MATH AND SCIENCE @ WORK

AP* CALCULUS Educator Edition

ASCENDING FROM THE MOON

Instructional Objectives

Students will

- use the chain rule to find the rates of change of two or more variables that are changing with respect to time; and
- investigate the relationship between the angle of elevation, the rate of change in the angle of elevation and time.

Degree of Difficulty

This problem is challenging because students need to recall and apply mathematical concepts from Algebra I, Geometry, and Trigonometry.

- For the average AP Calculus AB/BC student the problem is moderately difficult.

Background

This problem is part of a series of problems that apply math and science principles to human space exploration at NASA.

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

NASA is considering a new lander, which would be capable of landing a new generation of explorers on the surface of the Moon or perhaps even Mars.

Similar in design to the Apollo Lunar Excursion Module (LEM), this new lander would be much larger and would have the ability to carry four astronauts to the surface compared to the two-man Apollo LEM. The new lander would also have a much larger crew cabin volume, approximately 12 m^3 (424 ft^3), compared to the Apollo LEM, 6.65 m^3 (235 ft^3). Figure 1 shows a comparison of the new concept and the Apollo LEM.

Grade Level
11-12

Key Topic
Application of differentiation – related rates

Degree of Difficulty
Calculus AB: Moderate
Calculus BC: Moderate

Teacher Prep Time
5 minutes

Problem Duration
30-45 minutes

Technology
Graphing Calculator

Materials
Student Edition including:
- Background handout
- Problem worksheet
- Support diagrams

AP Course Topics
Derivatives:
- Application of derivatives
Computation of derivatives
- Chain Rule

NCTM Principles and Standards
- Geometry
- Problem Solving
- Connections

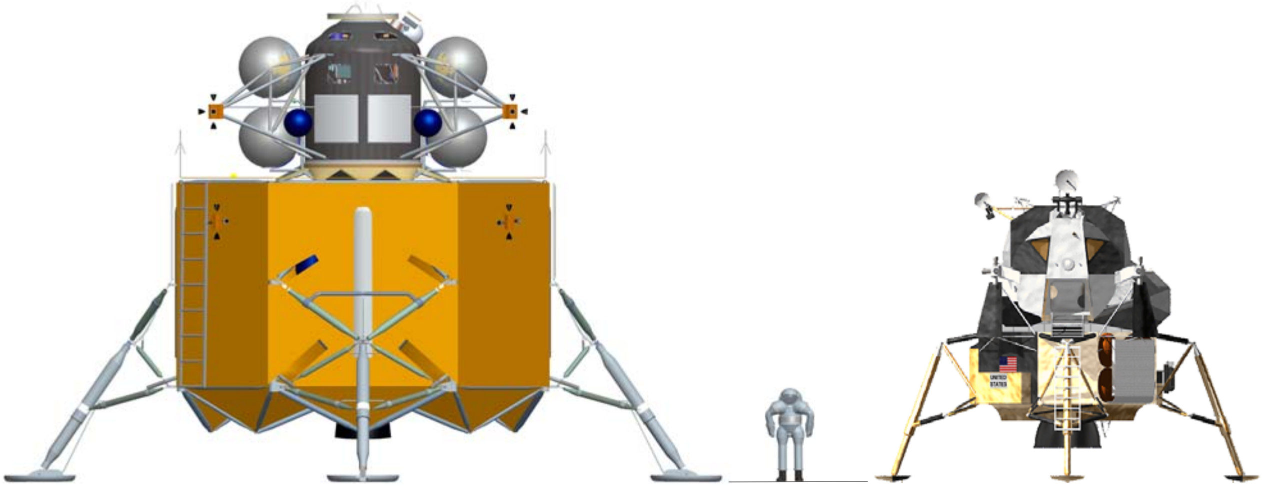


Figure 1: Comparison of the new lander (NASA concept) and the Apollo LEM (not to scale)

The entire lander (See Figure 2, below) would consist of a descent stage, an ascent stage, and a large cargo volume that can be occupied by habitation modules or cargo. The descent stage provides the capability to perform an orbital insertion and landing. It also serves as the launch platform for the ascent stage and as a “flat bed truck” that could transport large cargo to the surface. The ascent stage functions as the flight deck/crew cabin for landing on the surface. It would provide a limited number of hours of surface stay, and would allow the crew to ascend back to orbit.

