

Atomic-orbital Based Ab-initio simulations

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The **Nobel Prize** in **Chemistry 1998** was awarded to W. Kohn "for his development of the density-functional theory".



1923/3/9-2016/4/19

Applications:



New developments:

New functionals: more accurate Numerical methods: larger systems

The development of first-principles methods



Application of DFT for thousands of atoms



DNA containing 3439 atoms

J. Phys. : Condens. Matter **20** (2008) 294201 (CONQUEST)



Kohn-Sham equation

$$\left[-\frac{\nabla^2}{2m} + V_{Hxc}[n](r) + V_{ext}(r)\right] \varphi_i(r) = \varepsilon_i \varphi_i(r)$$

$$V_{Hxc}[n] = \frac{\delta E_{Hxc}[n]}{\delta n} \quad n(r) = \sum_{i}^{\text{occ}} |\varphi_i(r)|^2$$

KS equation maps the many-particle problem into a single-particle problem. The mapping is exact, if the energy functional is exact!



Solve Kohn-Sham equation

By choosing proper basis set, one can turn the Kohn-Sham equation to a matrix eigenvalue problem

$$|\psi_{i}\rangle = \sum_{q'} c_{i,q'} |q'\rangle$$

$$\sum_{q'} \langle q|H|q'\rangle c_{i,q'} = \varepsilon_{i} \sum_{q'} c_{i,q'} \langle q|q'\rangle$$

$$\sum_{q'} H(q,q') c_{i,q'} = \varepsilon_{i} \sum_{q'} S(q,q') c_{i,q'}$$

One of the central problems for designing efficient electronic structure algorithm is to find proper basis sets

All electron: APW, MTO Pseudopotential: Plane wave, atomic orbital, etc.

Popular basis sets



~ 500 - 1000 /atom



Plane wave basis (PWB)

- Undependent of atom positions, easy to implement.
- Forces can be easily calculated.
- Can be systematically improved by a single parameter (Ecut)
- Efficient for <100 atoms.
- Contraction of the second seco







Some DFT packages for thousands of atoms

CONQUEST (England and Japan)

ONETEP (England)

SIESTA (Spain)

OpenMX (Japan)

FHI-aims (Germany, FHI)











Basis : Numerical orbtials / finite element basis

Basis : real space grid/ Wannier functions

Basis : Numerical orbitals

Basis : Numerical orbitals

Basis : Numerical orbitals All electron code Bloch wave function in plane wave bases

$$\psi_{n\mathbf{k}}(\mathbf{r}) = \sum_{\mathbf{G}} e^{i(\mathbf{k}+\mathbf{G})\cdot\mathbf{r}} u_{n\mathbf{k}}(\mathbf{G})$$

Bloch wave function in numerical atomic orbital bases (LCAO)

$$\psi_{n\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}} \sum_{\alpha} c_{n\alpha,\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{R}} \phi_{\alpha} \left(\mathbf{r} - \boldsymbol{\tau}_{\alpha} - \mathbf{R}\right)$$
NAO

Atomic orbitals



Adapt to any symmetry!

Examples of atomic orbitals



How to improve the atomic basis sets



Atomic basis sets

Multi-zeta orbitals: Different radial wave functions for the same angular momentum

Polar orbitals: Orbitals with higher angular momentum than valence orbitals.

Systematically improvement: 'SZ', 'DZ', 'DZP', 'TZDP'...

SZ: Single-zeta (minimal basis set sp)

DZ: Double-zeta (2s2p)

DZP: Double-zeta + Polar orbital (2s2p1d)

TZDP: Triple-zeta + double Polar orbital (3s3p2d)

Taken Si as an example



How to construct atomic orbitals



J. Junquera et al, Phys Rev B **64**, 235111 (2001)

How to construct multi-zeta orbitals



Pablo Ordejon, Summer schoold of SIESTA, 2007

How to construct polarized orbitals

Method 2: Apply electric field

$$\left[-\frac{\nabla^2}{2} + V_{atom}(r) + V_{confine}(r) + E \cdot r\right] \psi_n = \varepsilon_n \psi_n$$

$$\int \text{Selection rule}$$

$$\left[\frac{1}{2r} \frac{\mathrm{d}^2}{\mathrm{d}r^2} r + \frac{(l+1)(l+2)}{2r^2} + V_r(l) - E_l\right] \phi_{l+1} = -\epsilon r \phi_l(r) c_{l+1}$$

J. Soler et al, J. phys: Condens. Matter. 14, 2745 (2002)

Compare method 1 and method 2



Fig. 3. d polarization orbitals for silicon for two different confinement conditions. a) Obtained with the electric-field polarization method and b) the confined d PAOs

Scheme (2) Scheme (1)

