Arithmetic of Shimura varieties over global fields

Fabrizio Andreatta

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On two mod p period maps: Ekedahl–Oort and fine Deligne–Lusztig stratifications

Consider the integral model S of a Shimura variety with good reduction, in mixed characteristic $0-p$. Caraiani and Scholze construct a perfectoid cover of the generic fiber of S and the so called Hodge-Tate period map, with target a suitable flag variety. In this talk I will compare the pull-back to this perfectoid cover of two stratifications. The first is the Ekedhal-Oort stratification on the mod p special fiber of S. The second is the fine Deligne-Lusztig stratification on the mod p special fiber of the flag variety.

Gregorio Baldi

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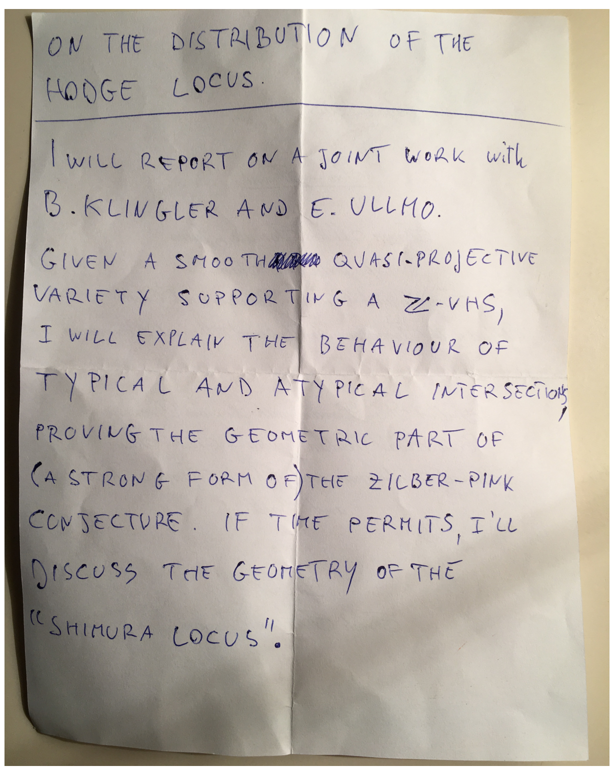

ON THE DISTRIBUTION OF THE HODGE LOCUS.

I WILL REPORT ON A JOINT WORK WITH B. KLINGLER AND E. ULLMO. GIVEN A SMOOTH QUASI-PROJECTIVE VARIETY SUPPORTING A $\mathbb{Z}$-VHS, I WILL EXPLAIN THE BEHAVIOUR OF TYPICAL AND ATYPICAL INTERSECTIONS, PROVING THE GEOMETRIC PART OF (A STRONG FORM OF) THE ZILBER-PINK CONJECTURE. IF TIME PERMITS, I'LL DISCUSS THE GEOMETRY OF THE "SHIMURA LOCUS".

Ana Caraiani

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On the cohomology of Hilbert modular varieties with torsion coefficients

After discussing a vanishing conjecture for the cohomology of locally symmetric spaces with torsion coefficients, I will give an overview of an approach to this conjecture for Shimura varieties, introduced in joint work with Peter Scholze. This approach relies on the geometry of the Hodge-Tate period morphism and on computing the cohomology of Igusa varieties. I will then specialise to the case of Hilbert modular varieties and describe a modified version of this approach that relies on the geometric Jacquet-Langlands results established by Tian-Xiao. This is joint work with Matteo Tamiozzo.

Laurent Fargues

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Geometrization of the local Langlands correspondence

I will speak about my joint work with Peter Scholze about the construction of the local Langlands correspondence for reductive groups over non-archimedean fields.

Javier Fresán

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Equidistribution of exponential sums over algebraic groups

I will discuss an ongoing work with Arthur Forey and Emmanuel Kowalski, in which we obtain an equidistribution theorem for discrete Fourier transforms of trace functions of perverse sheaves on a commutative algebraic group over a finite field. The proof relies on a generic vanishing theorem for twists of perverse sheaves, which allows for the construction of a tannakian category with convolution as tensor operation. If time permits, I will also explain how to compute the groups governing the equidistribution in a few interesting examples.

