Math 2280 - Assignment 3

Dylan Zwick

Spring 2013

Section 2.1 - 1, 8, 11, 16, 29

Section 2.2 - 1, 10, 21, 23, 24

Section 2.3 - 1, 2, 4, 10, 24

Section 2.1 - Population Models

2.1.1 Separate variables and use partial fractions to solve the initial value problem:

$$\frac{dx}{dt} = x - x^2 \qquad x(0) = 2.$$

$$\frac{dx}{x - x^2} = dt \Rightarrow \left(\frac{A}{x} + \frac{B}{1-x}\right) dx = dt$$

$$\Rightarrow \int \left(\frac{1}{x} + \frac{1}{1-x}\right) dx = \int dt$$

$$\ln x - \ln(1-x) = t + C$$

$$\ln\left(\frac{x}{1-x}\right) = t + C \Rightarrow \frac{x}{1-x} = Ce^t$$

$$x = (1-x)Ce^t \Rightarrow x(1 + Ce^t) = Ce^t$$

$$x(t) = \frac{Ce^t}{1 + Ce^t} \qquad x(0) = \frac{C}{1+C} = 2$$

$$\Rightarrow C = 2 + 2C \Rightarrow C = -2$$

$$x(t) = \frac{-2e^t}{1 - 2e^t} = \frac{2e^t}{2e^t - 1} \text{ or } \boxed{\frac{2}{2 - e^{-t}}}$$

More space, if necessary, for problem 2.1.1.

Not necessary.

2.1.8 Separate variables and use partial fractions to solve the initial value problem:

$$\frac{dx}{dt} = 7x(x-13) \qquad\qquad x(0) = 17.$$

$$\frac{dx}{7x(x-13)} = dt$$

$$\frac{A}{7x} + \frac{B}{x-13} = \frac{1}{7x(x-13)}$$

$$\frac{A(x-13) + B(7x)}{7x(x-13)} = \frac{1}{7x(x-13)}$$

$$Ax + 7Bx - 13A = 1 \Rightarrow A = -\frac{1}{13}$$

$$B = \frac{1}{91}$$

$$\Rightarrow \int \left(-\frac{1}{91x} + \frac{1}{91(x-13)} \right) dx = t + C$$

More space, if necessary, for problem 2.1.8.

$$-\frac{\ln x}{91} + \frac{\ln(x-13)}{91} = t + C$$

$$\Rightarrow \ln\left(\frac{x-13}{x}\right) = 91t + C$$

$$\Rightarrow \frac{x-13}{x} = Ce^{91t}$$

$$x - 13 = xCe^{91t}$$

$$x(1 - Ce^{91t}) = 13$$

$$x(t) = \frac{13}{1 - Ce^{91t}} \qquad x(0) = 17$$

$$x(0) = \frac{13}{1-C} = 17 \Rightarrow 13 = 17 - 17C$$

$$17C = 4 \quad C = 4/17$$

$$x(t) = \frac{13}{1 - \frac{4}{17}e^{91t}} = \boxed{\frac{221}{17 - 4e^{91t}}}$$

2.1.11 Suppose that when a certain lake is stocked with fish, the birth and death rates β and δ are both inversely proportional to $\sqrt{P}$.

(a) Show that

$$P(t) = \left(\frac{1}{2}kt + \sqrt{P_0}\right)^2.$$

(b) If $P_0 = 100$ and after 6 months there are 169 fish in the lake, how many will there be after 1 year?

a) $\beta = \dfrac{\beta_0}{\sqrt{P}}$ $\qquad \delta = \dfrac{\delta_0}{\sqrt{P}}$

$$\Rightarrow \frac{dP}{dt} = (\beta - \delta)P = (\beta_0 - \delta_0)\sqrt{P}$$

$$= k\sqrt{P} \quad \text{where } k = \beta_0 - \delta_0$$

$$\Rightarrow \frac{dP}{dt} = k\sqrt{P} \Rightarrow \frac{dP}{\sqrt{P}} = k\,dt$$

$$\Rightarrow 2\sqrt{P} = kt + C$$

$$\Rightarrow P(t) = \left(\tfrac{1}{2}kt + C\right)^2$$

$$P(0) = C^2 \Rightarrow C = \sqrt{P_0}$$

$$\Rightarrow \boxed{P(t) = \left(\tfrac{1}{2}kt + \sqrt{P_0}\right)^2}$$

More space, if necessary, for problem 2.1.11.

