ASHBOURNE COLLEGE

SoW for AS MATHS (new spec EDEXCEL) 2017/18

HT1 = 7 weeks
HT2 = 7 weeks
HT3 = 6 weeks
HT4 = 5 weeks
(Mock week included in above allocation)

The delivery of the course will follow this order:
HT1/2: Pure Maths & Statistics
HT3/4: Pure Maths & Mechanics

Break down of pure maths (100 hours in total):

- 2 hours of initial assessment
- 90 hours of teaching
- 8 hours of Mock Exam

Break down of applied maths (50 hours in total):

Statistics

- 24 hours of teaching
- 4 hours of Mock exam

Mechanics

- 18 hours of teaching
- 4 hours of Mock exam

The SoW is a framework for teachers to follow as a guide. The Mock exams are to be provisionally standardised.

Unit	Title	Estimated hours	Date
Section A – Statistics			
1	**Statistical sampling**	**2**	**HT1 week1**
A	Introduction to sampling terminology; Advantages and disadvantages of sampling		
B	Understand and use sampling techniques; Compare sampling techniques in context		
2	**Data presentation and interpretation**	**8**	**HT1 week 2-5**
A	Calculation and interpretation of measures of location; Calculation and interpretation of measures of variation; Understand and use coding		
B	Interpret diagrams for single-variable data; Interpret scatter diagrams and regression lines; Recognise and interpret outliers; Draw simple conclusions from statistical problems		
	Mock Exam	**2**	**HT1 Week 6**
3	**Probability:** Mutually exclusive events; Independent events	**2**	**HT1 week 7**
4	**Statistical distributions:** Use discrete distributions to model real-world situations; Identify the discrete uniform distribution; Calculate probabilities using the binomial distribution (calculator use expected)	**4**	**HT2 week 1-2**
5	**Statistical hypothesis testing**	**8**	**HT2 week 3-7**
A	Language of hypothesis testing; Significance levels		
B	Carry out hypothesis tests involving the binomial distribution		
	Mock exam	**2**	**HT2 week 6**
		28 hours	
Section B – Mechanics			
6	**Quantities and units in mechanics**		
A	Introduction to mathematical modelling and standard S.I. units of length, time and mass	**1**	**HT3 week 1**
B	Definitions of force, velocity, speed, acceleration and weight and displacement; Vector and scalar quantities	**1**	**HT3 week1**
7	**Kinematics 1 (constant acceleration)**		
A	Graphical representation of velocity, acceleration and displacement	**2**	**HT3 week2**
B	Motion in a straight line under constant acceleration; *suvat* formulae for constant acceleration; Vertical motion under gravity	**3**	**HT3 week 3-4**
8	**Forces & Newton's laws**		
A	Newton's first law, force diagrams, equilibrium, introduction to **i**, **j** system	**3**	**HT3 week4-6**
	Mock Exam	**2**	**HT3 Week 5**
B	Newton's second law, '$F = ma$', connected particles (no resolving forces or use of $F = \mu R$); Newton's third law: equilibrium, problems involving smooth pulleys	**3**	**HT4 week 1-2**
9	**Kinematics 2 (variable acceleration)**		
A	Variable force; Calculus to determine rates of change for kinematics	**3**	**HT4 week 2-3**
	Mock Exam	**2**	**HT4 week4**
B	Use of integration for kinematics problems i.e. $r = \int v\,dt$, $v = \int a\,dt$	**2**	**HT 4 Week 5**
		22 hours	

Break down of hours:

Statistics

- 24 hours of teaching
- 4 hours of Mock exam

Mechanics

- 18 hours of teaching
- 4 hours of Mock exam

Unit		Title	Estimated hours	Date
		Initial Assessment	**2**	**HT1 Week1**
1		**Algebra and functions**	**20**	**HT1 Week1-6**
	a	Algebraic expressions – basic algebraic manipulation, indices and surds		
	b	Quadratic functions – factorising, solving, graphs and the discriminants		
	c	Equations – quadratic/linear simultaneous		
	d	Inequalities – linear and quadratic (including graphical solutions)		
	e	Graphs – cubic, quartic and reciprocal		
	f	Transformations – transforming graphs – f(x) notation		
		Mock Exam	**2**	**HT1 Week 6**
2		**Coordinate geometry in the (x, y) plane**	**(9 see below)**	**See below**
	a	Straight-line graphs, parallel/perpendicular, length and area problems	**4**	**HT1 Week 7**
	b	Circles – equation of a circle, geometric problems on a grid	**5**	**HT2 Week 1-2**
3		**Further algebra**	**9**	**HT2 Week 2-4**
	a	Algebraic division, factor theorem and proof (proof can be taught at end)		
	b	The binomial expansion		
4		**Trigonometry**	**12**	**HT2 Week 4-7**
	a	Trigonometric ratios and graphs		
	b	Trigonometric identities and equations		
		Mock Exam	**2**	**HT2 Week 6**
5		**Vectors (2D)**	**11**	**HT3 Week 1-3**
	a	Definitions, magnitude/direction, addition and scalar multiplication		
	b	Position vectors, distance between two points, geometric problems		
6		**Differentiation**	**11**	**HT3 Week 3-6**
	a	Definition, differentiating polynomials, second derivatives		
	b	Gradients, tangents, normals, maxima and minima		
		Mock Exam	**2**	**HT3 Week 5**
7		**Integration**	**10**	**HT4 Week 1-3**
	a	Definition as opposite of differentiation, indefinite integrals of x^n		
	b	Definite integrals and areas under curves		
8		**Exponentials and logarithms:** Exponential functions and natural logarithms	**8**	**HT4 Week 3-5**
		Mock Exam	**2**	**HT4 Week 4**
			100 hours	

Break down of hours:

- 2 hours of initial assessment
- 90 hours of teaching
- 8 hours of Mock Exam

Pearson
Edexcel Level 3 Advanced Subsidiary GCE in Mathematics (8MA0)

One-year Scheme of Work

For first teaching from September 2017

Contents

Introduction

Content of this document is based on the accredited version of Pearson Edexcel Level 3 Advanced Subsidiary GCE in Mathematics (8MA0) and includes detailed scheme of work for pure and applied content of AS level Mathematics.

This scheme of work is based upon a one-year delivery model for AS level Mathematics.
It can be used directly as a scheme of work for the AS level Mathematics specification (8MA0).

The scheme of work is broken up into units and sub-units, so that there is greater flexibility for moving topics around to meet planning needs.

Each unit contains:

- Specification references
- Prior knowledge
- Keywords
- Notes.

Each sub-unit contains:

- Recommended teaching time, though of course this is adaptable according to individual teaching needs
- Objectives for students at the end of the sub-unit
- Teaching points
- Opportunities for problem-solving and modelling
- Common misconceptions and examiner report quotes (from legacy Specifications)
- Notes

Teachers should be aware that the estimated teaching hours are approximate and should be used as a guideline only.

Our free support for the AS and A level Mathematics specifications can be found on the Pearson Edexcel Mathematics website (quals.pearson.com/Alevelmaths2017) and on the Emporium (www.edexcelmaths.com).

AS level Mathematics
Paper 1: Pure Mathematics 62.5%, 2 hours, 100 marks
Paper 2: Statistics and Mechanics 37.5%, 1 hour 15 minutes, 60 marks

AS Mathematics pure content

Pure Mathematics

Unit		Title	Estimated hours
1		**Algebra and functions**	
	a	Algebraic expressions – basic algebraic manipulation, indices and surds	2
	b	Quadratic functions – factorising, solving, graphs and the discriminants	3
	c	Equations – quadratic/linear simultaneous	3
	d	Inequalities – linear and quadratic (including graphical solutions)	4
	e	Graphs – cubic, quartic and reciprocal	4
	f	Transformations – transforming graphs – f(x) notation	4
2		**Coordinate geometry in the (x, y) plane**	
	a	Straight-line graphs, parallel/perpendicular, length and area problems	4
	b	Circles – equation of a circle, geometric problems on a grid	5
3		**Further algebra**	
	a	Algebraic division, factor theorem and proof	4
	b	The binomial expansion	5
4		**Trigonometry**	
	a	Trigonometric ratios and graphs	5
	b	Trigonometric identities and equations	7
5		**Vectors (2D)**	
	a	Definitions, magnitude/direction, addition and scalar multiplication	5
	b	Position vectors, distance between two points, geometric problems	6
6		**Differentiation**	
	a	Definition, differentiating polynomials, second derivatives	5
	b	Gradients, tangents, normals, maxima and minima	6
7		**Integration**	
	a	Definition as opposite of differentiation, indefinite integrals of x^n	5
	b	Definite integrals and areas under curves	5
8		**Exponentials and logarithms:** Exponential functions and natural logarithms	8
			90 hours

AS Mathematics applied content

Statistics and Mechanics

Unit		Title	Estimated hours
Section A – Statistics			
1		**Statistical sampling**	
	a	Introduction to sampling terminology; Advantages and disadvantages of sampling	1
	b	Understand and use sampling techniques; Compare sampling techniques in context	1
2		**Data presentation and interpretation**	
	a	Calculation and interpretation of measures of location; Calculation and interpretation of measures of variation; Understand and use coding	3
	b	Interpret diagrams for single-variable data; Interpret scatter diagrams and regression lines; Recognise and interpret outliers; Draw simple conclusions from statistical problems	5
3		**Probability:** Mutually exclusive events; Independent events	2
4		**Statistical distributions:** Use discrete distributions to model real-world situations; Identify the discrete uniform distribution; Calculate probabilities using the binomial distribution (calculator use expected)	4
5		**Statistical hypothesis testing**	
	a	Language of hypothesis testing; Significance levels	3
	b	Carry out hypothesis tests involving the binomial distribution	5
			24 hours
Section B – Mechanics			
6		**Quantities and units in mechanics**	
	a	Introduction to mathematical modelling and standard S.I. units of length, time and mass	1
	b	Definitions of force, velocity, speed, acceleration and weight and displacement; Vector and scalar quantities	1
7		**Kinematics 1 (constant acceleration)**	
	a	Graphical representation of velocity, acceleration and displacement	2
	b	Motion in a straight line under constant acceleration; *suvat* formulae for constant acceleration; Vertical motion under gravity	3
8		**Forces & Newton's laws**	
	a	Newton's first law, force diagrams, equilibrium, introduction to $\mathbf{\underline{i}}$, $\mathbf{\underline{j}}$ system	3
	b	Newton's second law, '$F = ma$', connected particles (no resolving forces or use of $F = \mu R$); Newton's third law: equilibrium, problems involving smooth pulleys	3
9		**Kinematics 2 (variable acceleration)**	
	a	Variable force; Calculus to determine rates of change for kinematics	3
	b	Use of integration for kinematics problems i.e. $r = \int v\,dt$, $v = \int a\,dt$	2
			18 hours

AS Mathematics pure content

Pure Mathematics

Unit		Title	Estimated hours
1		**Algebra and functions**	
	a	Algebraic expressions – basic algebraic manipulation, indices and surds	**2**
	b	Quadratic functions – factorising, solving, graphs and the discriminants	**3**
	c	Equations – quadratic/linear simultaneous	**3**
	d	Inequalities – linear and quadratic (including graphical solutions)	**4**
	e	Graphs – cubic, quartic and reciprocal	**4**
	f	Transformations – transforming graphs – f(x) notation	**4**
2		**Coordinate geometry in the (x, y) plane**	
	a	Straight-line graphs, parallel/perpendicular, length and area problems	**4**
	b	Circles – equation of a circle, geometric problems on a grid	**5**
3		**Further algebra**	
	a	Algebraic division, factor theorem and proof	**4**
	b	The binomial expansion	**5**
4		**Trigonometry**	
	a	Trigonometric ratios and graphs	**5**
	b	Trigonometric identities and equations	**7**
5		**Vectors (2D)**	
	a	Definitions, magnitude/direction, addition and scalar multiplication	**5**
	b	Position vectors, distance between two points, geometric problems	**6**
6		**Differentiation**	
	a	Definition, differentiating polynomials, second derivatives	**5**
	b	Gradients, tangents, normals, maxima and minima	**6**
7		**Integration**	
	a	Definition as opposite of differentiation, indefinite integrals of x^n	**5**
	b	Definite integrals and areas under curves	**5**
8		**Exponentials and logarithms:** Exponential functions and natural logarithms	**8**
			90 hours

UNIT 1: Algebra and Functions

Return to overview

SPECIFICATION REFERENCES

2.1 Understand and use the laws of indices for all rational exponents

2.2 Use and manipulate surds, including rationalising the denominator

2.3 Work with quadratic functions and their graphs

The discriminant of a quadratic function, including the conditions for real and repeated roots

Completing the square

Solution of quadratic equations, including solving quadratic equations in a function of the unknown

2.4 Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation

2.5 Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions

Express solutions through correct use of 'and' and 'or', or through set notation

Represent linear and quadratic inequalities such as $y > x + 1$ and $y > ax^2 + bx + c$ graphically

2.6 Manipulate polynomials algebraically, including expanding brackets, collecting like terms and factorisation and simple algebraic division; use of the factor theorem

2.7 Understand and use graphs of functions; sketch curves defined by simple equations including polynomials, $y = \frac{a}{x}$ and $y = \frac{a}{x^2}$ (including their vertical and horizontal asymptotes)

Interpret algebraic solution of equations graphically; use intersection points of graphs to solve equations

2.8 Understand the effect of simple transformations on the graph of $y = \mathrm{f}(x)$ including sketching associated graphs:

$y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$

PRIOR KNOWLEDGE

GCSE (9-1) in Mathematics at Higher Tier

A4 Collecting like terms and factorising

N8 Surds

A19 Solving linear simultaneous equations

A18 Solving quadratic equations (by factorising and completing the square)

A22 Working with inequalities

Solving quadratic inequalities

A12 Functional notation and shapes of standard graphs (e.g. parabola, cubic, reciprocal)

N7 Rules of indices

KEYWORDS

Expression, function, constant, variable, term, unknown, coefficient, index, linear, identity, simultaneous, elimination, substitution, factorise, completing the square, intersection, change the subject, cross-multiply, power, exponent, base, rational, irrational, reciprocal, root, standard form, surd, rationalise, exact, manipulate, sketch, plot, quadratic, maximum, minimum, turning point, transformation, translation, polynomial, discriminant, real roots, repeated roots, factor theorem, quotient, intercepts, inequality, asymptote .

