Asset Pricing and Mispricing

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Abstract

In this paper we develop models for stock returns when stock prices are subject to stochastic mispricing errors. We show that expected rates of return depend not only on the fundamental risk that is captured by a standard asset pricing model, but also on the type and degree of asset mispricing, even when the mispricing is zero on average. Empirically, the mispricing induced return bias, proxied either by Kalman filter estimates or by volatility and variance ratio of residual returns, are shown to be significantly associated with realized risk adjusted returns.

"Does the joint hypothesis problem make empirical work on asset pricing models uninteresting?"

E.F. Fama (1991, p 1576)

1 Introduction

As Eugene Fama points out, tests of classical asset pricing models such as the CAPM, CCAPM, or ICAPM implicitly rely on an assumption of market efficiency which permits the substitution of realized returns for expected returns. However, there is increasing evidence that common stocks are mispriced relative to these models,[1] although the reasons for the pricing discrepancies remain in dispute. For example, de Bondt and Thaler (1985, 1987) find long run reversals of prior stock price changes which they interpret as corrections of prior over-reactions to news, while Jegadeesh and Titman (1993) among others find positive autocorrelation of individual stock returns at the 6-12 month horizon, which is consistent with the slow adjustment to firm specific news documented in a large number of studies. Jegadeesh and Titman (1995) also find evidence that stock prices tend to over-react to firm specific information. Lee and Swaminathan (2000) find that low (high) trading volume stocks tend to be under- (over-) valued by the market. Pastor and Stambaugh (2003), Acharya and Pedersen (2005) and Sadka (2006) show stock returns are affected by (or at least covary with) the state of stock market liquidity, while Amihud (2002) shows that unanticipated increases in market illiquidity reduce the level of stock prices. Lee *et al.* (1991) and Swaminathan (1996) (more circumspectly) argue that stock prices are affected by the state of 'sentiment'.

In this paper we show that, for securities which are subject to stochastic mispricing relative to a given asset pricing model, it is likely that either their *prices* will fail to be unconditionally rational or their *returns* will fail to be unconditionally rational, or both. By *unconditionally rational prices* we mean prices whose unconditional expectations are consistent with the fundamental asset pricing model, and by *unconditionally rational returns* we mean returns whose unconditional expectations are consistent with the fundamental

[1] French and Roll (1986) suggest that on average 4 to 12% of the daily return variance of common stock returns is due to mispricing.

asset pricing model. Thus a stock that on average trades at 100% of its fundamental value, but whose price fluctuates about the fundamental value, will have a return that is biased up relative to that predicted by the model that determines the fundamental value. This 'mispricing return bias' is the focus of this paper.

The basic intuition of our analysis follows immediately from Jensen's Inequality: price is a non-linear function of expected return, so that if one variable is subject to random error then the expectation of the other variable will be biased.[2] It is of course possible that neither prices nor expected returns are unconditionally rational. The bias in the expected returns due to mispricing is shown to depend on the volatility and first order autocorrelation of the mispricing. Unfortunately, the mispricing is not directly observable, and we must use proxies for the mispricing return bias. Our empirical tests reveal that portfolios formed on the basis of proxies for the mispricing return bias have significantly different returns after adjusting for risk using standard models.

The analysis in this paper has implications for studies that find significant relations between stock returns and variables that may be proxies for the mispricing related return bias we consider. Thus, measures of the cost of transacting such as the bid ask spread or Kyle's λ are likely to be positively associated with the magnitude of pricing errors since transactions costs impede arbitrage. This suggests that a part of the approximately 7% return differential between high and low liquidity portfolios documented in several studies[3] may be attributable to this mispricing return bias. Similarly, we show that the sensitivity of stock returns to variables that have common effects on stock prices, such as market liquidity or sentiment, is related to the mispricing return bias, so that a part of the the annual return premium of around 7.5% between high and low liquidity beta portfolios reported by Pastor and Stambaugh (2003) is likely to be attributable at least in part to the mispricing return bias. We also show that it is possible to generate a return premium of the type that Hou and Moskowitz (2005) have found to be associated with slow adjustment to (market-wide) information in a model in which prices are unconditionally rational but adjust slowly to new information and are subject to 'liquidity shocks', since these stocks will be subject to

[2] This is analogous to the point made by Cox *et al.* (1981) that expected holding period returns on bonds of all maturities cannot be equal for all holding periods, except under certainty.

[3] For an extensive survey of the research on liquidity and asset pricing see Amihud *et al.* (2005).

the mispricing return bias.

This paper follows an early literature on the implications of security mispricing for measuring rates of return, including Blume and Stambaugh (1983), and Roll (1983), who are concerned with the effects of daily auto-correlations and the bid-ask bounce on measured rates of return.[4] More recently Liu and Strong (2006) analyze the effects of portfolio rebalancing assumptions on reported returns.

The remainder of the paper is organized as follows. Section 2 presents a simple one period example which shows that, in the presence of random security mispricing, unconditionally rational prices are inconsistent with unconditionally rational returns. Section 3 analyzes the return bias introduced by random mispricing in a general intertemporal context. Section 4 presents examples of mispricing structures and uses them to analyze the return premia that have been found to be associated with transactions cost type variables, with systematic liquidity risk, and with slow adjustment to information. Section 5 presents empirical results relating returns to proxies for the mispricing return bias, and Section 6 concludes.

2 A Simple Example

Consider an asset whose payoff at the end of one period is $\tilde{X}$, and denote its fundamental price by P^*. Then

$$P^* = \frac{E[\tilde{X}]}{1+r^*} \tag{1}$$

where r^* is the equilibrium expected rate of return on the security according to some given pricing model. Let $P \equiv P^*\tilde{Z}$ denote the market price of the security, where $\tilde{Z}$ is a random variable which is independent of the payoff $\tilde{X}$. Then the mispricing of the security relative to the given model is written as $P^*(\tilde{Z}-1)$. If $\tilde{Z}$ has mean unity then we say that the price is unconditionally rational. Let $\tilde{R}$ denote the realized rate of return on the security. Then:

$$1+\tilde{R} = \frac{\tilde{X}}{P} = \frac{\tilde{X}}{P^*\tilde{Z}} = \frac{\tilde{X}(1+r^*)}{E[\tilde{X}]\tilde{Z}} \tag{2}$$

Taking expectations, we have:

$$E[1+\tilde{R}] = (1+r^*)E\frac{1}{\tilde{Z}} \neq 1+r^* \tag{3}$$

[4] See also Canina *et al* (1998).

Thus the unconditional expected return on the security is not equal to the equilibrium expected return, r^*, so long as the price is unconditionally rational so that $E[\tilde{Z}] = 1$. Conversely, if the returns are unconditionally rational so that $E[\tilde{R}] = r^*$, then $E[\tilde{Z}]$ cannot have mean unity so long as $\tilde{Z}$ has a strictly positive variance, and unconditionally rational prices imply that returns fail to be unconditionally rational.

3 General Structure

Now consider an arbitrary multi-period setting in which the security pays a dividend of $\tilde{D}_t$ at the end of period t and denote the market (fundamental) price at the beginning of the period by $P_t(P_t^*)$, where $P_t \equiv P_t^* Z_t$. We shall assume throughout that market prices are strictly positive and impose the further weak restriction that $z \equiv lnZ$, the log of the 'market pricing multiple', Z, is a stationary random variable.

Then we can write $1 + R_t$ the (gross) market rate of return on the security in period t as

$$1 + R_t \equiv \frac{P_{t+1}^* Z_{t+1} + D_t}{P_t^* Z_t} = \frac{P_{t+1}^*}{P_t^*}\frac{Z_{t+1}}{Z_t} + \frac{D_t}{P_t^* Z_t} \tag{4}$$

$$\equiv (1 + R_t^{*g})(1 + \frac{\Delta Z_t}{Z_t}) + \delta_t^*(1/Z_t) \tag{5}$$

where $\Delta Z_t \equiv Z_{t+1} - Z_t$, $R^{*g} \equiv (P_{t+1}^* - P_t^*)/P_t^*$ is the 'capital gain return' based on the fundamental price, and $\delta_t^* \equiv D_t/P_t^*$ is the dividend yield based on the fundamental price. Note that the return based on the fundamental price, R_t^* is equal to $R_t^{*g} + \delta_t^*$.

Then the market return is related to the fundamental return by:

$$R_t = R_t^* + R_t^{*g}\frac{\Delta Z_t}{Z_t} + \frac{\Delta Z_t}{Z_t} - \delta_t^*(1 - 1/Z_t) \tag{6}$$

Assume for simplicity that the mispricing variable $\tilde{Z}_t$ is independent of the (fundamental) dividend yield δ_t. Then, taking expectations in (6), the expected market return is related to the expected fundamental rate of return by:

$$E[R_t] = E[R_t^*] + E\left[R_t^{*g}\right] E\left[\frac{\Delta Z_t}{Z_t}\right] + cov\left(R_t^{*g}, \frac{\Delta Z_t}{Z_t}\right) + E\left[\frac{\Delta Z_t}{Z_t}\right] - E\left[\delta_t^*\right] E\left[1 - 1/Z_t\right] \tag{7}$$

The second and fifth terms in (6) are likely to be small since they involve products of returns or yields with the mispricing variable.[5] Therefore we shall ignore them in what follows, and write the relation between the expected market return and the expected fundamental return as:

$$
\begin{aligned}
E[R_t] &\approx E[R_t^*] + E\left[\frac{\Delta Z_t}{Z_t}\right] + cov\left(R_t^{*g}, \frac{\Delta Z_t}{Z_t}\right) &(8)\\
&\equiv E[R_t^*] + B_1 + B_2 &(9)
\end{aligned}
$$

where $B_1 \equiv E\left[\frac{\Delta Z_t}{Z_t}\right]$, $B_2 \equiv cov\left(R_t^{*g}, \frac{\Delta Z_t}{Z_t}\right)$, and $B \equiv B_1 + B_2$ denotes the *bias* in the expected return caused by mispricing.

If $B_2 > 0$, we shall say that the mispricing is associated with *over-reaction* since the pricing error tends to increase when fundamentals improve and to decrease when they deteriorate.

If $B_2 < 0$, we shall say that the mispricing is associated with *slow adjustment* since an increase (decrease) in the fundamental price is accompanied on average by a smaller proportional change in the market price.

If $B_2 = 0$, we shall say that the mispricing is *unrelated to fundamentals.*

Note that we can write $\Delta Z/Z = e^{\Delta z} - 1$, where $z \equiv lnZ$. Then it follows from the convexity of the exponential function and the assumed stationarity of z that $E[\Delta Z/Z] > 0$, and we have the following conditions on the sign of the bias, B, caused by mispricing:

Lemma 1 *If the mispricing is associated with over-reaction or is unrelated to news, then the bias is strictly positive.*

If the bias is associated with slow adjustment, then the sign of the bias is indeterminate.

In practice we shall find that B_2 is likely to be very small when the mispricing is due to slow adjustment, so that the mispricing return bias will be due mainly to B_1, and there is therefore a presumption that the total bias will be positive. Note that when we neglect

[5]Consider the magnitude of the bias associated with the dividend yield under the rational unconditional pricing assumption that $E[Z] = 1$. Then $1 - E[1/Z] \approx -\sigma_Z^2$, where σ_Z is the standard deviation of the stationary distribution of Z. Suppose that the standard deviation of the mispricing is 0.3 so that $\sigma_Z^2 = 0.09$. This would imply an annual dividend yield related bias of the order of 0.2% for a stock with a 2% (fundamental) dividend yield.

the dividend yield term the magnitude of the return bias is independent of $E[Z]$ so that mispricing will give rise to a return bias whether or not prices are unconditionally rational.

