = 6$

$\mathbf{a_n = 7 + 6(n - 1)}$

or

$\mathbf{a_n = 1 + 6n}$

$a_{25} = 7 + 6(25 - 1)$

$a_{25} = 7 + 6(24)$

$a_{25} = 151$

The 25th term is 151

4. Raymond works at a restaurant. He turns on the faucet to fill the sink with water so he can wash pots and pans. After one minute, there are 2.75 gallons of water in the sink. After two minutes, there are 5.5 gallons of water in the sink. After three minutes, there are 8.25 gallons of water in the sink.

a) If this pattern continues, write an explicit rule that can be used to find the number of gallons of water in the sink after *n* minutes.

n Minutes	1	2	3	4	5
a_n Gallons of H_2O	**2.75**	**5.5**	**8.25**	**11**	**13.75**

Explicit Rule: $a_n = 2.75 + 2.75(n - 1)$ **or**

Simplified Explicit

$a_n = 2.75 + 2.75(n - 1)$

$a_n = 2.75 + 2.75n - 2.75$

$\mathbf{a_n = 2.75n}$

b) How many gallons of water are in the sink after 6 minutes?

6 minutes $\rightarrow$ n = 6

$\mathbf{a_n = 2.75 + 2.75(n - 1)}$

$\mathbf{a_6 = 2.75 + 2.75(6 - 1)}$

$\mathbf{a_6 = 2.75 + 2.75(5)}$

$\mathbf{a_6 = 2.75 + 13.75}$

$\mathbf{a_6 = 16.5}$

After 6 minutes, there are 16.5 gallons of water in the sink.

Using a Table of Values

Enter explicit rule into y_1

2nd Graph to view table

n	a_n
5	13.75
6	16.5
7	19.25

c) When he turns off the water, there are 52 gallons of water in the sink. To the *nearest tenth* of a minute, determine how long he let the water run.

52 gallons of water $\rightarrow$ $a_n = 52$

$\mathbf{a_n = 2.75 + 2.75(n - 1)}$

$\mathbf{52 = 2.75 + 2.75(n - 1)}$

$\mathbf{52 = 2.75 + 2.75n - 2.75}$

$\mathbf{\frac{52}{2.75} = \frac{2.75n}{2.75}}$

18.9090… = n

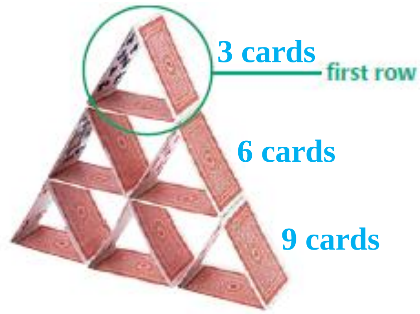
Raymond let the water run for about 18.9 minutes.

5. You are making a house of cards similar to the one shown.

a) Write an explicit rule that can be used to find the number of cards in the *nth* row.

n (row)	1	2	3
a_n (cards)	3	6	9

arithmetic → $a_1 = 3$ and $d = 3$



$a_n = 3 + 3(n - 1)$

b) What is the number of cards in the 14th row?

$a_n = 3 + 3(n - 1)$ **14th row → n = 14**

$a_{14} = 3 + 3(14 - 1)$
$= 3 + 3(13)$
$= 3 + 39$
$= 42$

There are 42 cards in the 14th row.

c) If there are 54 cards in the last row, how many rows were used to make the house of cards?

$a_n = 3 + 3(n - 1)$ **54 cards → $a_n = 54$**

$54 = 3 + 3(n - 1)$
$54 = 3 + 3n - 3$
$\frac{54}{3} = \frac{3n}{3}$
$18 = n$ **There were 18 rows.**

2nd Method to Solve for n
$54 = 3 + 3(n - 1)$
-3 -3
$\frac{51}{3} = \frac{3(n - 1)}{3}$
$17 = n - 1$
+1 +1
$18 = n$

6. Bacteria can multiply at an alarming rate. Each bacteria splits into two new bacteria each hour.

a) Create a table of values that represent the first four terms of the sequence described in the above scenario.

Geometric Sequence $r = 2$

Hours n	Number of Bacteria f(n)
1	1
2	2
3	4
4	8

b) Write an explicit rule to represent the sequence.

$f(1) = 1$ and $r = 2$ **$f(n) = 1(2)^{n-1}$**

c) If we start with one bacteria in the first hour, how many bacteria will we have by the end of one day?

There are 24 hours in one day.

$f(24) = 1(2)^{24-1}$
$f(24) = 1(2)^{23}$
$f(24) = 8,388,608$ **In one day, there will be 8,388,608 bacteria.**

Enter explicit rule into y_1
2nd Graph to view table

n	f(n)
9	256
10	512
11	1024

d) How many hours does it take to produce 512 bacteria?