1a. Algebraic expressions: basic algebraic manipulation, indices and surds (2.1) (2.2)	Teaching time 2 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to perform essential algebraic manipulations, such as expanding brackets, collecting like terms, factorising etc;
- understand and be able to use the laws of indices for all rational exponents;
- be able to use and manipulate surds, including rationalising the denominator.

TEACHING POINTS

Recap the skills taught at GCSE Higher Tier (9-1).

Emphasise that in many cases, only a fraction or surd can express the exact answer, so it is important to be able to calculate with surds.

Ensure students understand that $\sqrt{a} + \sqrt{b}$ is *not* equal to $\sqrt{a+b}$ and that they know that $a^{\frac{m}{n}}$ is equivalent to $\sqrt[n]{a^m}$ and that a^{-m} is equivalent to $\frac{1}{a^m}$.

Most students understand the skills needed to complete these calculations but make basic errors with arithmetic leading to incorrect solutions.

Questions involving squares, for example $(2\sqrt{3})^2$, will need practice.

Students should be exposed to lots of simplifying questions involving fractions as this is where most marks are lost in exams.

Recap the difference of two squares $(x + y)(x - y)$ and link this to $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) = x - y$, explaining the choice of term to rationalise the denominator.

Provide students with plenty of practice and ensure that they check their answers.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Include examples which involve calculating areas of shapes with side lengths expressed as surds. Exact solutions for Pythagoras questions is another place where surds occur naturally.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Common errors include: misinterpreting $(a\sqrt{b})^2$ as $(a + \sqrt{b})^2$; evaluating $(\sqrt{2})^2$ as 4 instead of 2; slips when multiplying out brackets; basic arithmetic errors; and leaving surds in the denominator rather than fully simplifying fractions. Two examples of errors with indices are, writing $\frac{1}{3x}$ as $3x^{-1}$and writing $\frac{4}{\sqrt{x}}$ as $4x^{\frac{1}{2}}$; these have significant implications later in the course (e.g. differentiation).

Many of these errors can be avoided if students carefully check their work and have plenty of practice.

NOTES

Make use of matching activities (e.g. Tarsia puzzles)

1b. Quadratic functions: factorising, solving, graphs and discriminants (2.3)	Teaching time 3 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to solve a quadratic equation by factorising;
- be able to work with quadratic functions and their graphs;
- know and be able to use the discriminant of a quadratic function, including the conditions for real and repeated roots;
- be able to complete the square. e.g. $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$;
- be able to solve quadratic equations, including in a function of the unknown.

TEACHING POINTS

Lots of practice is needed as these algebraic skills are fundamental to all subsequent work. Students must become fluent, and continue to develop thinking skills such as choosing an appropriate method, and interpreting the language in a question. Emphasise correct setting out and notation.

Students will need lots of practice with negative coefficients for x squared and be reminded to always use brackets if using a calculator. e.g. $(-2)^2$.

Include manipulation of surds when using the formula for solving quadratic equations. [Link with previous sub-unit.]

Where examples are in a real-life contexts, students should check that solutions are appropriate and be aware that a negative solution may not be appropriate in some situations.

Students must be made aware that this sub-unit is about finding the links between completing the square and factorised forms of a quadratic and the effect this has on the graph. Use graph drawing packages to see the effect of changing the value of the '+ c' and link this with the roots and hence the discriminant.

Start by drawing $y = x^2$ and add different x terms followed by different constants in a systematic way. Then move on to expressions where the coefficient of x^2 is not 1.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Links can be made with Unit 3a – Proof:

Proof by deduction: e.g. complete the square to prove that $n^2 - 6n + 10$ is positive for all values of n.

Disproof by counter-example: show that the statement "$n^2 - n + 1$ is a prime number for all values of n" is untrue.

The path of an object thrown can be modelled using quadratic graphs. Various questions can be posed about the path:

- When is the object at a certain height?
- What is the maximum height?
- Will it clear a wall of a certain height, a certain distance away?

Areas of shapes where the side lengths are given as algebraic expressions.

Proof of the quadratic formula. Working backwards, e.g. find a quadratic equation whose roots are $\frac{-5 \pm \sqrt{17}}{4}$

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

When completing the square, odd coefficients of x can cause difficulties. Students do not always relate finding the minimum point and line of symmetry to completing the square. Students should be provided with plenty of practice on completing the square with a wide range of quadratic forms.

Notation and layout can also be a problem; students must remember to show all the necessary working out at every stage of a calculation, particularly on 'show that' questions.

Examiners often refer to poor use of the quadratic formula. In some cases the formula is used without quoting it first and there are errors in substitution. In particular, the use of $-b \pm \frac{\sqrt{b^2-4ac}}{2a}$ so that the division does not extend under the "$-b$", is relatively common. Another common mistake is to think that the denominator is always 2. Also, students sometimes include x's in their expressions for the discriminant. Such methods are likely to lose a significant number of marks.

NOTES

Encourage the use of graphing packages or graphing Apps (e.g. Desmos or Autograph), so students can graph as they go along and 'picture' their solutions. You can link the discriminant with complex numbers if appropriate for students also studying Further Maths.

1c. Equations: quadratic/linear simultaneous (2.4)

Teaching time
3 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to solve linear simultaneous equations using elimination and substitution;
- be able to use substitution to solve simultaneous equations where one equation is linear and the other quadratic.

TEACHING POINTS

Simultaneous equations are important both in future pure topics but also for applied maths. Students will need to be confident solving simultaneous equations including those with non-integer coefficients of either or both variables.

The quadratic may involve powers of 2 in one unknown or in both unknowns, e.g. Solve $y = 2x + 3$, $y = x^2 - 4x + 8$ or $2x - 3y = 6$, $x^2 - y^2 + 3x = 50$.

Emphasise that simultaneous equations lead to a pair or pairs of solutions, and that both variables need to be found.

Make sure students practise examples of worded problems where the equations need to be set up.

Students should be encouraged to check their answers using substitution.

Sketches can be used to check the number of solutions and whether they will be positive or negative. This will be reviewed and expanded upon as part of the curve sketching topic.

Use graphing packages or graphing Apps (e.g. Desmos or Autograph), so students can visualise their solutions e.g. straight lines crossing an ellipse or a circle.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Simultaneous equations in contexts, such as costs of items given total cost, can be used. Students must be aware of the context and ensure that the solutions they give are appropriate to that context.

Simultaneous equations will be drawn on heavily in curve sketching and coordinate geometry.

Investigate when simultaneous equations cannot be solved or only give rise to one solution rather than two.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Mistakes are often due to signs errors or algebraic slips which result in incorrect coordinates. Students should be encouraged to check their working and final answers, and if the answer seems unlikely to go back and look for errors in their working. Examiners often notice that it is the more successful candidates who check their solutions.

Students should remember to find the values of both variables as stopping after finding one is a common cause of lost marks in exam situations. Students who do remember to find the values of the second variable must take care that they substitute into a correct equation or a correctly rearranged equations.

1d. Inequalities: linear and quadratic (including graphical solutions) (2.5)

Teaching time
4 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to solve linear and quadratic inequalities;
- know how to express solutions through correct use of 'and' and 'or' or through set notation;
- be able to interpret linear and quadratic inequalities graphically;
- be able to represent linear and quadratic inequalities graphically.

TEACHING POINTS

Provide students with plenty of practice at expressing solutions in different forms using the correct notation. Students must be able to express solutions using 'and' and 'or' appropriately, or by using set notation. So, for example:

$x < a$ **or** $x > b$ is equivalent to $\{x: x < a\} \cup \{x: x > b\}$

and $\{x: c < x\} \cap \{x: x < d\}$ is equivalent to $x > c$ **and** $x < d$.

Inequalities may contain brackets and fractions, but these will be reducible to linear or quadratic inequalities. For example, $\frac{a}{x} < b$ becomes $ax < bx^2$.

Students' attention should be drawn to the effect of multiplying or dividing by a negative value, this must also be taken into consideration when multiplying or dividing by an unknown constant.

Sketches are the most commonly used method for identifying the correct regions for quadratic inequalities, though other methods may be used. Whatever their method, students should be encouraged to make clear how they obtained their answer.

Students will need to be confident interpreting and sketching both linear and quadratic graphs in order to use them in the context of inequalities.

Make sure that students are also able to interpret combined inequalities. For example, solving

$ax + b > cx + d$

$px^2 + qx + r \geq 0$

$px^2 + qx + r < ax + b$

and interpreting the third inequality as the range of x for which the curve $y = px^2 + qx + r$ is below the line with equation $y = ax + b$.

When representing inequalities graphically, shading and correctly using the conventions of dotted and solid lines is required. Students using graphical calculators or computer graphing software will need to ensure they understand any differences between the conventions required and those used by their graphical calculator.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Financial or material constraints within business contexts can provide situations for using inequalities in modelling. For those doing further maths this will link to linear programming.

Inequalities can be linked to length, area and volume where side lengths are given as algebraic expressions and a maximum or minimum is given.

Following on from using a quadratic graph to model the path of an object being thrown, inequalities could be used to find the time for which the object is above a certain height.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students may make mistakes when multiplying or dividing inequalities by negative numbers.

In exam questions, some students stop when they have worked out the critical values rather than going on to identify the appropriate regions. Sketches are often helpful at this stage for working out the required region.

It is quite common, when asked to solve an inequality such as $2x^2 - 17x + 36 < 0$ to see an incorrect solution such as $2x^2 - 17x + 36 < 0 \Rightarrow (2x - 9)(x - 4) < 0 \Rightarrow x < \frac{9}{2}, x < 4$.

1e. Graphs: cubic, quartic and reciprocal (2.7)

Teaching time
4 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and use graphs of functions;
- be able to sketch curves defined by simple equations including polynomials;
- be able to use intersection points of graphs to solve equations.

TEACHING POINTS

Students should be familiar with the general shape of cubic curves from GCSE (9-1) Mathematics, so a good starting point is asking students to identify key features and draw sketches of the general shape of a positive or negative cubic. Equations can then be given from which to sketch curves.

Quartic equations will be new to students and they may benefit from initially either plotting graphs by hand or using a graphical calculator or graphing software to look at the shape of the curve.

Cubic and quartic equations given at this point should either already be factorised or be easily simplified (e.g. $y = x^3 + 4x^2 + 3x$) as students will not yet have encountered algebraic division.

The coordinates of all intersections with the axes will need to be found. Where equations are already factorised, students will need to find where they intercept the axes. Repeated roots will need to be explicitly covered as this can cause confusion.

Students should also be able to find an equation when given a sketch on which all intersections with the axes are given. To do this they will need to be confident multiplying out multiple brackets.

Reciprocal graphs in the form $y = \frac{a}{x}$ are covered at GCSE but those in the form $y = \frac{a}{x^2}$ will be new. When sketching reciprocal graphs such as $y = \frac{a}{x}$ and $= \frac{a}{x^2}$, the asymptotes will be parallel to the axes.

Intersecting points of graphs can be used to solve equations, a curve and a line and two curves should be covered. When finding points of intersection students should be encouraged to check that their answers are sensible in relation to the sketch.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to justify the number of solutions to simultaneous equations using the intersections of two curves.

Students can be given sketches of curves or photographs of curved objects (e.g. roller coasters, bridges, etc.) and asked to suggest possible equations that could have been used to generate each sketch.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

When sketching cubic graphs, most students are able to gain marks by knowing the basic shape and sketching it passing through the origin. Recognising whether the cubic is positive or negative sometimes causes more difficulty. Students sometimes fail to recognise the significance of a square factor in the factorised form of a polynomial.

When sketching graphs, marks can easily be lost by not labelling all the key points or labelling them incorrectly e.g. (0, 6) instead of (6, 0).