Consider now the determinants of the bias element B_1. Assume that the unconditional distribution of $\Delta z \equiv lnZ_{t+1} - lnZ_t$ is normal with parameters $(\mu_{\Delta z}, \sigma_{\Delta z})$. Since z is a stationary random variable, $\mu_{\Delta z} = 0$ and

$$B_1 = e^{\frac{1}{2}\sigma^2_{\Delta z}} - 1$$

Now we can always write:

$$\Delta z \equiv z_t - z_{t-1} = (\rho_1^z - 1)z_{t-1} + \eta_t$$

where ρ_1^z is the first order autocorrelation of z, and η_t is a zero mean normally distributed error term that is independent of z_{t-1}. Then:

$$B_1 = e^{(1-\rho_1^z)\sigma_z^2} - 1. \tag{10}$$

where σ_z^2 is the unconditional variance of z. Thus the mispricing return bias is decreasing in the first order autocorrelation of z; *ceteris paribus,* mispricing that is rapidly eliminated or even reversed will lead to a higher bias in expected returns. The bias is also increasing in the unconditional variance of the mispricing.

4 Three Models of Mispricing

Although there is a well developed theory of rational security pricing, there exists no canonical model of security *mispricing*. Therefore, in order to assess the effect of mispricing on measured security returns, we shall consider in turn three models of mispricing that have been analyzed in the literature and use them to analyze the mispricing return biases that may be associated with high transactions costs, with liquidity betas and with slow adjustment to information. The first model, which was developed by Poterba and Summers (1988), assumes that mispricing is independent of fundamentals, and follows a simple AR1 process. The second model assumes that mispricing is due to slow adjustment of market prices to new information. In this model, innovations in mispricing are correlated with fundamentals. In the third model, mispricing is associated with a market wide 'mispricing factor'.

4.1 The Poterba-Summers Model

Poterba and Summers (1988) assume that the logarithm of the market price, p_t, is related to the logarithm of the fundamental price, p_t^*, by

$$p_t = p_t^* + z_t \tag{11}$$

where the logarithm of the fundamental price, p_t^* follows a random walk:

$$p_t^* = p_{t-1}^* + \epsilon_t \tag{12}$$

and the logarithm of mispricing, z_t, follows an AR(1) process:

$$z_t = \phi_1 z_{t-1} + \eta_t \tag{13}$$

and ϵ_t and η_t are independent i.i.d. normal random variables with mean zero and variances, σ_ϵ^2 and σ_η^2, respectively.

Then, using equation (10) and noting that $\phi_1 = \rho_1^z$, B_1 may be written as:

$$B_1 = e^{(1-\phi_1)\sigma_z^2} - 1 \tag{14}$$

Provided that the mispricing is unrelated to news so that the innovations in mispricing, η, are uncorrelated with the innovations in the fundamental price, ϵ, B_2 is zero. Poterba and Summers assume that the innovations are uncorrelated. They set $\phi_1 = 0.98$ for monthly data on the market portfolio, which implies that innovations in mispricing have a half life of 2.9 years. Their calibrations yield values of σ_z for the market portfolio ranging from 0.1 to 0.38. These parameter values imply values of $B_1 \approx (1 - \phi_1)\sigma_z^2$ ranging from 0.02% to 0.28% per month, or 0.24% to 3.47% per year. We shall show in Section 5 that ϕ_1 and σ_z, and therefore the mispricing return bias B_1, differ across securities in predictable ways associated with size and other firm characteristics. The resulting mispricing return biases will create systematic patterns in the cross-section of deviations of average returns from equilibrium expected returns, r^*. For example, if for illiquid securities ϕ_1 is 0.90 per month so that 10% of mispricing tends to eliminated each month, and σ_z equals 0.20, then the annualized mispricing return bias for illiquid securities will appear to be 4.8% per year. If the most liquid securities have no mispricing so that for them $\sigma_z = 0$, then their return bias will be zero and the cross-sectional reward for illiquidity will appear to be 4.8%. If ϕ_1 for the illiquid securities is only 0.8, then with the same values of the other parameters the annualized cross-sectional return premium for illiquidity will appear to be 9.6%.

Thus, if illiquidity is associated with greater mispricing, the mispricing return bias may explain part of the very high risk-adjusted return differentials between portfolios chosen on the basis of their liquidity characteristics that researchers have documented in recent years. These return differentials seem to be too high to be explained by the costs of transaction alone. For example, the estimates of Brennan and Subrahmanyam (1996, Table 4) imply that the annualized return differential between the highest and lowest liquidity quintiles of NYSE stocks is 6.6% when (il)liquidity is measured by Kyle's (1985) λ, while Amihud and Mendelson (1986, fn. 19) report an annual risk adjusted return differential of 7% between the extremes of 7 NYSE portfolios formed on the basis of the bid-ask spread. Liu (2006) finds an annual risk-adjusted return differential of over 9% between the extremes of 10 NYSE/AMEX portfolios formed on the basis of the number of days on which no trade takes place, and argues that a systematic liquidity risk factor constructed from this measure of (il)liquidity explains size, book-to-market, cash flow-to-price and divididend yield return anomalies. These high return differentials associated with illiquidity are surprising because theoretical analyses of portfolio strategies under transaction costs by Constantinides (1986) and Vayonnos (1998) suggest that while proportional transactions costs affect trading frequency they have only small effects on prices.[6]

It is likely that illiquidity is associated with mispricing. Pontiff (1996), Gemmill and Thomas (2002), and Kumar and Lee (2006) for example show that other costs of arbitrage (primarily residual risk) are associated with mispricing. Chordia *et al* (2006) find that post earnings announcement drift is concentrated in highly illiquid securities. Hong *et al.* (2000) find that momentum strategies work particularly well for firms that are followed by a small number of analysts for their size, while Brennan and Subrahmanyam (1998) find that analyst following is positively related to liquidity and Brennan *et al* (1993) find that analyst coverage is positively related to the speed of adjustment of prices to market wide information. Thus firms whose costs of trading are high are likely to be especially prone to

[6] See also Lo *et al* (2001) and Huang (2003).

mispricing, both because such firms are typically followed by few if any investment analysts, and because the costs of trading deter those who would attempt to profit from temporary mispricing.[7] If illiquidity is associated with mispricing in general, then our analysis suggest that the mispricing return bias will be highest for illiquid firms that are costly to trade; this may explain part of the very high returns associated with illiquidity.

Other explanations that have been offered for the high return premium for illiquidity include Novy-Marx (2004) who argues that illiquidity is likely to be associated with high expected returns, not because the expected returns are a reward for bearing illiquidity, but because both high returns and illiquidity are likely to be caused by risk factors that may be omitted from the standard asset pricing models. Similarly, Johnson (2006) shows that expected returns and liquidity could covary without liquidity causing expected returns, because they are jointly determined in equilibrium.

4.2 A Model of Slow Adjustment to Information

The possibility that stock prices adjust slowly to new information has been recognized at least since Dimson (1979), and there is an extensive literature starting with Ball and Brown (1968) documenting slow adjustment to earnings news.[8] Brennan *et al* (1993) show that stocks that adjust slowly to *market wide* information tend to be smaller and to be followed by fewer analysts, and Hou and Moskowitz (2005) show that the most delayed firms command a large returns premium that is not explained by risk, size, or liquidity. Jackson and Johnson (2006) show that momentum can be attributed to slow adjustment to news about future earnings. Slow adjustment to new information implies mispricing and therefore a mispricing return bias. This may help to explain the slow adjustment return premium of Hou and Moskowitz (2005).

In order to assess the magnitude of the bias in returns that is caused by slow adjustment

[7]However, Sadka and Scherbina (2004) find evidence of the systematic *overpricing* of firms with a high dispersion of analyst forecasts (presumably due to the Miller (1977) effect) and argue that this is also associated with high costs of transacting.

[8]See Bernard (1993) for a more recent survey of the evidence.

to new information, we shall use the following model of stock prices:

$$P^*_{t+1} = P^*_t\left[1 + R_f + \beta(R_m - R_f) + \epsilon_{t+1}\right] \tag{15}$$

$$P_{t+1} = \kappa P^*_{t+1} + (1-\kappa)P_t \tag{16}$$

Equation (15) in which R_m, and R_f are the market return and risk free rate and ϵ is a mean zero error term, implies that the fundamental price, P^*, satisfies the CAPM. Equation (16) implies that the market price, P, adjusts slowly to the fundamental price so long as $\kappa < 1$.[9]

Market and fundamental prices and returns were simulated 5000 times for 200 months from equations (15) and (16) using randomly selected sequential monthly market returns and risk free rates from the CRSP file for the period January 1926 to December 2003, and normally distributed idiosyncratic error terms ϵ with annualized volatilities of 15% and 30%. $z_t \equiv ln(P_t/P^*_t)$, the log mispricing factor was calculated for each month and the elements of the mispricing return bias were calculated using equation (9). The results, which are reported in Panel A of Table 1, suggest that slow adjustment *alone* is not sufficient to generate a significant bias in returns. For example, even with $\sigma_\epsilon = 0.30$, a value of κ as low as 0.70 generates an average value of σ_z of only 0.03 and a total annual bias, B, of the order of 1%. But for these parameter values the average autocorrelation of monthly stock returns, ρ^R_1, is 0.35 which is much too high to be consistent with the data. Higher values of the bias can be generated with lower values of the adjustment coefficient, κ, but only at the cost of even more implausibly high values of the return autocorrelation.

However, if we add additional proportional *iid* noise, ξ_t, to the market price so that equation (16) becomes:

$$P_{t+1} = \kappa P^*_{t+1} + (1-\kappa)P_t + P_t\xi_t \tag{17}$$

then it is possible to generate a bias in realized returns which is similar to the return premium for slow adjustment found by Hou and Moskowitz (2005) with reasonable values of the return autocorrelation.

[9] Amihud and Mendelson (1987) use a similar partial adjustment model.

Panel B shows results for $\sigma_\xi > 0$. Now it is easy to generate an annualized bias, B, of the order of 2.5-4.3% while keeping the autocorrelation of returns, ρ_1^R, in a range of $\pm$ 0.05. Most of the bias is due to the element $B_1 \equiv E[dZ/Z]$, and B_2 is close to zero in all cases, despite the fact that slow adjustment implies that the innovation in Z is negatively correlated with the innovation in the rational price, P^*.

The table also reports the average coefficients, α and β, from regressions of the simulated stock excess returns on the market excess returns. The average values of α tend to be close to the estimates of the total bias, B.

Panel C of the table shows that when $\kappa = 1$ so that there is no slow adjustment, the mispricing return bias remains, but now the return autocorrelation is an implausible -0.22. Thus it is the combination of slow adjustment with the market price noise that enables us to generate a significant mispricing return bias while maintaining the first order autocorrelation of returns at plausible levels. Hou and Moskowitz (2005) report an annualized risk adjusted return spread between high and low delay decile portfolios of the order of 16% *per annum* and most of this is due to the exceptionally high risk adjusted returns on the high delay portfolio as the mispricing return bias hypothesis would predict. We do not suggest that the mispricing return bias accounts for all of the apparent slow adjustment premium and we have not attempted to obtain parameters that would generate the whole observed premium while preserving plausible autocorrelations for stock returns.

4.3 A Model of Systematic Mispricing

To this point we have considered only idiosyncratic mispricing, and have not considered situations in which there are pervasive factors that cause mispricing relative to a given rational pricing model.