$f(n) = 512$ **In 10 hours, there will be 512 bacteria.**

7. Write a recursive rule for each of the following sequences.

a) -7, -3, 1, 5…

Arithmetic: +4

$a_n = a_{n-1} + 4$ ← *The nth term is found by adding four to the previous term.*

$a_1 = -7$ ← *Always state the first term of the sequence when writing a recursive rule.*

b) $\frac{1}{2}$, -1, 2, -4…

Geometric: x -2

$a_n = a_{n-1} \bullet -2$ ← *The nth term is found by multiplying the previous term by negative two.*

$a_1 = \frac{1}{2}$ ← *Always state the first term of the sequence when writing a recursive rule.*

$a_n = -2a_{n-1}$; $a_1 = \frac{1}{2}$ ← *An equivalent way to write the recursive rule above.*

8. If $a_n = 5a_{n-1} - 2$ and $a_1 = -1$ then find a_4.

Recursive Rule: **$a_n = 5\,a_{n-1} - 2$; $a_1 = -1$**

Translation: The nth term of the sequence = 5 times the previous term minus 2

The first term of the sequence = -1

$a_2 = 5(a_1) - 2$	**$a_3 = 5(a_2) - 2$**	**$a_4 = 5(a_3) - 2$**	***Short Notation***
$a_2 = 5(-1) - 2$	**$a_3 = 5(-7) - 2$**	**$a_4 = 5(-37) - 2$**	$5(-1) - 2 = $ **-7**
$a_2 = -5 - 2$	**$a_3 = -35 - 2$**	**$a_4 = -185 - 2$**	$5(-7) - 2 = $ **-37**
$a_2 = -7$	**$a_3 = -37$**	**$a_4 = -187$**	$5(-37) - 2 = $ **-187**
$a_2 = -7$	**$a_3 = -37$**		

-1, -7, -37, -187

The fourth term of the sequence is -187

9. If $f(1) = 10$ and $f(n) = -3f(n - 1) + 1$ then find $f(5)$.

Recursive Rule: **$f(n) = -3\,f(n - 1) + 1$; $f(1) = 10$**

Translation: The nth term of the sequence = -3 times the previous term plus 1

The first term of the sequence = 10

$f(2) = -3f(1) + 1$	**$f(3) = -3f(2) + 1$**	**$f(4) = -3f(3) + 1$**	**$f(5) = -3f(4) + 1$**
$f(2) = -3(10) + 1$	**$f(3) = -3(-29) + 1$**	**$f(4) = -3(88) + 1$**	**$f(5) = -3(-263) + 1$**
$f(2) = -30 + 1$	**$f(3) = 87 + 1$**	**$f(4) = -264 + 1$**	**$f(5) = 789 + 1$**
$f(2) = -29$	**$f(3) = 88$**	**$f(4) = -263$**	**$f(5) = 790$**
$f(2) = -29$	**$f(3) = 88$**	**$f(4) = -263$**	

Short Notation

$-3(10) + 1 = $ **-29**

$-3(-29) + 1 = $ **88**

$-3(88) + 1 = $ **-263**

$-3(-263) + 1 = $ **790**

10, -29, 88, -263, 790

The fifth term in the sequence is 790

10. Four students were given the following problem:

Dave mowed his lawn yesterday and the newly cut grass stands 1 inch tall. Every day the grass grows about 0.2 inch. Write a function rule that can determine the height, $f(n)$, of the grass in n days.

The function rules created by the four students are listed below. Analyze each function rule and determine if the student is correct or incorrect. Explain your reasoning.

Create a table of values to make sense of the situation. Use your table to determine if the relationship is arithmetic or geometric. Write an **explicit** and **recursive** rule that models the table and compare your rules to the rules generated by students A – D.

n **number of days**	0	1	2	3	4	5
***f*(*n*)** **height of grass**	1	1.2	1.4	1.6	1.8	2

Explicit
$f(1) = 1.2 \quad d = 0.2$
$f(n) = 1.2 + 0.2(n - 1)$

Recursive
$f(n) = f(n - 1) + 0.2$
$f(1) = 1.2$

Student A: $f(n) = 0.2n + 1$

Student A is correct.
The function rule is written in y = mx + b form. The rate of change is 0.2 which indicates that grass grows 0.2 inches per day. The y-intercept is 1 which indicates the starting height of the grass. This rule models the table shown above.

Student B: $f(n) = 1(0.2)^{n-1}$

Student B is incorrect.
The function rule represents a geometric sequence. The above situation is a linear relationship. There is a constant rate of change (common difference). The rule does not model the table shown above.

Student C: $f(n) = f(n - 1) + 0.2$ and $f(0) = 1$

Student C is correct.
The function rule is written recursively. It states that the nth term is found by adding 0.2 to the previous term ($f(n - 1)$). The rule also states that the term before the first term is 1. This term represents the starting height of the grass.

Check: $f(n) = f(n - 1) + 0.2$
$f(1) = f(1 - 1) + 0.2$
$f(1) = f(0) + 0.2$
$f(1) = 1 + 0.2$
$f(1) = 1.2$

Student D: $f(n) = 1.2 + 0.2(n - 1)$

Student D is correct.
The function rule is the explicit formula for the sequence. The first term of the sequence displayed by the table above is 1.2 and the common difference is 0.2. This rule models the table.