E. Artacho et al, Phys. Stat. Sol. (b) 215, 809 (1999)

Test atomic basis on bulk Si



J. Junquera et al, Phys Rev B **64**, 235111 (2001)

Problems of previous orbital generation methods

1. Complicated especially for multi-zeta and polar orbtials.

2. The orbitals are not fully optimized.

Our Method: spillage minimization

What is "Spillage":
$$S = \frac{1}{N_n} \sum_{n=1}^{N_n} \langle \Psi_n | 1 - \hat{P}(\Psi_n) \rightarrow \text{Reference}_{\text{states}}$$

 $\hat{P} = \sum_{\mu\nu} |\phi_{\mu}\rangle S_{\mu\nu}^{-1} \langle \phi_{\nu}|$ Local orbital space



Cup: Hilbert Space spanned by reference states (plane wave).

Hilbert Space spanned by Local orbitals

Hilbert space left : spillage

Ice Block: atomic orbitals

D. S. Portal, et al. Solid State Communications <u>95</u>, 685 (1995)

Radial functions



Mohan Chen, G-C Guo, and Lixin He, J. Phys.: Cond. Matt. 22 445501 (2010)

Steps to get the orbitals

$$f_{\mu,l}(\mathbf{r}) = \begin{cases} \sum_{q} c_{\mu q} j_l(qr), & r < r_c \\ 0 & r \ge r_c \end{cases}$$

- 1. Choose the angular momentum you want to use.
- 2. Choose the radii of the orbitals



Mohan Chen, G-C Guo, and Lixin He, J. Phys.: Cond. Matt. 22 445501 (2010)

Reference system: dimers

Molecule made of two identical atoms: "dimer"



No bias!



 Choose a set of dimers with different bond lengths.

 Calculate the energies and wave functions

 ◆ 4~5 dimers are enough to get good orbitals



Systematically generate atomic basis

$$\hat{P}^{(1)} = \sum_{\mu\nu} |\phi_{\mu}^{(1)}\rangle S_{\mu\nu}^{-1} \langle \phi_{\nu}^{(1)}|$$

$$|\Psi_{n}^{(2)}\rangle = (1 - \hat{P}^{(1)})|\Psi_{n}^{(1)}\rangle$$

$$\hat{P}^{(1)}|\Psi_{n}^{(2)}\rangle = 0$$
Input parameters of the atomic orbitals
Input parameters of the atomic orbitals
All zeta orbitals and polar orbitals are obtained from the

orbitals are obtained from same procedure!

The radial functions of Carbon (6.0 Bohr)



SCF with:

- Carbon dimers of 5 bond lengths.
 LDA
- Energy cutoff = 100 Ry

Radial functions have oscillation!

Unphysical + worsen the transferability of the basis

Optimize the orbital shape

Minimize "kinetic energy" of the orbitals

$$\min\left[\mathrm{T}_{\mu}(c_{\mu q})\right] = \min\left[\sum_{q} c_{\mu q}^{2} q^{2} / 2 + \kappa\right]$$

$$\kappa = \begin{cases} 0, & S/S_0 - 1 < \Delta \\ \infty, & S/S_0 - 1 > \Delta \end{cases}$$
 (0.002~0.005 is enough)

S: Current Spillage

S0: Spillage before minimize kinetic energy.

Mohan Chen, G-C Guo, and Lixin He, J. Phys.: Cond. Matt. 22 445501 (2010)

Optimized radial functions for Carbon



Spillage vs. Total energy



Spillage can describe very well the convergence of the total energy!

Balance the accuracy of different elements



RESEARCH ARTICLE

DFT METHODS

Reproducibility in density functional theory calculations of solids

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New methods Mutual agreement

Old methods Different values

Stefaan Cottenier et al., Science 351, 1415 (2016)



Stefaan Cottenier et al., Science 351, 1415 (2016)

DZP bases



LCAO-DZP vs. PW $\langle \Delta \rangle \approx 2.9 \text{ meV}$ NC(SG15) $\langle \Delta \rangle \approx 1.4 \text{ meV}$ LCAO-DZP vs. WIEN2K $\langle \Delta \rangle \approx 3.0 \text{ meV}$ PAW $\langle \Delta \rangle \approx 0.6 - 1.6 \text{ meV}$ (no f orbitals for 3d, 4d elements)USPP $\langle \Delta \rangle \approx 0.5 - 6.3 \text{ meV}$

TZDP bases



Egg-Box effects

Translation symmetry broken by grid points!



SIESTA used a special filtering procedure to reduce the eggbox effects

P-F Li et al. Computational Materials Science 112 (2016) 503–517

Egg-Box effects – why?

Spherical Bessel functions:



Our scheme allow the atomic orbitals localized both in real space and in **G** space!



