Assessment of the Gaussian Covariance Approximation over an Earth-Asteroid Encounter Period

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Outline

- ***Summary***
- ***Background***
  - Observability and Orbit Determination
  - Assessing Impact Risk
- ***Motivation and Approach***
- ***Results***
- ***Conclusions and Future Work***

Summary

- ***Previous analysis examined the use of Mahalanobis distance for assessing an asteroid's impact risk to the Earth***
  - Assumed the asteroid's state uncertainty (covariance matrix) remained Gaussian over the encounter

- ***This analysis examines the validity of that assumption and attempts to identify conditions where this assumption breaks down***
  - Identifies an assessment metric, characteristic scale ratio:

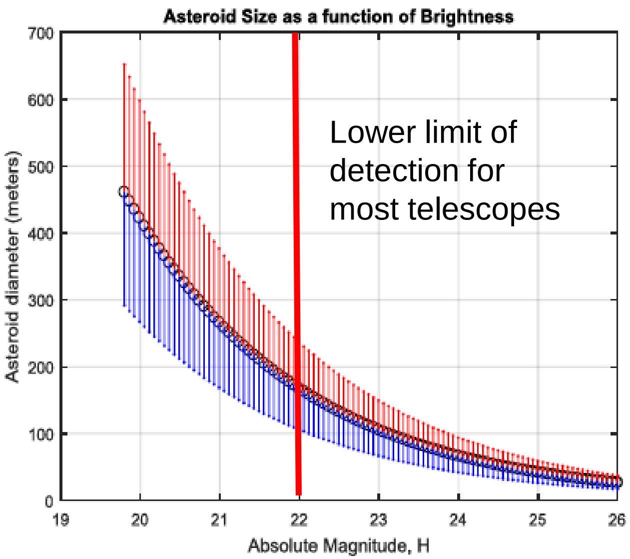
$$R_{sc} = \frac{\max\left(eigenvalue(P, just\ prior\ to\ the\ Earth\ encounter)\right)}{\min(nominal\ miss\ distance)}$$

  - Where P is the asteroid's 3x3 position covariance matrix

Observability and Orbit Determination

- ***In 2005, the US Congress directed a survey to find 90% of all near-Earth objects larger than 140 meters in diameter by 2020***
  - To date, we've only discovered approximately 28%
  - Driven by limited observability of small celestial objects

$$D = \frac{1329}{\sqrt{\alpha}} 10^{-0.2H}$$

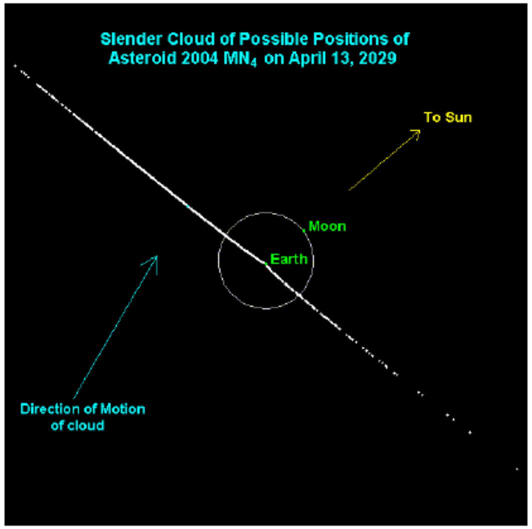


- ***Limited observability leads to short observation periods***
  - Often < 1% of an asteroid's orbit

- ***Poor observations result in large initial uncertainties in an asteroid's orbit energy and velocity***

Assessing Impact Risk

- ***Impact predictions may span years or even decades***
- ***Propagating large initial velocity uncertainties over long spans results in incredibly large position uncertainties at the Earth-encounter period***
- ***Two common metrics used for assessing risk:***
  - Probability of collision (P_c)
  - Mahalanobis distance (D_{MH})
- ***Large uncertainties cause P_c computations to return negligible values***
  - Also susceptible to "false-positives" from Pc-roll-off
- ***D_{MH} computations must assume that the covariance remains Gaussian throughout the encounter period***