b)

$$P(t) = \left(\tfrac{1}{2}kt + \sqrt{100}\right)^2$$

$$= \left(\tfrac{1}{2}kt + 10\right)^2$$

$$P(6) = \left(\tfrac{1}{2}k(6) + 10\right)^2 = 169$$

$$\Rightarrow 3k + 10 = 13 \Rightarrow k = 1$$

So,

$$P(t) = \left(\tfrac{1}{2}t + 10\right)^2$$

$$P(12) = \left(\tfrac{1}{2}(12) + 10\right)^2 = 16^2$$

$$= \boxed{256 \text{ fish}}$$

2.1.16 Consider a rabbit population $P(t)$ satisfying the logistic equation $dP/dt = aP - bP^2$. If the initial population is 120 rabbits and there are 8 births per month and 6 deaths per month occuring at time $t = 0$, how many months does it take for $P(t)$ to reach 95% of the limiting population M?

$$a(120) = 6 \Rightarrow a = \frac{8}{120} = \frac{1}{15}$$

$$b(120)^2 = 8 \Rightarrow b = \frac{6}{120^2} = \frac{1}{2400}$$

$$\frac{dP}{dt} = \frac{1}{15}P - \frac{1}{2400}P^2$$

$$\frac{dP}{dt} = \frac{P}{15} - \frac{P^2}{2400} \Rightarrow \int 2400\left(\frac{dP}{160P - P^2}\right) = \int dt$$

$$2400 \int \left(\frac{A}{P} + \frac{B}{160-P}\right) dP = t + C$$

$$A(160-P) + BP = 1 \Rightarrow A = \frac{1}{160} \quad B = \frac{1}{160}$$

$$\Rightarrow 15 \int \left(\frac{1}{P} + \frac{1}{160-P}\right) dP = t + C$$

$$15 \ln\left(\frac{P}{160-P}\right) = t + C$$

$$\Rightarrow P/160-P = Ce^{t/15}$$

More space, if necessary, for problem 2.1.16.

Solving for P we get

$$P(1+Ce^{t/15}) = 160Ce^{t/15}$$

$$P(t) = \frac{160Ce^{t/15}}{1+Ce^{t/15}} = \frac{160}{1+Ce^{-t/15}}$$

$$P(0) = 120 = \frac{160}{1+C} \Rightarrow C = \frac{1}{3}$$

$$P(t) = \frac{480}{3+e^{-t/15}} \qquad M = \frac{480}{3} = 160$$

is limiting population

95% of M is 152

$$152 = \frac{480}{3+e^{-t/15}} \Rightarrow e^{-t/15} = \frac{480}{152} - 3$$

$$t = -15\ln\left(\frac{480}{152} - 3\right) = \boxed{27.69 \text{ months}}$$

2.1.29 During the period from 1790 to 1930 the U.S. population $P(t)$ (t in years) grew from 3.9 million to 123.2 million. Throughout this period, $P(t)$ remained close to the solution of the initial value problem

$$\frac{dP}{dt} = 0.03135P - 0.0001489P^2, \qquad P(0) = 3.9.$$

(a) What 1930 population does this logistic equation predict?

(b) What limiting population does it predict?

(c) Has this logistic equation continued since 1930 to accurately model the U.S. population?

[This problem is based on the computation by Verhulst, who in 1845 used the 1790-1840 U.S. population data to predict accurately the U.S. population through the year 1930 (long after his own death, of course).]

a) Using the logistic population formula from the textbook:

$$P(t) = \frac{(210.54)(3.9)}{3.9+(206.64)e^{-.03135t}}$$

$$P(140) \approx \boxed{122.0 \text{ million}}$$

More space, if necessary, for problem 2.1.29.

b)

$$M = \frac{0.03135}{.0001489} = \boxed{210.54 \text{ million}}$$

So, about 210.5 million

c) $\boxed{\text{No.}}$ The current U.S. population is above 300 million.

Section 2.2 - Equilibrium Solutions and Stability

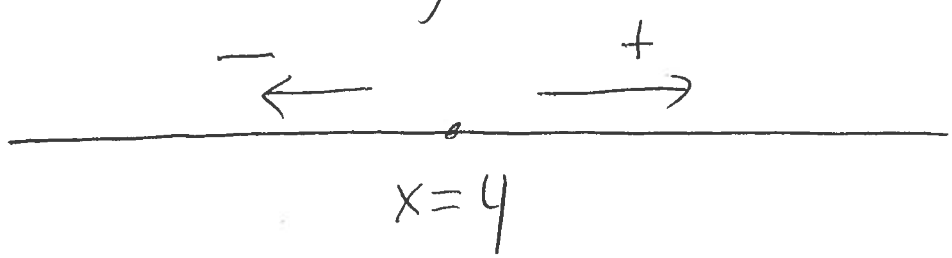
2.2.1 - Find the critical points of the autonomous equation

$$\frac{dx}{dt} = x - 4.$$

Then analyze the sign of the equation to determine whether each critical point is stable or unstable, and construct the corresponding phase diagram for the differential equation. Next, solve the differential equation explicitly for $x(t)$ in terms of t. Finally, use either the exact solution or a computer-generated slope field to sketch typical solution curves for the given differential equation, and verify visually the stability of each critical point.

