

6 Decimals and ratio

MASTER

Check
P149

Strengthen
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Extend
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Test
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6.1 Ordering decimals and rounding

You will learn to:

- Round numbers to an appropriate degree of accuracy
- Order positive and negative numbers, including decimals.



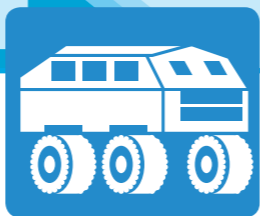
Why learn this?

We don't often use precise values in our day-to-day conversations.

Fluency

7.4, 7.7, 7.1, 7.6, 7.8

- Which of these decimals are closer to 7 and which are closer to 8?
- How do you decide whether to round up or down?
- Which symbol, < or >, should go between the numbers 7.4 and 7.7?



Explore

How many votes were cast in the X Factor final?

CONFIDENCE

Warm up

Exercise 6.1

- Round each number to the nearest 100.
a 245 b 878 c 495
d 523 e 1449 f 67
- Write each number in words.
a 4013 b 23527 c 146005 d 1529400
- Rearrange these numbers in *ascending* order.
27, 14, 103, -11, 83, 10.1, -10.1, 38.9
- Round each number to the nearest 1000.
a 2455 b 5199 c 12875
d 45812 e 546848 f 623399
- Round each number to the nearest 10000.
a 84562 b 47487 c 9458
d 48099 e 754397 f 873822
- Real** This table shows the total attendance at five Premier League football teams' grounds in the 2007/08 season. Round each value to the nearest 100 000.

Team	Actual attendance
Arsenal	1 141 335
Aston Villa	760 560
Chelsea	786 549
Everton	702 142
Liverpool	827 111

Q3 Literacy hint

Ascending order means in order of size with the lowest number first.

Key point

To round to the nearest 10 000, look at the digit in the thousands column.

Key point

To round to the nearest 100 000, look at the digit in the ten thousands column.

- Round each number to two decimal places.
a 2.536 b 7.489 c 5.083
d 6.199 e 45.157 f 23.007
- Write each set of decimal numbers in *ascending* order.
a 1.093, 0.08666, 1.232, 0.20071, 0.1258
b 4.227, 4.051, 4.234, 4.735, 3.292
c 0.71113, 0.0732, 7.001, 0.7499, 7.0932
- Rearrange these numbers in *descending* order.
24.457, 25.645, 22.961, 24.833, 25.622
- Rearrange each set of numbers in *ascending* order.
a -8.12, -0.89, -5.76, -3.11, -1.88
b -0.125, -0.845, -0.149, -0.135, -0.0122
c -0.033, -0.0309, -0.0342, -0.0325, 0.0324

Worked example

Write 1662682 as a decimal number of millions to one **decimal place**.

1662682 = 1.662682 million — Write as a decimal number of millions.

1.7 million — Round to 1 decimal place (1 d.p.)

Key point

To round a decimal to two decimal places (2 d.p.), look at the digit in the third decimal place.

Key point

When ordering decimals, look at the place value of each digit.

$0.3 = \frac{3}{10}$, $0.03 = \frac{3}{100}$
So 0.3 is larger than 0.03.

Q8 Literacy hint

Ascending order means getting bigger. **Descending order** means getting smaller.

Key point

To save writing all the zeros, you can write
1 000 000 as 1 million
2 500 000 as 2.5 million.

- Real** The table shows the populations of 10 capital cities in Europe. Write each population as a decimal number of millions to one decimal place.

City	Actual population
Moscow	11 541 000
London	8 174 100
Berlin	3 520 000
Madrid	3 233 527
Rome	2 792 508
Paris	2 268 265
Budapest	1 728 718
Vienna	1 552 789
Prague	1 227 332
Dublin	1 045 769

- Round each number to three decimal places.
a 4.5391 b 29.7965 c 69.0852
d 85.8008 e 72.7576 f 3.2567

Key point

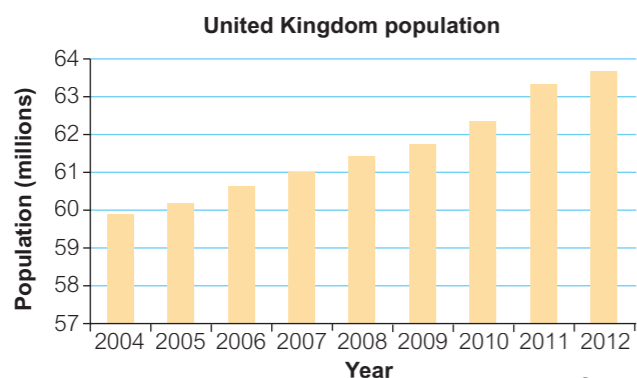
To round a decimal to three decimal places, look at the digit in the fourth decimal place.

- Reasoning / Real** In a restaurant the tips are divided equally between the workers. Work out how much each worker receives each day.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Total tips (£)	55	68	71	86.50	94	124.50	100
Number of workers	6	7	6	8	7	12	9

Discussion Did you round up or down? Explain.

14 **Real** The graph shows the population of the UK between 2004 and 2012.



Source: ONS

- a Describe what happened to the UK population between 2004 and 2012.
 b What was the population in 2005 to the nearest million?
 c In which years was the population 61 million to the nearest million?
- 15 Copy and complete these. Put the correct sign, $<$ or $>$, between each pair of numbers.
 a $1.064 \square 1.022$ b $6.242 \square 6.224$
 c $7.737 \square 7.739$ d $0.06852 \square 0.06812$
- 16 Rearrange these numbers in *descending* order.
 $-0.029, -0.0205, -0.092, -0.0925, -0.052,$
 $-0.0209, -0.0592, -0.095, -0.0529$
- 17 Work out the length of one side of a square with perimeter
 a 10 cm b 24.3 cm
 c 13.65 cm d 1.526 km
 Round all your answers to an appropriate degree of accuracy.
- 18 Copy and complete these. Put the correct sign, $<$ or $>$, between each pair of numbers.
 a $-2.078 \square -2.087$ b $-8.27 \square -8.72$
 c $-6.26 \square -6.25$ d $-0.0532 \square -0.0530$

Key point

For most calculations, an appropriate degree of accuracy is a value you can measure accurately.

Investigation **Real / Finance**

Petrol and diesel are sold by the litre. The price is often given to one decimal place. For example, you might see petrol at 132.9p per litre. Actual prices need to be rounded when the customer has finished pumping fuel.

- 1 Choose some volumes of petrol in whole numbers of litres. Will the price need to be rounded up or down?
 2 Why do you think petrol stations give the price as a decimal number of pennies?

Part 1 hint

Try 2 litres, 5 litres, 10 litres.