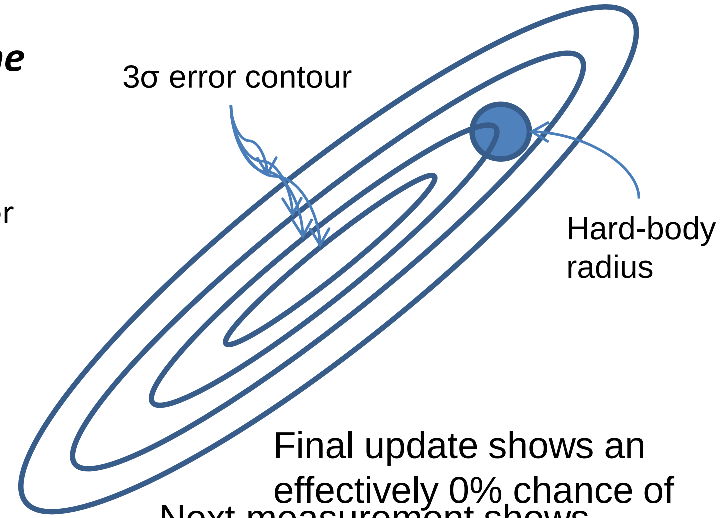
Courtesy of NASA JPL Near Earth Object Program
neo.jpl.nasa.gov/news/news146.html

P_c -roll-off

- ***An asteroid-Earth impact is a stochastic determination***
  - Measurement of the asteroid's state (range/range-rate measurements) has associated error
  - Error in the asteroid's state (position/velocity) is directly correlated to these measurement errors
  - Propagation of the state forward in time thus requires propagating these errors forward in time as well
  - As such, the "success" of impact mitigation must include some stochastic measure based on the associated state error
- ***Collision Probability (Pc) is generally considered to be the "standard" metric***
  - Restricted to Cartesian space via the hard-body radius
  - Susceptible to "Pc-roll-off" which could provide false positives or delayed reactions to true-positives
- ***Mahalanobis distance***
  - Unitless and scale-invariant
  - Deconstructs state uncertainties into sigma contours
  - Not susceptible to "roll-off" phenomena

$$\overrightarrow{dR} = \overrightarrow{r_{Earth}} - \overrightarrow{r_{asteroid}}$$

$$D_{Mahalanobis} = \sqrt{\overrightarrow{dR}^T * [P_{pos}]^{-1} * \overrightarrow{dR}}$$



Motivation and Approach

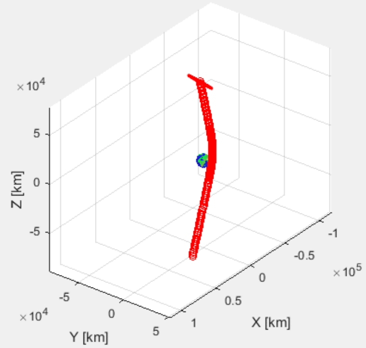
- ***Previous work examined using D_{MH} to assess impact risk***
  - Used only the state transition matrix to propagate the covariance across the Earth-encounter
  - Found that the matrix orientation was greatly affected after the encounter
    - Suggests that the matrix experienced a gravitational gradient over the encounter period
      - This gradient could likely lead to non-Gaussian characteristics

- ***The Gaussian assessment was performed by comparing a Monte Carlo sampling of the initial covariance to the propagated matrix***
  - A covariance quality factor (C_{QF}) was defined as the fraction of Monte Carlo samples that remained inside the appropriate σ-contour

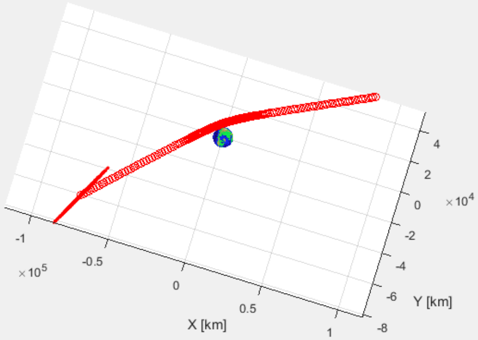
- ***C_{QF} was then compared to the characteristic scale ratio (R_{sc})***

Gravity-Gradient “Torquing” (6,300 km miss)

