AVL trees

Today

- AVL delete and subsequent rotations
- Testing your knowledge with interactive demos!

AVL tree

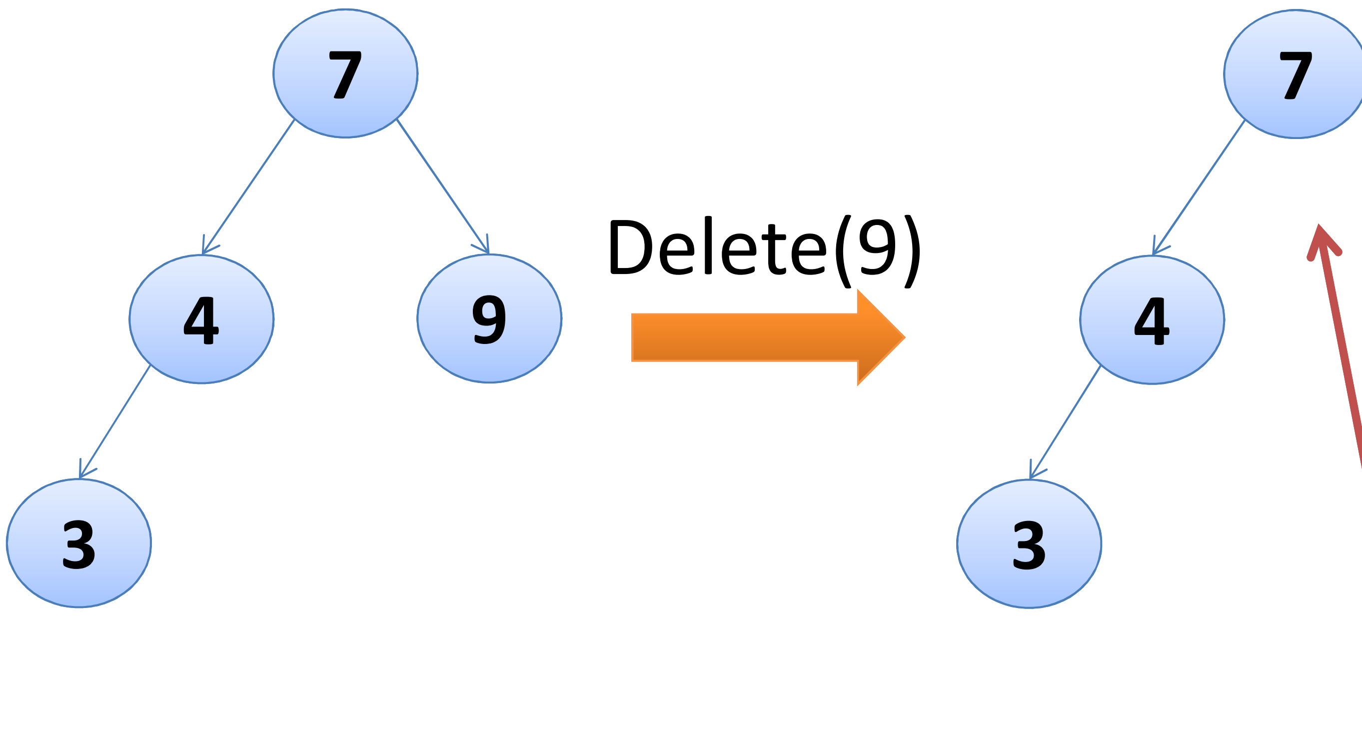
- Is a binary search tree
- Has an additional ***height constraint***:
  - For each node x in the tree, Height(x.left) differs from Height(x.right) by at most 1
- I promise:
  - If you satisfy the ***height constraint***, then the **height of the tree is O(lg n)**.
  - (Proof is easy, but no time! =])

AVL tree

- To be an AVL tree, must **always:**
  - (1) Be a ***binary search tree***
  - (2) Satisfy the ***height constraint***