- 19 **Explore** How many votes were cast in the X Factor final? Is it easier to explore this question now you have completed the lesson? What further information do you need to be able to answer this?
- 20 **Reflect** In this lesson you have been doing lots of work with decimals. Imagine someone had never seen a decimal point before. How would you define it? How would you describe what it does? Write a description in your own words. Compare your description with others in your class.

6.2 Place-value calculations

You will learn to:

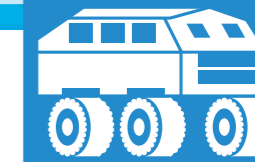
- Multiply larger numbers
- Multiply decimals with up to two decimal places
- Multiply any number by 0.1 and 0.01



Why learn this?
 Metric measurements use decimals. You need to calculate with decimals to find lengths and areas.

Fluency

- What does the '1' represent in 0.1 and 0.01?
- How do you write 0.3 and 0.07 as fractions?



Explore

Does multiplying one number by another always make it bigger?

CONFIDENCE

Warm up

Exercise 6.2

- 1 Work out
- | | | | |
|-----------------|------------------|------------------|-------------------|
| a 45×7 | b 53×28 | c 32×17 | d 267×15 |
| _____ | _____ | _____ | _____ |
- 2 Work out
- | | | | |
|------------------|--------------------|-----------------|-------------------|
| a 63×10 | b 182×100 | c $430 \div 10$ | d $4300 \div 100$ |
|------------------|--------------------|-----------------|-------------------|
- 3 Estimate these by rounding one or both numbers.
- | | | | |
|-------------------|------------------|---------------------|-------------------|
| a 50×0.8 | b 5.3×7 | c 19.9×0.5 | d 134×11 |
|-------------------|------------------|---------------------|-------------------|
- 4 Copy and complete.
- | | |
|--------------|---------------------------------|
| 137 | |
| $\times 245$ | |
| _____ | $\leftarrow 137 \times 5$ |
| _____ | $\leftarrow 137 \times 40$ |
| _____ | $\leftarrow 137 \times 200$ |
| _____ | \leftarrow Add these together |

Worked example

Work out 2.6×3.2
 Estimate: $3 \times 3 = 9$

$$\begin{array}{r} 2.6 \\ \times 3.2 \\ \hline 5.2 \\ + 7.80 \\ \hline 8.32 \end{array}$$

Use a standard method to work out 26×32

Use your estimated answer to see where to put the decimal point.

$2.6 \times 3.2 = 8.32$



5 Work out

- a 3.7×2.2 b 2.5×4.2 c 7.22×3.1
 d 3.46×8.9 e 8.94×0.32 f 4.04×8.2

Discussion For each part, count the number of digits after the decimal point in both numbers in the question. Do the same for the answer. What do you notice?

6 **Real** A car can travel 13.8 kilometres on 1 litre of petrol. How far can it travel on 8.8 litres of petrol?

7 Follow these steps to work out 3.26×5.12

- a Estimate the answer.
 b Work out 326×512
 c Decide where to position the decimal point.

8 Use the multiplication facts given to work out the answers.

- a $12 \times 17 = 204$. Work out 1.2×1.7
 b $36 \times 14 = 504$. Work out 3.6×0.14
 c $108 \times 4 = 432$. Work out 10.8×0.04
 d $36 \times 72 = 2592$. Work out 0.36×7.2

9 Work out

- a 36×0.1 b $36 \div 10$ c 45×0.1
 d $45 \div 10$ e 107×0.1 f $107 \div 10$

Discussion What do you notice?

10 Work out

- a 8.6×0.1 b 11.6×0.1 c 0.53×0.1

11 a Copy and complete.

$29 \times 1 = \square$
 $29 \times 0.1 = \square$
 $29 \times 0.01 = \square$

b **Reasoning** What division calculation is equivalent to ' $\times 0.01$ '?

12 Work out

- a 3621×0.01 b 4568×0.01 c 88.6×0.01
 d 11.6×0.01 e 534×0.01 f 683×0.01

13 **Problem-solving** A factory makes 3.5 silk flowers every second.

- a Each flower uses 60.3 cm of silk. How many metres of silk are used in one minute?
 b Each flower has a 0.325 m wire stem. A hotel orders 275 silk flowers. What length of wire is needed?

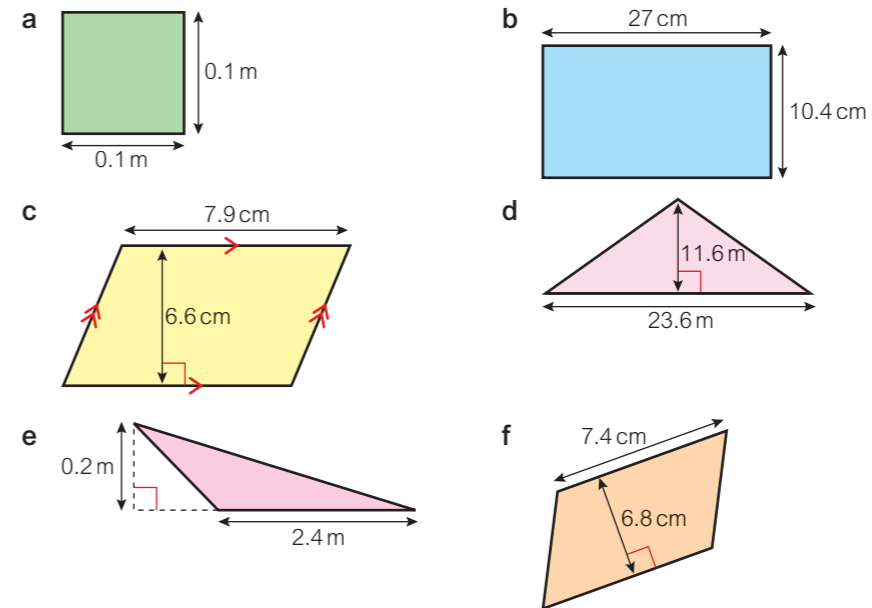
Q5 hint

Estimate first.

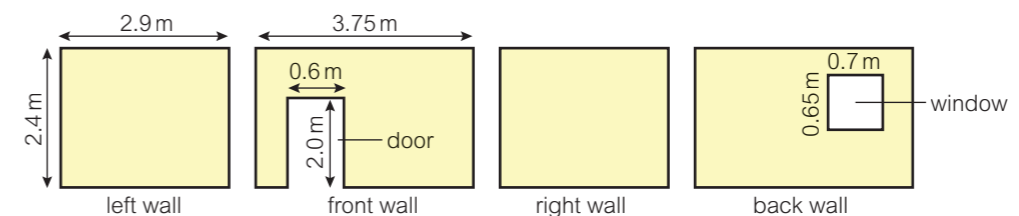
Q8a hint

The answer will have the digits 204. Where do you put the decimal point?