1f. Transformations: transforming graphs (2.8)

Teaching time
4 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand the effect of simple transformations on the graph of $y = \mathrm{f}(x)$;
- be able to sketch the result of a simple transformation given the graph of any function $y = \mathrm{f}(x)$.

TEACHING POINTS

Transformations to be covered are: $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$ and $y = \mathrm{f}(ax)$.

Students should be able to apply one of these transformations to any of the functions listed and sketch the resulting graph:

quadratics, cubics, quartics, reciprocals, $y = \frac{a}{x^2}$, $\sin x$, $\cos x$, $\tan x$, e^x and a^x.

Students will need to be able to transform points and asymptotes both when sketching a curve and to give either the new point or the equation of the line.

Given a curve or an equation that has been transformed students should be able to state the transformation that has been used.

Links can be made with sketching specific curves. Students should be able to sketch curves like $y = (x - 3)^2 + 2$ and $y = \frac{2}{x-3} + 2$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Examples can be used in which the graph is transformed by an unknown constant and students encouraged to think about the effects this will have.

The use of graphing packages or graphing Apps (e.g. Desmos or Autograph) can be invaluable here.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

One of the most common errors is translating the curve in the wrong direction for $\mathrm{f}(x + a)$ or $\mathrm{f}(x) + a$. Students sometimes also apply the wrong scale factor when sketching $\mathrm{f}(ax)$.

Other errors involve algebraic mistakes and incomplete sketches, or sketches without key values marked.

Students should be encouraged to check any answers they have calculated against their sketches to check they make sense

NOTES

Dynamic geometry packages can be used to help students investigate and visualise the effect of transformations.

UNIT 2: Coordinate geometry in the (*x*, *y*) plane

Return to overview

SPECIFICATION REFERENCES

2.7 Understand and use proportional relationships and their graphs

3.1 Understand and use the equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$
Gradient conditions for two straight lines to be parallel or perpendicular
Be able to use straight line models in a variety of contexts

3.2 Understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$
Completing the square to find the centre and radius of a circle
Use of the following properties:

- the angle in a semicircle is a right angle
- the perpendicular from the centre to a chord bisects the chord
- the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point

PRIOR KNOWLEDGE

Algebraic manipulation covered so far

- Simultaneous equations
- Completing the square

GCSE (9-1) in Mathematics at Higher Tier

A9 Equation of a line
Parallel and perpendicular lines

G20 Pythagoras

A14 Conversion graphs

R10 Calculating the proportionality constant *k*

G10 Circle theorems

KEYWORDS

Equation, bisect, centre, chord, circle, circumcircle, coefficient, constant, diameter, gradient, hypotenuse, intercept, isosceles, linear, midpoint, parallel, perpendicular, proportion, Pythagoras, radius, right angle, segment, semicircle, simultaneous, tangent.

2a. Straight-line graphs, parallel/perpendicular, length and area problems (3.1) (2.7)

Teaching time
4 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and use the equation of a straight line;
- know and be able to apply the gradient conditions for two straight lines to be parallel or perpendicular;
- be able to find lengths and areas using equations of straight lines;
- be able to use straight-line graphs in modelling.

TEACHING POINTS

Students should be encouraged to draw sketches when answering questions or, if a diagram is given, annotate the diagram.

Equations can be given or asked for in the forms $y = mx + c$ and $ax + by + c = 0$ where a, b and c are integers. Students will need to be familiar with both forms, so questions should be asked where different forms are given or required in the answer. Given either form, students should be able to find the intercepts with the axes and the gradient. The x-intercept often causes students more difficulty, so will need more practice, but is useful for sketches and questions involving area or perimeter.

Students should be able to find the equation of a line given the gradient and a point, either the formula $y - y_1 = m(x - x_1)$ can be used or the values substituted into $y = mx + c$. To find the equation of a line from two points the gradient can be found and then one of the previous two methods used or the formula $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$ can be used. If this formula is used, care needs to be taken to ensure that the y-values are substituted into the correct places and that negative signs are taken into account. It should be emphasised that in the majority of cases, the form $y - y_1 = m(x - x_1)$ is far more efficient and less prone to errors than other methods.

The gradient conditions for parallel and perpendicular lines may be remembered from GCSE (9-1), but are still worth revising. They need to be well understood as they are used further when dealing with circles and in differentiation. Students should be able to identify whether lines are parallel, perpendicular or neither and find the equation of a parallel or perpendicular line when given a point on the line.

The length of a line segment is found by using Pythagoras' theorem, which can be written as the formula $d = \sqrt{(x - x_1)^2 + (y_2 - y_1)^2}$. This can be linked to proof with students being encouraged to show how to go from Pythagoras to the formula. Answers to length and distance questions are likely to be given in surd form, giving further practice in simplifying surds. Students should be encouraged to give answers in exact form unless specified otherwise.

Make shapes using lines and the axes; students can then be asked to find the area or perimeter of composite shapes. Answers should be given in exact form to practise combining and simplifying surds.

Real-life situations such as conversions can be modelled using straight-line graphs, this is likely to be familiar from GCSE (9-1) Mathematics.

Students should also be familiar with finding the relationship between two variables and expressing this using the proportion symbol $\propto$ or using an equation involving a constant (k). This can be extended to straight-line graphs through the origin with a gradient of k. Students should be able to calculate and interpret the gradient.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

To help students see how much information is given in the equation of a line, a good activity is to give an equation and ask students to find everything they know about that line, e.g. the intercepts, a point on the line, the gradient, a sketch, a parallel line, etc.

Students can be given sketches and asked to suggest equations that would/would not work.

Modelling with straight-line graphs gives the opportunity to collect data that can then be plotted and a line of best fit used to find an equation. It might also be possible to compare the data to a theoretical model. Students should be encouraged to consider strengths and limitations of modelling.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

In exams, students should be encouraged to quote formulae before using them. This allows method marks to be awarded even if arithmetical slips are made or incorrect values substituted.

Questions may specify a particular form for an answer (for example integer coefficients). Emphasise to students the importance of following these instructions carefully so as not to lose marks.

Students should be encouraged to draw diagrams while working on solutions as this often results in fewer mistakes and can act as a sense check for answers. At the same time, where diagrams are given in questions, students should be aware that these are not to be relied upon and ‘spotting’ answers by looking at a diagram without providing evidence to support this will not gain full marks. However, candidates should be encouraged to use any diagrams provided to help them answer the question.

The usual sorts of algebraic and numerical slips cause marks to be lost and students should be encouraged to carefully check their working. A common error is to incorrectly calculate the gradient of a straight line when it is given in the form $ax + by + c = 0$, so students should be encouraged to practice this technique.

NOTES

Dynamic geometry programs can be used to make changes and observe the effect, helping students to discover and visualise the effect of changing equations.

2b. Circles: equation of a circle, geometric problems on a grid (3.2)

Teaching time
5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to find the midpoint of a line segment;
- understand and use the equation of a circle;
- be able to find points of intersection between a circle and a line;
- know and be able to use the properties of chords and tangents.

TEACHING POINTS

Drawing sketches or annotating given diagrams will help students to understand the question in many cases and so should be encouraged.

Students should be able to find the midpoint given two points from GCSE (9-1) Mathematics. This can be built upon to find the coordinate of a point given the midpoint and one of the end points. The midpoint can be used to find the perpendicular bisector, recapping the work from straight-line graphs.

The equation of the circle $(x-a)^2+(y-b)^2=r^2$ can be derived from Pythagoras' theorem, giving students the opportunity to look at proof.

Students should be able to find the radius and the coordinates of the centre of the circle given the equation of the circle, and vice versa. Students should be familiar with the equations $x^2+y^2+2fx+2gy+c=0$ and $(x-a)^2+(y-b)^2=r^2$. 'Complete the square' method should be used to factorise the equation into the more useful form. Students will need practice within this context to ensure that they are confident with the algebraic manipulation needed, in particular mistakes are often made with the signs and forgetting the constant term.

Circle theorems from GCSE (9-1) Mathematics can be used in questions so a quick recap could be useful and then they should be incorporated into questions. Examples of this include: finding the equation of the circumcircle of a triangle with given vertices; or finding the equation of a tangent using the perpendicular property of tangent and radius.

Simultaneous equations can be used to find the points of intersection between a circle and a straight line. Students can also be asked to show that a line and circle do not intersect, for which the discriminant can be used. Finding intersections with the axes should also be covered.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

The conditions in which a circle and a line intersect can be investigated, with students justifying which will and will not intersect.

Investigate finding the equation of a circle given 3 points on its circumference.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Most errors when completing the square to find the equation of a circle involve the constant term. Students may forget to subtract it or perhaps add it instead. Having found the equation, when giving the coordinates of the centre students must take care to get the signs the right way round as marks are easily lost by getting this wrong.

When substituting into equations to find the intersections with axes, students sometimes substitute for the wrong variable, for example substituting $y = 0$ when trying to find the intersection with the y-axis. Another error is substituting the entire bracket $(x – a)$ for 0 rather than just x.

When finding the equation of a tangent to a point on the circle, typical errors are: finding the gradient of the radius; finding a line parallel to the radius; and finding a line through the centre of the circle.

UNIT 3: Further algebra

Return to overview

SPECIFICATION REFERENCES

2.6 Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem

1.1 Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including: proof by deduction, proof by exhaustion, disproof by counter-example

4.1 Understand and use the binomial expansion of $(a + bx)^n$ for positive integer n; the notations $n!$ and ${}_nC_r$; link to binomial probabilities

PRIOR KNOWLEDGE

Algebraic manipulation covered so far

- Factorising quadratics
- notation

GCSE (9-1) in Mathematics at Higher Tier

A4 Expanding brackets

A2 Substitution

A6 Proof

KEYWORDS

Binomial, coefficient, probability, proof, assumptions, deduction, exhaustion, disproof, counter-example, polynomials, factorisation, quadratic, cubic, quartic, conjecture, prediction, rational number, implies, necessary, sufficient, converse, fully factorise, factor, expand, therefore, conclusion.

3a. Algebraic division, factor theorem and proof (2.6) (1.1)

Teaching time
4 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to use algebraic division;
- know and be able to apply the factor theorem;
- be able to fully factorise a cubic expression;
- understand and be able to use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion;
- be able to use methods of proof, including proof by deduction, proof by exhaustion and disproof by counter-example.

TEACHING POINTS

When using algebraic division, only division by $(ax + b)$ or $(ax – b)$ will be required.

Different methods for algebraic division should be considered depending on students' prior experience and preferred ways of working. Whichever method is used, clear working out should be shown.

Equations in which the coefficient of x or x^2 is 0 for example $x^3 + 3x^2 - 4$ or $2x^3 + 5x - 20$ will need additional explanation and practice.

Students should know that if $f(x) = 0$ when $x = a$, then $(x – a)$ is a factor of $f(x)$. Questions in the form $(ax + b)$ should be covered.

Where a negative is being substituted into the equation the distinction between $(-2)^2$ and -2^2 will be important especially when students are using a calculator as examiners often comment on the fact that students will sometimes evaluate $(-2)^2$ as -4.

Factor theorem can be used to find an unknown constant. For example: Find a given that $(x - 2)$ is a factor of $x^3 + ax^2 - 4x + 6$. Two conditions can also be given in order to form simultaneous equations to solve.

When fully factorising a cubic, emphasis should be placed on choosing appropriate values. The final answer may need to be written as a factorised cubic or, alternatively, as the solutions to an equation which can then be used to sketch the curve. Students sometimes use the roots of a polynomial equation to help them factorise but this method must be used with care. Questions sometimes use the word 'hence' and so students must be careful which method they chose in these cases.

This is an excellent opportunity to review curve sketching by asking students to give a sketch following factorisation.

Students should be familiar with basic proofs from GCSE (9-1) Mathematics this knowledge can be built upon to look at the different types of proof. Students will need to understand how to set out each type of proof; the correct conventions in language and layout should be encouraged.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

The factor theorem can be introduced through investigation by substituting different values and checking against division to look for patterns.

Proof gives the opportunity to review previous concepts in a different way for example coordinate geometry. Proof will also be included in later topics.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

The majority of errors seen in exam questions are not due to misunderstanding the method, but instead arithmetic and algebraic mistakes. For example, incorrect simplification of terms – especially those involving fractions; mistakes with negative numbers; and writing expressions rather than equations.

Students should be aware that long division is not always the best or quickest method to use and sometimes results in some complicated algebra.

When using the factor theorem, stress the importance of checking the value that is substituted; a common error is to use, for example, f(1) rather than f(–1).

You should also emphasise the importance of fully factorising expressions, as a fairly significant number of students stop when they have reached one linear factor and a quadratic factor.

3b. The binomial expansion (4.1)

Teaching time
5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use the binomial expansion of $(a + bx)^n$ for positive integer n;
- be able to find an unknown coefficient of a binomial expansion.

TEACHING POINTS

Students should initially be introduced to Pascal's triangle, which can be used to expand simple brackets.

Students will need to be familiar with factorials and the ${}_nC_r$ notation.

Introduce the formal binomial expansion in the same way as the formula booklet and discuss the various terms to ensure all students understand.