Suppose that market prices at time t, $P_{it}(i = 1, \cdots, n)$ are related to the fundamental prices P_{it}^* by:

$$P_{it} = P_{it}^* Z_i \equiv P_i^* e^{\gamma_i z_{mt}} \tag{18}$$

where z_{mt} is a market wide state variable, and γ_i is the sensitivity of asset i's mispricing to the market wide state variable.

Then, neglecting dividend payments, we have:

$$1 + R_{it} = \frac{P_{i,t+1}}{P_{it}} = \frac{P^*_{i,t+1}}{P^*_{it}} \frac{e^{\gamma_i z_{m,t+1}}}{e^{\gamma_i z_{m,t}}} \tag{19}$$

$$= (1 + R^*_{it}) e^{\gamma_i \Delta z_m} \approx (1 + R^*_{it})[1 + \gamma_i \Delta z_m + \frac{1}{2}(\gamma_i \Delta z_m)^2] \tag{20}$$

where R^*_{it} is the rate of return based on the fundamental price. Taking expectations in (20) under the assumption of joint normality:

$$E[R_{it}] \approx E[R^*_{it}] + E[R^*_{it}]E[\gamma_i \Delta z_m + \frac{1}{2}(\gamma_i \Delta z_m)^2] + E[\gamma_i \Delta z_m + \frac{1}{2}(\gamma_i \Delta z_m)^2] + cov\,(R^*_{it}, \gamma_i \Delta z_m) \tag{21}$$

Assuming stationary mispricing, $E[\Delta z_m] = 0$. Then, neglecting the second term in (21), the expected market return is related to the fundamental return by:

$$E[R_{it}] \approx E[R^*_{it}] + \frac{1}{2}\gamma_i^2 E[(\Delta z_m)^2] + \gamma_i cov\,(R^*_{it}, \Delta z_m) \tag{22}$$

Thus, it is natural to define the return bias associated with systematic mispricing for stock i as B_i, where:

$$B_i = \frac{1}{2}\gamma_i^2 E[(\Delta z_m)^2] + \gamma_i \rho_{i,\Delta z_m} \sigma_i \sigma_{\Delta z_m} \equiv B_{1,i} + B_{2,i} \tag{23}$$

In equation (23), $\rho_{i,\Delta z_m}$ is the correlation between the *fundamental* return and the innovation in the state variable, z_{mt}, while γ_i measures the sensitivity of the *mispricing* to the state variable. If we think of z_m as being a variable such as market liquidity (rather than *il*liquidity) or (positive) sentiment, then it is natural to think of the correlation between the fundamental return and the innovation in the state variable, $\rho_{i,\Delta z_m}$, as being positive.[10] We shall refer to state variables for which $\rho_{i,z_m} > 0, \forall i$, as *positively correlated with fundamentals*. Similarly, we expect mispricing to be positively associated with market liquidity or sentiment, so that $\gamma_i > 0$. Then we have the following result:

Lemma 2 Systematic Mispricing *For state variables that are positively correlated with fundamentals, the return bias created by systematic mispricing is an increasing function of γ_i, for $\gamma_i > 0$.*

[10] Amihud (2002) shows that market returns are negatively associated with *innovations* in *il*liquidity and positively associated with the *level* of *il*liquidity, and Pastor and Stambaugh (2003) report that the average correlation between market returns and innovations in their measure of liquidity is 0.36. The correlation is 0.52 in negative return months and only 0.03 in positive return months.

The return bias can also be written in terms of the first order autocorrelation and volatility of z as

$$B_{1,i} = (1-\rho_1^z)\gamma_i^2\sigma_z^2 \tag{24}$$

$$B_{2,i} = \gamma_i\rho_{i,\Delta z_m}\sigma_i\sqrt{2(1-\rho_1^z)}\sigma_z \tag{25}$$

The evidence of Chordia, Roll and Subrahmanyam (2000) shows that there is common variation in liquidity across stocks. Pastor and Stambaugh (2003) show that the common variation in (*il*)liquidity is related to common variation in stock prices as equation (18) implies. They also find a surprisingly high return premium associated with systematic illiquidity risk, which is the covariance of a security's price change or return with changes in the level of market (il)liquidity and corresponds to our variable γ_i: they report a 7.5% risk adjusted return differential between stocks with high and low exposures to aggregate market liquidity, while Acharya and Pedersen (2005) calculate the difference between the expected returns of the least and most liquid of 25 portfolios at 4.6% per year. Sadka (2006) finds an annualized risk-adjusted return spread of 6.12% between high and low liquidity beta decile portfolios where liquidity is measured by an empirical version of the Kyle's λ.[11]Downing *et al.* (2006) report that the average return on illiquid municipal bonds exceeds that on liquid bonds by 11.5% annually after adjusting for market, default and interest rate risks. Chacko (2006) reports a risk-adjusted reward to liquidity beta for corporate bonds of around 3% *per month*.

There is also evidence that high illiquidity forecasts high market returns. For example, Jones (2002) finds that the market return is increasing in the bid-ask spread of the previous year and decreasing in the share turnover of the previous year, and Amihud (2002) also reports that illiquidity predicts future market returns. Consistent with the finding that expected returns are increasing in illiquidity, Chordia *et al.*(2001), Amihud (2002), Jones (2002), and Pastor and Stambaugh (2003) find a positive contemporaneous relation between market returns and *changes* in market liquidity. This is consistent with market liquidity being a state variable which determines the level of stock prices relative to some normal

[11]Both Pastor and Stambaugh and Sadka argue that systematic liquidity risk can explain a significant fraction of the momentum effect, and Sadka finds that it also explains a significant fraction of the the 'post-earnings announcement drift' phenomenon.

level that would prevail under normal liquidity conditions; when liquidity is poor prices are low and expected returns are high and conversely for good liquidity. This time variation in market prices relative to 'normal' prices is likely to induce a return premium associated with the sensitivity to the state variable controlling the pricing fluctuations that is due to the mispricing return bias. Thus, our analysis can potentially account for the risk premium associated with systematic liquidity risk.

Table 2 reports calculations of the components of the return bias for selected parameter combinations of ρ_1^z, γ_i, $\rho_{i,\Delta z_m}$ and $\sigma_i = 0.30$. In this table σ_z is set equal to unity without loss of generality. Then γ_i, the sensitivity to the market wide state variable, is the unconditional standard deviation of mispricing, so that $\gamma_i = 0.01$ implies a 1% standard deviation of the mispricing variable for firm i. Note first that, in contrast to the results obtained with slow adjustment of prices where the bias component B_2 was close to zero, it can now be significant if innovations in the state variable (liquidity) are correlated with rational returns, and may even exceed B_1 if the correlation is sufficiently high.

There is considerable variation in the time series properties of the aggregate liquidity measures that have been reported in the literature. Using monthly data, Pastor and Stambaugh (2003) report that their measure has a first order autocorrelation of 0.22, Acharya and Pedersen (2005) report a value of 0.87, and the first order autocorrelation of the Sadka (2006) liquidity measure is 0.71.[12] Amihud (2002) reports a value of 0.87 for *annual* data.

Panel A shows that if aggregate liquidity is primarily a high frequency phenomenon, as suggested by the Pastor and Stambaugh (2003) estimate of the first order autocorrelation then, even if liquidity has only a modest effect on stock prices, it can cause a fairly large return bias. For example, a stock whose liquidity related mispricing has a standard deviation (γ_i) of 5% will have an annualized return bias of 3.96% if the correlation of its rational return with innovations in liquidity is 0.25. For the higher Sadka (Acharya and Pedersen) value of ρ_1^z used in Panel B(C) which implies greater persistence in liquidity, the return bias is significantly lower for a given combination of mispricing volatility and $\rho_{i\Delta z_m}$. The annualized bias drops to 1.86%(1.05%) for $\gamma_i = 5\%$ and $\rho_{i,\Delta z_m} = 0.25$. However, if liquidity

[12]Private communication. This is the autocorrelation of the variable component of liquidity. The autocorrelation of the fixed component of liquidity is 0.97. Sadka finds that sensitivity only to the variable component of liquidity is priced.

is a relatively persistent variable then higher values of γ_i are more plausible, and some liquidity sensitive stocks (e.g. $\gamma_i = 0.1$) may have return biases of the order of 4-7%. Panel D shows that if aggregate liquidity is highly persistent as suggested by Amihud (2002), then the return bias due to mispricing is likely to be fairly small. For example, if the standard deviation of liquidity related mispricing is 10% and the correlation between innovations in liquidity and the market return is as high as 0.5, then the annual return bias is only 0.89%, and this is the maximum bias for the range of parameter values considered.

The increasing monotonic relation between γ_i the systematic liquidity risk of a security i and its return bias is consistent with the findings of Pastor and Stambaugh, Acharya and Pedersen. Moreover it seems likely that γ_i and $\rho_{i\Delta z_m}$ will be positively related: stocks whose prices are highly sensitive to liquidity (high γ_i) will also be those whose overpricing tends to increase when the rational return is high (high $\rho_{i\Delta z_m}$) since liquidity is positively related to market returns. This positive association between γ_i and $\rho_{i\Delta z_m}$ will tend to increase the apparent liquidity risk premium as measured by the slope of a simple regression of the return bias on the systematic liquidity risk γ_i. However, a complication in applying this analysis to the prior empirical findings is that both Pastor and Stambaugh and Acharya and Petersen report *negative* values for the liquidity betas of a substantial fraction of their portfolios and the mononotonicity of the mispricing return bias can only be guaranteed for *positive* γ_i.[13] It is possible that the negative liquidity betas are an artefact of including in the multiple regression in which they are estimated the returns on the Fama-French (FF) portfolios HML and SMB which themselves have exposure to the liquidity state variable.

5 Empirical Analysis

In this section we present evidence that risk adjusted returns are related to proxies for the mispricing return bias. We focus on the bias element $B_1 \approx (1-\rho_1^z)\sigma_z^2$, and assume that B_2 is zero, because $B_2 \neq 0$ implies non-zero correlation between the innovations in mispricing and fundamental returns, which causes identification problems in our Kalman filter estimation of B_1; this is discussed in Watson (1986), and Harvey (1989). We use two approaches to proxy

[13]Indeed, since we have argued that $B_{2,i}$ is likely to be small, the return bias is likely to be decreasing in γ_i for negative values of γ_i.

for B_1. First, we use a Kalman filter to estimate the bias, assuming that the mispricing follows a simple AR1 process. The AR1 assumption is restrictive, and does not allow for positive short term autocorrelation in returns: as a result, our estimation algorithm did not converge for a significant number of stocks.[14] Therefore our second approach uses the volatility and variance ratio of residual returns to proxy for mispricing. The variance ratio, which is approximately a linear combination of sample autocorrelations (Cochrane, 1988), does not require us impose any structure on the mispricing process.

5.1 Data

The primary data that we use are the monthly returns on all stocks registered on the NYSE, AMEX and NASDAQ from January 1962 to December 2004, which are taken from CRSP. We include only common shares, and exclude preferred stocks, ADR's, REIT's, etc. To alleviate the potential influence of 'stale prices', we include only observations with positive trading volume and with valid month-end closing prices. We also filter out penny stocks. We use as factors in the fundamental asset pricing model monthly returns on the 3 Fama-French factors, and the momentum factor of Carhart (1997); these, together with 1-month Tbill returns, are taken from Ken French's website.[15] In order to construct predicted mispricing return bias estimates, we use data on book values from COMPUSTAT, and on prices, market capitalization and turnover from CRSP. Finally, in some of our regressions we use data on analyst following and the dispersion of analysts' forecasts of earnings 1 to 2 quarters ahead, which are taken from IBES.