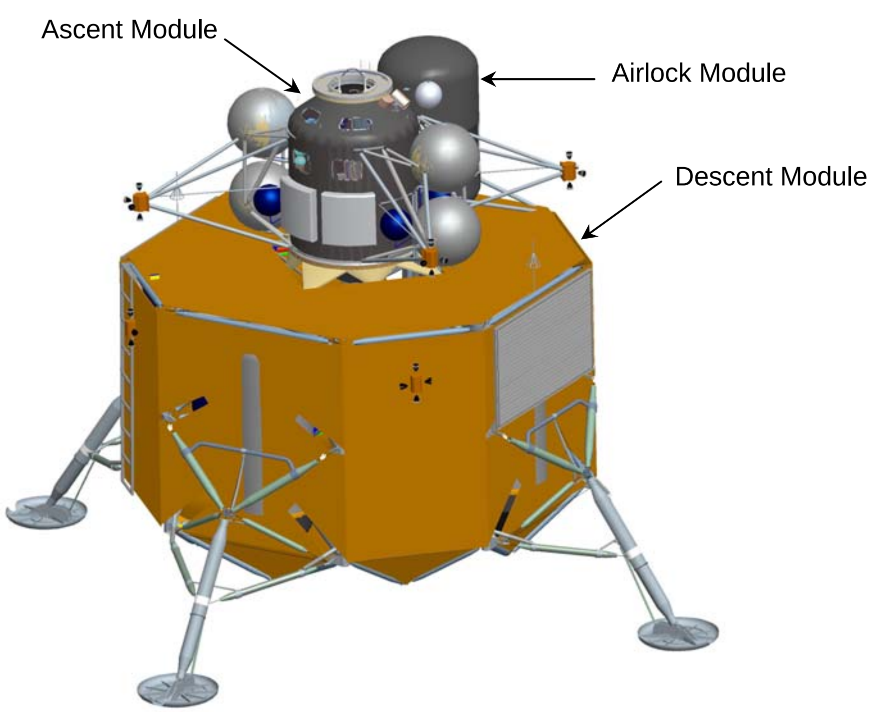


Figure 2: New lander (NASA Concept)

AP Course Topics

Derivatives

- Applications of derivatives:
  - Modeling rates of change.
- Computations of derivatives:
  - Chain rule and implicit differentiation

NCTM Principles and Standards

Geometry

- Use trigonometric relationships to determine lengths and angle measures.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.

Connections

- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

Problem

An astronaut, with camera at ground level, near a lunar outpost on the Moon is filming the liftoff of the ascent portion of the lander that is rising vertically according to the position equation $y(t)=0.683t^2$, where $y(t)$ is measured in meters and t is measured in seconds. The astronaut holding the camera is 1000 m from the base of the lander. See Figure 3.