Setting out work clearly and logically will be invaluable in helping students to achieve the final answer and also to spot mistakes if necessary.

Where there is a coefficient of x (other than 1) students will need to be reminded that the power applies to the whole term, not just the x, and that answers must be simplified appropriately. Negative and fractional coefficients will also need practice.

The limitations of the binomial expansion should be discussed.

Students should practice finding the coefficient of a single term, they should also be able to deal with setting up simple algebraic equations to find unknown constants.

Use of the binomial expansion can be linked to basic probability and approximations.

[Links can also be made with the statistics work in A level Mathematics.]

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students can be encouraged to discover the link between Pascal's triangle and the expansion of simple brackets.

Students could look at find the general term of a particular expansion.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Marks are most commonly lost in exam questions because of errors in expanding terms. For example not including the coefficient when calculating, say, $(ax)^2$; not simplifying terms fully; sign errors; and omitting brackets. Good notation will help to avoid many of these mistakes.

When writing expansions which involve unknown constants, some students fail to also include the x's in their expansion.

When using their expansions to work out the value of a constant, a significant number of students do not understand that the coefficient does not include the x or x^2 part and so are often unable to form an equation in the unknown alone.

Questions often go on to ask students to use their binomial expansion to evaluate a number raised to a power. For example, evaluating $(1.025)^8$ by substituting $x = 0.025$ into an expansion for $(1 + x)^8$. Students should be advised that simply using their calculator to evaluate $(1.025)^8$ will gain no marks as it is not answering the question.

NOTES

Be aware of an alternative notation such as $\binom{n}{r}$ and ${}^{n}C_{r}$.

UNIT 4: Trigonometry

Return to overview

SPECIFICATION REFERENCES

5.1 Understand and use the definitions of sine, cosine and tangent for all arguments; the sine and cosine rules; the area of a triangle in the form $\frac{1}{2}ab\sin C$

5.2 Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity

5.3 Understand and use $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Understand and use $\sin^2\theta + \cos^2\theta = 1$

5.4 Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle

PRIOR KNOWLEDGE

Algebra covered so far

- Basic algebraic manipulation
- Quadratics
- Graph transformations

GCSE (9-1) in Mathematics at Higher Tier

G20 Pythagoras' Theorem

Trigonometry in right-angled triangles

G22 The sine rule

The cosine rule

G23 The area of a triangle

G15 Bearings

KEYWORDS

Sine, cosine, tangent, interval, period, amplitude, function, inverse, angle of elevation, angle of depression, bearing, degree, identity, special angles, unit circle, symmetry, hypotenuse, opposite, adjacent, intercept.

4a. Trigonometric ratios and graphs (5.1) (5.2)

Teaching time
5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use the definitions of sine, cosine and tangent for all arguments;
- understand and be able to use the sine and cosine rules;
- understand and be able to use the area of a triangle in the form $\frac{1}{2}ab\sin C$;
- understand and be able to use the sine, cosine and tangent functions; their graphs, symmetries and periodicity.

TEACHING POINTS

Students should be shown the x and y coordinates of points on the unit circle can be used to give cosine and sine respectively.

Use of trigonometric ratios will have been covered at GCSE (9-1) Mathematics; questions should now be focused more on multi-step problems and questions set in context.

When using the sine rule the ambiguous case should be covered.

Links to proof can be made, for example proving the area of a triangle.

Students should be encouraged to write down any formulae they will be using before substituting in the numbers.

Students should be able to solve questions in various contexts; these could include coordinate geometry or real-life situations. Questions may involve bearings, which may not be well remembered from GCSE so should be reviewed. Students should be encouraged to check that their answers are realistic as this check can show up errors.

When completing multi-step questions emphasise to students that they should show all working out and use the answer function on their calculators to avoid rounding errors. It can be a useful teaching point to divide the class asking one side to round all answers and the other to keep values stored in their calculator to show how this affects the final answer.

The unit circle can again be used to show how the trigonometric graphs are formed. Characteristics such as the period and amplitude should be discussed. Knowledge of graphs of curves with equations such as $y = \sin x$, $y = \cos (x + 30)$, $y = \tan 2x$ is expected so this is a good opportunity to recap transformations.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Use of the graphs can be linked to modelling situations such as yearly temperatures, wave lengths and tidal patterns.

Proof of the sine and cosine rules.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students occasionally assume that triangles given in exam questions are right-angled and so use right-angled trigonometric ratios rather than the sine and cosine rules.

A frequently seen error in these questions is students using the cosine rule to calculate an incorrect angle, sometimes despite having drawn a correctly labelled diagram. This indicates a lack of understanding of how the labelling of edges and angles on a diagram relates to the application of the cosine rule formula.

4b. Trigonometric identities and equations (5.3) (5.4)

Teaching time
7 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to solve trigonometric equations within a given interval
- understand and be able to use $\tan\theta = \frac{\cos\theta}{\sin\theta}$
- Understand and use $\sin^2\theta + \cos^2\theta = 1$

TEACHING POINTS

When solving trigonometric equations, finding multiple values within a range can initially be illustrated using the graphs of the functions. The decision can then be made whether to move on to using CAST diagrams or continue using graphs. Whichever method is used students will need plenty of practice in identifying all values within the limits correctly.

Intervals with negative solutions as well as positive solutions should be used.

Students should be able to solve equations such as $\sin(x + 70°) = 0.5$ for $0 < x < 360°$; $3 + 5\cos 2x = 1$ for $–180° < x < 180°$; and $6\cos 2x + \sin x − 5 = 0$ for $0 < x < 360°$, giving their answers in degrees.

Students should be comfortable factorising quadratic trigonometric equations and finding all possible solutions. It should be noted that in some cases only one of the factorisations will give solutions but in most case there will be two sets of solutions. Situations where one answer is equal to zero can cause some confusion with students then not looking for further solutions. This sort of example should be covered in class. For example, the equation, $\sin\theta(3\sin\theta + 1) = 0$ will often be simplified to just $3\sin\theta + 1 = 0$, resulting in the loss of solutions to the original equation.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Following from the previous section, if graphs are used to model situations then the equations can be used to find values at given points.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Common errors include: not finding values in the given range; finding extra, incorrect, solutions; not going on to find the values of x and instead leaving the values for, say $2x$ or $x + 30$; algebraic slips when rearranging the equation; and not giving answers to the correct degree of accuracy. The loss of accuracy in the final answers to trigonometric equations is common and often results in the unnecessary loss of marks.

Sketches of the trigonometric functions are often helpful to check all solutions have been found.

UNIT 5: Vectors (2D)

Return to overview

SPECIFICATION REFERENCES

9.1 Use vectors in two dimensions

9.2 Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form

9.3 Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations

9.4 Understand and use position vectors; calculate the distance between two points represented by position vectors

9.5 Use vectors to solve problems in pure mathematics and in context, (including forces)

PRIOR KNOWLEDGE

Covered so far

- Surds

GCSE (9-1) in Mathematics at Higher Tier

G24 Vectors

KEYWORDS

Vector, scalar, magnitude, direction, component, parallel, perpendicular, modulus, dimension, ratio, collinear, scalar product, position vectors.

5a. Definitions, magnitude/direction, addition and scalar multiplication (9.1) (9.2) (9.3)

Teaching time
5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to use vectors in two dimensions;
- be able to calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form;
- be able to add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.

TEACHING POINTS

Students need to be familiar with column vectors and with the use of **i** and **j** vectors in two dimentions.

Students should be able to find a unit vector in the direction of $\boldsymbol{a}$, and be familiar with the notation $|\boldsymbol{a}|$.

The triangle and parallelogram laws of addition should be known and students should be able to use them. Students should understand that vectors are commutative.

Where answers are given in surds they should be simplified if possible.

When performing operations on vectors this should also be understood geometrically, diagrams will be helpful here. Students should be able to use given diagrams but also draw their own in order to assist with questions.

Students should understand and be able to use the conditions for parallel vectors.

Use the classroom floor as a 2-dimensional grid to help students visualise vectors. Use the position of students in the room to illustrate concepts.

Consider vectors in the real world, e.g. ask students to think of everyday phenomena that have a magnitude and direction e.g. forces, velocities, displacements.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students can prove vectors are parallel to demonstrate their reasoning skill.

Given particular vectors, students can investigate places they can or cannot reach, for example the knights problem on a chessboard.

Consider an aircraft landing in a cross-wind – what direction does it need to fly?

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students sometimes make mistakes when manipulating vectors in **i** and **j** form and should be encouraged to use column vectors when possible.

5b. Position vectors, distance between two points, geometric problems (9.4) (9.5) | **Teaching time** 6 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use position vectors;
- be able to calculate the distance between two points represented by position vectors;
- be able to use vectors to solve problems in pure mathematics and in context, (including forces).

TEACHING POINTS

Students should know and be able to use $\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

Students should be able to calculate the distance between two points (x_1, y_1) and (x_2, y_2) using the formula $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$.

Use the ratio theorem to find the position vector of a point C dividing AB in a given ratio.

Use familiar shapes to illustrate the difference between 2 vectors and vector addition, e.g. parallelogram, rectangle.

When solving problems using vectors only pure contexts are covered.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Finding position vector of the fourth corner of a shape (e.g. parallelogram) *ABCD* with three given position vectors for the corners *A*, *B* and *C*.

Use regular polygons to find vectors connecting different vertices and to illustrate the ratio theorem.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Examiners comment that students understand the simple basics of vectors but are unable to deal with the complexity of ratios. Students should be given plenty of practice in identifying points that divide line segments in a particular ratio both externally and internally.

UNIT 6: Differentiation

Return to overview

SPECIFICATION REFERENCES

7.1 Understand and use the derivative of f(x) as the gradient of the tangent to the graph of y = f(x) at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change

Sketching the gradient function for a given curve

Second derivatives

Differentiation from first principles for small positive integer powers of x

7.2 Differentiate x^n, for rational values of n, and related constant multiples, sums and differences

7.3 Apply differentiation to find gradients, tangents and normals, maxima and minima and stationary points

Identify where functions are increasing or decreasing

PRIOR KNOWLEDGE

Covered so far

- Solving quadratics
- Coordinate geometry
- Proof
- Function notation
- Indices

GCSE (9-1) in Mathematics at Higher Tier

N8 Fractions

G16 Area of 2D shapes

Volume and surface area of 3D shapes

A5 Rearranging equations

KEYWORDS

Differentiation, derivative, first principles, rate of change, rational, constant, tangent, normal, increasing, decreasing, stationary point, maximum, minimum, integer, calculus, function, parallel, perpendicular.

6a. Definition, differentiating polynomials, second derivatives (7.1) (7.2)

Teaching time
5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use the derivative of f(x) as the gradient of the tangent to the graph of y = f(x) at a general point (x, y);
- understand the gradient of the tangent as a limit and its interpretation as a rate of change;
- be able to sketch the gradient function for a given curve;
- be able to find second derivatives;
- understand differentiation from first principles for small positive integer powers of x;
- be able to differentiate x^n, for rational values of n, and related constant multiples, sums and differences.

TEACHING POINTS

Students should know that $\frac{\mathrm{d}y}{\mathrm{d}x}$ is the rate of change of y with respect to x.

Knowledge of the chain rule is not required.

The notation f′(x) may be used for the first derivative and f″(x) may be used for the second order derivative.

Students should be able to identify maximum and minimum points as points where the gradient is zero.

Cover the use of the second derivative to establish the nature of a turning point.

Students should be able to sketch the gradient function f′(x) for a given curve y = f(x), using given axes and scale. This could involve speed and acceleration for example.

Students should know how to differentiate from first principles. Students should be able to use, for n =2 and n = 3, the gradient expression $\lim_{h\to 0}\left(\frac{(x+h)^n-x^n}{h}\right)$. The alternative notations $h \to 0$ rather than $\delta x \to 0$ are acceptable.

Students will need to be confident in algebraic manipulation of functions to ensure that they are in a suitable format for differentiation. For example, students will be expected to be able to differentiate expressions such as $(2x + 5)(x - 1)$ and $\frac{x^2+3x-5}{4x^{\frac{1}{2}}}$, for $x > 0$. Mistakes are easily made with negative and/or fractional indices so there should be plenty of practice with this.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Maxima and minima problems set in the context of a practical problem, e.g. minimising the materials required to make a container of a particular shape. The open box problem – simple cases and the general case.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Algebraic manipulation, particularly where surds are involved, can cause problems for students. For example, when multiplying out brackets and faced with $-4\sqrt{x}\times-4\sqrt{x}$ common incorrect answers are $-4\sqrt{x}$, $\pm16\sqrt{x}$ and $\pm16x^{\frac{1}{4}}$. Similarly, when dividing by $\sqrt{x}$, some students think that $\frac{x}{\sqrt{x}} = 1$.

6b. Gradients, tangents, normals, maxima and minima (7.3)

Teaching time
6 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to apply differentiation to find gradients, tangents and normals, maxima and minima and stationary points;
- be able to identify where functions are increasing or decreasing.