5.2 AR1 Estimates of Mispricing

In order to identify mispricing, the 'fundamental return', R^*, was assumed to follow an ex-post version of the Fama-French 3-factor model (FF3):

$$R^*_{i,t} - R_{F,t} = \alpha_i + b_i(R_{M,t} - R_{F,t}) + c_i SMB_t + d_i HML_t + \epsilon_{i,t} \tag{26}$$

[14] We also developed an estimate of the bias based on an AR2 process for the mispricing. See Khil and Lee (2002). The empirical results for this model are qualitatively similar to those for the AR1 process. However, they are less significant, which is probably due to the difficulty of identifying the parameters of the more complex model.

[15] $http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html$

where $R^*_{i,t}$ is the fundamental return on stock i in month t, $R_{F,t}$ is the riskless interest rate, and $R_{M,t}, SMB_t, HML_t$ are the Fama-French factors. Then the market return, $R_{i,t}$ is given by:

$$R_{i,t} - R_{F,t} = \alpha_i + b_i(R_{M,t} - R_{F,t}) + c_i SMB_t + d_i HML_t + e_{i,t} \tag{27}$$

where $e_{i,t} = z_{i,t} - z_{i,t-1} + \epsilon_{i,t}$, and $z_{i,t}$ is (approximately) the log of the mispricing factor at time t.

The log of the mispricing was assumed to follow the AR(1) process in Equation (13). Then, following Khil and Lee (2002), a Kalman filter was used to estimate the logarithm of the mispricing factor, z_t, and the parameters of the mispricing process, ϕ_1 and σ_η, from the FF3 residual returns, $e_{i,t}$. The observation equation for the Kalman filter is:

$$e_t = z_t - z_{t-1} + \epsilon_t \tag{28}$$

and the transition equation is $z_t = \phi_1 z_{t-1} + \eta_t$. Given the estimates from the Kalman filter, the estimated mispricing return bias is:

$$\hat{B}_1 \approx (1 - \hat{\phi}_1)\hat{\sigma}_z^2 = \frac{\hat{\sigma}_\eta^2}{1 + \hat{\phi}_1} \tag{29}$$

Details of the Kalman filter algorithm are given in the Appendix.

The parameters of the mispricing process and the mispricing return bias, $\hat{B}_1$, were estimated in January of each year from 1967 to 2004 for all stocks with at least 36 monthly returns using the FF3 residual returns estimated over the previous 60 months as available.

Panel A of Table 3 reports the results of regressing the Kalman filter estimates of the first order autocorrelation, ϕ_1, the variance of the mispricing variable, σ_z^2, and the resulting bias, B_1, on firm characteristics that may be expected to influence mispricing. The characteristics that we consider are firm size, book-to-market ratio, share price, share turnover, the logarithm of the number of analysts following the firm, and the dispersion of the analyst earnings forecasts for the next 1 to 2 quarters. Only firms that are followed by at least two analysts are included in the regressions.

Firm size is positively associated with the persistence, and negatively associated with the variance, of mispricing, so that the net effect of firm size on the return bias is negative.

Book-to-market ratio is negatively associated with the variance of misprcing, and therefore with the return bias. As a result, small growth firms tend to have more variable mispricing and therefore a higher mispricing return bias. A higher stock price reduces the persistence of mispricing (insignificantly), and reduces the volatility of mispricing; the net effect is that higher stock prices reduce the mispricing return bias.[16] The main effect of turnover is to increase variance of the mispricing, and therefore the return bias.[17] We had expected the number of analysts following a stock to reduce the persistence and variability of mispricing, and the dispersion of analysts forecast to increase the variability of mispricing. While the signs of the coefficients on the number of analysts are consistent with these expectations, they are not significant, and the net effect of the number of analysts on the return bias is insignificant. On the other hand, the dispersion of analysts' forecast is significantly associated with variability of the mispricing and therefore with the return bias, and the direction of the effect is consistent with prior intuition.

In order to assess the relation between our estimates of the mispricing return bias and risk-adjusted returns, stocks were assigned in January of each year from 1967 to 2004 to one of ten equal size portfolios according to the current mispricing return bias estimate, $\hat{B}_1$, after winsorizing by excluding stocks whose bias estimates fell into the top decile[18]. An equal investment was assumed to made in each stock in the portfolios at the beginning of the year and no rebalancing was assumed within the year.[19] The first portfolio allocation occurs at the end of December 1966, and the last at the end of of December 2003. The post-ranking returns were then linked across time, yielding a time series of returns for each decile from January 1967 to December 2004. On average, there are over 180 stocks within each portfolio, and at no time is the number of stocks in any portfolio less than 98. Stocks

[16]Kumar and Lee(2006) report that the returns on small firms, lower priced firms, firms with lower institutional ownership, and value firms, have higher loadings on a measure of retail investor sentiment (which induces transient mispricing).

[17]To address the issue of inter-dealer trading in OTC markets, we multiply the NASDAQ volume by 0.6, following Atkins and Dyl (1997).

[18]The results were similar without winsorization.

[19]Since mispricing is most likely to found among small stocks, we use an equal weighting scheme to compensate for the over-representation of large, liquid, and closely followed stocks that are less likely to be subject to mispricing. Acharya and Pedersen (2005), Amihud (2002), and Chordia *et al.* (2000) adopt a similar strategy in their studies of liquidity and asset pricing. Liu and Strong (2006) show that monthly rebalancing can lead to significant biases in average returns, especially for small, low price, value and loser stocks.

for which Kalman Filter did not converge were assigned to Portfolio 'NCV'. The 10-1 spread corresponds to a zero-investment portfolio that is long in the high bias portfolio and short in the low bias portfolio.

Panel A of Table 4 reports the characteristics of the portfolios. Betas of the portfolios were obtained by regressing excess returns on the portfolios on the three FF factors and the momentum factor of Carhart (1997). The high bias portfolios tend to have higher loadings on the market and on SMB, and lower loadings on HML. Mispricing return bias is strongly related to firm size as we found in Table 3, which is also consistent with the pattern of loadings on SMB: the relation is almost perfectly monotonic and the firms in the high bias portfolio are less than 1/10th of the size of firms in the low bias portfolio. The lower loadings of high bias portfolios on HML are also consistent with the finding in Table 3 that return bias is higher for growth firms. The size composition of decile portfolios is consistent with the relation between firm size and return bias reported in Table 3. There is no relation between the loadings on MOM and bias. The average firm in Portfolio NCV has characteristics that are close to the average of all firms, except for β_{HML}, which is close to that of the high bias portfolio.

The average estimated AR1 coefficient, ϕ_1, varies almost monotonically from 0.07 for the low bias portfolio to 0.26 for the high bias one. The average volatility of mispricing, σ_z, is monotonic, ranging from 1.26% for the low bias portfolio to 10.93% for the high bias portfolio. Finally, the estimated volatility of the fundamental return, σ_ϵ, is monotonically increasing across portfolios, so that the firms with the most (fundamental) idiosyncratic risk tend to be those most subject to mispricing. The estimated annualized mispricing return bias runs from 14 bp to over 7%. Note that expression (29) implies that the estimated bias is non-negative. For the first six portfolios the bias estimates are moderate, reaching 1.76% for portfolio 6. However, they increase rapidly for the last four portfolios, more than doubling between portfolios 8 and 10.

The excess returns on the decile portfolios were regressed in turn on the excess market returns, on the 3-factor FF model, and on the Carhart 4-factor model. The intercepts from these regressions provide estimates for risk-adjusted returns, which are reported in Panel B. There is a clear tendency for the risk-adjusted returns to increase in the bias estimates for all three risk adjustment benchmarks. The correlation between the return bias estimates and both the FF3 and the FF4 adjusted returns is 0.95. The spread in FF3 adjusted return between the high and low bias portfolios is 4.44% per year, and is statistically significant.

The power of our test is limited by the errors in the Kalman filter estimates of the return bias. Therefore, following Pastor and Stambaugh (2003), we also constructed portfolios based on predicted values of the mispricing return bias. The original Kalman filter estimates were projected onto a set of firm characteristics, which explained more than 70% of the variation in the bias estimates. The predicted biases used for portfolio formation at the end of year t were calculated using parameters estimated by ordinary least squares on all data up to that date, so that there is no look-ahead bias in the portfolio formation procedure.

The results of using the predicted return bias as a sorting variable were quite similar to those reported in Table 4. The new portfolio formation procedure reduced the spread in the estimated return bias from 7.21% per year to 4.27% but, consistent with the more efficient portfolio formation method, the realized risk adjusted return spread rose from 4.44% per year to 8.20%.

Thus there is strong evidence that commonly used measures of excess return at the monthly frequency are significantly affected by the mispricing return bias that we have identified. As a robustness check we shall examine whether other proxies for the mispricing return bias are also associated with excess risk-adjusted returns.

5.3 Variance Ratio and Volatility of Residual Returns

Equation (10) shows that the return bias, B_1, is increasing in the volatility of mispricing σ_z, and decreasing in the first order autocorrelation ρ_1^z. Unfortunately, as we have seen, the mispricing variable, z, is not directly observable and therefore these parameters can be inferred only by making strong assumptions about the stochastic process of the mispricing variable. These assumptions are unlikely to be satisfied in practice. Therefore, as a robustness check, in this section we adopt a more informal approach to proxying for the mispricing return bias.

Define the k-month variance ratio for (FF3) residual returns by $VR(k) \equiv \left(var(e^k)/k\right)/var(e^1)$, where e^k is the cumulative residual return over k months. First, we observe that in the absence of mispricing, the FF3 residuals will be serially independent, so that $VR(k) = 1$. To the extent that there is transient mispricing, the variance ratio will be less than 1, and the stronger is the mean reversion in mispricing, the lower will the ratio. This suggests using the variance ratio of residual returns as a proxy for ρ_1^z. The volatility of residual returns, σ_e, depends on both the idiosyncratic volatility of the fundamental returns (σ_ϵ) and the variability of mispricing (σ_z). Therefore we can think of residual return volatility (σ_e) as a noisy signal of the volatility of mispricing (σ_z). Hence we expect the mispricing return bias to decrease in the variance ratio and to increase in residual return volatility.

Our analysis is based on the 24-month variance ratio.[20] The results in Panel B of Table 3 provide support for these conjectures. The variance ratio is negatively associated with the persistence of the mispricing that is estimated using the AR1 Kalman filter, while residual return volatility is positively associated with the estimated variability of mispricing, and both variables are highly significant in explaining the estimated mispricing bias. Since it is likely that the AR1 model is misspecified, it is possible that the variance ratio and residual return volatility will be better proxies for the unobservable true mispricing return bias.

10 portfolios were formed each year based on the 24-month residual variance ratio estimated over the previous 60 months, $VR(24)$. Only stocks with at least 36 monthly returns within the past 5 years were considered. The variance of the one month residual returns was estimated using the previous 5 years' FF3 residuals. The variance of the sum of 24 months' residuals, $var(e^{24})$, was estimated as the average of the squared sum of residuals over each 24-month period in the previous 5 years. Thus, the residuals over months 1-24 were summed and squared, similarly with the residuals for months 2-25,..,37-60, and the average of these was taken as the estimate of the 24-month variance. Securities were then assigned to one of 10 portfolios according to the variance ratio estimate.