$x - 4 = 0 \Rightarrow x = 4$ is the critical point.

Phase diagram



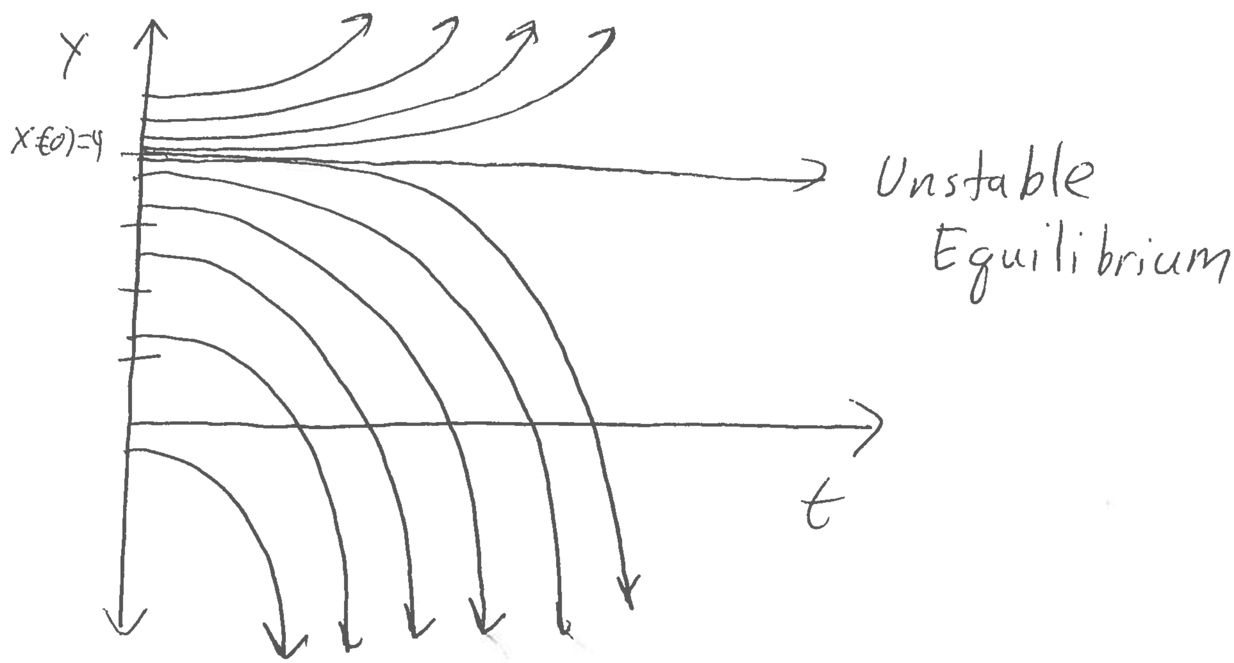
Unstable.

More space, if necessary, for problem 2.2.1.