TEACHING POINTS

Students should be able to use differentiation to find equations of tangents and normals at specific points on a curve. This reviews and extends the earlier work on coordinate geometry.

Maxima, minima and stationary points can be used in curve sketching. Problems may be set in the context of a practical problem. This could bring in area and volume from GCSE (9-1) Mathematics as well as using trigonometry.

Students will need plenty of practice at setting up equations from a given context, in some cases this may include showing that it can be written in a particular form. Where students are given the answer to work towards they must be aware that they need to work forwards showing all steps clearly rather than starting with the answer and working backwards.

Students need to know how to identify when functions are increasing or decreasing. For example, given that $f'(x) = x^2 - 2 + \frac{1}{x^2}$, , prove that f($x$) is an increasing function.

Use graph plotting software that allows the derivative to be plotted so that students can see the relationship between a function and its derivative graphically.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Differentiation can be linked to many real-world applications, there can be discussion with students about contexts and the validity of solutions.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students may have difficult differentiating fractional terms such as $\frac{8}{x}$ if they are unable to rewrite this as $8x^{-1}$ before differentiating.

When working out the equations of tangents and normal, some students mix the gradients and equations up and end up substituting in the wrong place.

Questions involving finding a maximum or minimum point do require the use of calculus and attempts using trial and improvement will receive no marks.

When finding a stationary point, some students use inequalities as their condition rather than equating their derivative to zero. Another error is to differentiate twice and solve $f''(x) = 0$.

When applying differentiation in context, students should be ensure they give full answers and not just a partial solution. For example if asked to find the volume of a box they must not stop after finding the side length.

UNIT 7: Integration

Return to overview

SPECIFICATION REFERENCES

8.1 Know and use the Fundamental Theorem of Calculus

8.2 Integrate x^n (excluding $n = -1$), and related sums, differences and constant multiples

8.3 Evaluate definite integrals; use a definite integral to find the area under a curve

PRIOR KNOWLEDGE

Covered so far

- Algebraic manipulation
- Differentiation

KEYWORDS

Calculus, differentiate, integrate, reverse, indefinite, definite, constant, evaluate, intersection.

7a. Definition as opposite of differentiation, indefinite integrals of x^n (8.1) (8.2)

Teaching time
5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- know and be able to use the Fundamental Theorem of Calculus;
- be able to integrate x^n (excluding $n = -1$), and related sums, differences and constant multiples.

TEACHING POINTS

Integration can be introduced as the reverse process of differentiation. Students need to know that for indefinite integrals a constant of integration is required.

Similarly to differentiation, students should be confident with algebraic manipulation. For example, the ability to integrate expressions such as $\frac{1}{2}x^2 - 3x^{-\frac{1}{2}}$ and $\frac{(x+2)^2}{x^{\frac{1}{2}}}$ is expected. Introduce students to the integral sign; this can be useful in setting work out clearly on these sorts of questions and will be used later in definite integration.

Given $f'(x)$ and a point on the curve, students should be able to find an equation of the curve in the form $y = f(x)$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to explain the need for the $+ c$ in indefinite integration.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students sometimes have difficulty when integrating expressions involving negative indices. Forgetting to add $+ c$ when working out indefinite integrals is also a very common mistake.

7b. Definite integrals and areas under curves (8.3)

Teaching time
5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to evaluate definite integrals;
- be able to use a definite integral to find the area under a curve.

TEACHING POINTS

It is important that students show their working out clearly as mistakes are easily made when putting values into a calculator. Students should also be encouraged to check their answers. Calculators that perform numerical integration can be used as a check, but a full method will be needed.

Students will be expected to understand the implication of a negative answer from indefinite integration.

Links can be made with curve sketching in questions where students need to find the points of intersection with the x-axis for a curve in order to find the limits of integration.

Areas can be made up of a combination of a curve and a line so further links can be made to coordinate geometry.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Discuss the implication of a negative answer to encourage students reasoning skills.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Lack of algebraic fluency can cause problems for some students, particularly when negative/fractional indices are involved or when a negative number is raised to a power. Arithmetic slips are also a common cause of lost marks, often when negative numbers are substituted and subtracted after integration.

Students are generally more successful if they expand any brackets before attempting to integrate the function.

UNIT 8: Exponentials and logarithms
Exponential functions and natural logarithms
(6.1) (6.2) (6.3) (6.4) (6.5) (6.6) (6.7)

Teaching time
8 hours

Return to overview

SPECIFICATION REFERENCES

6.1 Know and use the function a^x and its graph, where a is positive

Know and use the function e^x and its graph

6.2 Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications

6.3 Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$

Know and use the function $\ln x$ and its graph

Know and use $\ln x$ as the inverse function of e^x

6.4 Understand and use the laws of logarithms:

$$\log_a x + \log_a y = \log_a (xy)$$

$$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$$

$k\log_a x = \log_a x^k$(including, for example, $k = -1$ and $= -\frac{1}{2}$)

6.5 Solve equations of the form $a^x = b$

6.6 Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y

6.7 Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models

PRIOR KNOWLEDGE

Covered so far

- Indices

GCSE (9-1) in Mathematics at Higher Tier

R16 Compound interest

KEYWORDS

Exponential, exponent, power, logarithm, base, initial, rate of change, compound interest

OBJECTIVES

By the end of the sub-unit, students should:

- know and be able to use the function $a^x$and its graph, where *a* is positive;
- know and be able to use the function e^x and its graph;
- know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications;
- know and be able to use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$;
- know and be able to use the function $\ln x$ and its graph;
- know and be able to use ln *x* as the inverse function of e^x ;
- understand and use the laws of logarithms:

  $\log_a x + \log_a y = \log_a (xy)$

  $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$

  $k\log_a x = \log_a x^k$(including, for example, k =–1 and $k = -\frac{1}{2}$)
- be able to solve equations of the form $a^x = b$;
- be able to use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for *x* and *y*;
- understand and be able to use exponential growth and decay in modelling, giving consideration to limitations and refinements of exponential models.

TEACHING POINTS

When sketching the graph of a^xstudents should understand the difference in shape between *a* < 1 and *a* > 1.

Explain to students that e^x is a special case of a^x. Graphs of the function e^x should include those in the form $y = e^{ax+b} + c$.

Students should realise that when the rate of change is proportional to the *y*-value, an exponential model should be used.

An ability to solve equations of the form $e^{ax+b} = p$ and $\ln (ax + b) = q$ is expected.

Students can use the laws of indices to prove the laws of logarithms and show that $\log_a a = 1$.

In solving equations students may use the change of base formula. Solving equations questions may be in the form $2^{3x-1} = 3$.

Students should be able to plot log *y* against log *x* and obtain a straight line where the intercept is $\log_a$ and the gradient is *n* and plot log *y* against *x* to obtain a straight line where the intercept is log *k* and the gradient is log *b*. There should be discussion about why this is an appropriate model and why it is only an estimate.

Contexts for modelling should could include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth. Students should be familiar with terms such as initial, meaning when *t* = 0. They may need to explore the behaviour for large values of *t* or to consider whether the range of values predicted is appropriate. Consideration of a second improved model may be required.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students can look at different models for population growth using the exponential function.

Use graphing software to investigate varying the parameters of a population model.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Errors seen in exam questions where students have to sketch exponential curves include: stopping the curve at $x = 0$; getting the wrong y-intercept; and believing the curve levels off to $y = 1$ for $x < 0$.

When using laws of logs to answer proof or 'show that' questions, students must show all the steps clearly and not have jumps in their working out.

AS Mathematics applied content
Section A: Statistics

Unit		Title	Estimated hours
1		**Statistical sampling**	
	a	Introduction to sampling terminology; Advantages and disadvantages of sampling	**1**
	b	Understand and use sampling techniques; Compare sampling techniques in context	**1**
2		**Data presentation and interpretation**	
	a	Calculation and interpretation of measures of location; Calculation and interpretation of measures of variation; Understand and use coding	**3**
	b	Interpret diagrams for single-variable data; Interpret scatter diagrams and regression lines; Recognise and interpret outliers; Draw simple conclusions from statistical problems	**5**
3		**Probability:** Mutually exclusive events; Independent events	**2**
4		**Statistical distributions:** Use discrete distributions to model real-world situations; Identify the discrete uniform distribution; Calculate probabilities using the binomial distribution (calculator use expected)	**4**
5		**Statistical hypothesis testing**	
	a	Language of hypothesis testing; Significance levels	**3**
	b	Carry out hypothesis tests involving the binomial distribution	**5**
			24 hours

UNIT 1: Statistical sampling

Return to overview

SPECIFICATION REFERENCES

1.1 Understand and use the terms 'population' and 'sample'
Use samples to make informal inferences about the population
Understand and use sampling techniques, including simple random sampling and opportunity sampling.
Select or critique sampling techniques in the context of solving a statistical problem, including understanding that different samples can lead to different conclusions about the population

PRIOR KNOWLEDGE

GCSE (9–1) in Mathematics at Higher Tier

S1 Infer properties of populations or distributions from a sample, while knowing the limitations of sampling

S5 Apply statistics to describe a population

KEYWORDS

Population, census, sample, sampling unit, sampling frame, simple random sampling, stratified, systematic, quota, opportunity (convenience) sampling.

1a. Introduction to sampling terminology; Advantages and disadvantages of sampling (1.1)

Teaching time
1 hour

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use the terms 'population' and 'sample';
- know how to use samples to make informal inferences about the population;
- be able to describe advantages and disadvantages of sampling compared to census.

TEACHING POINTS

This section is a great opportunity to introduce the large data set to look at a population of data and discuss reasons for sampling from it.

Students will be expected to be able to comment on the advantages and disadvantages associated with a census and a sample.

Discuss in context the meanings of populations and samples. Look at data from populations and samples, initially using data from the sample to make inferences about the population before then checking the data for the population.

Discuss the advantages and disadvantages of sampling making sure to include time, cost etc.

Ensure students are given the opportunity, and are able, to give full and thorough answers within the context of the question.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

The biggest opportunity here is introducing students to the large data set and starting to get them familiar with the data included in it.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Some students confuse sample sizes and population sizes, but the recurring problem is not giving answers in context. Candidates need to be clear about the difference between sample sizes and population sizes.

1b. Understand and use sampling techniques; Compare sampling techniques in context (1.1)

Teaching time
1 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use sampling techniques;
- be able to describe advantages and disadvantages of sampling techniques;
- be able to select or critique sampling techniques in the context of solving a statistical problem;
- understand that different samples can lead to different conclusions about the population.

TEACHING POINTS

Students will also be expected to be familiar with different types of sampling including simple random, stratified, systematic, quota and opportunity (convenience) sampling.

Students will gain a more thorough understanding of the types of sampling if the advantages and disadvantages alongside the method used for each type are understood. They will then be more able to select an appropriate technique for a given statistical problem and be able to critique a technique which has been used.

Give students the opportunity to use the techniques they learn about on the large data set.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Again, this is a perfect opportunity to use the large data set and discuss how different samples from the same data set could lead to different conclusions.

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

Students need to be able to describe the sampling techniques clearly and will lose marks if they are not sufficiently precise.

As always, answers must be given using the context of the question and not simply be quoted from text books in a general form.

UNIT 2: Data presentation and interpretation

Return to overview

SPECIFICATION REFERENCES

2.1 Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency

Connect to probability distributions

2.2 Interpret scatter diagrams and regression lines for bivariate data, including recognition of scatter diagrams which include distinct sections of the population (calculations involving regression lines are excluded)

Understand informal interpretation of correlation

Understand that correlation does not imply causation

2.3 Interpret measures of central tendency and variation, extending to standard deviation
Be able to calculate standard deviation, including from summary statistics

2.4 Recognise and interpret possible outliers in data sets and statistical diagrams
Select or critique data presentation techniques in the context of a statistical problem
Be able to clean data, including dealing with missing data, errors and outliers

PRIOR KNOWLEDGE

GCSE (9–1) in Mathematics at Higher Tier

S2 Interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data and know their appropriate use

S3 Construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use

S4 Interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers), quartiles and inter-quartile range

S6 Use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends while knowing the dangers of so doing

KEYWORDS

Histogram, box plot, probability density function, cumulative distribution function, continuous random variable, scatter diagram, linear regression, explanatory (independent) variables, response (dependent) variables interpolation, extrapolation, product moment correlation coefficient (PMCC), mean, median, mode, variance, standard deviation, range, interquartile range, interpercentile range, outlier, skewness, symmetrical, positive skew, negative skew.

2a. Calculation and interpretation of measures of location; Calculation and interpretation of measures of variation; Understand and use coding (2.3) (2.4)

Teaching Time
3 Hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to calculate measures of location, mean, median and mode;
- be able to calculate measures of variation, standard deviation, variance, range and interpercentile range;
- be able to interpret and draw inferences from summary statistics.

TEACHING POINTS

The calculation of the mean, median and mode should be recapped from GCSE however the focus now is on students using calculators to do the calculations. Check understanding of the terminology and teach calculator methods.