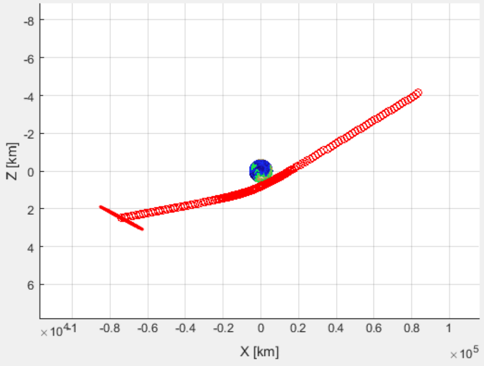
Earth-relative Trajectory



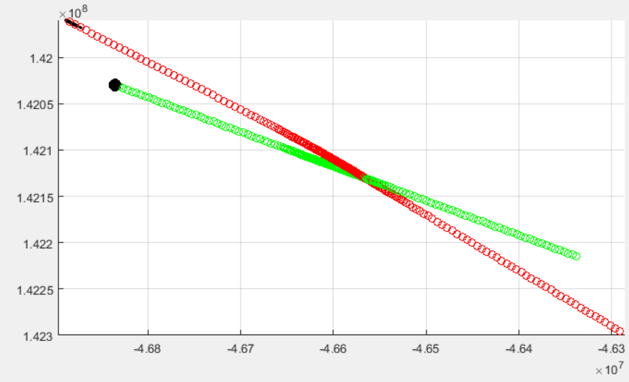
Earth-relative Trajectory



Earth-relative Trajectory



Heliocentric Trajectory



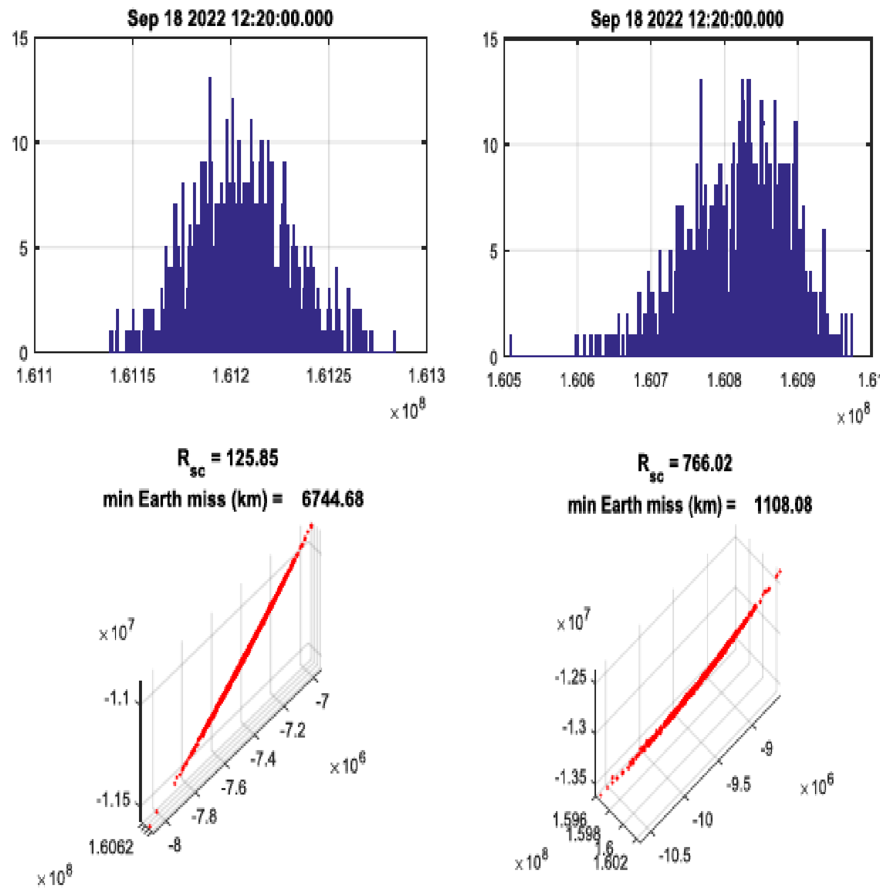
Ensure Mission Success

Results

- ***As expected, the cases that exhibited smaller matrices and passed further from the Earth showed better Gaussian behavior***