14 Work out the area of each shape.



15 **Real / Problem-solving** Anita is planning to paint the walls of her living room.



Anita needs 0.1 litres of paint to paint each 1 m^2 . How much paint will she need to paint all the walls?

16 **Explore** Does multiplying one number by another always make it bigger?

Choose some sensible numbers to help you explore this situation. Then use what you have learned in this lesson to help you answer the question.

17 **Reflect** Look back at Q5. At the end of this question you discussed a mathematical 'rule'.

The rule tells you where to put the decimal point in the answer when multiplying decimals. Write the 'rule' in your own words. Why do you think the rule was at the end of the question and not at the beginning? What would you do to multiply two decimals, if you couldn't remember the rule?

6.3 Calculations with decimals

You will learn to:

- Add and subtract decimals of any size
- Multiply and divide by decimals
- Divide by 0.1 and 0.01

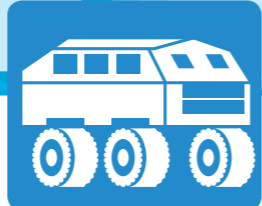


Why learn this?

We need to calculate using decimals when dealing with money and measurements.

Fluency

- Work out
- $57 - 14 - 23$
 - $63 - 12 - 31$



Explore

Why does a sharp axe cut better than a blunt one? Think about the area of the cutting surface.

Worked example

Work out $67.8 \div 1.2$

$$\times 10 \left(\begin{array}{r} 1.2 \overline{)67.8} \\ \underline{12} \\ 57 \\ \underline{60} \\ 78 \\ \underline{72} \\ 60 \\ \underline{60} \\ 0 \end{array} \right) \times 10$$

1.2 has one decimal place, so multiply both numbers by 10.

$$\begin{array}{r} 56.5 \\ 12 \overline{)678.0} \\ \underline{60} \\ 78 \\ \underline{72} \\ 60 \\ \underline{60} \\ 0 \end{array}$$

Work out the division.

Check: $12 \times 56.5 \approx 10 \times 60 = 600$

Key point

To divide by a decimal, multiply both numbers by a power of 10 (10, 100, ...) until you have a whole number to divide by. Then work out the division.

Q6f hint

You will need to work out the second decimal place and then round, rather than just stopping at the first decimal place.

Q8a hint

Keep the decimal points in line.

$$\begin{array}{r} 3241 \\ 306.192 \\ + 2.308 \\ \hline \end{array}$$

- 6 Work these out using a written method. Give your answers to one decimal place where appropriate.
- a $18.9 \div 0.09$
 - b $39 \div 0.75$
 - c $131.72 \div 0.37$
 - d $348 \div 5.8$
 - e $43.32 \div 0.3$
 - f $82.3 \div 6.25$
 - g $367 \div 2.4$
 - h $0.556 \div 3.6$
 - i $72.5 \div 0.7$

Discussion 'Dividing a number by a number less than 1 gives you an answer larger than the first number.' Is this statement true?

- 7 **STEM** A scientist has 27.9g of substance X. He needs to divide it into samples for testing. Each testing dish holds 2.4g. How many testing dishes does the scientist need?

- 8 Work out
- a $3241 + 306.192 + 2.308$
 - b $806.5 - 21.33 - 95$
 - c $3150.14 - 88.6 + 27.2031$
 - d $3096 + 108.7 + 0.204 - 3.14$

- 9 **STEM / Problem-solving** Suzie is testing a beaker of water. She removes these samples for analysis.

A	B	C	D
2.13ml	0.005ml	3.075ml	0.321ml

- a There is 32.4 ml in the beaker after samples A and B are removed. How much water was originally in the beaker?
- b How much water is left in the beaker after all samples are removed?

- 10 **Real / Problem-solving** A skateboard factory makes boards from sheets of plywood. The factory checks the area of plywood wasted each week. One week the total waste was 28.75m² over the five days the factory was open.

Day	Waste
Monday	4.35m ²
Wednesday	5.4m ²
Thursday	6.14m ²

- a How much plywood was wasted on Tuesday and Friday? On Friday 2.4m² more plywood was wasted than on Tuesday.
- b How much was wasted on Tuesday?

Exercise 6.3

- 1 **Finance** Billy has been checking his bank statement.

Date		Paid in	Paid out	Balance
16/09/2013	Start balance			£125.68
	Water bill		£23.75	
	Electricity		£17.29	
	Lotto	£10.00		
	Mobile		£15.99	
	Wages	£256.75		
17/09/2013	End balance			

What is the balance of Billy's account after his wages are paid in?

- 2 Work out
- a 4.83×2.7
 - b 2.45×3.32
- 3 Use a written method to calculate
- a $3 \overline{)294}$
 - b $23 \overline{)943}$

Q1 Literacy hint

Your **bank balance** is the amount of money in your account.



- 4 Work out
- a $36 \div 12$ and $3.6 \div 1.2$
 - b $72 \div 8$ and $7.2 \div 0.8$
 - c $484 \div 4$ and $4.84 \div 0.04$
 - d $625 \div 25$ and $6.25 \div 0.25$
- Discussion** What do you notice? How does this help you work out $8.1 \div 0.9$ and $0.64 \div 0.08$ without a calculator?

- 5 Work out
- a $6.3 \div 0.7$
 - b $4.8 \div 0.6$
 - c $12.1 \div 1.1$
 - d $0.28 \div 0.07$
 - e $0.9 \div 0.03$
 - f $14.4 \div 0.12$

6.4 Ratio and proportion with decimals

You will learn to:

- Use ratios involving decimals
- Solve proportion problems.



Why learn this?

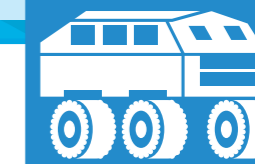
Increasing or decreasing quantities in proportion does not always give us whole numbers.

Fluency

3 : 5 6 : 9 5 : 16 14 : 21

12 : 19 30 : 45 4 : 9

Which of these ratios are equivalent to 2 : 3?

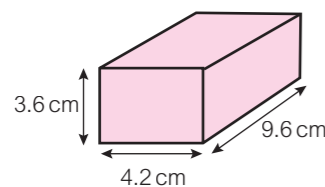


Explore

Why do some old TV programmes have space at the sides of the screen?

- 11 Work out
- a 2.724×3.25 b 4.59×2.764
 c 8.91×5.126 d 7.261×9.28
 e 6.903×0.425 f 23.241×7.26

- 12 a Work out the volume of this cuboid.



- b Another cuboid has a volume of 35.52 m^3 . Its length is 4 m and its width is 2.4 m. What is its height?