Students require an understanding of measures of variation too and should be able to use their calculators to calculate the variance and standard deviation. They should be able to use the statistic $S_{xx} = \sum (x - \bar{x})^2 = \sum x^2 - \frac{\left(\sum x\right)^2}{n}$. Students are expected to use standard deviation $= \sqrt{\frac{S_{xx}}{n}}$ but equivalents including spreadsheet formula ($s = \sqrt{\frac{S_{xx}}{n-1}}$) will be accepted.

The data may be discrete or continuous, grouped or ungrouped, and students need to be able to interpret these summary statistics clearly and be able to make inferences from them. Significance tests will not be expected.

Coding for both mean and standard deviation needs to be covered. Be clear that students need to be able to uncode both mean and standard deviation. Emphasise that the standard deviation is unaffected by the addition or subtraction of constants.

Students are expected to be able to use linear interpolation to calculate percentiles from grouped data.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

There is opportunity for further use of the large data set here. Summary statistics of elements from the data set can be calculated and then used to compare and interpret for both location and variation statistics.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

When calculating the mean, of grouped data some student may divide by the number of groups rather than the number of items of data, they may also use class widths in the calculation rather than the mid-points.

When finding the standard deviation, the most common error is forgetting to take the square root (perhaps because they are not clear about the difference between variance and standard deviation). Some students waste time by ignoring given values and recalculating Σfx and Σfx^2.

Difficulties with coding are due to a lack of understanding about how coding affects the mean and standard deviation, and poor algebraic skills. Students sometimes substitute for the wrong variable, fail to solve equations correctly or get the order of operations the wrong way around.

Students should be reminded that they must be precise in their use of language and use the correct terms such as 'median'. 'range' or 'inter-quartile range' rather than the more general 'average' and 'spread'. Students should also remember to use accurate values throughout calculations to avoid losing marks due to premature rounding.

NOTES

Students are expected to know the different notation for population summary statistics (μ, σ^2, σ) and sample summary statistics $(\bar{x}, s^2, s)$.

2b. Interpret diagrams for single-variable data; Interpret scatter diagrams and regression lines; Recognise and interpret outliers; Draw simple conclusions from statistical problems (2.1) (2.2) (2.4)

Teaching time
5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- know how to interpret diagrams for single variable data;
- know how to interpret scatter diagrams and regression lines for bivariate data;
- recognise the explanatory and response variables;
- be able to make predictions using the regression line and understand its limitations;
- understand informal interpretation of correlation;
- understand that correlation does not imply causation;
- recognise and interpret possible outliers in data sets and statistical diagrams;
- be able to select or critique data presentation techniques in the context of a statistical problem;
- be able to clean data, including dealing with missing data, errors and outliers.

TEACHING POINTS

Students should be familiar with and be able to interpret histograms, frequency polygons, box and whisker plots and cumulative frequency diagrams. These should have been covered at GCSE but it is worth a recap for consistency of methods. Also cover calculating summary statistics from diagrams, including the mean and standard deviation from a histogram.

For bivariate data students should understand the terms explanatory and response variables and know where each is placed on the axes of a scatter diagram. This is particularly important as variables other than y and x could be used.

Students are not expected to know, calculate or understand the regression line formula. Students will need to understand the use of interpolation when using a regression line equation to make predictions within the range of values of the explanatory variable and they need to understand the dangers of extrapolation (predictions outside the range), again variables other than y and x could be used.

Students will be expected to describe the correlation on a scatter diagram in terms of positive, negative or no correlation and strong or weak but no calculations need to be made. Values from calculations will not be given for interpretation.

Outliers will need to be identified and interpreted from data sets and statistical diagrams. Any rules to be used will be given in the question, for example $Q_1 - 1.5 \times IQR$, $Q_3 + 1.5 \times IQR$.

Students will be expected to select an appropriate diagram or critique the choice of one which is used. They should also be able to clean data by identifying possible outliers (box plots and scatter diagrams). They may also be asked to fill in missing data using a regression line.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Again all of the diagrams and techniques used in this unit could be modelled using data from the large data set.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Knowing how to interpret statistics students have calculated is sometimes found challenging, and often discriminates between students in exam questions. Full and clear reasons for interpretations and decisions need to be given for marks to be awarded.

Many students have difficulties calculating the sizes of bars in histograms, as commented on by one examiner: 'Most were able to state the correct width of the bar but few used frequency densities correctly to find the height, some finding the frequency density of but then calculating $\frac{1}{3} \times 2.5$ rather than $2.5 \div \frac{1}{3}$. Some identified that 1.5 cm^2 represented 10 customers but were then unable to use this correctly to find the height … some students had an incorrect class width because they did not realize that the lower class boundary was 70 not 69.5.'

UNIT 3: Probability Mutually exclusive events; Independent events (3.1)	Teaching time 2 hours

Return to overview

SPECIFICATION REFERENCES

3.1 Understand and use mutually exclusive and independent events when calculating probabilities Link to discrete and continuous distributions

PRIOR KNOWLEDGE

GCSE (9–1) in Mathematics at Higher Tier

P1 Record, describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees

P2 Apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments

P3 Relate relative expected frequencies to theoretical probability, using appropriate language and the 0–1 probability scale

P4 Apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one

P6 Enumerate sets and combinations of sets systematically, using tables, grids

P7 Construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities

P9 Tree diagrams and Venn diagrams

KEYWORDS

Sample space, exclusive event, complementary event, discrete random variable, continuous random variable, mathematical modelling, independent, mutually exclusive, Venn diagram, tree diagram.

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use mutually exclusive and independent events when calculating probabilities;
- be able to make links to discrete and continuous distributions.

TEACHING POINTS

Tree and Venn diagrams should have been covered at GCSE but will need to be recapped as one way of looking at probabilities.

The focus at this level is on independent and mutually exclusive events in probability calculations. Students should be confident in the definitions of both independent and mutually exclusive events and how to use their properties to solve real-life probability problems.

Cover showing independence but be aware that the use of set notation is not required at AS level. At this level this is done by showing the product of the probabilities of two events gives the probability of both events occurring together. Understanding of conditional probability is not expected at AS level.

Students do not need to be aware of probability density functions however they should understand that probability is represented by the area under a curve in a continuous distribution. This could be mentioned here and comparisons drawn by using the binomial model as a bar chart in the next unit.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Include questions of the type where A and B are independent which use Venn diagrams and informal use of the addition rule but where both $P(A)$ and $P(A \cap B)$ for example are unknown; the solution relies on a knowledge of independence. (Set notation not required)

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students may confuse ‘independent’ and ‘mutually exclusive’.

Using a diagram almost always helps students to answer probability questions. When drawing a Venn diagram, students should remember to include a box defining the universal set.

UNIT 4: Statistical Distributions **Use discrete distributions to model real-world situations; Identify the discrete uniform distribution; Calculate probabilities using the binomial distribution (calculator use expected) (4.1)**	**Teaching time** 4 hours

Return to overview

SPECIFICATION REFERENCES

4.1 Understand and use simple, discrete probability distributions (calculation of mean and variance of discrete random variables is excluded), including the binomial distribution, as a model; calculate probabilities using the binomial distribution

PRIOR KNOWLEDGE

An understanding of probability from the previous unit and the awareness that the area under a curve will be looked at again in this unit.

GCSE (9–1) in Mathematics at Higher Tier

N1 Order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, and ≥

KEYWORDS

Binomial, probability, discrete distribution, discrete random variable, uniform, cumulative probabilities.

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use simple, discrete probability distributions, including the binomial distribution;
- be able to identify the discrete uniform distribution;
- be able to calculate probabilities using the binomial distribution.

TEACHING POINTS

Students will be expected to model real-world situations by using simple discrete probability distributions. They should know and be able to recognise a discrete uniform distribution; look at equally likely outcomes such as numbers on a dice.

The only specific distribution students are expected to use as well as understand is the binomial distribution. Students will be expected to comment critically on how appropriate a given probability model may be for a situation.

The notation $X \sim \mathrm{B}(n, p)$ may be used, so you should ensure students are familiar with this from the outset. Make sure the properties of the binomial are clear for all students, so that they know a fixed number of trials is needed, there are only two possible outcomes per trial and the outcome of each trial is independent.

Once the binomial distribution has been introduced link back to thinking about probability being the area under a curve. Use a bar chart for discrete binomial distributions and show how this would smooth into a curve if it were a continuous distribution. Another teaching point for this concept of area could come from considering the discrete uniform distribution as bars of equal width; it looks like a rectangle, like the continuous uniform distribution.

Students need to calculate probabilities using the binomial distribution for both individual and cumulative probabilities. Calculator use is expected for all of this, so time needs to be spent making sure students are competent in the use of these calculator functions.

The bar chart model mentioned earlier helps students distinguish between for example $\mathrm{P}(X < 2)$ and $\mathrm{P}(X \le 2)$, also to understand $\mathrm{P}(X \ge 6) = 1 - \mathrm{P}(X \le 5)$. Explain this is due to the binomial being a discrete distribution. This is essential when manipulating before using the calculator to find probabilities. Encourage students to shade the bars required to help with this understanding.

Emphasise the importance of reading questions carefully. The probability of success can be worded negatively in the question for example ‘the probability of people failing their driving test first time is 0.6’.

Students are not expected to be able to calculate the mean and variance of discrete random variables.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Look at a wide variety of real-world scenarios and model using a number of different distributions to ensure students are fluent in their comments on the appropriateness of a particular distribution.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

The most common difficulty is with manipulating inequalities: 'A significant number of students were unable to cope with the expression $P(5 \leq X < 11)$. There were students who translated this expression into the more convenient form $P(5 \leq X \leq 10)$ and then in turn transformed this into an equivalent form that can be applied to the table of cumulative probabilities: $P(X \leq 10) - P(X \leq 4)$. However, there were also many instances of incorrect versions such as: $P(X < 11) - P(X \geq 5)$, $P(X \leq 10) + P(X \geq 5)$, $P(X \leq 10) - (1 - P(X \geq 5))$ and $P(X \leq 11)$ – either $P(X \leq 5)$ or $P(X \leq 4)$.'

In a similar vein, students have a tendency to write, for example, $P(X > 2)$ as $1 - P(X \leq 1)$ instead of $1 - P(X \leq 2)$.

NOTES

It would be good for understanding to see what the random variable looks like in pdf form and table form although this is not explicitly in the specification.

UNIT 5: Statistical hypothesis testing

Return to overview

SPECIFICATION REFERENCES

5.1 Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, p-value

5.2 Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context

Understand that a sample is being used to make an inference about the population and appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis

PRIOR KNOWLEDGE

An understanding of how to calculate binomial probabilities and using samples from populations from previous units.

KEYWORDS

Hypotheses, significance level, one-tailed test, two-tailed test, test statistic, null hypothesis, alternative hypothesis, critical value, critical region, acceptance region, p-value, binomial model, accept, reject, sample, inference.

5a. Language of hypothesis testing; Significance levels (5.1)

Teaching time
3 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to apply the language of statistical hypothesis testing, developed through a binomial model.

TEACHING POINTS

The concept of a hypothesis could be introduced initially by posing some hypotheses yourself. You may wish to make reference to the large data set again and say for example 'the daily maximum temperature was higher in Hurn than Heathrow in May 1987'.

Following this introduce the null and alternative hypotheses and their respective notation H_0 and H_1. Discuss how to move from statements like the one above to using the language of the binomial distribution in terms of looking at p, the probability of success.

The focus of this sub-unit is the language used in terms of hypothesis testing, but a scenario must be set. You may wish to use an example like 'the number of 6s thrown in 50 throws of a dice', students could carry out this experiment and you could use their results to form a variety of tests which would cover all of the terminology without actually carrying out the tests. Make sure you save these examples to be tested in the next sub-unit.

All of the terms from the keywords section should be thoroughly discussed and understood before attempting to carry out a hypothesis test.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Use a wide variety of scenarios from the real world and invite students to offer their own scenarios; discuss their suitability for hypothesis testing.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Emphasise the importance of stating hypotheses clearly using the correct notation.

Similarly, correct notation is important when describing the critical region: 'There were still a few students using incorrect notation for critical regions: $P(X \leq 1)$, for example, is not a critical region: it is a probability.'

NOTES

The expected value of the binomial distribution being np needs to be appreciated for a two-tailed test.

5b. Carry out hypothesis tests involving the binomial distribution (5.2)	**Teaching time** 5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context;
- understand that a sample is being used to make an inference about the population;
- appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis.

TEACHING POINTS

Once all the terminology that has been discussed in the previous sub-unit is fully understood, you can go back to the examples you used and conduct the hypothesis tests. Carry out the tests both by finding the critical value to compare with your test statistic and by finding the probability (p-value) of the test statistic and comparing it with the critical region. Ensure students are competent with both methods. Make sure hypotheses are always written clearly in terms of ρ, the probability of success.

Spend time making sure that students can write clear and concise conclusions in the given context of the questions.

When using a sample of data, ensure students understand, what it infers about the population itself.

Type I errors are not part of the specification, but it is important that students understand what the significance level of a test actually means. Discuss carefully that rejecting the null hypothesis may actually be incorrect and the significance level is the probability of this. Also cover 'the actual significance level of a test' with students.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Again, use a wide variety of scenarios from the real world and make sure all conclusions are written in these contexts.