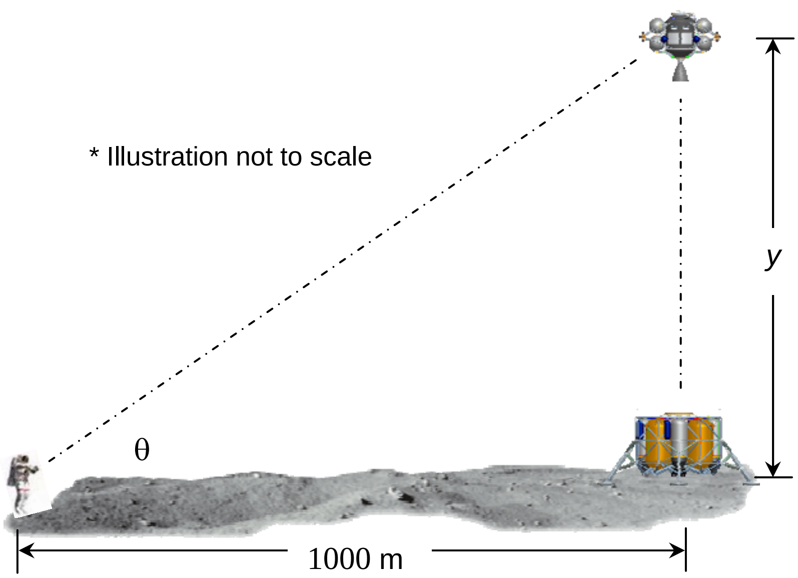


Figure 3: Problem Diagram

A. Using the Chain Rule, find the rate of change in the angle of elevation of the camera at 10 seconds after liftoff. Express your answer in rad/s and deg/s.

B. Describe the relationship verbally and graphically between the angle of elevation and time.

C. Use the graphing calculator to determine the time, *t*, when the rate of change in the angle of elevation is a maximum.

D. What is the angle of elevation when the rate of change in the angle of elevation is a maximum? Express your answer in radians and degrees.

Solution Key

A. Using the Chain Rule, find the rate of change in the angle of elevation of the camera at 10 seconds after liftoff. Express your answer in rad/s and deg/s.

Given:

$$t = 10\text{ s}$$

$$y = 0.683t^2$$

$$y = 0.683(10\text{ s})^2$$

$$y = 68.3\text{ m}$$

Find: $\dfrac{d\theta}{dt}$

You can relate y and θ by the equation,

$$\tan\theta = \frac{y}{1000}$$

or

$$\theta = \tan^{-1}\frac{y}{1000}$$

Remember, $y(t) = 0.683t^2$*. Therefore, you can relate t and* θ *by the equation,*

$$\theta = \tan^{-1}\left(\frac{0.683t^2}{1000}\right)$$

or

$$\theta = \tan^{-1}\left(0.000683t^2\right)$$

To find $\frac{d\theta}{dt}$ use the chain rule: $\frac{d\theta}{dt} = \frac{du}{dt}\cdot\frac{d\theta}{du}$

Let $u = 0.000683t^2$

Thus $\theta = \tan^{-1}(u)$

$$\frac{du}{dt} = 0.00136t$$

$$\frac{d\theta}{du} = \frac{1}{1+u^2} \quad or \quad \frac{d\theta}{du} = \frac{1}{1+\left(0.000683t^2\right)^2}$$

$$\frac{d\theta}{dt} = 0.00136t\cdot\frac{1}{1+\left(0.000683t^2\right)^2}$$

$$\frac{d\theta}{dt} = \frac{0.00136t}{1+\left(0.000683t^2\right)^2}$$

When t = 10 s,

$$\frac{d\theta}{dt} = \frac{0.00136\,(10\text{ s})}{1+\left(0.000683\,(10\text{ s})^2\right)^2}$$

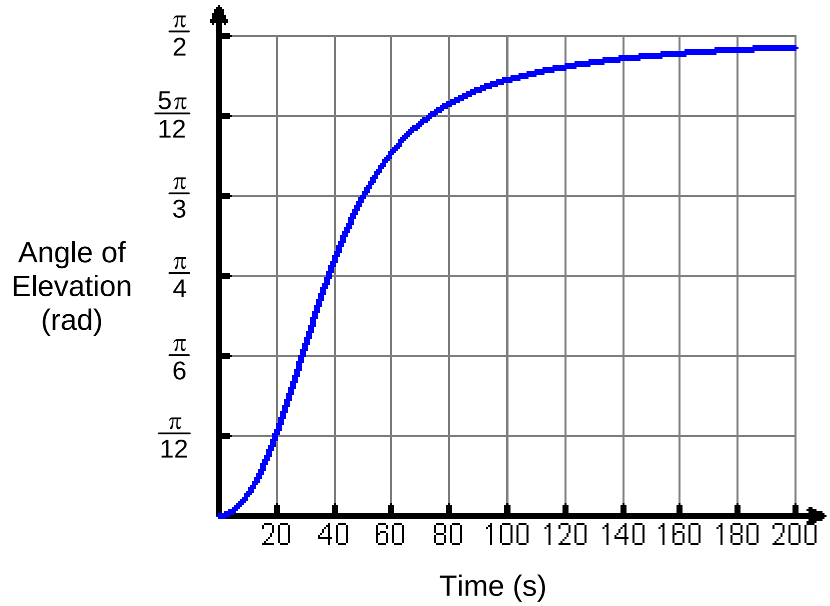
$$\frac{d\theta}{dt} = 0.0136\text{ rad/s or } 0.779\text{ deg/s}$$