$$\frac{dx}{x-4} = dt \Rightarrow \int \frac{dx}{x-4} = \int dt$$

$$\Rightarrow \ln(x-4) = t + C \Rightarrow x - 4 = Ce^t$$

$$x(t) = Ce^t + 4$$



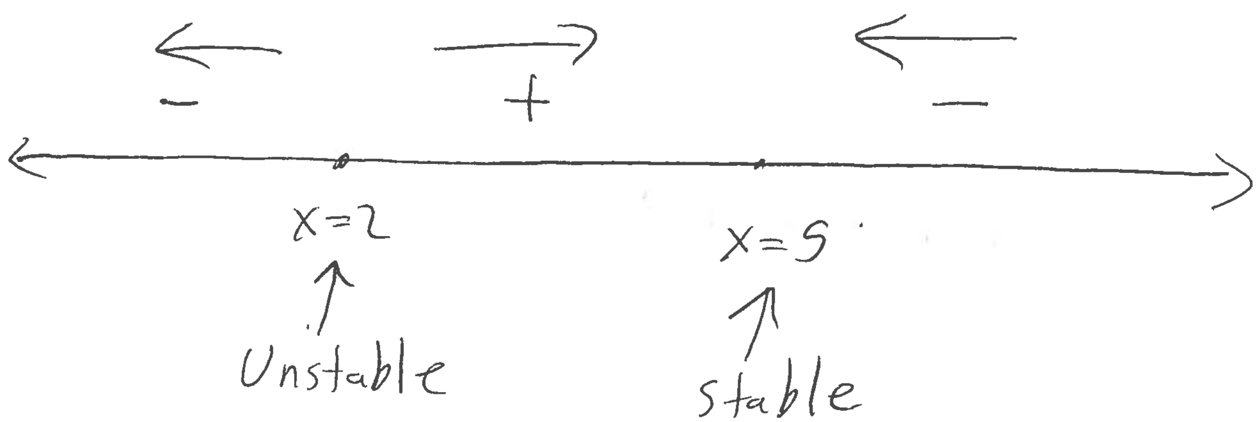
2.2.10 Find the critical points of the autonomous equation

$$\frac{dx}{dt} = 7x - x^2 - 10.$$

Then analyze the sign of the equation to determine whether each critical point is stable or unstable, and construct the corresponding phase diagram for the differential equation. Next, solve the differential equation explicitly for $x(t)$ in terms of t. Finally, use either the exact solution or a computer-generated slope field to sketch typical solution curves for the given differential equation, and verify visually the stability of each critical point.

$7x - x^2 - 10 = (5-x)(x-2)$

So, equilibrium points where $\frac{dx}{dt} = 0$ are $x = 5$ and $x = 2$.



More space, if necessary, for problem 2.2.10.

$$\int \frac{dx}{(5-x)(x-2)} = \int dt$$

$$A(x-2)+B(5-x)=1$$
$$-2A+5B=1$$
$$A-B=0 \Rightarrow A=B$$
$$A=B=\frac{1}{3}$$

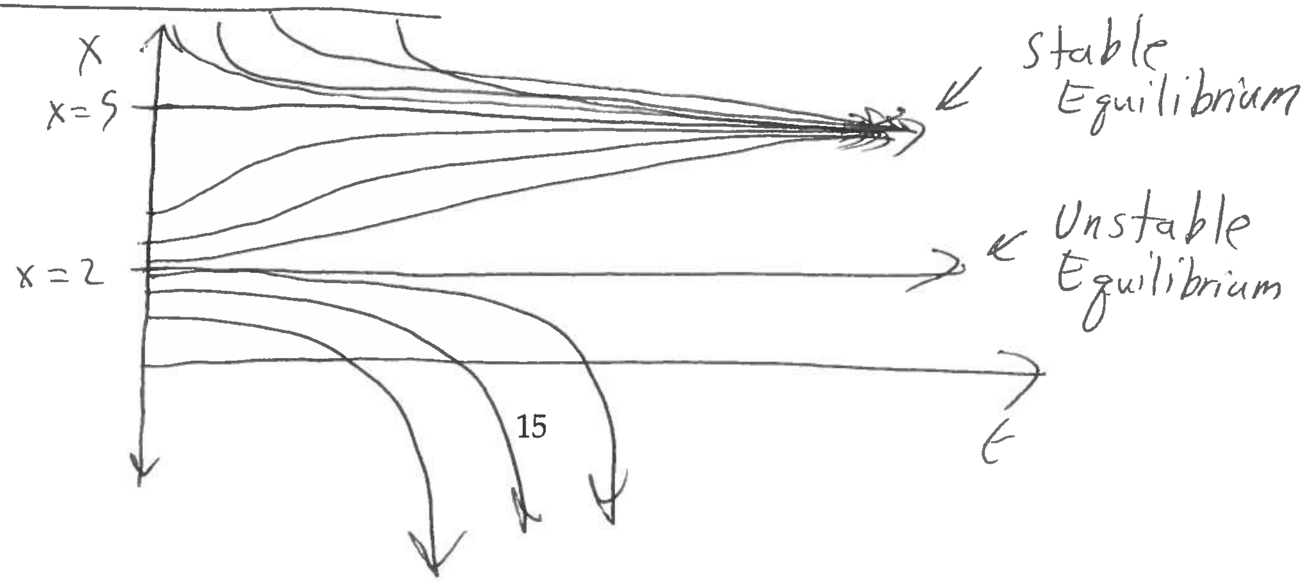
$$\int \left(\frac{A}{5-x} + \frac{B}{x-2}\right)dx = \int dt$$