The characteristics of the portfolios are reported in Table 5. The estimates of the variance ratios for individual stocks are very noisy, so that the sample selection bias involved in sorting on this variable causes the spread of portfolio average variance ratio estimates to be very wide, ranging from 0.12 to 2.04 as compared with the value of unity implied by the

[20] Results obtained using 12 and 36 month variance ratios were similar.

iid assumption. Nevertheless, it is striking that for 8 out of the 10 portfolios the estimate of $VR(24)$ is less than unity; this, together with the fact that the average residual autocorrelation is negative for 9 portfolios, suggests a widespread tendency for residual returns to reverse themselves, which is consistent with transient mispricing. There is relatively little difference in the average residual variances of the stocks in the portfolios except for Portfolio 1 whose residual variance is about 33% higher than that of the other portfolios. Firm size tends to decrease with the variance ratio, but the relation is not as marked as it was for the previous two portfolio formation methods. As conjectured, the (FF3) risk adjusted returns are decreasing in the variance ratio, with a correlation of -0.89. The spread in *risk-adjusted* returns between the high and low VR portfolios is similar to that reported in Table 4 for portfolios formed on AR1 return bias estimates. However, the t-statistic on the spread in the (FF3) risk adjusted returns is now 3.91, instead of 2.29.

In order to determine whether risk adjusted returns decrease in variance ratio, holding constant the residual variance, 25 portfolios were formed each year. First the stocks were sorted into quintiles based on the estimated variance of the residual returns. Then within each variance quintile the stocks were further sorted into quintiles based on $VR(24)$.[21] The time series of returns on the resulting 25 portfolios were calculated as in the previous case assuming equal investments in each stock and annual rebalancing. Risk-adjusted returns were then calculated as before and the results are reported in Table 6. We focus on the results for returns that are adjusted for risk using the 3 and 4 factor models, which are quite similar.

The relation between risk adjusted returns and the variance ratio depends strongly on the residual variance. As the residual variance increases, the relation becomes stronger and is highly significant for the two highest residual variance quintiles. The spread in the risk adjusted returns for the highest residual variance quintile is 9.12% per year with a t-statistic of 4.51. When a 'σ_e^2-neutral' portfolio is formed for each VR category by averaging the returns across the five σ_e^2 quintiles, the spread between the (FF3) risk adjusted return on the low and high VR portfolios is 3.36% per year with $t-$statistic of 4.44.

In order to explore the effect of the residual variance holding constant the variance

[21]Similar results were obtained when sorting simultaneously on these two variables.

ratio, the portfolios were reformed by sorting first on the variance ratio and then on the residual variance. The results are reported in Table 7. Only for the lower VR quintiles is there any evidence that residual variance is associated with risk adjusted returns after controlling for VR. For the lowest VR quintile the (FF3) risk adjusted return increases monotonically in the residual variance, and the spread between the risk-adjusted returns on the high and low σ_e^2 portfolios is 9.32% per year which is significant at 1% level.[22] Note that we should not expect a relation between risk-adjusted returns and residual variance for the high variance ratio quintile portfolios since these portfolios are unlikely to be subject to significant mispricing.

Finally, we repeat the analysis, sorting first on firm size and then on the variance ratio. The results are reported in Table 8. For all but the 2nd size quintile, the spread in risk adjusted returns between low and high VR portfolios is positive and significant. For the large firm quintile, the spread is less than 3% per year and for the small one, the spread is over 5% per year, which is consistent with the greater mispricing volatility among small firms. For all levels of the variance ratio the risk-adjusted return in highest on small firms by a wide margin. This is consistent with the volatility of mispricing being largest for small firms.

6 Conclusion

In this paper we have shown that when market prices differ from fundamental prices because of stochastic pricing errors, a bias in average returns is created due to Jensen's inequality. The bias has two components. The first component is decreasing in the persistence of the pricing errors and increasing in their volatility, while the second is equal to the covariance of the fundamental return with the innovation in the proportional mispricing. Three specific models of the pricing error were considered. The first assumes that the (log of) relative mispricing follows an AR(1) process that is uncorrelated with fundamentals. It is shown that if the volatility of the mispricing is 20% and 10% of the mispricing is eliminated each

[22]Spiegel and Wang (2005) report that at the monthly frequency expected returns are increasing in the level of conditional idiosyncratic risk measured relative to the FF 3-factor model, while Ang *et al.* (2006a,b) find that at the daily frequency the relation is reversed in the US and most foreign markets. Neither study considers the variance ratio.

month, then the annualized return bias is of the order of 5% per year. It was argued that, to the extent that measures of individual stock illiquidity proxy for mispricing, it is likely that a significant portion of the very high return premium for illiquidity that has been estimated in previous studies may be due to the mispricing return bias.

The second model assumes that market prices respond slowly to innovations in the fundamentals that determine the equilibrium price. Simulations show that when this is the only departure from market efficiency the mispricing return bias is likely to be small. However, if some additional noise is introduced into the market price then it is possible to generate a mispricing return bias of the order of 4.5% per year while preserving a plausibly small first order autocorrelation in returns. Thus the mispricing return bias has the potential to explain a significant portion of the return premium associated with slow price adjustment to new information that has been identified by Hou and Moskowitz (2005).

The third model of mispricing assumes that mispricing for individual securities responds to a common market-wide factor such as liquidity. In this model the mispricing return bias in increasing in the responsiveness of mispricing to this factor, which corresponds to the 'liquidity beta' estimated in previous studies. It is shown that, depending on the persistence of liquidity, the mispricing return bias could account for a significant portion of the return premium that has been found to be associated with liquidity betas.

In order to estimate the mispricing return bias for individual securities we assume an AR(1) process for the (log of) relative mispricing, and apply a Kalman filter to the residuals from the Fama-French 3 factor model assuming that mispricing is unrelated to fundamentals. The estimated variability of mispricing is found to be negatively related to firm size, B/M ratio, stock price, and number of analysts following the firm; it is positively related to share turnover and the dispersion of analysts' earnings forecasts. The persistence of mispricing is positively related to firm size and (insignificantly) to B/M ratio, turnover and dispersion of analysts' forecasts. Persistence is negatively related to share price and number of analysts following. Consequently, under the (strong) assumption that mispricing is independent of fundamentals, the mispricing return bias is negatively related to firm size, B/M ratio, share price and (insignificantly) analyst following. It is positively related to share turnover and the dispersion of analysts' forecasts.

When 10 portfolios are formed on the basis of the estimated mispricing return bias, the FF3 risk adjusted returns have a correlation of 0.95 with the estimated mispricing return bias, and the spread in the risk adjusted returns between the highest and lowest bias portfolios is 4.44% per year with t-statistic of 2.29. We also constructed portfolios based on predicted values of the sorting parameter (bias), where the predictions were obtained by projecting the Kalman filter estimates onto firm characteristics. As expected, with the more efficient sorting procedures the portfolios display a wider range in risk-adjusted returns. Thus there is significant evidence that risk adjusted returns are affected by the mispricing return bias that we have analyzed.

As a robustness check, we also form portfolios based on the 24-month variance ratios from the Fama-French 3-factor model. Consistent with our hypothesis, we find that risk-adjusted returns are significantly higher on low variance ratio portfolios, and when we sort first on residual return volatility and then on variance ratio we find that the effect is more pronounced for the high residual volatility groups of portfolios. This is consistent with the hypothesis that the variance ratio of residual returns is a good proxy for the first order autocorrelation of mispricing and that the volatility of residual returns is a proxy for the volatility of mispricing. However, when we form portfolios first on the variance ratio and then on residual volatility, we find only weak evidence, for the low variance ratio quintile portfolios, that risk-adjusted returns increase with residual volatility, holding contant the variance ratio.

Our results suggest caution for researchers who attempt to measure the effects on risk-adjusted returns of variables such as liquidity which may be good proxies for mispricing, and of variables such as liquidity betas which measure the amplitude of price fluctuations about fundamental value. Such variables are likely to proxy for the mispricing return bias. Finally, we observe that the mispricing return bias is primarily a 'high frequency' phenomenon. It is present in monthly returns but is likely to be attenuated in quarterly or annual returns.

Appendix

A. Kalman Filter Algorithm for an AR1 z process

Assume that the logarithm of the mispricing component, z_t, follows a AR1 process, the transition equation can then be denoted as

$$\alpha_t = T_t \alpha_{t-1} + w_t \text{ with } w_t \sim N(0, Q_t) \tag{A1}$$

where $\alpha_t = [z_t]$, $T_t = [\phi_1]$, $w_t = [\eta_t]$,$Q_t = \left[\sigma_\eta^2\right]$.

The observation equation is based on the FF3 risk adjusted returns,e_t, $t = 1, 2, ...T$, and is given

$$e_t = \mu + s_t' \alpha_t + \epsilon_t \tag{A2}$$

with $s_t = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$

At the end of each year from 1967 to 2003, for each stock, a Kalman filter is fitted to the FF3 adjusted returns over the past 60 months, following a 2-stage iteration process. The first stage is the prediction stage. At time $t-1$, the optimal predictor, $\alpha_{t|t-1}$, and the associated covariance, $P_{t|t-1}$, are given by

$$\alpha_{t|t-1} = T_t \alpha_{t-1|t-1} \tag{A3}$$

$$P_{t|t-1} = T_t P_{t|t-1} T_t' + Q_t \tag{A4}$$

The second stage is the updating stage. When e_t becomes observable at time t, we can calculate the prediction errors given by the predicted parameter in the first stage, $v_t = e_r - e_{t|t-1} = s_t'(\alpha_t - \alpha_{t|t-1}) + \epsilon_t$, with mean of 0, and variance of $s' P_{t|t-1} s_t + \sigma_\epsilon^2$. Define, $f_t \equiv s_t' P_{t|t-1} s_t + \sigma_\epsilon^2$, the parameters then are updated as follows:

$$\alpha_{t|t} = \alpha_{t|t-1} + P_{t|t-1} s_t (e_t - s_t' \alpha_{t|t-1}) f_t^{-1} \tag{A5}$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1} s_t s_t' P_{t|t-1} f_t^{-1} \tag{A6}$$

and $P_{t|t-1} s_t f_t^{-1}$ is also known as "Kalman Gain". The log likelihood function of the observations (e_1, e_2, ... e_t) can be calculated as

$$L = -\frac{T}{2} log(2\pi) - \frac{1}{2} \sum_{t=1}^{T} log(f_t) - \frac{1}{2} \sum_{t-1}^{T} \frac{v_t^2}{f_t} \tag{A7}$$

Following Campbell (1989), initial guesses of α_0 and P_0 are set to zero and $\frac{\sigma_\eta^2}{1-\phi_1^2}$, respectively. We maximize the above log likelihood function to get the final parameter estimates. Based on the estimated parameters, the bias is calculated by

$$\hat{B}_1 = e^{(1-\hat{\phi}_1)\hat{\sigma}_z^2} - 1 = e^{\frac{\hat{\sigma}_\eta^2}{1+\hat{\phi}_1}} - 1 \tag{A8}$$

.

B. Simplified Kalman Filter for Portfolio Construction

In Appendix A, Kalman filter estimation iterates over the parameters ϕ_1, σ_η^2, and σ_ϵ^2. To reduce the number of estimated parameters, we resort to the following moments conditions in FF3 risk adjusted stock returns, e_t. The first order autocorrelation condition of this residual stock returns ρ_1^e states

$$\rho_1^e \quad \equiv \quad \rho_{e_t, e_{t-1}} = \frac{(2\rho_1^z - \rho_2^z - \rho_0^z)\sigma_z^2}{\sigma_e^2} \tag{B1}$$

and the variance for this residual return is denoted as σ_e^2.