- 13 Work out
- a $15 \div 0.1$ b $2.6 \div 0.1$ c $85.3 \div 0.01$
 d $572 \div 0.01$ e $7.6 \div 0.01$ f $0.3 \div 0.1$

Q11a hint

Set out in columns, e.g.

$$\begin{array}{r} 2724 \\ \times 325 \\ \hline \end{array}$$

Q12b Strategy hint

Make a sketch.



CONFIDENCE

Exercise 6.4

- 1 Write each ratio in its simplest form.
- a 8 : 4 b 12 : 3 c 15 : 25
 d 4 : 18 e 7 : 49 f 40 : 60
- 2 a Share £20 in the ratio 2 : 3.
 b Share £36 in the ratio 4 : 5.
 c A piece of rope 24m long is cut in the ratio 5 : 3. How long is each piece of rope?

Worked example

Share £114 between Alice, Bert and Chen in the ratio 5 : 2 : 1.

$$5 + 2 + 1 = 8 \text{ parts}$$

$$£114 \div 8 = £14.25 \text{ per part}$$

$$\text{Alice: } 5 \times £14.25 = £71.25$$

$$\text{Bert: } 2 \times £14.25 = £28.50$$

$$\text{Chen: } 1 \times £14.25 = £14.25$$

$$\text{Check: } £71.25 + £28.50 + £14.25 = £114$$

First find out how many parts there are in total.

Find out how much one part is worth.

Multiply the amount that one part is worth by each value in the ratio.

- 3 Share each quantity in the ratio given.
- a £108 in the ratio 2 : 3 : 4 b £486 in the ratio 1 : 3 : 5
 c £510 in the ratio 1 : 2 : 3 d £242 in the ratio 1 : 2 : 3 : 5
 e 429m in the ratio 2 : 3 : 6 f 468kg in the ratio 3 : 6 : 7
 g 591 km in the ratio 1 : 2 : 4 : 5 h £1032 in the ratio 3 : 5 : 9
- Discussion** How should you round when working with ratios in money? What about kg? Why?

- 4 Simplify each ratio into a whole number ratio in its simplest form.
- a 40 : 28.5 b 70 : 51.2 c 25.5 : 17 d 28.6 : 5.15

Topic links: Multiplying and dividing by 10 and 100, Metric measures, Imperial measures

Investigation

Reasoning

- 1 Choose a number. Carry out these operations on your number.
 $\times 100$ $\times 10$ $\times 0.1$ $\times 0.01$ $\div 100$ $\div 10$ $\div 0.1$ $\div 0.01$
- 2 Repeat part 1 with another number.
- 3 Are any of these operations equivalent? Use your answers to parts 1 and 2 to complete these rules.
 $\times 100$ is equivalent to \square
 \square is equivalent to $\div 0.1$
 \square is equivalent to $\div 10$
 $\times 0.01$ is equivalent to \square
- 4 What do you think the rules are for
 a $\times 0.001$ b $\div 0.001$?
 Test your rules.



- 14 **Explore** Why does a sharp axe cut better than a blunt one? Look back at the maths you have learned in this lesson. How can you use it to answer this question?

- 15 **Reflect**
- a What happens when you divide a positive number by a number between 0 and 1?
 b What happens when you multiply a positive number by a number between 0 and 1?
 c Write your own 'What happens when ...?' question and answer it.

Q15 hint

- a Look back at some of the calculations you did in Q5.
 b Look back at some of the calculations you did in lesson 6.2.

Q4a hint

Simplify using powers of 10. 28.5 has one decimal place, so multiply both sides of the ratio by 10, then simplify.

$$\begin{array}{l} \times 10 \left(\begin{array}{l} 40 : 28.5 \\ \hline 400 : 285 \end{array} \right) \times 10 \\ \div \square \left(\begin{array}{l} 80 : \square \end{array} \right) \div \square \end{array}$$

Warm up

Explore

Reflect

6.5 STEM: Using ratios

You will learn to:

- Solve engineering problems using ratio and proportion
- Use unit ratios.



Why learn this?

Most machines have gears, and gears depend on ratios.

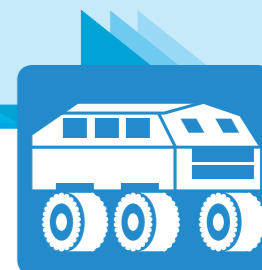
Fluency

Simplify each ratio.

4 : 7 4 : 8 6 : 16

5 : 20 3 : 5 24 : 28

Can all the ratios be simplified?



Explore

How do mountain bikes get up steep hills?

CONFIDENCE

- 5 **Real / Problem-solving** 2p coins used to be made from a mix of copper, tin and zinc in the ratio 95 : 3.5 : 1.5.
- A 2p coin had a mass of 7 g. What were the masses of copper, tin and zinc in the coin?
 - Sally had £1 in 2p pieces. What was the total mass of the coins?

- 6 **Real** Turquoise paint is made by mixing blue, green and yellow in the ratio 2.5 : 1.4 : 0.1.

Copy and complete the table to show how much of each colour is needed to make the quantities shown.

Size	Blue	Green	Yellow
1 litre			
1.5 litres			
2.5 litres			

- 7 **Real / Reasoning** A photo-printing service offers the following picture sizes:

6 × 4 inches, 7 × 5 inches, 8 × 6 inches, 10 × 8 inches, 12 × 8 inches.

A digital camera takes photographs in the ratio 3 : 2.

Which sizes of photo can be printed from this camera?

- 8 **STEM** The aspect ratio describes the ratio 'width : height' of an image.

Most modern televisions have an aspect ratio of 16 : 9.

How high would screens be with these widths?

a 32 cm b 30.5 cm c 41.7 cm d 44.3 cm

How wide would screens be with these heights?

e 27 cm f 17.5 cm g 26.4 cm h 35.2 cm

- 9 **Reasoning** The triathlon is a race where competitors swim, cycle and run. Four recognised lengths of race are shown in the table below.

Race	Swim	Cycle	Run
Sprint	0.75 km	20 km	5 km
Olympic	1.5 km	40 km	10 km
Half Ironman	1.9 km	90 km	21.1 km
Ironman	3.8 km	180.2 km	42.2 km

- What proportion of the Sprint triathlon is running?
- Cycling is Tom's strongest sport. Which race or races would give him the best chance of winning?

- 10 **Explore** Why do some old TV programmes have space at the sides of the screen?

What have you learned in this lesson to help you answer this question? What other information do you need?

11 Reflect

- Look back at Q5a. Write all the steps you took to work out the answer.
- Look back at Q9. Write the steps you took to work out the answer.
- Lou says, 'Question 5 was about ratio. A ratio compares one part to another part. Question 9 was about proportion. A proportion compares one part to the whole thing.' Is Lou correct?

Q6 hint

Simplify the ratio into whole numbers. Then share the amount of paint in the new ratio.