$$\frac{1}{3} \int \left(\frac{1}{5-x} + \frac{1}{x-2}\right)dx = \int dt$$

$$\Rightarrow \frac{1}{3} \ln\left(\frac{x-2}{5-x}\right) = t + C \Rightarrow \frac{x-2}{5-x} = Ce^{3t}$$

$$x-2 = (5-x)Ce^{3t} \Rightarrow \boxed{x(t) = \frac{2+5Ce^{3t}}{1+Ce^{3t}}}$$

Solution Curves



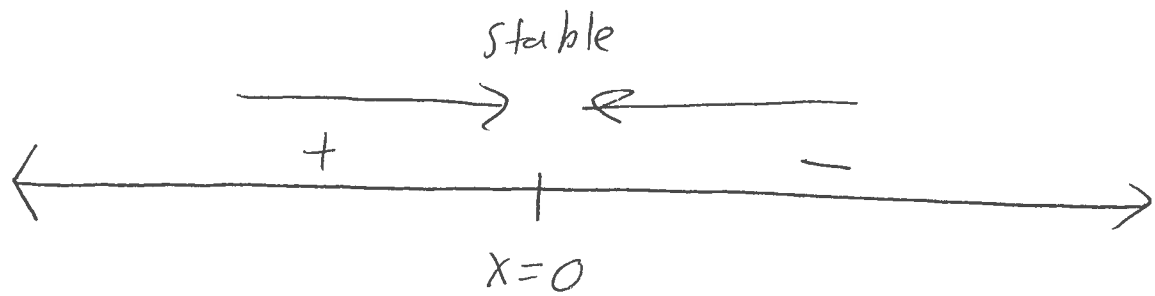
2.2.21 Consider the differential equation $dx/dt = kx - x^3$.

(a) If $k \leq 0$, show that the only critical value $c = 0$ of x is stable.

(b) If $k > 0$, show that the critical point $c = 0$ is now unstable, but that the critical points $c = \pm\sqrt{k}$ are stable. Thus the qualitative nature of the solutions changes at $k = 0$ as the parameter k increases, and so $k = 0$ is a bifurcation point for the differential equation with parameter k.

The plot of all points of the form (k, c) where c is a critical point of the equation $x' = kx - x^3$ is the "pitchform diagram" show in figure 2.2.13 of the textbook.

$kx - x^3 = x(k - x^2)$ ~~If~~ The roots are $x = 0$, and $x = \pm\sqrt{k}$. If $k \leq 0$ the $\pm\sqrt{k}$ is imaginary or also 0, and we have only one real root.



$kx - x^3 < 0$ for $k \leq 0$, ~~$x \neq 0$~~ $x > 0$

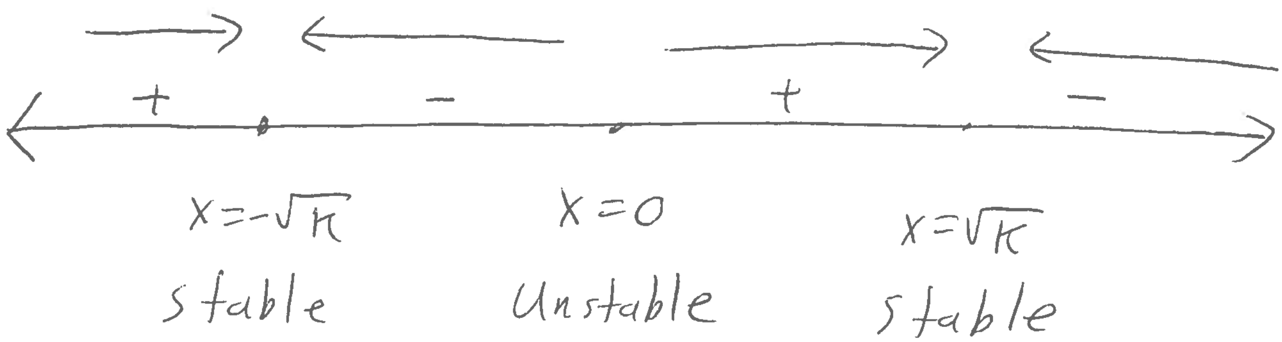
$kx - x^3 > 0$ for $k \leq 0$, $x < 0$

More space, if necessary, for problem 2.2.21.

b) For $k > 0$ there are three distinct real roots

$$kx - x^3 = -x(x+\sqrt{k})(x-\sqrt{k})$$

~~So, as k~~ The roots are $x = 0$, $x = \pm\sqrt{k}$



2.2.23 Suppose that the logistic equation $dx/dt = kx(M-x)$ models a population $x(t)$ of fish in a lake after t months during which no fishing occurs. Now suppose that, because of fishing, fish are removed from the lake at a rate of hx fish per month (with h a positive constant). Thus fish are "harvested" at a rate proportional to the existing fish population, rather than at the constant rate of Example 4 from the textbook.