✓Molecules

Bond length, vibration frequency, atomization energy

✓ Solid (bulk)

Lattice constants, cohesive energy, bulk modulus, structure

✓ Surface reconstruction

Test on bulk Si



J. Junquera et al, Phys Rev B 64, 235111 (2001)

Semiconductors

			a					В	$\overline{}$	
Compound	SZ	DZP	TZDP	\mathbf{PW}	Expr.	SZ	DZP	TZDP	\mathbf{PW}	Expr.
GaAs	10.67	10.50	10.49	10.48	$10.68^{\rm a}$	69	78	77	77	75.57°
GaP	10.28	10.11	10.11	10.10	10.30^{a}	82	92	93	93	89^{d}
GaSb	11.54	11.38	11.37	11.36	11.52^{a}	49	59	58	57	57^{d}
InAs	11.40	11.27	11.28	11.28	11.45^{a}	63	66	65	65	60^{d}
InP	11.07	10.94	10.94	10.93	11.09^{a}	78	79	79	80	$71^{\rm d}$
InSb	12.33	12.05	12.05	12.07	$12.24^{\rm a}$	41	50	49	50	$47^{\rm d}$
AlAs	10.88	10.63	10.62	10.59	$10.70^{\rm a}$	67	76	76	76	$77^{\rm d}$
AlP	10.50	10.26	10.25	10.21	10.33^{a}	64	87	88	89	86^{d}
AlSb	11.83	11.58	11.57	11.54	11.59^{a}	48	55	56	56	58^{d}
Ge	10.82	10.68	10.61	10.61	10.70°	57	67	71	71	77.20°
Si	10.59	10.28	10.25	10.23	10.26^{b}	74	94	94	94	99^{b}
Ce	6.78	6.67	6.67	6.67	6.75^{b}	436	470	467	466	$442^{\rm b}$



晶格常数平均误差(Bohr): 0.218 (SZ), 0.026 (DZP), 0.015 (TZDP) 体弹性模量平均误差(GPa): 12.3 (SZ), 1.4 (DZP), 0.4 (TZDP) DZP 可以准确预测半导体的结构性质

Tests on ZnO



Tests on AI (metal)









SC



bcc

hcp

Correct energy orders and energy differences!

LCAO works well for metals and insulators _°

Electronic structure interpolation via atomic orbitals

Mohan Chen, G-C Guo, and Lixin He, J. Phys.: Cond. Mat. 23 325501 (2011)

Motivation : Large k sampling problem

The number of k points used to calculate anomalous Hall conductivity in bcc Fe:

Self consistent – 20, 000 k points Berry curvature , larger than 400,000 – 2,000,000 k points



Yugui Yao et al, Phys. Rev. Lett 92, 037204 (2004)

First Principle Calculation of Anomalous Hall Conductivity in Ferromagnetic bcc Fe

Band interpolation via Wannier functions

Maximally localized wannier functions (MLWF)

Minimize the spread functional:

(a)



MLWF

Xinjie Wang et al, Phys. Rev. B 74, 195118 (2006)

Our Method: atomic basis



 $H_{\mu\nu}(\mathbf{R}_{n}) = \langle \phi_{\mu0} | \hat{H} | \phi_{\nu n} \rangle$, Calculate once for all k-points $H_{\mu\nu}(\mathbf{k}) = \sum_{\mathbf{R}_{\mathbf{n}}} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{R}_{\mathbf{n}}} H_{\mu\nu}(\mathbf{R}_{\mathbf{n}}),$ $\Psi_{i\mathbf{k}}(\mathbf{r}) = \sum c_{i\mu}(\mathbf{k})\phi_{\mu}(\mathbf{r}-\mathbf{r}_{\mu}-\mathbf{R}_{\mathbf{n}})e^{i\mathbf{k}\cdot\mathbf{R}_{\mathbf{n}}}.$ $\mathbf{V} \qquad O_{ii}(\mathbf{k},\mathbf{q}) = \left\langle \psi_{i,\mathbf{k}} \left| \hat{O} \right| \psi_{i,\mathbf{q}} \right\rangle$ $O_{ij}(\mathbf{k},\mathbf{q}) = \sum e^{-i\mathbf{k}\cdot\mathbf{R}_{\mathbf{n}}+i\mathbf{q}\cdot\mathbf{R}_{\mathbf{m}}}c^*_{i\mu}(\mathbf{k})c_{j\nu}(\mathbf{q})\langle\phi_{\mu\mathbf{n}}|\hat{O}|\phi_{\nu\mathbf{m}}\rangle.$

Test: Na (bcc), Al (fcc)



Test: BaFe₂As₂



Mohan Chen, G-C Guo, and Lixin He, J. Phys.: Cond. Mat. 23 325501 (2011)