Q8 hint

A screen 16 cm wide would be 9 cm tall.

Q9a hint

First find the total distance of the race. Then write the proportion for 'run' as a fraction, and simplify.

Q11c hint

Use your steps for Q5a and Q9 to help you.

Exercise 6.5: Engineering ratios

- Divide each quantity in the ratio given.
 - 567 kg in the ratio 5 : 1
 - 486 metres in the ratio 3 : 2
 - £7816 in the ratio 2 : 3 : 5
- Simplify each ratio into a whole number ratio in its simplest form.
 - 5.2 : 4.5
 - 8.2 : 6.3
 - 8.5 : 2.25
 - 2.56 : 1.37
- Wood's metal is an alloy made from bismuth, lead, tin and cadmium. Mixing these amounts will make 1 kg of Wood's metal.

bismuth	500 g
lead	250 g
tin	125 g
cadmium	125 g

How much of each metal is needed to make 2.5 kg?

Worked example

A new TV has aspect ratio of 16 : 9. Express this as a **unit ratio**. Give your answer to two decimal places.

$$\div 9 \left(\begin{array}{l} 16 : 9 \\ \hline 1.78 : 1 \end{array} \right) \div 9$$

Divide both sides of the ratio by the smallest number, 9

- Write each ratio as a **unit ratio**. Give each answer to a maximum of two decimal places.
 - 9 : 5
 - 11 : 4
 - 17 : 33
 - 11 : 23

Key point

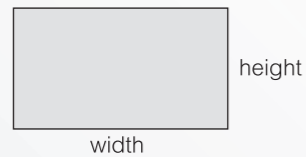
You can compare ratios by writing them as **unit ratios**. In a unit ratio, one of the two numbers is 1.

Warm up

Explore

Reflect

- 5 **Real** Over the years, images have been shown in many different rectangular shapes, usually expressed as aspect ratios, width : height.



- a Convert each aspect ratio to a unit ratio.
- i 5 : 3 (European widescreen)
 - ii 3 : 2 (35 mm film)
 - iii 8 : 5 (computer screen)
 - iv 4 : 3 (cathode ray tube TV)
 - v 37 : 20 (US widescreen)
 - vi 12 : 5 (cinema widescreen)
- b Which of these ratios shows the widest picture?

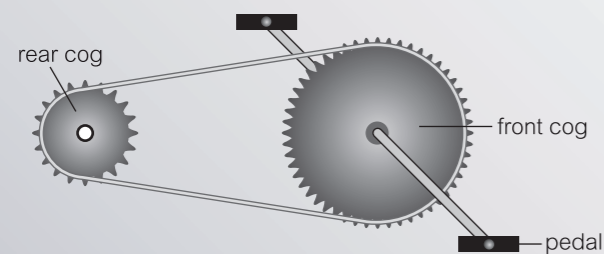


- 6 **Real / STEM** Engine performance can be compared by looking at the ratio of power to weight. A high power-to-weight ratio means a car will accelerate (or perform) well. Find the ratio of power to weight for each of these cars as a unit ratio.

Car	Power (kW)	Weight (tonne)	Power : weight (unit ratio)
Chevrolet Corvette	476	1.51	315 : 1
Caparo T1	429	0.47	
Caterham Superlight R500	196	0.51	
Ariel Atom 500	373	0.55	
Ferrari F12	544	1.63	
Porsche GT2RS	456	1.37	

Discussion Which car has the best performance?

- 7 **Real / STEM** Most modern bikes have a variety of gears, with a number of different-sized cogs. A road-racing bike has a front cog at the pedals with 53 teeth and a choice of 5 cogs at the rear.



One turn of the pedals turns the front cog once.

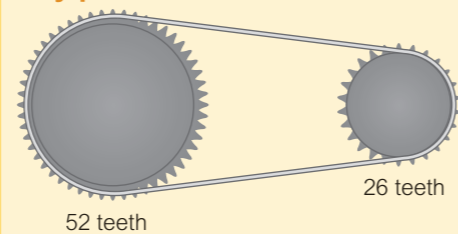
Copy and complete the table to work out the number of turns the rear wheel will make when the pedals are turned once for different gears.

Front cog teeth	53	53	53	53	53
Gear	1	2	3	4	5
Rear cog teeth	32	25	19	14	11
Ratio of front teeth to rear teeth	53 : 32				
Unit ratio	1.66 : 1				
Number of rear wheel turns per turn of the pedals	1.66				

Q6 hint

Use a calculator to work out $\frac{\text{power}}{\text{weight}}$ and round to a whole number.

Key point



In engineering, gears are used to change speeds. These two cogs are connected by a chain and have equal sized teeth. Each turn of the large cog makes the small cog turn twice, because $1 \times 52 = 2 \times 26$.

- 8 **Real / STEM** Some cyclists prefer fixed wheel bikes, with no gears. Typically a front cog has 50 teeth and a rear cog has 20 teeth.

- a What is the ratio of front cog teeth to rear cog teeth?
b How many times does the rear wheel turn for every turn of the pedals?

A typical road bike travels 195.3 cm for every rotation of the rear wheel.

- c How many times must a cyclist turn the pedals to travel 1 km?

- 9 **Real / STEM** Although not as visible, cars use gears in the same way as bikes. Different gear ratios (number of turns in the engine : number of turns in the wheels) make the wheels travel different distances for each revolution in the engine.

In a typical car each revolution of the wheels is about 2 m.

Gear	Turns in the engine : turns in the wheels
1st	2.97 : 1
2nd	2.07 : 1
3rd	1.43 : 1
4th	1 : 1
5th	0.84 : 1
6th	0.56 : 1

- a Explain why 6th gear is the fastest gear.
b How many revolutions of the engine does it take to travel 1 km in 6th gear?

- 10 **Real / Problem-solving** Clocks and watches with hands also have gears.

What is the gear ratio of the minute hand to the second hand?

- 11 **Explore** How do mountain bikes get up steep hills?

Is it easier to explore this question now you have completed the lesson? What further information do you need to be able to answer this?

- 12 **Reflect** In this lesson you answered lots of real problem-solving questions. This is different from some other lessons in this unit where you worked out lots of calculations (as in lesson 6.2). Which type of lesson do you like best? Explain.

Q9 Literacy hint

A revolution is a full rotation of 360° .



6 Check up

Log how you did on your Student Progression Chart.

Ordering and rounding

- Copy and complete these. Put the correct sign, < or >, between each pair of numbers.
 - $7.152 \square 7.251$
 - $4.0531 \square 4.0501$
 - $0.6091 \square 0.6901$
- Write each number as a decimal number of millions to 1 decimal place.
 - 7 500 000
 - 4 250 000
 - 85 650 000
- Rearrange these decimal numbers in *ascending* order.
5.9281 5.90113 5.0982 5.9408
- Rearrange these temperatures in *descending* order.
 -30.5°C -31.03°C -31.3°C -30.01°C
- Round each number to three decimal places.
 - 7.1335
 - 108.44958

Place-value calculations

6 $81 \times 56 = 4536$

Use this multiplication fact to work out these.