COMMON MISCONCEPTIONS/ EXAMINER REPORT QUOTES

The most common error in these sorts of questions include not writing a clear conclusion in the context of the question. Students either omit the context or sometimes fail to give any conclusion to their calculations.

AS Mathematics applied content
Section B – Mechanics

Unit		Title	Estimated hours
6		**Quantities and units in mechanics**	
	a	Introduction to mathematical modelling and standard S.I. units of length, time and mass	1
	b	Definitions of force, velocity, speed, acceleration and weight and displacement; Vector and scalar quantities	1
7		**Kinematics 1 (constant acceleration)**	
	a	Graphical representation of velocity, acceleration and displacement	2
	b	Motion in a straight line under constant acceleration; *suvat* formulae for constant acceleration; Vertical motion under gravity	3
8		**Forces & Newton's laws**	
	a	Newton's first law, force diagrams, equilibrium, introduction to **i**, **j** system	3
	b	Newton's second law, '$F = ma$', connected particles (no resolving forces or use of $F = \mu R$); Newton's third law: equilibrium, problems involving smooth pulleys	3
9		**Kinematics 2 (variable acceleration)**	
	a	Variable force; Calculus to determine rates of change for kinematics	3
	b	Use of integration for kinematics problems i.e. $r = \int v \, dt$, $v = \int a \, dt$	2
			18 hours

UNIT 6: Quantities and units in mechanics

Return to overview

SPECIFICATION REFERENCES

6.1 Understand and use fundamental quantities and units in the S.I. system: length, time, mass.
Understand and use derived quantities and units: velocity, acceleration, force, weight

PRIOR KNOWLEDGE

GCSE (9-1) in Mathematics at Higher Tier

R1 Change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts

R11 Use compound units such as speed, rates of pay, unit pricing, density and pressure

A14 Plot and interpret graphs (including reciprocal graphs and exponential graphs) and graphs of non-standard functions in real contexts to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

A15 Calculate or estimate gradients of graphs and area under graphs (including quadratic and non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts

KEYWORDS

Modelling, smooth, rough, light, inelastic, inextensible, particle, rigid body, mass, weight, rod, plane, lamina, length, distance (m), displacement (m), velocity (m s^{-1}), speed (m s^{-1}), acceleration (m s^{-2}), force (N), retardation (m s^{-2}), newtons (N), scalar, vector, direction, magnitude, (normal) reaction, friction, tension, thrust, compression

NOTES

There may not be a direct examination question on this topic. However, the modelling process and fluent knowledge of the S.I. units is a vital pre-requisite that underpins the rest of the mechanics course.

6a. Introduction to mathematical modelling and standard S.I units of length, time and mass (6.1)

Teaching time
1 hour

OBJECTIVES

By the end of the sub-unit, students should:

- understand the concept of a mathematical model, and be able to abstract from a real-world situation to a mathematical description (model);
- know the language used to describe simplifying assumptions;
- understand the particle model;
- be familiar with the basic terminology for mechanics;
- be familiar with commonly-made assumptions when using these models;
- be able to analyse the model appropriately, and interpret and communicate the implications of the analysis in terms of the situation being modelled;
- understand and use fundamental quantities and units in the S.I. system: length, time and mass;
- Understand that units behave in the same way as algebraic quantities, e.g. meters per second is $m/s = m \times 1/s = ms^{-1}$.

TEACHING POINTS

Begin by asking students 'What is mechanics?' Lead them to the idea that mechanics is a branch of applied mathematics that deals with motion and the forces producing motion.

Students need to be comfortable with the idea that mathematics is used to model real life and need to become familiar with the modelling cycle:

> mechanics problem → create a mathematical model (using diagrams, general principles or formulae) → solve the model → refer back to the original problem → refine the model

[Link with the data-handling cycle]

It is important for students to get a 'feel' for mechanics at this early stage in order to support later work.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Examples of problems that may be solved in this way include:

- How far apart should the cameras be within an average speed zone?
- At what angle should you hold an umbrella to keep snow off you?

Some examples of simplifying assumptions for these problems include:

- treating the car as a particle
- motion is in a straight line
- snow falls vertically.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students can generally correctly state assumptions, but they need to make sure that any assumptions or statements about the model relate directly to the context they are considering. For example they could make the comment ‘the resistance will not be constant’ more specific by saying ‘resistance will increase as velocity increases’.

NOTES

The particle (point mass) model is introduced here, i.e. the body has no size but does have mass, so rotation is ignored and the forces all act at one point.
The language of simplifying assumptions (light, smooth, uniform, inextensible, thin, rigid etc) is mostly introduced in subsequent sections.

6b. Definitions of force, velocity, speed, acceleration, weight and displacement; Vector and scalar quantities (6.1)	Teaching time 1 hour

OBJECTIVES

By the end of the sub-unit, students should:

- understand and use derived quantities and units: velocity, acceleration, force, weight;
- know the difference between position, displacement and distance;
- know the difference between velocity and speed, and between acceleration and magnitude of acceleration;
- know the difference between mass and weight (including gravity);
- understand that there are different types of forces.

TEACHING POINTS

Revise GCSE (9-1) in Mathematics compound units for speed and acceleration and make sure that students are comfortable converting from one unit to another, e.g. from km h^{-1} into m s^{-1}.

Define the vector quantities displacement and velocity as the vector versions of distance and speed respectively.

Begin by walking across the room and explaining the difference between position (referred to a fixed origin), displacement (a vector measured from any position) and distance (a scalar quantity for the total movement). Then move onto discussing speed (the rate at which an object covers distance) and velocity (the rate of change of displacement or speed in a certain direction.

Mention the special acceleration (for a falling object) due to gravity. In this course, this value is assumed to be a constant g, usually 9.8 m s^{-2} though it does vary in the real world.

This could be a good opportunity to dispel common misconceptions around weight and mass. Make it clear that mass is the amount of 'stuff' something is made of, is a scalar and is fixed (in kg), whereas weight is a force of attraction between an object and the centre of the earth and can vary depending on gravity and is measured in newtons. Hence weight = mass × gravity (or $W = mg$).

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Show some basic force diagrams (as an introduction to Unit 8a) to illustrate different types of forces such as weight, reaction and tension (all in newtons).

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

As mentioned above, students may mix up mass and weight and their related units. Some struggle to use the correct vocabulary e.g. for velocity and displacement. It is important to be really clear when giving the definitions and to always use the correct vocabulary in discussions.

NOTES

Defining the units of acceleration as 'metres per second per second' helps explain the concept of rate of change of speed. Show that m/s/s is algebraically equivalent to ms^{-2}. It may also help to think about it in terms of 'how many metres per second of speed is the object gaining every second?'

UNIT 7: Kinematics 1 (constant acceleration)

Return to overview

SPECIFICATION REFERENCES

7.1 Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration

7.2 Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time and interpretation of gradient; velocity against time and interpretation of gradient and area under the graph

7.3 Understand, use and derive the formulae for constant acceleration for motion in a straight line

8.3 Understand and use weight and motion in a straight line under gravity; gravitational acceleration, *g*, and its value in S.I. units to varying degrees of accuracy

PRIOR KNOWLEDGE

GCSE (9-1) in Mathematics at Higher Tier

R1 Change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts

R11 Use compound units such as speed, rates of pay, unit pricing, density and pressure

A2 Substitute numerical values into formulae and expressions, including scientific formulae

A5 Understand and use standard mathematical formulae; rearrange formulae to change the subject

A14 Plot and interpret graphs (including reciprocal graphs and exponential graphs) and graphs of non-standard functions in real contexts to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

A15 Calculate or estimate gradients of graphs and area under graphs (including quadratic and non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts

A17 Solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation)

A18 Solve quadratic equations (including those that require rearrangement) algebraically by factorising, by completing the square and by using the quadratic formula

AS Mathematics – Pure Mathematics content

3.1 Gradient (See Unit 2a of the SoW)

KEYWORDS

Distance (m), displacement (m), speed (m s^{-1}), velocity (m s^{-1}), acceleration (m s^{-2}), retardation (m s^{-2}), deceleration (m s^{-2}), scalar, vector, 2D, linear, area, trapezium, gradient, equations of motion, gravity, constant, 9.8 m s^{-2}, vertical.

NOTES

The guidance on the specification document states that graphical solutions to problems may be required. This section assumes *constant* acceleration; hence the graphical approach involves linear line segments and the familiar equations of linear motion *suvat*, formulae for constant acceleration. (N.B. 'equation of motion' refers to $F = ma$, and is nothing to do with these formulae).

The guidance also states that derivation of constant acceleration formulae may use knowledge of sections 7.2 and/or 7.4 (Unit 9).

Kinematics 2 (Unit 9) analyses particles' motion under a variable force, hence a variable acceleration. The mathematical model for this requires calculus which is covered in AS Mathematics – Pure Mathematics content, see SoW Units 6 and 7.

The usual value for g in this course is $9.8\ \mathrm{m\,s^{-2}}$, but some questions may specify a different value. Students may assume that g is constant, but should be aware that it is not a universal constant but depends on location.

7a. Graphical representation of velocity, acceleration and displacement (7.1) (7.2)	Teaching time 2 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to draw and interpret kinematics graphs, knowing the significance (where appropriate) of their gradients and the areas underneath them.

TEACHING POINTS

Introduce this topic by making links to the GCSE (9-1) in Mathematics prior knowledge for distance–time (travel) and speed–time graphs. Kinematics is the analysis of a particle's motion without reference to the resultant force that caused that motion.

Stress that forces causing the motion of the body in this section are *constant*, therefore acceleration is constant and this results in a *straight line* travel speed-time or velocity-time graph.

Extend the ideas to displacement by considering a particle which moves in reverse direction back beyond the starting point.

For a velocity–time graph, consider the units for the area of a unit square 1 m s^{-1} by 1 s. The 's' cancels, leaving 'm', therefore the area represents the displacement.

Discuss and interpret graphs that model real situations. For example, the distance–time graph for a particle moving with constant speed, the velocity–time graph for a particle with constant acceleration.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Throw an object straight (vertically) up in the air. Time the flight and estimate the greatest height, to scale the graphs correctly, and keep for possible later use. Draw the displacement–time and velocity–time graphs (upward direction positive and initial velocity non-zero). If the object is caught at the same height at which it was thrown, what is the average velocity for the motion?

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Many students can draw a velocity time graph with the correct shape, but do not always label the required speeds and times clearly on the axes. Students often tend to add a scale (for example 4, 8, 12, 16, …) unnecessarily, rather than just indicating the initial and final speeds.

Candidates are able to find distance travelled and the acceleration from velocity–time graphs and can find an average speed, but some struggle with the vocabulary of velocity and displacement.

NOTES

This unit can be linked with Unit 7b by drawing a general velocity–time graph for a particle with initial velocity (u), final velocity (v), taking time (t) moving under constant acceleration (a). The gradient of the line is $\frac{v-u}{t} = a$, which rearranges to $v = u + at$. Finding the area under the graph in 3 different ways will lead to 3 of the other *suvat* formulae but $v^2 = u^2 + 2as$ will have to be derived by eliminating t between two of them.

7b. Motion in a straight line under constant acceleration; *suvat* formulae for constant acceleration; Vertical motion under gravity (7.3) (8.3)

Teaching time
3 hours

OBJECTIVES

By the end of the sub-unit, students should:

- recognise when it is appropriate to use the *suvat* formulae for constant acceleration;
- be able to solve kinematics problems using constant acceleration formulae;
- be able to solve problems involving vertical motion under gravity.

TEACHING POINTS

Make links back to Unit 7a and contrast the previous graphical approach with this algebraic approach. Note that there are five quantities, s, u, v, a and t (four vectors and one scalar) and each formula relates four of them hence there are five formulae. The formulae that must be derived and learnt are:

- $v = u + at$
- $s = \frac{(u+v)t}{2}$
- $s = ut + \frac{1}{2}at^2$
- $v^2 = u^2 + 2as$
- $s = vt - \frac{1}{2}at^2$

These formulae are only valid for *constant* acceleration in a straight line (and are referred to as the *suvat* formulae).

When solving problems, write down known variables and the variable(s) to be found – this should help to identify which one (or more, as some problems will involve simultaneous equations) of the *suvat* formulae to select. Emphasise to students the need to make sure units are compatible.

Model the good practice of drawing a diagram to illustrate the situation whenever possible, especially when considering vertical motion under gravity. This will encourage students to draw their own diagrams.

Mark the positive direction on the diagram and take acceleration due to gravity (g) to be 9.8 m s^{-2} unless directed otherwise. Students may assume that g is constant, but they should be aware that g is not a universal constant but depends on location.