(a) If $0 < h < kM$, show that the population is still logistic. What is the new limiting population?

(b) If $h \geq kM$, show that $x(t) \to 0$ as $t \to \infty$, so the lake is eventually fished out.

a) $$\frac{dx}{dt} = kx(M-x) - hx$$

$$= x(kM-h) - kx^2 = kx\left(\left(M-\frac{h}{k}\right)-x\right)$$

Still logistic with limiting population $\boxed{M-\frac{h}{K}}$.

b) If $h \geq kM$ the solution to the logistic population equation will be:

More space, if necessary, for problem 2.2.23.

$$P(t) = \frac{\left(M - \frac{h}{k}\right) P_0}{P_0 + \left(M - \frac{h}{k} - P_0\right) e^{-k\left(M - \frac{h}{k}\right) t}}$$

~~Define~~ $\cancel{N = kM - h \leq 0 \Rightarrow N \geq 0}$

$\cancel{\;\; = h - kM}$

$$\cancel{P(t) = \frac{-N P_0}{k P_0}}$$

Define $N = \frac{h}{k} - M \geq 0$

$$P(t) = \frac{-N P_0}{P_0 - (P_0 + N) e^{kNt}} = \frac{N P_0}{(P_0 + N) e^{kNt} - P_0}$$

$$\lim_{t \to \infty} \frac{N P_0}{(P_0 + N) e^{kNt} - P_0} = 0.$$

So, the lake is eventually fished out.

Note: If $h = kM$ then

$\frac{dx}{dt} = -kx^2 \Rightarrow \quad \frac{dx}{x^2} = -k\,dt \Rightarrow -\frac{1}{x} = -kt + C$

$\Rightarrow x(t) = \frac{1}{kt + C} = \frac{P_0}{1 + P_0 k t}$ also goes to 0 as $t \to \infty$.

2.2.24 Separate variables in the logistic harvesting equation

$$dx/dt = k(N - x)(x - H)$$

and then use partial fractions to derive the solution given in equation 15 of the textbook (also appearing in the lecture notes).

$$\int \frac{dx}{(N-x)(x-H)} = \int k\,dt$$

$$\int \left(\frac{A}{N-x} + \frac{B}{x-H}\right) d\cancel{t}x = \int k\,dt$$

$$A(x-H) + B(N-x) = 1$$

$$\Rightarrow A - B = 0 \Rightarrow A = B$$

$$-AH + BN = 1 \Rightarrow A(N-H) = 1 \quad A = \frac{1}{N-H} = B$$

$$\frac{1}{N-H} \int \left(\frac{1}{N-x} + \frac{1}{x-H}\right) dx = \int k\,dt$$

$$\frac{1}{N-H} \ln\left(\frac{x-H}{N-x}\right) = kt + C$$

$$\ln\left(\frac{x-H}{N-x}\right) = k(N-H)t + C$$

More space, if necessary, for problem 2.2.24.

$$\frac{x-H}{N-x} = C e^{k(N-H)t}$$

$$x - H = (N-x)\, C e^{k(N-H)t}$$

$$x\left(1 + C e^{k(N-H)t}\right) = H + N C e^{k(N-H)t}$$

$$x(t) = \frac{H + N C e^{k(N-H)t}}{1 + C e^{k(N-H)t}}$$

$$x(0) = x_0 = \frac{H + NC}{1 + C} \Rightarrow x_0 + C x_0 = H + NC$$

$$\Rightarrow C(x_0 - N) = H - x_0 \Rightarrow C = \frac{H - x_0}{x_0 - N}$$

$$x(t) = \frac{H + N\left(\frac{H - x_0}{x_0 - N}\right) e^{k(N-H)t}}{1 + \left(\frac{H - x_0}{x_0 - N}\right) e^{k(N-H)t}} \cdot \frac{(x_0 - N)\, e^{-k(N-H)t}}{(x_0 - N)\, e^{-k(N-H)t}}$$

$$= \frac{H(x_0 - N)\, e^{-k(N-H)t} + N(H - x_0)}{(H - x_0) + (x_0 - N)\, e^{-k(N-H)t}} \times \frac{-1}{-1}$$