- Suppose we start with an AVL tree, then delete as if we're in a regular BST.
- Will the tree be an AVL tree after the delete?
  - (1) It will still be a BST…
  - (2) Will it satisfy the ***height constraint***?
- (Not covering insert, since you already did in class)

BST Delete breaks an AVL tree

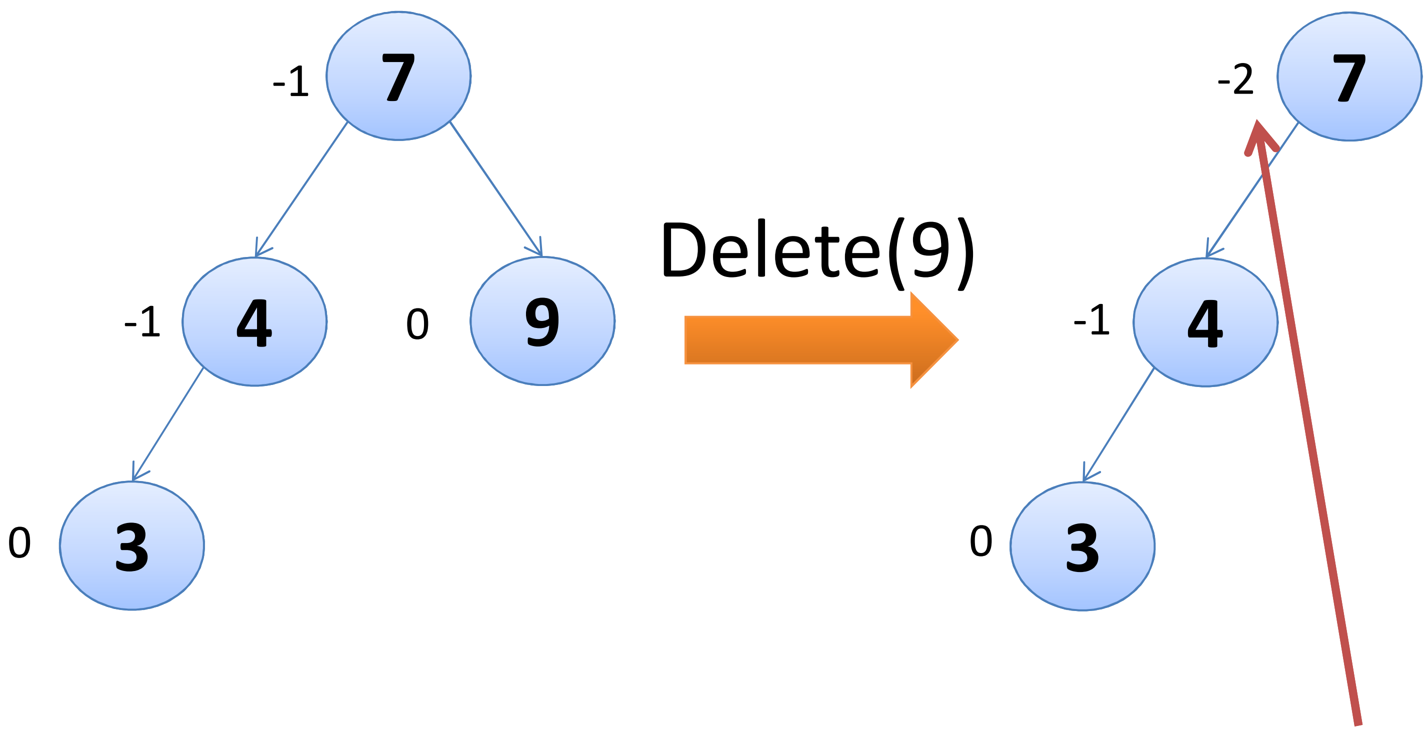


h(left) > h(right)+1
so **NOT** an AVL tree!