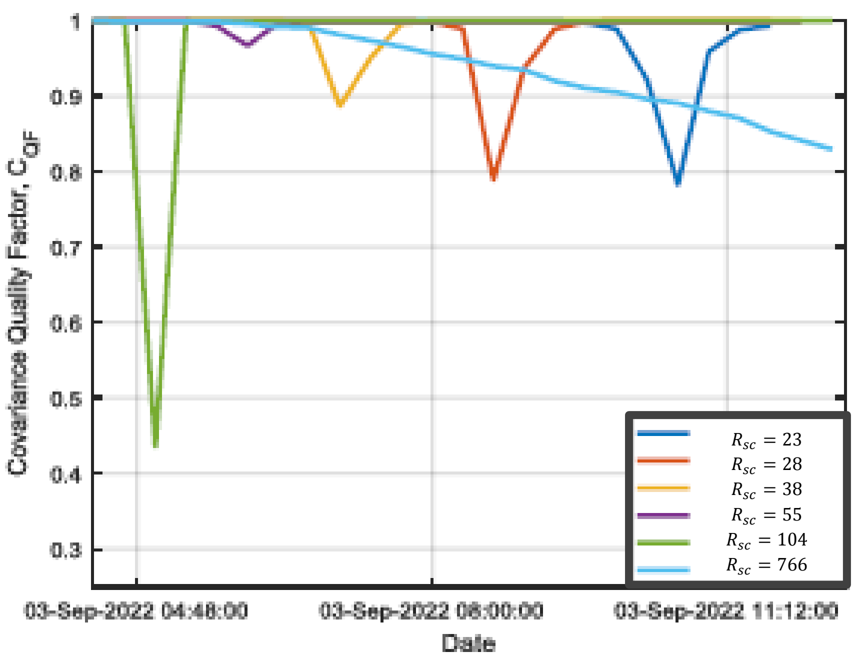
- ***However, the Gaussian characteristics were tolerant of cases where the covariance matrix was greater than 150x larger than the minimum achieved miss distance***

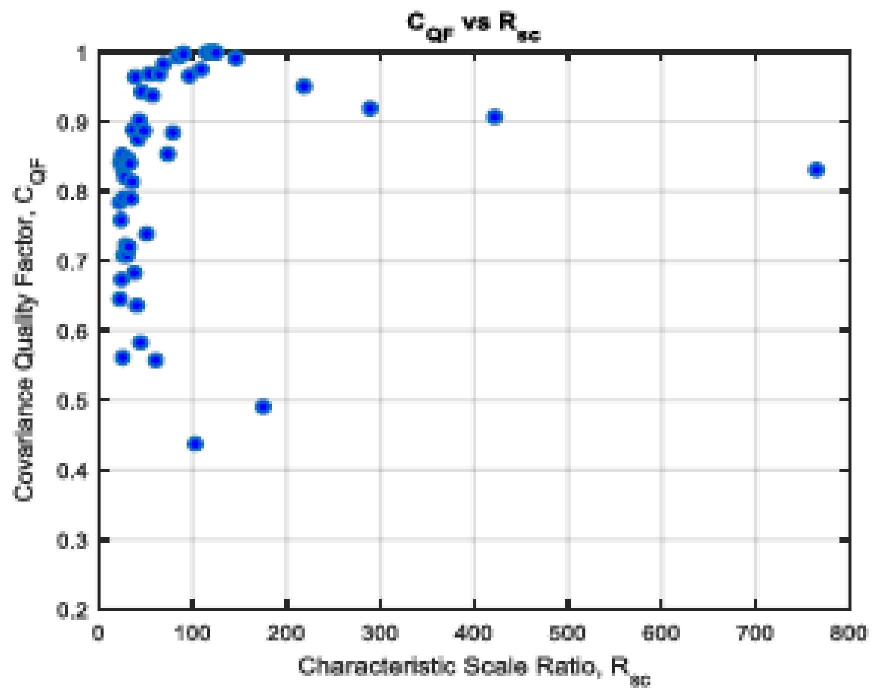
- ***Additionally, for cases where*** $R_{sc} \lesssim 200$***, the covariance matrix appears to "rebound" back to a Gaussian distribution shortly after the encounter***



Results

6σ-contour Agreement





Conclusions and Future Work

- ***For impact scenarios where $R_{sc} \leq 150$, the Mahalanobis distance is a valid metric to assess the risk that an asteroid poses to Earth***
  - Suspect propagation step-size is obfuscating minimum quality factor, but covariance matrices appear to "rebound" when $R_{sc} \leq 150$
  - D_{MH} is not susceptible to "roll-off" phenomena, so may be preferred over P_c for these cases

- ***Future work needs to address the peaked minima shown in the previous slide***
  - Likely due to step-size granularity used to propagate the asteroid across the Earth encounter

- ***Future work will also need to address different relative orbit geometries***
  - Cases shown here were generated by perturbing the hypothetical impact scenario of 2015PDC
  - Perturbations to this orbit were applied at different points in its trajectory

Questions?