If an object is thrown upwards and upwards is taken as being positive then $a = -9.8$ m s^{-2}. Explain that the velocity is zero at the greatest height and there is symmetry in the path (up and down to the same point) due to the fact that we model air resistance as being negligible.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

One of the more demanding problems is when two objects are released (or dropped) at different times, say 2 seconds apart, and students are asked to find the common position when one catches-up or passes the other. Students may find it difficult to select the times (values of t) to assign in the equations; they may need guiding towards t and $(t - 2)$ or t and $(t + 2)$.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students are generally able to use *suvat* formulae in 2D to find unknown heights, velocities etc. However, students sometimes ignore the significance of a negative value for velocity, acceleration or displacement and don't refer their answer back to the original problem. They need to recognise that $s = -3$ m means the object is 3 m *below* its starting point in the negative direction i.e. s is effectively a coordinate. This is where a diagram helps students understand the physics of the situation.

NOTES

End the section by looking forwards to Kinematics 2 (Unit 9) with a problem illustrating a variable acceleration e.g. $v = 2t^2 + 3t$. Explain that this will give a curved velocity–time graph so the *suvat* formulae will not work and instead we may need to find the gradient of the tangent and the area under the curve (Link back to GCSE (9-1) in Mathematics at Higher Tier or to calculus in AS Mathematics – Pure Mathematics content, see SoW Unit 7) You could also consider how to analyse motion in 2D (or 3D); this will be addressed during Kinematics 2 (Unit 9).

UNIT 8: Forces and Newton's laws

Return to overview

SPECIFICATION REFERENCES

8.1 Understand the concept of a force; understand and use Newton's first law.

8.2 Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2D (**i**, **j**) vectors).

8.4 Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line; application to problems involving smooth pulleys and connected particles.

PRIOR KNOWLEDGE

- Modelling and definitions/assumptions from the introduction in Unit 6

GCSE (9-1) in Mathematics at Higher Tier

A19 Solve two simultaneous equations in two variables (linear/linear or linear/quadratic) algebraically; find approximate solutions using a graph

AS Mathematics – Pure Mathematics content

10.1 – 10.5 Vectors in 2D (See SoW Unit 5)

KEYWORDS

Force, newtons, mass, weight, gravity, tension, thrust, compression, air resistance, reaction, driving force, braking force, resultant, force diagram, equilibrium, inextensible, light, negligible, particle, smooth, uniform, pulley, string, retardation, free particle.

NOTES

This section does *not* contain any resolving of forces into perpendicular components, nor does it consider the use of the coefficient of friction for frictional forces.

8a. Newton's first law, force diagrams, equilibrium, introduction to i, j system of vectors (8.1)	Teaching time 3 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand the concept of a force; understand and use Newton's first law.

TEACHING POINTS

Relate this topic back to the different types of forces defined in Unit 6b.

Newton said '*An object continues in state of rest or uniform motion unless acted on by an external force*.' Hence one can define a force as something which causes a body to accelerate. Explain to students that 'no force acting' means a body will either be stationary or be moving with constant velocity (i.e. acceleration = zero). This is why in outer space an object keeps moving at constant speed once pushed (there are no forces to speed it up, slow it down or stop it moving.)

So, an object at rest or constant velocity $\Rightarrow$ no resultant force; an object changing speed or direction $\Rightarrow$ resultant force. This will lead to Newton's second law in the next section.

Newton also stated '*When an object A exerts a force on another object B there is an equal and opposite reaction force of B on A*.' Explain that if a book is on a smooth, horizontal table, the forces acting on the book are the Weight, W (vertically down) and the normal reaction, R (always at 90° to the *surface* of contact). Assuming the table surface material is strong enough to hold the full weight of the book, the two forces balance each other and there is no resultant force. The book does not move, hence it is in equilibrium.

Ask questions such as: If the book has a mass of 5 kg, what is its weight? Therefore, what would the magnitude of the normal reaction be to guarantee equilibrium?

Draw different examples of force diagrams to illustrate: weight, reaction, tension (in strings), thrust (in rods), compression (in light rods, springs) etc.

To illustrate thrust, balance a book on a ruler.In which direction is the thrust force acting?

Introduce the **i - j** notation. The forces can be given in **i - j** form or as column vectors. Questions on equilibrium will be limited to perpendicular forces so the sum of the forces must be $0\mathbf{i} + 0\mathbf{j}$ for equilibrium.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

You could extend to rough inclined planes and show all the forces balancing to provide equilibrium. Resolving forces is not in the AS course (covered in A Level Mathematics – Mechanics content, see SoW Unit 5a).

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students are often good at drawing force diagrams, but common errors are omitting arrowheads, incorrectly labelling (e.g. 4 kg rather than 4g) and missing off the normal reaction.

Students can easily be confused by the vocabulary, e.g. mixing up 'resultant' and 'reaction'.

NOTES

Resolving forces is not in the AS course and equilibrium problems will not require forces to be resolved. Scenarios will be restricted to forces in two perpendicular directions, or simple cases of forces, given as 2D vectors.

Resolving forces and the concept of a friction force (which opposes relative motion) is covered in A level Mathematics – Mechanics section, see SoW Unit 5.

8b. Newton's second law, '$F = ma$', connected particles (no resolving forces or use of $F = \mu R$); Newton's third law: equilibrium, problems involving smooth pulleys (8.2) (8.4)

Teaching time
3 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2D (**i**, **j**) vectors.);
- understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line; application to problems involving smooth pulleys and connected particles.

TEACHING POINTS

Newton stated, '*Where there is a force, there is an acceleration (or deviation from uniform motion) and the force is proportional to the acceleration*'. Therefore $F \propto a$, and choosing the constant to suit the motion units gives $F = ma$. (Newton's second law). This is known as the 'equation of motion'.

Explain to students that if they sum all the effects of the forces acting, in a particular direction, this will be equal to the mass x the acceleration in that direction. This process is called resolving the forces in that direction e.g. resolving horizontally, or R(→) for short. It's usually best to resolve IN the direction of the acceleration and/or perpendicular to the direction of the acceleration.

When resolving always take the positive direction as the direction of the acceleration and put all the forces on one side of the equation and (mass x acceleration) on the other side.

When working on connected particles problems (such as trains or pulley systems) explain to students that they should consider the whole system as well as the separate parts. Applications to be covered are lift problems, car and caravan type questions and connected particles passing over a smooth pulley. Consider both pulley scenarios: a pulley with both stings hanging vertically; and a pulley at the end of a horizontal table.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

For the connected particle problems, discuss, the assumptions from Unit 6a, i.e. smooth pulley, inextensible string, same tension in the string. Extend the questions so that (for a pulley question) the particle moving down eventually hits the table and the string goes slack. This means the particle moving up continues as a 'free' particle so we now apply the equations of motion with $a = -9.8 \text{ m s}^{-2}$.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Pulleys: In past exam questions, most students used an equation of motion for each particle with very few 'single equation' solutions. Students may also mistakenly take the acceleration to be equal to g rather than the value obtained in the question.

2 Vehicles: In exam questions of a car-and-trailer type, students may consider the car and trailer as a single system. Common errors when resolving are: to add a tension force (when there is no rope): to consider the weight; or to confuse the positive and negative directions.

NOTES

Starting with the ‘winner’ is most useful when the dynamics move into more complicated questions later in the course (e.g. inclined planes and resolving forces).

UNIT 9: Kinematics 2 (variable acceleration)

Return to overview

SPECIFICATION REFERENCES

7.4 Use calculus in kinematics for motion in a straight line.

PRIOR KNOWLEDGE

GCSE (9-1) in Mathematics at Higher Tier

A11 Identify and interpret roots, intercepts, turning points of quadratic functions graphically; deduce roots algebraically and turning points by completing the square

A14 Plot and interpret graphs (including reciprocal graphs and exponential graphs) and graphs of non-standard functions in real contexts to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

A15 Calculate or estimate gradients of graphs and area under graphs (including quadratic and non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts

AS Mathematics – Pure Mathematics content

7, 8 Differentiation and integration of polynomials (See Units 6 and 7 of the SoW)

KEYWORDS

Distance, displacement, velocity, speed, constant acceleration, variable acceleration, retardation, deceleration, gradient, area, differentiate, integrate, rate of change, straight-line motion, with respect to time, constant of integration, initial conditions.

NOTES

All the functions in this section are functions of time, so the differentials and integrals are always with respect to time.

9a. Variable force; Calculus to determine rates of change for kinematics (differentiation) (7.4)

Teaching time
3 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to use calculus (differentiation) in kinematics to model motion in a straight line for a particle moving with variable acceleration;
- understand that gradients of the relevant graphs link to rates of change;
- know how to find max and min velocities by considering zero gradients and understand how this links with the actual motion (i.e. acceleration = 0).

TEACHING POINTS

Start by stating that the *suvat* formulae from Unit 7 can only be used when acceleration is constant and the motion is in a straight line. This means the speed-time or velocity-time graphs are made up of straight lines.

Draw the graph of say, $v = 2t^2 + 2t + 1$ (for $t > 0$). This is part of a parabola where the gradient is increasing so as time passes the object is accelerating more quickly. As acceleration is not constant, the *suvat* formulae will not work for this model.

Make links (using AS Pure Mathematics calculus) to the rate of change of velocity explaining that $\frac{\mathrm{d}v}{\mathrm{d}t}$ = gradient = acceleration. This idea that the gradient of a velocity–time graph gives acceleration should be familiar from previous work in Unit 7 and also from GCSE (9-1) in Mathematics.

Summarise the situation by talking about, velocity as the rate of change of displacement and acceleration as the rate of change of velocity.

Express these statements in the notation of calculus: $v = \frac{\mathrm{d}s}{\mathrm{d}t}$ and $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$.

Students will also need to relate the fact that the gradient = 0 at the max or min point to this mathematical model i.e. if $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$, then acceleration = 0, so the particle must be at max or min velocity, as it cannot accelerate (or get any faster or slower) any more at this point in time.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

You could extend the calculus approach to relate double differentiation and signs of $\frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$ to indicate if it is a min or max displacement.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students who draw sketches of the situation are often more successful in reaching the correct solution, so you should continue to encourage this wherever possible.

Students often ignore or don't recognise the difference between displacement and distance and so may end up discarding negative values without considering how they should be interpreted.

NOTES

The level of calculus will be consistent with the contents of AS Pure Mathematics.

The specification states the following:– $v = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}$, $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2}$ using ‘$\boldsymbol{r}$’ to represent displacement ‘s’.
r will become the vector notation of displacement when we later analyse 2D kinematics using the **i**, **j** system (A level Mathematics – Mechanics section, see SoW Unit 8).

9b. Use of integration for kinematics problems i.e. $\boldsymbol{r} = \int \boldsymbol{v}\,\mathrm{d}t$, $\boldsymbol{v} = \int \boldsymbol{a}\,\mathrm{d}t$ (7.4)	Teaching time 2 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to use calculus (integration) in kinematics to model motion in a straight line for a particle moving under the action of a variable force;
- understand that the area under a graph is the integral, which leads to a physical quantity;
- know how to use initial conditions to calculate the constant of integration and refer back to the problem.

TEACHING POINTS

Return to the graph of $v = 2t^2 + 2t + 1$ (for $t > 0$) introduced at the start of Unit 9a.

From earlier work in Unit 7 and from GCSE (9-1) in Mathematics, students should know that the area under a velocity–time graph equals the displacement.

Remind students that, from their work for Pure Mathematics, the area under a curve can be found using integration. This means that the integral of the velocity expression (with respect to time) gives the displacement.

By linking integration with the reverse of differentiation, displacement and velocity can be found by integrating expressions for velocity and acceleration respectively:

$$\boldsymbol{r} = \int v\,\mathrm{d}t \text{ and } v = \int a\,\mathrm{d}t$$

(Again 's' can be used in place of '$\boldsymbol{r}$' for straight line motion in this section)

Move on to explain that the constant of integration, c needs to be found by referring back to the problem and using some (usually initial) information about the body. For example knowing that the particle starts from O at rest means that when $t = 0$ (initially), $s = 0$ (at O) and $v = 0$ (at rest). These values can be substituted to calculate c.

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

Students need to be able to know when to differentiate and/or integrate and how acceleration = 0 gives a maximum velocity so questions like the following are useful.

A particle moves so that it's motion is modelled by the following equation, $v = 6t(3 - t)$ m s^{-1}.

Find: **a** the times when it is at rest, **b** its maximum velocity, **c** an expression for its acceleration, **d** the total distance it travels between the times it is stationary.

Extension: Starting with constant a, students can derive the earlier equations of uniform motion. Stress constants of integration, which produce u in $v = u + at$ and the s_0 (s when $t = 0$) in $s = ut + \frac{1}{2}at^2 + s_0$.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students can easily forget that if the velocity becomes negative, for example when a particle stops and changes direction, they need to split the integral to calculate distance rather than displacement.

NOTES

The following diagram may help students decide whether to differentiate or integrate to solve a problem.

'**d**' for the **d**own arrow means '**d**ifferentiate'. Hence, down from 's' gives 'v' or $\frac{\mathrm{d}s}{\mathrm{d}t} = v$.

Integration is the opposite of differentiation so up is integrate, so up from 'a' gives 'v' or integral of 'a' with respect to t gives 'v'.

↓ s

diff ↓ v ↑ int

a ↑