Transferability of the bases



Absorption spectrum of Si



Dielectric function:

$$\epsilon_2(\omega) = \frac{2e^2\pi}{\Omega\epsilon_0} \sum_{\mathbf{k},\mathbf{c},\mathbf{v}} |\langle \Psi_{\mathbf{k},\mathbf{c}} | \hat{\mathbf{e}} \cdot \mathbf{r} | \Psi_{\mathbf{k},\mathbf{v}} \rangle|^2 \delta \Big[E_{\mathbf{c}}(\mathbf{k}) - E_{\mathbf{v}}(\mathbf{k}) - \hbar\omega \Big]$$

ABACUS project

- Atomic-orbital Based Ab-initio
 Computation at UStc
- Written in C++
- Start at 2007
- first version v1.0.0 released at2015.10
- v1.0.1 released at 2016.12
- v1.1 will be released in 2018

http://abacus.ustc.edu.cn/



http://abacus.ustc.edu.cn

ATOMIC-ORBITAL BASED AB-INITIO COMPUTATION AT USTC

Today is: Tuesday the 10th of October, 2017

HOME FEATURES	TEAM PUBLICATIONS	RESOURCES	DOWNLOAD	FAQS	FORUM			
News	ABACUS (Atomic-orbtial Based Ab-initio Computation at UStc) is an open-source computer code package aiming for large-scale electronic-structure simulations from first principles, developed at the Key Laboratory of Quantum Information, University of Science and Technology of China (USTC) - C-omputer Network and Information Center, Chinese of Academy (CNIC of CAS).							
16.12.19 ABACUS V1.0.1	ABACUS currently provides the following features and functionalities:							
ABACUS version1.0.1 is updated for download!	 Ground-state total energy calculations using Kohn-Sham (KS) density functional theory (DFT) with local-density or generalized gradient approximations (LDA/GGAs). Brillouin zone sampling using the Monkhorst-Pack special k-points. Geometry relaxation with both Conjugated Gradient (CG) and BFGS methods. Semi-emperical van der Waals energy correction using the Grimme DFT-D2 scheme. NVT molecular dynamics simulation using the Nose-Hoover thermostat. 							
	More about >>							





pseudopotential library



download



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- Plane waves/Atomic orbital dual bases sets
- Normal conserving pseudopotential (UPF format)
- Functionals: LDA, GGA, spins, van der Waals (DFT-D2)
- Electronic structures, structural relaxation
- Molecular dynamics (NVE, NVT)
- stress and cell optimization
- ◆ Real-time TDDFT (cooperate with 孟胜)
- Berry phase
- Interface to Wannier90
- spin-orbital coupling
- Hybrid functional (HSE and PBE0)
- RPA, GW, optical properties
 - ◆ Transports (cooperate with 夏柯, 袁喆)

Will be included in the next release

Example: LiSn metal liquid



Lithium Tokamak Experiment (LTX), Princeton Plasma Physics Laboratory (PPPL) Liquid metals: promising materials are lithium, tin, gallium, etc.

Advantages:

no irreversible erosion no neutron damage no heat overload

Nuclear fusion rector wall protection materials MD simulation system size: 200 – 500 atoms



Structure properties of solid tin

	a_0	c_0/a_0	V_0	ΔE	B_0	Method
β -tin	5.780	0.537	26.15	0.000	58	FP (PW)
	5.786	0.538	26.23	0.000	57	FP (LCAO)
	5.831	0.546	27.07	-	-	EXP^{45}
	5.8119	0.543	26.65	-	-	EXP^{46}
	-	-	-	-	57.037	EXP^{47}
	-	-	-	-	57.9	EXP^{48}
α -tin	6.442	-	33.41	-0.019	43	FP (PW)
	6.445	-	33.47	-0.055	43	FP (LCAO)
	6.483	-	34.05	-	-	EXP^{45}
	-	-	-	-	42.617	EXP^{47}
	-	-	-	-	54	EXP^{49}
bct	3.933	0.846	25.73	0.045	53	FP (PW)
	3.920	0.844	25.42	0.047	54	FP (LCAO)
bcc	3.664		24.60	0.095	70	FP (PW)
	3.658		24.47	0.130	71	FP (LCAO)



Diffusion coefficient of pure Sn metal liquid



Deuterium diffusion in Sn metal liquid





- 1. D does not form bonds with Sn;
- 2. D does not form bonds with other D;
- 3. Good for D distraction.





LiSnD liquid slab

- 1、ABACUS: LCAO-DZP
- 2、BOMD-NVT;
- 4、 Time step: 0.2 fs, total time: 20 ps (100,000 steps);
- $5\,{\scriptstyle \times}\,$ Li : Sn $\,$ = 1:4, T=573 K,673 K and 873 K ,

deuterium for 10 and 100.









(a) At 573 K and 673 K, D₂ dimer may escape to the vacuum.

(a) At 873 K, small mount of LiD dimer may also escape to vacuum.

In prepartion





Collaborators

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