Recall

$$\sigma_z{}^2 \quad = \quad \frac{\sigma_\eta{}^2}{1+\phi_1} \tag{B2}$$

Combine this with the *Yule-Walker* condition for the autocorrelations in z:

$$\rho_1^z \quad = \quad \phi_1 \tag{B3}$$

$$\rho_2^z \quad = \quad \phi_1^2 \tag{B4}$$

$$\ldots$$

Plug all these into the ρ_1^e equation, and we get:

$$\rho_1^e \sigma_e^2 \quad = \quad \frac{(\phi_1 - 1)\sigma_\eta^2}{\phi_1 + 1} \tag{B5}$$

hence, σ_η^2 can be expressed as a function of ϕ_1, while ρ_1^e and σ_e^2 are measured with the sample moments:

$$\sigma_\eta^2 = \frac{\rho_1^e \sigma_e^2 (\phi_1 + 1)}{\phi_1 - 1} \tag{B6}$$

Furthmore, recall $e_t = z_t - z_{t-1} + \epsilon_t$, hence, we have

$$\begin{aligned} \sigma_e^2 &= \sigma_{\Delta z}^2 + \sigma_\epsilon^2 && \text{(B7)} \\ &= 2(1-\phi_1)\sigma_z^2 + \sigma_\epsilon^2 && \text{(B8)} \\ &= \frac{2\sigma_\eta^2}{1+\phi_1} + \sigma_\epsilon^2 && \text{(B9)} \end{aligned}$$

Therefore, σ_ϵ^2 can also be expressed as a function of ϕ_1:

$$\sigma_\epsilon^2 = \sigma_e^2 \left(1 - \frac{2\rho_1^e}{\phi_1 - 1}\right) \tag{B10}$$

By now, the free parameter set is reduced to just ϕ_1 for the Kalman filter estimation. This substantially reduces the estimation time.

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Table 1:Simulation Estimates of Mispricing Return Bias in a Model with Slow Adjustment of Prices and Random Shocks to Market Prices.
Market prices P_t adjust slowly to fundamental prices P_t^* which themselves follow an ex-post form of the CAPM:

$$\begin{aligned} P_{t+1}^* &= P_t^* \left[1 + R_f + \beta(R_{mt} - R_{ft}) + \epsilon_{t+1}\right] \\ P_{t+1} &= \kappa P_{t+1}^* + (1-\kappa)P_t + P_t\xi_{t+1} \end{aligned}$$

Market returns, R_{mt}, and risk free interest rates, R_{ft}, are randomly chosen samples of 200 consecutive months of CRSP market and risk free returns from the period January 1926 to December 2003. ϵ_{t+1} and ξ_{t+1} are simulated normal variates. The table reports average values from 5000 simulations for a stock with $\beta = 1$. The volatilities $\sigma_\epsilon, \sigma_\xi$, and σ_R, which are in *per cent* per year, are annualized by multiplying by $\sqrt{12}$. ρ_1^R is the first order autocorrelation of the simulated stock return: $R_{t+1} \equiv P_{t+1}/P_{t-1}$. σ_z and ρ_1^z are the estimated standard deviation and first order autocorrelation of the mispricing variable, $z \equiv ln(P/P^*)$. B, B_1 and B_2, which are in *per cent* per year, are the annualized estimates of the mispricing return bias and its components calculated from the expressions in equation (9). α, which is annualized and in *per cent* per year, and β are the estimated coefficients from the regression of the simulated stock excess returns on the market excess return.

Simulation Parameters			Simulated Return Properties								
κ	σ_ϵ	σ_ξ	σ_R	ρ_1^R	σ_z	ρ_1^z	B_1	B_2	B	α	β
Panel A: $\sigma_\xi = 0.0$											
0.98	30.00	0.00	36.83	0.07	.00	0.065	0.00	0.00	0.00	0.10	0.98
0.90	30.00	0.00	34.10	0.15	0.01	0.14	0.13	-0.01	0.12	-0.15	0.91
0.80	30.00	0.00	31.06	0.25	0.02	0.24	0.50	-0.02	0.48	-0.19	0.83
0.70	30.00	0.00	28.10	0.35	0.03	0.34	1.09	-0.01	1.08	-0.22	0.74
0.98	15.00	0.00	26.20	0.12	0.01	0.12	0.26	0.00	0.26	0.05	0.98
0.95	15.00	0.00	25.60	0.15	0.00	0.15	0.08	0.00	0.08	0.08	0.96
0.90	15.00	0.00	24.40	0.20	0.01	0.19	0.07	0.00	0.07	0.39	0.92
0.80	15.00	0.00	22.30	0.30	0.02	0.29	0.23	-0.00	0.23	0.93	0.83
0.70	15.00	0.00	20.20	0.39	0.02	0.38	0.51	-0.01	0.50	1.58	0.74
Panel B: $\sigma_\xi > 0.0$											
0.98	15.00	0.15	34.10	-0.13	0.04	0.01	2.33	0.00	2.33	2.29	0.98
0.95	15.00	0.15	33.30	-0.11	0.04	0.04	2.72	0.00	2.71	2.39	0.96
0.90	15.00	0.15	32.10	-0.07	0.04	0.09	2.75	-0.01	2.74	2.51	0.91
0.80	15.00	0.15	29.90	-0.02	0.05	0.20	2.74	-0.02	2.72	2.82	0.83
0.70	15.00	0.15	27.80	0.04	0.05	0.31	2.82	-0.02	2.80	3.28	0.74
0.70	17.00	0.20	33.00	-0.05	0.06	0.30	4.25	-0.02	4.23	4.72	0.74
0.65	17.00	0.20	31.90	-0.03	0.07	0.35	4.62	-0.03	4.59	4.77	0.69
Panel C: $\kappa = 1, \sigma_\xi > 0.0$											
1.00	17.00	0.20	40.26	-0.22	0.06	-.0.01	4.04	0.00	4.04	4.06	1.00

Table 2: Mispricing Return Bias due to Systematic Mispricing.

The standard deviation of the state variable, σ_{z_m}, is set equal to unity so that the standard deviation of mispricing for stock i is γ_i. The standard deviation of the fundamental residual return σ_i is set equal to 30% per year. The return bias is expressed in *per cent* per year. In Panels A, B, and C, the annualized return bias is calculated by multiplying the monthly return bias by 12. In Panel D, the annual return bias is calculated directly from the first order autocorrelation of annual returns.

	Panel A: $\rho_1^z = 0.22$ M Pastor/Stambaugh 2003			Panel B: $\rho_1^z = 0.71$ M Sadka, 2006			Panel C: $\rho_1^z = 0.87$ M Acharya/Pedersen, 2002			Panel D: $\rho_1^z = 0.87$ A Amihud, 2002		
	B1(annualized, %)			B1(annualized, %)			B1(annualized, %)			B1(annualized, %)		
		$\rho_{i\Delta z_m}$			$\rho_{i\Delta z_m}$			$\rho_{i\Delta z_m}$			$\rho_{i\Delta z_m}$	
	0.00	0.25	0.50	0.00	0.25	0.50	0.00	0.25	0.50	0.00	0.25	0.50
$\gamma_i = 0.01$	0.09	0.09	0.09	0.03	0.03	0.03	0.02	0.02	0.02	0.00	0.00	0.00
0.02	0.37	0.37	0.37	0.14	0.14	0.14	0.06	0.06	0.06	0.01	0.01	0.01
0.05	2.34	2.34	2.34	0.87	0.87	0.87	0.39	0.39	0.39	0.03	0.03	0.03
0.10	9.36	9.36	9.36	3.48	3.48	3.48	1.56	1.56	1.56	0.13	0.13	0.13
	B2(annualized, %)			B2(annualized, %)			B2(annualized, %)			B2(annual, %)		
		$\rho_{i\Delta z_m}$			$\rho_{i\Delta z_m}$			$\rho_{i\Delta z_m}$			$\rho_{i\Delta z_m}$	
	0.00	0.25	0.50	0.00	0.25	0.50	0.00	0.25	0.50	0.00	0.25	0.50
$\gamma_i = 0.01$	0.00	0.32	0.65	0.00	0.20	0.40	0.00	0.13	0.26	0.00	0.04	0.08
0.02	0.00	0.65	1.30	0.00	0.40	0.79	0.00	0.26	0.53	0.00	0.08	0.15
0.05	0.00	1.62	3.24	0.00	0.99	1.98	0.00	0.66	1.32	0.00	0.19	0.38
0.10	0.00	3.24	6.49	0.00	1.98	3.96	0.00	1.32	2.65	0.00	0.38	0.76
	B1+B2(annualized, %)			B1+B2(annualized, %)			B1+B2(annualized, %)			B1+B2(annual, %)		
		$\rho_{i\Delta z_m}$			$\rho_{i\Delta z_m}$			$\rho_{i\Delta z_m}$			$\rho_{i\Delta z_m}$	
	0.00	0.25	0.50	0.00	0.25	0.50	0.00	0.25	0.50	0.00	0.25	0.50
$\gamma_i = 0.01$	0.09	0.42	0.74	0.03	0.23	0.43	0.02	0.15	0.28	0.00	0.04	0.08
0.02	0.37	1.02	1.67	0.14	0.53	0.93	0.06	0.33	0.59	0.01	0.08	0.16
0.05	2.34	3.96	5.58	0.87	1.86	2.85	0.39	1.05	1.71	0.03	0.22	0.41
0.10	9.36	12.60	15.85	3.48	5.46	7.44	1.56	2.88	4.21	0.13	0.51	0.89

Table 3: Determinants of the Mispricing Return Bias
The coefficients are the time series averages of the coefficients from cross-sectional regressions of security mispricing return bias estimates on firm characteristics. $t-$statistics are computed using standard errors computed from the time series of the coefficients and take account of heteroscedasticity and autocorrelation using a Newey-West adjustment with 4 lags.
$\hat{\phi}_1$, $\hat{\sigma_z^2}$ and $\hat{B}_1$ are estimated from the residuals from Fama-French 3-factor regressions using a Kalman filter and assuming an AR1 process for mispricing. Only firms with at least 2 analysts are included in the regressions. *Size* is the market value of equity at the end of the year. *B/M* is the book-to-market ratio calculated from the year end market value and the most recent book equity that is available at least two quarters before the year end. *Price* is the share price at the end of the year. *Turnover* is the ratio of the average number of shares traded per month to the number of shares outstanding during the last quarter of the year. NASDAQ turnover is multiplied by 0.6. *#Analysts* is the number of investment analysts following the firm at the end of the year. *DISP* is the dispersion of the analysts' earnings forecasts for the next 1-2 quarters. $VR(24)$ and σ_e^2 are the 24-month variance ratio and variance of residual returns from a Fama-French 3-factor regressions over the previous 60 months.