Equation 2.2.15 from the text.

$$= \boxed{\frac{N(x_0 - H) - H(x_0 - N)\, e^{-k(N-H)t}}{(x_0 - H) - (x_0 - N)\, e^{-k(N-H)t}}}$$

Section 2.3 - Acceleration-Velocity Models

2.3.1 The acceleration of a Maserati is proportional to the difference between 250 km/h and the velocity of this sports car. If the machine can accelerate from rest to 100 km/h in 10s, how long will it take for the car to accelerate from rest to 200 km/h?

$$\frac{dv}{dt} = k(250-v)$$

$$\int \frac{dv}{250-v} = \int k\,dt$$

$$\Rightarrow \quad -\ln(250-v) = kt + C$$

$$\Rightarrow \quad 250 - v = Ce^{-kt}$$

$$\Rightarrow \quad v(t) = 250 - Ce^{-kt} \qquad v(0) = 0 = 250 - C$$

$$\Rightarrow C = 250$$

$$v(t) = 250(1 - e^{-kt})$$

$$v(10) = 250(1 - e^{-k10}) = 100$$

$$\Rightarrow \quad 1 - e^{-10k} = \frac{2}{5} \Rightarrow \ln(e^{-10k}) = \ln\left(\frac{3}{5}\right)$$

$$\Rightarrow \quad k = \frac{\ln(5) - \ln(3)}{10} \approx .05108$$

More space, if necessary, for problem 2.3.1.

$$200 = 250\left(1 - e^{-.05108t}\right)$$

$$\Rightarrow \frac{4}{5} = 1 - e^{-.05108t}$$

$$\Rightarrow e^{-.05108t} = \frac{1}{5} \Rightarrow t = \frac{\ln(5)}{.05108}$$

$$\boxed{t \approx 31.55}$$

2.3.2 Suppose that a body moves through a resisting medium with resistance proportional to its velocity v, so that $dv/dt = -kv$.

(a) Show that its velocity and position at time t are given by

$$v(t) = v_0 e^{-kt}$$

and

$$x(t) = x_0 + \left(\frac{v_0}{k}\right)(1 - e^{-kt}).$$

(b) Conclude that the body travels only a finite distance, and find that distance.

a) $\frac{dv}{dt} = -kv \Rightarrow \int \frac{dv}{v} = -\int k\,dt$

$\Rightarrow \ln v = -kt + C \Rightarrow v(t) = Ce^{-kt}$

$v(0) = v_0 = C \Rightarrow$

$v(t) = v_0 e^{-kt}$

$x(t) = \int v_0 e^{-kt} = -\frac{v_0}{k} e^{-kt} + C$

$x(0) = x_0 = -\frac{v_0}{k} + C \Rightarrow C = x_0 + \frac{v_0}{k}$

$\Rightarrow x(t) = x_0 + \frac{v_0}{k} - \frac{v_0}{k} e^{-kt} = \boxed{x_0 + \left(\frac{v_0}{k}\right)(1 - e^{-kt})}$

More space, if necessary, for problem 2.3.2.

b) $$\lim_{t \to \infty} x(t) = x_0 + \frac{v_0}{k}\left(1 - e^{-k\infty}\right)$$

$$= \boxed{x_0 + \frac{v_0}{k}}$$

2.3.4 Consider a body that moves horizontally through a medium whose resistance is proportional to the *square* of the velocity v, so that

$$dv/dt = -kv^2.$$

Show that

$$v(t) = \frac{v_0}{1 + v_0 kt}$$

and that

$$x(t) = x_0 + \frac{1}{k}\ln(1 + v_0 kt).$$

Note that, in contrast with the result of Problem 2, $x(t) \to \infty$ as $t \to \infty$. Which offers less resistance when the body is moving fairly slowly - the medium in this problem or the one in Problem 2? Does your answer seem to be consistent with the observed behaviors of $x(t)$ as $t \to \infty$?

$$\frac{dv}{v^2} = -k\,dt \Rightarrow \int \frac{dv}{v^2} = -\int k\,dt$$

$$-\frac{1}{v} = -kt + C \Rightarrow \frac{1}{v} = kt + C$$

$$\Rightarrow v(t) = \frac{1}{kt + C} \qquad v(0) = v_0 = \frac{1}{C} \Rightarrow C = \frac{1}{v_0}$$

$$v(t) = \frac{1}{kt + \frac{1}{v_0}} = \boxed{\frac{v_0}{1 + v_0 kt}}$$

More space, if necessary, for problem 2.3.4.