Ziyang Gao

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A proof of the Uniform Mordell—Lang Conjecture

Let A be an abelian variety and let X be a subvariety, both defined over $\bar{\mathbb{Q}}$. For any finite rank subgroup Γ of $A(\bar{\mathbb{Q}})$, the famous Mordell—Lang Conjecture predicts that each component of $X \cap \Gamma$ is a coset of A. This conjecture is proved by Faltings and one also needs a result of Hindry to handle division points. The Uniform Mordell—Lang Conjecture predicts that the number of irreducible components concerned above is bounded solely in terms of dim A, deg X and the rank of Γ. The question was posed by Mazur and David—Philippon. Recently this conjecture is proved in a series of work (Dimitrov - Gao - Habegger, Kühne; Gao - Ge - Kühne). In this talk I will report this proof. I will focus on the case of rational points on curves and then explain how to generalize this method to the general case. This is a joint project with Vesselin Dimitrov, Philipp Habegger; Tangli Ge, Lars Kühne.

Luis Garcia

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Kudla-Millson theta series and variations of Hodge structure

Kudla and Millson have introduced certain theta series valued in the cohomology of locally symmetric spaces. When integrated on compactly supported homology classes, they give rise to modular forms whose Fourier coefficients can be interpreted as intersection numbers. In some explicit examples the integral over non-compact classes has been computed and has been shown to be a mock modular form. I will discuss an alternative construction of these theta series. I will explain how, to a polarised variation of Hodge structure V of weight two over a base S we attach a differential form $\theta_V(\tau)$ on S depending on a parameter τ in the upper half plane, and transforming like a non-holomorphic modular form in τ. I will also report on work in progress in understanding the boundary behaviour of these classes for one-variable degenerations of Hodge structure in terms of the mixed Hodge structure at the boundary.

Valentin Hernandez

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The Infinite Fern in higher dimensions

In general, deformations spaces of residual Galois representation are quite mysterious objects. It is natural to ask if there is at least enough modular points in their generic fiber X. A related question is the density of the p-adic modular forms, which forms a fractal-like object called the Infinite Fern. In dimension 2, in most cases Gouvêa and Mazur proved that this infinite fern is Zariski dense in X. In higher dimension we look at *polarized* Galois representation, and the analogous question becomes much more complicated. Chenevier explained a strategy by looking for *good* (called generic) points in Eigenvarieties, studied the analogous local (p-adic) question and solved the case of dimension 3. Recently Breuil-Hellmann-Schraen studied the local Infinite Fern at well behaved crystalline points, and Hellmann-Margerin-Schraen, under strong Taylor-Wiles hypothesis, managed to prove the density of the (global) Infinite Fern (in a union of connected components) in all dimensions using the *patched* Eigenvariety. In this talk I would like to explain how to only use the local geometric input to deduce the analogous density result without using the Taylor-Wiles hypothesis, but using another kind of *good* points as in Chenevier's strategy. This is a joint work with Benjamin Schraen.

Lars Kühne

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Equidistribution and Uniformity in Families of Curves

I will present an equidistribution result for families of (non-degenerate) subvarieties in a (general) family of abelian varieties. This extends a result of DeMarco and Mavraki for curves in fibered products of elliptic surfaces. Using this result, one can deduce a uniform version of the classical Bogomolov conjecture for curves embedded in their Jacobians, namely that the number of torsion points lying on them is uniformly bounded in the genus of the curve. This has been previously only known in a few select cases by work of David–Philippon and DeMarco–Krieger–Ye. Furthermore, one can deduce a rather uniform version of the Mordell-Lang conjecture by complementing a result of Dimitrov–Gao–Habegger: The number of rational points on a smooth algebraic curve defined over a number field can be bounded solely in terms of its genus and the Mordell-Weil rank of its Jacobian. Again, this was previously known only under additional assumptions (Stoll, Katz–Rabinoff–Zureick-Brown). All these results have been recently generalized beyond curves in joint work with Ziyang Gao and Tangli Ge.