	$\hat{\phi}_1$(%)		$\hat{\sigma_z^2}$		$\hat{B}_1$	
	coeff	t-stat	coeff	t-stat	coeff	t-stat
Panel A:1976-2003						
Const	10.09	6.36	99.93	4.19	6.32	4.88
log(Size)	1.13	2.87	-10.14	-3.62	-0.65	-3.97
B/M	0.28	0.46	-6.51	-2.67	-0.41	-3.37
Price	-0.02	-1.10	-0.18	-2.49	-0.01	-6.06
Turnover	0.04	0.74	2.83	5.84	0.12	6.73
log(#Analysts)	-0.81	-1.62	-1.28	-0.65	-0.04	-0.47
DISP	2.11	1.35	14.40	3.25	0.52	2.08
$AdjR^2$ (%)	0.17		4.49		16.20	
Panel B:1966-2003						
Const	17.09	23.95	7.27	1.01	1.13	3.71
VR(24)	-4.50	-12.62	-7.95	-0.98	-1.34	-6.30
σ_e^2	0.00	0.22	0.37	15.84	0.02	17.63
$AdjR^2$ (%)	0.38		19.08		58.76	

Table 4: Properties of Equally-Weighted Decile Portfolios formed Each Year from January 1967 to December 2004 on Kalman Filter Estimates of the Mispricing Return Bias
Mispricing Return Bias, which is winsorized at 90% level and annualized and in *per cent*, is equal to $(1-\phi_1)\sigma_z^2$, where ϕ_1 and σ_z are the first order autocorrelation and volatility of mispricing which are estimated from Fama-French 3-factor residuals using a Kalman filter and assuming an AR(1) process for mispricing. The $\beta's$ are the loadings of the portfolio returns on the 3 Fama-French and the Carhart Momentum factors. *Size* is the time series mean portfolio size in billion \$. σ_z and σ_ϵ, the volatility of the (log) mispricing variable (z) and the monthly volatility of the fundamental return (ϵ), are quoted in *per cent*. Panel B reports raw returns and the intercepts (α) from regressions of excess returns on the market excess returns (CAPM), the 3 Fama-French factors (FF3), and the 3 Fama-French factors plus the Carhart Momentum factor (FF4). The returns and $\alpha's$ are in *per cent* per month, and the t-statistics are adjusted for autocorrelation and heteroscedasticity. Stocks for which the Kalman filter fail to converge are included in Portfolio NCV.

	Port Low Bias	2	3	4	5	6	7	8	9	Port High Bias	Spread 10 − 1	Port NCV
Panel A: Portfolio Characteristics												
β_{mkt}	0.89	0.92	0.94	0.99	0.99	1.01	1.03	1.05	1.05	1.05	0.16	1.01
t-statistic	36.78	41.25	41.03	47.91	46.91	42.22	43.60	39.90	36.86	32.47	4.07	57.70
β_{SMB}	0.42	0.43	0.47	0.56	0.67	0.77	0.88	1.01	1.21	1.42	1.00	0.86
t-statistic	15.08	9.14	10.56	16.60	14.04	17.32	19.21	20.76	25.13	24.44	14.80	29.18
β_{HML}	0.44	0.45	0.44	0.44	0.38	0.39	0.36	0.35	0.31	0.23	-0.21	0.22
t-statistic	11.19	9.20	9.81	10.56	8.58	8.37	7.72	5.63	5.50	2.99	-2.13	7.14
β_{MOM}	-0.02	-0.03	-0.05	-0.02	-0.02	-0.02	-0.04	-0.06	-0.06	-0.05	-0.02	-0.06
t-statistic	-0.79	-0.76	-1.38	-0.63	-0.67	-0.67	-1.18	-1.49	-1.20	-0.77	-0.34	-1.46
Size	2.33	2.08	1.90	1.34	1.02	0.88	0.55	0.36	0.24	0.15		0.99
ϕ_1	0.07	0.03	0.05	0.08	0.12	0.14	0.16	0.20	0.22	0.26		
σ_z	1.26	2.12	2.81	3.41	4.09	4.85	5.66	6.72	8.26	10.93		
σ_ϵ	7.33	7.39	7.41	7.74	8.14	8.36	9.02	9.61	10.29	11.45		
Mispricing Return Bias (% p.a.)	0.14	0.40	0.66	0.96	1.32	1.76	2.34	3.20	4.59	7.35		
Number of Stocks												
Mean	187	188	188	187	186	185	185	185	185	184		
Mininum	103	102	105	101	101	101	98	101	102	103		

	Port Low Bias	2	3	4	5	6	7	8	9	Port High Bias	Spread 10 − 1	Port NCV
Panel B:Monthly Returns, (%)												
Raw Ret	1.32	1.37	1.39	1.34	1.41	1.48	1.62	1.66	1.78	1.94	0.62	1.43
t-statistic	5.95	6.02	5.64	5.23	5.33	5.21	5.26	5.09	4.95	4.86	2.48	4.70
Capm α	0.42	0.46	0.45	0.38	0.43	0.48	0.59	0.61	0.71	0.83	0.41	0.40
t-statistic	3.22	3.58	3.22	2.76	2.99	3.20	3.46	3.37	3.21	3.37	1.95	2.45
FF3 α	0.10	0.13	0.12	0.04	0.10	0.14	0.25	0.25	0.33	0.48	0.37	0.13
t-statistic	1.35	1.98	1.40	0.59	1.45	2.00	3.35	3.06	3.49	3.33	2.29	1.80
FF4 α	0.13	0.16	0.17	0.06	0.13	0.16	0.29	0.31	0.39	0.52	0.40	0.19
t-statistic	1.74	2.65	2.01	0.93	1.81	2.16	3.23	3.45	3.38	3.29	2.38	2.07

Table 5: Properties of Equally-Weighted Decile Portfolios formed Each Year from January 1967 to December 2004 on the Variance Ratio of Residual Returns
The 24 month variance ratio is estimated each year from the residuals of the residuals from Fama-French 3-factor model regressions estimated over the previous 60 months. The portfolio returns run from January 1967 to December 2004.
$VR(24)$ is the average of the 24-month variance ratios which are estimated from Fama-French 3-factor residuals over the previous 60 months The $\beta's$ are the loadings of the portfolio returns on the 3 Fama-French and the Carhart Momentum factors. *Size* is the time series mean firm size in billion \$. σ_e^2 and *AR1* are the average variance and first order autocorrelation of the residuals from Fama-French 3-factor regressions over the previous 60 months. σ_e^2 is in *(per cent)*^{2} per month. Panel B reports raw returns and the intercepts (α) from regressions of excess returns on the market excess returns (CAPM), the 3 Fama-French factors (FF3), and the 3 Fama-French factors plus the Carhart Momentum factor (FF4). The returns and $\alpha's$ are in *per cent* per month, and the t-statistics are adjusted for autocorrelation and heteroscedasticity.

	High VR	2	3	4	5	6	7	8	9	Low VR	Spread $10-1$
Panel A: Portfolio Characteristics											
β_{mkt}	1.05	1.02	1.02	1.00	0.98	1.01	0.98	1.00	1.00	0.94	-0.12
t-statistic	43.50	45.99	49.79	42.21	44.85	49.06	53.91	45.45	39.36	33.65	-3.41
β_{SMB}	0.99	0.90	0.87	0.90	0.82	0.84	0.84	0.83	0.78	0.95	-0.04
t-statistic	26.10	25.22	22.94	22.06	18.27	32.03	27.82	22.73	16.83	24.85	-0.90
β_{HML}	0.18	0.28	0.31	0.30	0.31	0.35	0.33	0.36	0.35	0.34	0.16
t-statistic	5.07	6.97	8.55	6.82	7.96	9.65	9.59	8.89	7.46	7.14	3.48
β_{MOM}	-0.05	-0.08	-0.07	-0.05	-0.06	-0.04	-0.02	0.00	-0.02	0.03	0.08
t-statistic	-1.70	-1.74	-1.79	-1.06	-1.79	-0.98	-0.56	-0.15	-0.47	0.74	2.40
Size	0.79	0.86	0.96	1.00	1.06	1.09	1.09	1.04	1.17	1.03	
VR(24)	2.04	1.13	0.83	0.65	0.52	0.41	0.33	0.26	0.19	0.12	
AR1	0.01	-0.04	-0.06	-0.08	-0.09	-0.11	-0.11	-0.13	-0.14	-0.18	
σ_e^2	225.25	163.59	153.00	151.95	145.69	140.27	137.75	135.20	134.84	135.48	
Number of Stock											
Mean	287	287	288	288	289	288	289	289	289	289	
Mininum	159	158	161	164	158	160	161	163	159	158	
Pane B: Monthly Returns (%)											
Raw Ret	1.38	1.42	1.49	1.57	1.56	1.55	1.55	1.61	1.67	1.71	0.32
t-statistic	4.16	4.59	4.88	5.18	5.39	5.33	5.33	5.42	5.68	5.78	3.47
Capm α	0.31	0.39	0.46	0.54	0.56	0.53	0.55	0.61	0.67	0.72	0.41
t-statistic	1.70	2.36	2.82	3.20	3.33	3.38	3.47	3.77	4.19	4.10	4.89
FF3 α	0.04	0.08	0.14	0.23	0.25	0.20	0.23	0.28	0.35	0.38	0.34
t-statistic	0.47	1.03	2.00	2.82	2.90	2.94	3.23	4.01	4.42	4.31	3.91
FF4 α	0.10	0.16	0.21	0.28	0.31	0.24	0.25	0.28	0.36	0.35	0.26
t-statistic	0.97	1.68	2.37	2.71	3.36	2.86	3.09	3.56	4.13	3.79	2.93

Table 6: Properties of Equally-Weighted 5 by 5 Portfolios Sorted on VAR and Variance Ratios from January 1967 to December 2004. At the end of each year from 1966 to 2003, stocks are sorted into σ_e^2 quintile portfolios, where σ_e^2 is the variance of the residuals from Fama-French 3-factor model regressions estimated over the previous 60 months. Within each σ_e^2 quintile, they are further sorted into 5 variance ratio (VR) portfolios, where VR is defined as the ratio between the actual and the implied (assuming zero autocorrelation) variances of FF3 residuals over a 24-month period. The table reports raw returns and the intercepts ($\alpha's$) from regressions of excess returns on the market excess returns (CAPM), the 3 Fama-French factors (FF3), and the 3 Fama-French factors plus the Carhart Momentum factor (FF4). The returns and $\alpha's$ are in *per cent* per month, and the t-statistics are adjusted for autocorrelation and heteroscedasticity.