- 8.1×56
 - 0.81×560
 - 56×8100
- Work out
 - 708×0.1
 - 41×0.01
 - 6.11×0.01
 - Jane says that she can use an equivalent calculation to find the answer to $4.03 \div 0.1$.
What calculation could she do?
 - Work out
 - $734 \div 0.1$
 - $174 \div 0.01$
 - $253 \div 0.01$
 - To paint an area of 1 m^2 , you need 0.1 litres of emulsion paint.
What is the maximum area that you can paint with 1 litre of paint?

Decimal calculations

- Ollie bought these items.
 - Milk £1.48
 - Bacon £2.75
 - Bread 89p
 - Juice £1.68
 - Low fat spread £1.49
 What is the total cost?

- Serpil has £456.56 in her bank account.
She pays her water bill of £21.69 and her phone bill of £15.99.
A shop refunds her £42.25.
How much is in her bank account now?
- Work out
 - $506.23 - 71.6 + 28.603$
 - $4999 + 235.6 + 0.037 - 34.89$
- Work out
 - 6.8×4.3
 - 1.25×8.6
 - 3.46×2.18
- Work out
 - $64 \div 0.8$
 - $38 \div 2.5$
 - $185 \div 1.25$

Ratio and proportion with decimals

- Write each ratio in its simplest form.
 - 10 : 2.5
 - 4.8 : 3
- Share each quantity in the ratio given.
 - 6.5 kg in the ratio 2 : 3
 - 451 litres in the ratio 2 : 4 : 5
 - £1000 in the ratio 1 : 3 : 5
- A small pot of custard has 3.3 g of protein, 18 g of carbohydrate and 7.1 g of fat.
 - What proportion of the custard is fat?
 - A large pot weighs 3.5 times as much.
How many grams of protein does the large pot contain?
- Write each ratio as a unit ratio.
 - 7 : 5
 - 5 : 18
- How sure are you of your answers? Were you mostly
 - Just guessing
 - Feeling doubtful
 - Confident
 What next? Use your results to decide whether to strengthen or extend your learning.

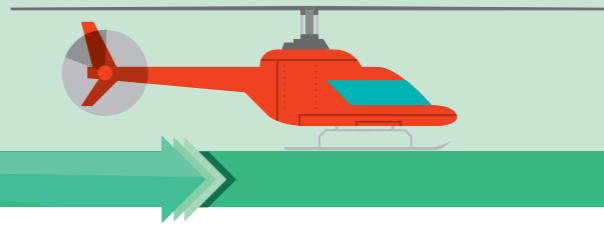
Challenge

- A string factory makes 1563.25 m of string each day.
One ball of string uses 6.5 m.
 - How many balls of string does the factory make in one day?
 - How many balls of string does the factory make in a week (Monday to Friday)?
- Work out $1 \div 0.7$
Write your answer to six decimal places.
Repeat for $2 \div 0.7$, $3 \div 0.7$, $4 \div 0.7$, and so on.
What do you notice?
What happens if you work out $1 \div 1.4$, $2 \div 1.4$, and so on?

6 Strengthen

You will:

- Strengthen your understanding with practice.



Ordering and rounding

- 1 Round each number to the nearest 1000.

a 14 526 b 47 851 c 39 205 d 83 764

- 2 The number of cars entering the London Congestion Zone is recorded each day during the week.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Number of cars	174 567	158 211	162 421	143 896	136 491	168 504	123 855

Round each number to the nearest 10 000.

- 3 Round each number to one decimal place.

a 3.67 b 14.56 c 2.06 d 3.65

- 4 Round each number to two decimal places.

a 5.128 b 4.865 c 12.476 d 26.048

- 5 **Real** In a time trial in a velodrome (cycling track), riders complete 1 km on their own as fast as they can. Here are the times (in seconds) for seven riders.

A 54.194, B 53.696, C 55.103, D 53.656, E 54.725, F 59.308, G 50.514

Who came first, second and third in this race?

- 6 Write each number as millions.

Parts **a** and **d** have been done for you.

a 2 000 000 = 2 million b 8 000 000 = million
 c 12 000 000 = million d 8 600 000 = 8.6 million
 e 7 400 000 = million f 15 700 000 = million

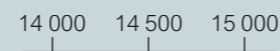
- 7 **Real** These are Sunday night TV viewing figures.

Programme	Viewers
Downton Abbey	9623 145
By Any Means	3450 238
Countryfile	6285 016
The Crane Gang	9268 18
X Factor	9528 586

Round each number to a decimal number of millions to one decimal place.

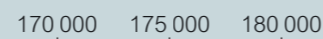
Q1a Strategy hint

Draw a number line to help.



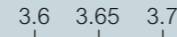
Q2 Strategy hint

Draw a number line to help.



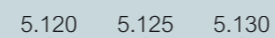
Q3a Strategy hint

You can use number lines for decimals too.



Q4a Strategy hint

Use a number line.



Q5 Strategy hint

List the times in a column with the decimal points lined up. Look at the whole number parts first, then the tenths, ...

Q7 hint

9623 145 = 9.623 145 million
 = 9. million (1 d.p.)

- 8 Rearrange these numbers in *ascending* order (smallest first).

7.29, 7.88, 7.605, 7.325, 7.52, 7.22, 7.292, 7.50, 7.4, 7.61
 7.22, 7.29, ... 7.88

- 9 Rearrange these numbers in *descending* order (largest first).

-7.13, -6.68, -4.80, -1.48, -7.3, -0.98, -1.62, -5.05, -4.2, -2.18

- 10 A satellite tracking device measures distances on Earth in kilometres to four decimal places.

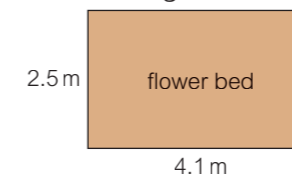
Distance A 18.8177 km
 Distance B 17.2264 km
 Distance C 15.8191 km
 Distance D 15.0941 km
 Distance E 12.6015 km

Round each distance to three decimal places.

- 11 Copy and complete these. Put the correct sign, < or >, between each pair of decimal numbers.

a 6.6 6.13 b 4.4 4.51 c 6.5 6.405
 d 5.1 5.368 e 5.21 5.201 f 15.45 15.445

- 12 The diagram shows a rectangular flower bed.



- a Which is the best estimate to use for the calculation 2.5×4.1 ?