Keerthi Madapusi Pera

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Derived special cycles on Shimura varieties

In previous work with B. Howard, we constructed certain cycle classes, indexed by positive semi-definite lattices, in the rational Chow groups of integral models for orthogonal Shimura varieties. In the generic fiber, this construction is due to S. Kudla, and in line with his expectations, we showed that these cycles classes can be organized into Fourier expansions for Siegel modular forms. The construction is K-theoretic, quite indirect, and presents various technical issues when one attempts to generalize it to other contexts. In this talk, I will explain how a 'derived' perspective helps give a uniform definition of these cycles with the right (virtual) codimension. This simultaneously resolves issues related to the semi-definiteness of the lattice as well as issues arising in the special fiber from the divisibility of the discriminant of the lattice, and also upgrades the rational classes constructed in our work with Howard to classes in the integral Chow group.

Vincent Pilloni

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Higher Hida theory for Siegel varieties

In higher Coleman theory we introduced certain local cohomology groups on Shimura varieties which generalize p-adic or overconvergent modular forms and can be used to describe the coherent cohomology of Shimura varieties in caracteristic zero. We are now trying to define an integral version of this theory. This is joint work (in progress) with G. Boxer.

Olivier Taïbi

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New tools for the computation of discrete series multiplicities for classical groups over $\mathbb{Z}$

In 2014 I gave an algorithm to compute certain orbital integrals for the unit element of the unramified Hecke algebra of a p-adic classical group. This allowed me to compute the geometric side of Arthur's "L^2 Lefschetz" trace formula giving averaged discrete series multiplicities in the level one discrete automorphic spectrum of classical groups over $\mathbb{Z}$ of rank at most 6. Equivalently, this gives (non-trivially) explicit formulas for the number of self-dual level one cuspidal algebraic regular automorphic representations of general linear groups over $\mathbb{Q}$ in dimension at most 13. This followed work of Gaëtan Chenevier and David Renard using definite classical groups over $\mathbb{Z}$. More recently Gaëtan Chenevier and I introduced a new method, using the Weil explicit formula, to explicitly compute the geometric side of the above trace formula without computing any orbital integral, up to rank 8 (i.e. up to dimension 17). In this talk I will recall these methods and explain how to bring them together and other tools to handle cases of even higher rank (up to dimension 24). This is part of a long-term project with Gaëtan Chenevier.

Emmanuel Ullmo

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Special subvarieties of non-arithmetic ball quotients and Hodge Theory

The lecture will report on a joint work with Gregorio Baldi. Let $\Gamma \subset PU(n,1)$ be a lattice. Then Γ acts on B_n the complex unit ball of dimension n and the quotient $S\Gamma = \Gamma\backslash B_n$ is a quasi projective variety by a result of Baily-Borel if Γ is arithmetic and by a result of Mok if Γ is not arithmetic. We prove that, if $S\Gamma$ contains infinitely many maximal complex totally geodesic subvarieties, then Γ is arithmetic. We first show that $S\Gamma$ can be embedded in a period domain for polarised integral variations of Hodge structures and interpret totally geodesic subvarieties as unlikely intersections, then we use some Ax-Schanuel type result in this context due to Bakker and Tsimerman to conclude the proof. A similar result is also obtained by Bader, Fisher, Miller and Stover using super-rigidity techniques. We also prove a version of Ax-Schanuel Conjecture for $S\Gamma$ viewed as hermitian locally symmetric space by itself.

Pol Van Hoften

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Monodromy and irreducibility of Igusa varieties

Igusa varieties are smooth varieties in characteristic p arising naturally as covers of certain subvarieties (central leaves) of Shimura varieties, for example of the ordinary locus of the modular curve. The l-adic cohomology of Igusa varieties acts as a bridge between the cohomology of Rapoport-Zink spaces (local) and the cohomology of Shimura varieties (global), and it is therefore an interesting object of study. In this talk I will discuss recent joint work with Luciena Xiao Xiao, where we compute the 0th cohomology group. This is more or less equivalent to determining the irreducible components of Igusa varieties, and our results generalize results of Hida and Chai-Oort. Our strategy combines recent work of D'Addezio on monodromy of compatible local systems with a generalization of a method of Hida, and the Honda-Tate theory for Shimura varieties of Hodge type of Kisin - Madapusi Pera - Shin.

Paul Ziegler

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The tautological ring of a Shimura variety

The tautological ring of a Shimura variety of Hodge type is the subring of its Chow ring generated by the Chern classes of automorphic vector bundles. I will talk about joint work with Torsten Wedhorn on this ring as well as its characteristic p variant. The later is strongly related to the question of understanding the cycle classes of Ekedahl-Oort strata in the Chow ring.