	Raw Returns (%)						t-statistic					
	Hi VR	2	3	4	Lo VR	Spread (5-1)	Hi VR	2	3	4	Lo VR	Spread (5-1)
Lo σ_e^2	1.23	1.27	1.23	1.27	1.21	-0.03	6.42	6.80	6.70	6.68	6.09	-0.38
2	1.34	1.33	1.33	1.41	1.43	0.09	5.43	5.36	5.41	5.94	5.86	1.26
3	1.41	1.41	1.52	1.50	1.61	0.20	4.75	4.78	5.25	5.02	5.40	2.54
4	1.57	1.65	1.73	1.65	1.86	0.29	4.14	4.37	4.82	4.48	5.07	3.04
Hi σ_e^2	1.52	1.62	2.09	2.06	2.30	0.78	3.39	3.53	4.58	4.31	4.78	4.95
σ_e^2 neutral	1.42	1.46	1.60	1.59	1.70	0.28	4.74	4.88	5.45	5.30	5.60	4.84
	CAPM Adj. Returns (%)						t-statistic					
	Hi VR	2	3	4	Lo VR	Spread (5-1)	Hi VR	2	3	4	Lo VR	Spread (5-1)
Lo σ_e^2	0.41	0.45	0.41	0.43	0.36	-0.05	3.07	3.54	3.45	3.60	2.81	-0.65
2	0.40	0.40	0.40	0.48	0.50	0.10	2.70	2.80	2.80	3.58	3.59	1.36
3	0.39	0.39	0.52	0.50	0.61	0.22	2.37	2.48	2.96	2.76	3.48	2.87
4	0.45	0.53	0.63	0.56	0.79	0.34	2.04	2.42	2.95	2.59	3.45	3.64
Hi σ_e^2	0.32	0.43	0.92	0.88	1.16	0.84	1.18	1.50	2.90	2.69	3.43	5.52
σ_e^2 neutral	0.39	0.44	0.58	0.57	0.69	0.30	2.47	2.80	3.54	3.51	4.04	5.31
	FF3 Adj. Returns (%)						t-statistic					
	Hi VR	2	3	4	Lo VR	Spread (5-1)	Hi VR	2	3	4	Lo VR	Spread (5-1)
Lo σ_e^2	0.12	0.15	0.13	0.16	0.07	-0.05	1.24	1.73	1.59	1.92	0.82	-0.64
2	0.07	0.07	0.04	0.16	0.13	0.06	0.75	0.72	0.44	1.83	1.56	0.81
3	0.07	0.02	0.13	0.13	0.23	0.16	0.90	0.24	1.55	1.35	2.90	2.06
4	0.08	0.18	0.34	0.21	0.44	0.36	0.72	1.88	3.33	1.83	3.77	3.67
Hi σ_e^2	0.05	0.12	0.60	0.54	0.82	0.76	0.30	0.67	2.90	2.36	3.41	4.51
σ_e^2 neutral	0.09	0.12	0.26	0.25	0.37	0.28	1.21	1.92	3.81	3.81	4.56	4.44
	FF3+MOM Adj. Returns (%)						t-statistic					
	Hi VR	2	3	4	Lo VR	Spread (5-1)	Hi VR	2	3	4	Lo VR	Spread (5-1)
Lo σ_e^2	0.16	0.18	0.14	0.17	0.10	-0.06	1.94	2.27	2.02	2.32	1.19	-0.75
2	0.11	0.13	0.09	0.19	0.20	0.09	1.34	1.53	1.09	2.58	2.57	1.12
3	0.13	0.10	0.20	0.20	0.24	0.11	1.40	1.21	2.25	2.23	2.76	1.32
4	0.12	0.23	0.38	0.23	0.48	0.36	0.94	1.84	3.15	1.74	3.35	3.72
Hi σ_e^2	0.24	0.34	0.71	0.61	0.84	0.60	1.20	1.44	2.78	2.29	3.33	3.83
σ_e^2 neutral	0.14	0.19	0.31	0.27	0.37	0.23	1.73	2.30	3.67	3.34	4.22	4.28

Table 7: Properties of Equally-Weighted 5 by 5 Portfolios Sorted on Variance Ratios and σ_e^2 from January 1967 to December 2004. At the end of each year from 1966 to 2003, stocks are sorted into variance ratio (VR) quintile portfolios, where VR is defined as the ratio between the actual and the implied (assuming zero autocorrelation) variances of the residuals from Fama-French 3-factor model regressions over a 24 month period estimated over the previous 60 months. Within each VR quintile, they are further sorted into 5 σ_e^2 portfolios, where σ_e^2 is the variance of FF3 residuals estimated over the previous 60 months. The table reports raw returns and the intercepts ($\alpha's$) from regressions of excess returns on the market excess returns (CAPM), the 3 Fama-French factors (FF3), and the 3 Fama-French factors plus the Carhart Momentum factor (FF4). The returns and $\alpha's$ are in *per cent* per month, and the t-statistics are adjusted for autocorrelation and heteroscedasticity.

	Raw Returns (%)						t-statistic					
	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)
Hi VR	1.22	1.29	1.55	1.49	1.43	0.21	5.98	4.91	4.68	3.80	3.08	0.59
2	1.29	1.33	1.47	1.71	1.84	0.55	6.91	5.35	4.85	4.50	4.02	1.49
3	1.22	1.29	1.48	1.61	2.06	0.84	6.63	5.30	5.33	4.55	4.57	2.23
4	1.26	1.41	1.56	1.64	2.01	0.76	6.79	5.87	5.33	4.62	4.34	1.93
Lo VR	1.21	1.40	1.59	1.77	2.34	1.13	6.21	6.03	5.69	4.96	4.90	2.79
VR Neutral	1.24	1.34	1.53	1.65	1.94	0.70	6.63	5.56	5.27	4.55	4.27	1.91
	CAPM Adj. Returns (%)						t-statistic					
	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)
Hi VR	0.37	0.33	0.49	0.34	0.22	-0.15	2.75	2.18	2.49	1.53	0.78	-0.49
2	0.47	0.40	0.44	0.58	0.67	0.20	3.71	2.69	2.75	2.59	2.21	0.66
3	0.40	0.36	0.49	0.52	0.90	0.50	3.25	2.65	2.87	2.48	2.87	1.53
4	0.43	0.49	0.56	0.57	0.84	0.42	3.58	3.54	3.16	2.77	2.78	1.26
Lo VR	0.38	0.48	0.62	0.72	1.20	0.82	2.90	3.55	3.89	3.19	3.56	2.32
VR Neutral	0.41	0.41	0.52	0.55	0.77	0.36	3.39	3.05	3.22	2.64	2.60	1.18
	FF3 Adj. Returns (%)						t-statistic					
	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)
Hi VR	0.09	0.00	0.18	0.04	-0.05	-0.14	0.89	0.04	1.95	0.36	-0.25	-0.66
2	0.17	0.07	0.08	0.23	0.35	0.18	1.96	0.74	1.09	2.11	1.72	0.78
3	0.11	0.01	0.10	0.19	0.58	0.47	1.32	0.08	1.09	1.92	2.78	1.96
4	0.14	0.16	0.18	0.23	0.49	0.35	1.71	1.74	1.83	2.35	2.42	1.42
Lo VR	0.08	0.15	0.21	0.34	0.86	0.78	0.94	1.69	2.46	2.94	3.62	2.87
VR Neutral	0.12	0.08	0.16	0.21	0.45	0.33	1.49	0.97	2.40	2.46	2.46	1.54
	FF3+MOM Adj. Returns (%)						t-statistic					
	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)	Lo σ_e^2	2	3	4	Hi σ_e^2	Spread (5-1)
Hi VR	0.13	0.05	0.20	0.09	0.21	0.08	1.51	0.57	1.82	0.70	0.98	0.37
2	0.22	0.11	0.18	0.29	0.51	0.29	2.87	1.41	1.96	2.08	1.96	1.09
3	0.12	0.07	0.17	0.26	0.71	0.60	1.50	0.89	1.83	2.32	2.69	2.07
4	0.17	0.18	0.25	0.27	0.53	0.36	2.28	2.38	2.76	2.11	2.38	1.43
Lo VR	0.10	0.21	0.23	0.35	0.88	0.78	1.16	2.53	2.63	2.59	3.51	2.87
VR Neutral	0.14	0.12	0.21	0.25	0.57	0.43	2.13	1.84	2.87	2.30	2.55	1.77

Table 8: Properties of Equally-Weighted 5 by 5 Portfolios Sorted on Size and Variance Ratios from January 1967 to December 2004 At the end of each year from 1966 to 2003, stocks are sorted into Size quintile portfolios according to the market value of equity. Within each size quintile, they are further sorted into 5 variance ratio (VR) portfolios, where VR is defined as the ratio between the actual and the implied (assuming zero autocorrelation) variances of the residuals from Fama-French 3-factor model regressions over a 24 month period estimated over the previous 60 months. The table reports raw returns and the intercepts ($\alpha's$) from regressions of excess returns on the market excess returns (CAPM), the 3 Fama-French factors (FF3), and the 3 Fama-French factors plus the Carhart Momentum factor (FF4). The returns and $\alpha's$ are in *per cent* per month, and the t-statistics are adjusted for autocorrelation and heteroscedasticity.

	Raw Returns (%)						t-statistic					
	Hi VR	2	3	4	Lo VR	Spread (5-1)	Hi VR	2	3	4	Lo VR	Spread (5-1)
Small	1.87	2.13	2.18	2.17	2.27	0.40	4.67	5.37	5.51	5.42	5.62	3.74
2	1.44	1.50	1.50	1.56	1.52	0.08	4.13	4.36	4.67	4.73	4.89	0.76
3	1.17	1.26	1.33	1.38	1.47	0.31	3.55	4.18	4.66	4.84	5.23	2.84
4	1.07	1.16	1.20	1.24	1.34	0.27	3.83	4.32	4.72	5.12	5.43	2.34
Large	0.90	1.07	1.00	1.13	1.13	0.23	3.76	4.86	4.66	5.33	5.29	2.35
Size Neutral	1.40	1.53	1.56	1.60	1.67	0.28	4.40	5.08	5.37	5.46	5.71	4.34
	CAPM Adjusted Returns (%)						t-statistic					
	Hi VR	2	3	4	Lo VR	Spread (5-1)	Hi VR	2	3	4	Lo VR	Spread (5-1)
Small	0.83	1.11	1.15	1.14	1.26	0.43	3.05	3.93	4.02	4.13	4.31	4.18
2	0.34	0.44	0.46	0.52	0.50	0.16	1.65	2.11	2.28	2.55	2.68	1.63
3	0.06	0.20	0.29	0.37	0.46	0.40	0.38	1.31	1.88	2.42	2.99	4.20
4	0.01	0.15	0.21	0.25	0.37	0.36	0.07	1.35	1.81	2.34	3.16	3.33
Large	-0.11	0.10	0.04	0.17	0.18	0.29	-1.71	1.56	0.61	2.78	2.31	3.02
Size Neutral	0.34	0.51	0.55	0.60	0.68	0.34	2.04	3.14	3.38	3.75	4.12	5.87
	FF3 Adjusted Returns (%)						t-statistic					
	Hi VR	2	3	4	Lo VR	Spread (5-1)	Hi VR	2	3	4	Lo VR	Spread (5-1)
Small	0.30	0.56	0.60	0.60	0.75	0.45	1.92	3.51	3.58	3.81	4.48	4.12
2	-0.02	0.03	0.07	0.11	0.09	0.11	-0.21	0.34	0.72	1.33	1.26	1.08
3	-0.20	-0.12	-0.02	0.04	0.13	0.32	-2.39	-1.58	-0.27	0.55	1.82	3.44
4	-0.16	-0.08	-0.07	0.03	0.12	0.28	-2.23	-1.02	-0.95	0.42	1.34	2.34
Large	-0.17	0.02	-0.04	0.07	0.06	0.23	-2.76	0.33	-0.72	1.47	0.98	2.45
Size Neutral	0.05	0.19	0.22	0.27	0.36	0.31	0.64	2.77	3.25	4.15	4.66	4.94
	FF3+MOM Adjusted Returns (%)						t-statistic					
	Hi VR	2	3	4	Lo VR	Spread (5-1)	Hi VR	2	3	4	Lo VR	Spread (5-1)
Small	0.43	0.70	0.71	0.65	0.78	0.34	2.50	3.50	3.69	3.80	4.28	3.24
2	0.02	0.03	0.12	0.13	0.10	0.08	0.19	0.26	1.28	1.41	1.28	0.72
3	-0.15	-0.06	0.01	0.04	0.14	0.29	-1.87	-0.75	0.07	0.52	1.77	3.12
4	-0.10	0.01	-0.03	0.07	0.09	0.20	-1.32	0.11	-0.40	1.01	1.10	1.73
Large	-0.11	0.04	0.00	0.10	0.10	0.21	-1.96	0.58	-0.06	1.94	1.55	2.28
Size Neutral	0.11	0.24	0.28	0.29	0.35	0.24	1.31	2.61	3.39	3.90	4.19	4.11