B. Describe the relationship verbally and graphically between the angle of elevation and time.

It is a Tangential Relationship. The angle of elevation approaches $\frac{\pi}{2}$ *radians or 90°.*

$$\theta = \tan^{-1}\left(\frac{0.683t^2}{1000}\right)$$

Angle of Elevation vs. Time

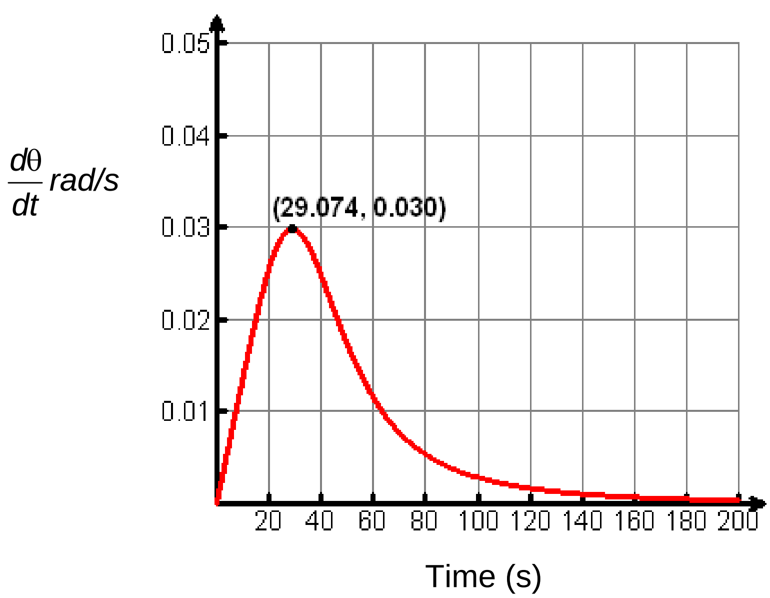


C. Use the graphing calculator to determine the time, *t*, when the rate of change in the angle of elevation is a maximum.

Graph the first derivative of θ *that was found in #1 then find the maximum.*

$$\frac{d\theta}{dt} = \frac{0.001366t}{1+\left(0.000683t^2\right)^2}$$

Rate of Change of Angle of Elevation vs. Time



The maximum is found at t = 29.074 *seconds*

D. What is the angle of elevation when the rate of change in the angle of elevation is a maximum? Express your answer in radians and degrees.

$$\theta = \tan^{-1}\left(\frac{y}{1000}\right)$$

$$\theta = \tan^{-1}\left(\frac{0.683t^2}{1000}\right)$$

$$\theta = \tan^{-1}\left(\frac{0.683(29.074)^2}{1000}\right)$$

$$\theta = 0.524 \text{ radians or } 30°$$

Scoring Guide

Suggested 10 points total to be given.

Question		Distribution of points
A	*4 points*	1 expresses θ as a composite function of t 1 find $\frac{d\theta}{dt}$ using the chain rule 2 correct answer in rad/s and in deg/s
B	*2 point*	1 point for the verbal description 1 point for the graphical description
C	*2 points*	1 point for the graph of $\frac{d\theta}{dt}$ 1 point for the time that gives a maximum value
D	*2 points*	2 correct answer in rad and in deg

Contributors

Thanks to the subject matter experts for their contributions in developing this problem:

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