$$x(t) = \int \frac{v_0}{1+v_0 kt} = \frac{v_0}{v_0 k} \ln(1+v_0 kt) + C$$

$$= C + \frac{1}{k} \ln(1+v_0 kt)$$

$$x(0) = x_0 = C + 0 \Rightarrow C = x_0 \text{ and}$$

$$\boxed{x(t) = x_0 + \frac{1}{k} \ln(1+v_0 kt)}$$

For $|v|<1$ we have $v^2<|v|$ and so the ~~dist~~ drag is <u>smaller</u> for fairly small values of v. This is why the distance can go forever, and is not finite.

2.3.10 A woman bails out of an airplane at an altitude of 10,000 ft, falls freely for 20s, then opens her parachute. How long will it take her to reach the ground? Assume linear air resistance $\rho v\ ft/s^2$, taking $\rho = .15$ without the parachute and $\rho = 1.5$ with the parachute. (*Suggestion*: First determine her height above the ground and velocity when the parachute opens.)

First, calculate the distance traveled in the first 20 seconds

$$\frac{dv}{dt} = g - \rho_1 v$$

$$\int \frac{dv}{g - \rho_1 v} = \int dt$$

$$\Rightarrow -\frac{\ln(g - \rho_1 v)}{\rho_1} = t + C$$

$$\Rightarrow \ln(g - \rho_1 v) = -\rho_1 t + C$$

$$\Rightarrow g - \rho_1 v = Ce^{-\rho_1 t}$$

$$v(t) = \frac{g}{\rho_1} - Ce^{-\rho_1 t}$$

$$v(t) = 0 \Rightarrow C = \frac{g}{\rho_1}$$

More space, if necessary, for problem 2.3.10.

$$v(t) = \frac{g}{\rho_1}\left(1 - e^{-\rho_1 t}\right)$$

$$x(t) = \int \frac{g}{\rho_1}\left(1 - e^{-\rho_1 t}\right) dt$$

$$= \frac{g}{\rho_1}\left(t + \frac{e^{-\rho_1 t}}{\rho_1}\right) + C$$

$$x(0) = x_0 = \frac{g}{\rho_1^2} + C \Rightarrow C = x_0 - \frac{g}{\rho_1^2}$$

$$x(t) = x_0 + \frac{g}{\rho_1}\left(t + \frac{1}{\rho_1}\left(e^{-\rho_1 t} - 1\right)\right)$$

$x_0 = 10,000 \qquad t = 20$

$$x(20) = 10,000 - \frac{32.2}{.15}\left(20 + \frac{1}{.15}\left(e^{-.15(20)} - 1\right)\right)$$

$$= 7,060.527 \text{ ft.}$$

Now, we must find the time for the rest of the distance

$$0 = 7,060.527 - \frac{32.2}{1.5}\left(t + \frac{1}{1.5}\left(e^{-1.5t} - 1\right)\right)$$

$$\Rightarrow t + \frac{1}{1.5}\left(e^{-1.5t} - 1\right) = 7,060.527\left(\frac{1.5}{32.2}\right)$$

$e^{-1.5t} \approx 0$ for even moderately large values of t.

$$t \approx 7,060.527\left(\frac{1.5}{32.2}\right) + \frac{1}{1.5} \approx 329 s$$

Total time $= 329s + 20s = 349s \approx$ $\boxed{5 \text{ min } 49 \text{ sec}}$

2.3.24 The mass of the sun is 329,320 times that of the earth and its radius is 109 times the radius of the earth.

(a) To what radius (in meters) would the earth have to be compressed in order for it to become a *black hole* - the escape velocity from its surface equal to the velocity $c = 3 \times 10^8 m/s$ of light?

(b) Repeat part (a) with the sun in place of the earth.

a)

$$3 \times 10^8 \, m/s = \sqrt{\frac{2 G M_e}{R}}$$

$$\Rightarrow R = \frac{2 G M_e}{c^2} = \frac{2 \left(6.67 \times 10^{-11} \frac{N m^2}{kg^2}\right)\left(5.972 \times 10^{24} kg\right)}{\left(3 \times 10^8 m/s\right)^2}$$

$$\approx .00885 m = \boxed{.885 cm}$$

Wow!

b)

$$R = \frac{2 G M_s}{c^2} = \frac{2 G M_e}{c^2}(329{,}320)$$

$$\approx 2{,}915 m = \boxed{2.915 km}$$