3×4.1

2.5×4

3×4

- b Work out 25×41

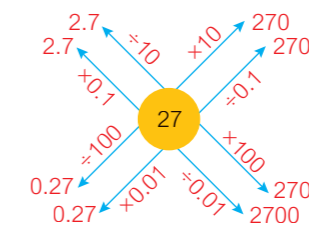
- c Use your answers to parts **a** and **b** to work out 2.5×4.1 to give you the area of the flower bed.

- 13 Petrol costs £1.37 per litre.

How much does 24.5 litres of petrol cost?

Place-value calculations

- 1 Here is a spider diagram for 27 showing the links between multiplying and dividing by powers of 10.



Draw a spider diagram like this for 157.

- 2 Draw a similar spider diagram for each of these numbers.

a 57 b 101 c 45.2 d 2.8

- 3 Multiply each number by 0.1

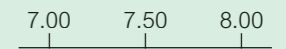
a 9.06 b 4.73 c 6.43

- 4 Multiply each number by 0.01

a 3.42 b 1.14 c 7.36
 d 6.214 e 57.972 f 61.03

Q8 hint

Use a number line to help.



Q10 hint

Look at the 4th decimal place to decide whether to round up or down.

Q11 hint

Decide which number is greater. Put the wider end of the symbol next to the greater number.

Q11a Strategy hint

Look at the whole numbers first, then the tenths, then the hundredths, ...

Q12a Strategy hint

What is easy to multiply but close to the original numbers?

Q13 hint

- Estimate.
- Work out 137×245 .
- Put in the decimal point.
- Round answers in pounds to two decimal places.

6 Extend

You will:

- Extend your understanding with problem-solving.



- 1 **Real** The tables show the heights of the world's highest mountains (in feet). Round each height to the nearest thousand feet.

Mountain	Height (feet)
Everest	29021
K2	28244
Kangchenjunga	28162
Lhotse	27932
Makalu	27758

Mountain	Height (feet)
Cho Oyu	26899
Dhaulagiri	26788
Manaslu	26775
Nanga Parbat	26650
Annapurna	26538

Discussion How useful is this rounded data?

- 2 **Problem-solving** Donna, Shakira and Myles are going out for a meal. They decide to put their money together. Donna has £13.50, Shakira has £18.20 and Myles has £22.75. The prices of their food are given in the table below.

Item	Donna	Shakira	Myles
drink	£1.95	£1.95	£1.95
starter	£2.95	£2.95	£2.50
main	£6.95	£7.75	£7.95
dessert	£3.50	£3.50	£3.50

- How much is the total bill?
 - Could Donna afford to pay for all her own food if they hadn't pooled their money?
 - They leave a tip of 10% of the bill. How much is this?
 - How much money do they have left?
 - They share the remaining money equally between them. How much do they get each?
- 3 **Real** The tables show the drainage areas (in km²) of 10 river basins.

River basin	Drainage area (km ²)
Nile	3254555
Amazon	6144727
Yangtse	1722155
Mississippi	3202230
Yenisei	2554482

River basin	Drainage area (km ²)
Yellow River	945000
Ob	2970000
Parana	2582672
Congo	3730000
Amur	1929981

Write each area in millions to one decimal place. The first one has been done for you.

Nile: 3.3 million km²

Discussion Do you think some of these are rounded values? Explain.

- 4 The tables show the amounts of money spent by a local council.

Item	Amount
road maintenance	£14454001
salaries	£11771908
community services	£7912748
building works	£7003404

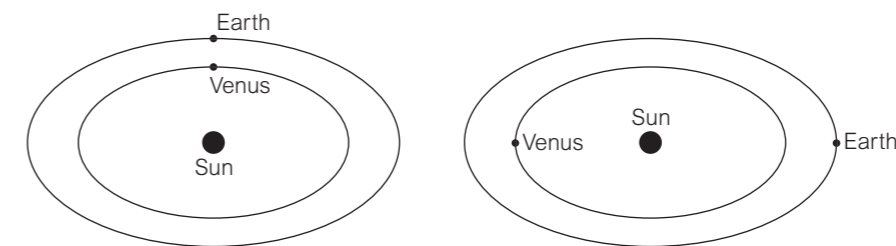
Item	Amount
media services	£6746849
waste recycling	£4444025
health services	£4251390
housing	£3334303

- Rewrite each amount in millions to one decimal place.
 - Draw a bar chart to show this data.
- 5 **Modelling / Problem-solving** A football pitch can be any length between 90m and 120m, and any width between 45m and 90m to the nearest metre. What is the minimum area of a football pitch?

Q5 hint

90m to the nearest metre could be as short as 89.5m

- 6 **Reasoning** The nearest planet to Earth is Venus. Both planets orbit the Sun. The orbit of Earth varies between 147 million km and 152 million km away from the Sun. The orbit of Venus varies between 107 million km and 109 million km away from the Sun. These measurements are given to the nearest million km.



Assume that both planets lie in the same plane and orbit the Sun at different speeds.

- What is the closest possible distance between them?
 - What is the furthest possible distance between them?
- 7 **Real / Finance** Bank statements show overdrawn balances as negative numbers. These students are all overdrawn. Two bank balances are missing.

Student	Bank balance (£)
Lily	-65.94
Mia	
Freya	-72.31
Maya	-12.62
Arjan	-12.84

Student	Bank balance (£)
Josh	-47.15
Luke	-17.03
Ali	-22.67
Junior	-5.82
Lincoln	

- Mia owes the most. Write a possible balance for Mia's account.
 - Lincoln owes the least. Write a possible balance for Lincoln's account.
- 8 Liquid medicines can be measured in centilitres or millilitres.
- Change these doses from ml to cl.
 - 25ml
 - 50ml
 - 60ml
 - 125ml
 - A bottle contains 1 litre of medicine. How many of each dose from part a could you get from the bottle?

Q8 hint

10ml = 1cl
1 litre = 1000ml

9 STEM Prescription medicine doses are measured in grams and milligrams.

A high dose tablet of ibuprofen has 600mg of active ingredient.

a How much is this in grams?

A tablet with 600mg of active ingredient weighs 2.4g in total.

b How much of the tablet is *not* active ingredient?

10 Real Nurses frequently carry out calculations using ratios to convert between units.

A doctor prescribes 200mg of ibuprofen.

The medicine is in a container that has 500mg of ibuprofen dissolved in 40ml of water.

How much of the liquid should the nurse give to the patient so they take the correct dose of ibuprofen?

11 Reasoning Work out

a $105 \div 5$

b 105×0.2

c $425 \div 5$

d 425×0.2

e Copy and complete.

$\div 10$ is equivalent to $\times 0.1$

$\div 5$ is equivalent to \square

$\div 0.2$ is equivalent to \square

$\div 2$ is equivalent to \square

$\div 4$ is equivalent to \square

12 Real 5g of grass seed covers a 10m by 10m square.

a How many m^2 will 5g cover?

b How many grams do you need to cover a football pitch that is 110m \times 60m?

13 Problem-solving Ramiz is thinking of assembling a bike from spare parts bought from an online retailer.

The prices of the main items are given in these tables.

Part	Price
frame	£495.00
wheels (each)	£112.49
gears	£37.99
brakes (each)	£53.75
saddle	£20.99

Part	Price
seat pillar	£47.36
handlebars	£39.96
tyres (each)	£43.75
chain	£11.89
inner tubes (each)	£4.49

a How much would making such a bike cost?

Postage and packing adds 10% to the price.

b How much will it cost to have all the components delivered?

A similar new bike in a bike shop costs £1150.

c Which would be cheaper, and by how much?

14 A cereal box is 19.6cm wide, 7.2cm deep and 27.5cm high.

a What is the volume of the cereal box?

b All three dimensions are halved. What is the ratio of the volume of the small box to the volume of the original one?

15 8km is approximately 5 miles.

a How many miles is each km?

b How many km is each mile?

Q9 hint

1000mg = 1g

Q11e hint

Use your answers from parts **a** to **d** to help you.

16 Finance

a On a particular day £200 is worth €229.

i How much is £1 worth in euros?

ii How much is €1 worth in pounds?

b On another day £50 is worth \$79.

i How much is £1 worth in dollars?

ii How much is \$1 worth in pounds?

17 Use suitable equivalent calculations to work out these.

a 3.5×62

b 1.6×125

c 2.25×848

d 1.5×4682

e 1.8×4235

f 6.25×488



18 Use a calculator to work out

a $4.2^2 \times (3.6 + 1\frac{1}{2})$

b $\frac{9}{4} \times 3.5 + 8.4^2$

c $6^3 + 4.2^2 + 1.1^2 + \frac{4}{5}$

d $(2.6 + 3.2)^2 \times (\frac{3}{4} + 1.12)^2$

19 Finance Banks use interest rates as a way to charge people for borrowing money.

The charge you pay is a percentage of the amount you borrow.

For example, Clare borrows £10 000 at an interest rate of 5% per year.

At the end of the year she is charged $\text{£}10\,000 \times \frac{5}{100} = \text{£}10\,000 \times 0.05 = \text{£}500$ interest.

Ajmal has borrowed £200 000 from the bank to buy a house, at an interest rate of 3.2% per year.

a How much interest will he pay if he borrows the money for a year?

b He pays the bank £850 per month. How much does he owe the bank at the end of the first year?

20 Copy and complete these. Put the correct sign, < or >, between each pair of numbers.

a $-30.58 \square -33.9$

b $-23.69 \square -18.93$

c $-85.93 \square -66.47$

d $-13.87 \square -82.57$

e $-66.43 \square -25.07$

f $-40.02 \square -25.83$

g $-39.93 \square -39.929$

h $-4.59 \square 4.61$

Investigation

Estimate the volume and the surface area of a typical adult.

Use a cuboid as a model.

You may wish to use measuring equipment to help you.

Use a sensible degree of accuracy for all your measurements and calculations.

Modelling



21 Reflect What kind of jobs might need the maths skills you have used in these Extend lessons?

Look back at the questions to help you. For example, Q19 asked you to work out interest on a loan. Someone working as a financial advisor needs these skills.

Q17a hint

3.5 is equivalent to $\frac{7}{2}$
Multiplying by 3.5 is the same as multiplying by 7 and dividing by 2.

6 Unit test

Log how you did on your
Student Progression Chart.

1 Round each amount to two decimal places.

- a £66.255 b £134.0875 c £236.625

2 This table shows the distance between London and four other large cities.
Round each distance to the nearest 1000 km.

From	To	Distance
London	Auckland	18 327 km
London	Tokyo	9 582 km
London	Buenos Aires	11 102 km
London	Los Angeles	8 778 km

3 Round each number to three decimal places.

- a 4.7913 b 37.0004 c 21.4897

4 Work out

- a $26.1 + 9.65$ b $10 - 1.72$ c $9.4 + 6.57 - 11.46$

5 Work out

- a 345×0.62 b 3.5×0.15 c 0.05×0.64

6 Long rolls of cloth need to be cut in the ratio 5 : 1 : 2.
How long is the longest piece of cloth from a roll 48 m long?7 Rearrange these numbers in *ascending* order.
45.39, 45.18, 45.275, 45.33, 66.5, 66.39

8 Work out

- a $36 \div 0.1$ b $419 \div 0.01$ c $4.8 \div 0.6$
d $48 \div 0.08$ e $8.4 \div 0.2$ f $0.63 \div 0.3$

9 Rearrange these numbers in *ascending* order.
-9.31, -9.78, -9.57, -9.3, -9.53, -9.511, -9.9

10 Simplify each ratio.

- a 12 : 16.8
b 1.5 : 7.5

11 Sophie mixes acid and water in the ratio 2 : 5.2.
She makes 288 ml of the mixture.
How much acid and how much water did she mix?

12 Ben makes orange paint by mixing red, yellow and white paint in the ratio 20 : 16 : 1.5.

How much of each colour does he need to make 1.5 litres of orange paint?

13 $471 \times 34 = 16014$

Use this multiplication fact to work out

- a 4.71×0.34 b 0.471×34
c 47.1×0.034 d 0.471×0.34

14 50 inches is about the same distance as 127 cm.

What is the ratio of inches to cm?

Give your answer as a unit ratio.

15 Work out

- a 54.18×6.7 b $78.03 \div 1.7$

16 Copy and complete these. Put the correct sign, < or >, between each pair of numbers.

- a $40.43 \square 58.57$ b $68.6 \square 66.79$ c $87.62 \square 87.43$
d $-7.62 \square -7.7$ e $-6.145 \square -6.154$ f $-9.803 \square -9.088$

17 John's savings account pays 2.5% interest per year.

John has £500 in savings.

How much interest will he have earned after 1 year?

Challenge

0.12	0.86	1.188	12.5
5.04	27.5	9	0.7
11.3	6.3	0.1	33
51.3	2.97	10.7	10.8

Each of the numbers in the blue rectangle can be made by adding, subtracting, multiplying or dividing some or all of these decimal numbers.

0.3 0.4 1.4 9.9 3.6 6.2 5.7

a You can use each number a maximum of once in each calculation.
Make as many of the numbers from the blue rectangle as you can.
Keep a note of the calculations you do to avoid duplication.

b Following the same rules:

What is the highest number you can make?

What is the lowest number you can make?

What is the number closest to zero you can make?

19 **Reflect** Look back at the questions in this unit test.

Which took the shortest time to answer? Why?

Which took the longest time to answer? Why?

Which took the most thought to answer? Why?