Replacing the height constraint with balance factors

- Instead of thinking about the heights of nodes, it is helpful to think in terms of ***balance factors***
- The balance factor $bf(x) = h(x.right) - h(x.left)$
  - bf(x) values -1, 0, and 1 are allowed
  - If $bf(x) < -1$ or $bf(x) > 1$ then tree is **NOT AVL**

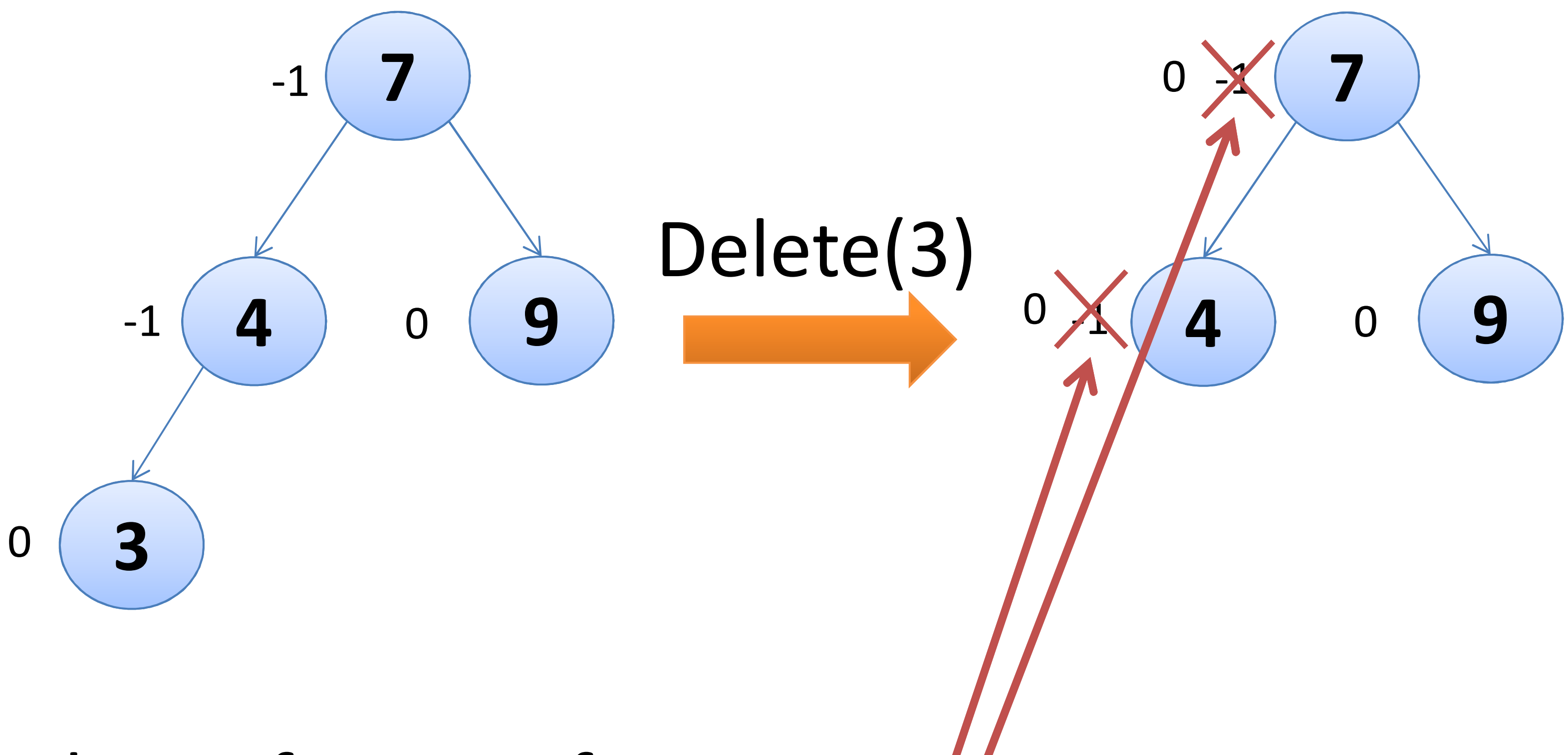
Same example with **bf(x), <u>not</u> h(x)**



bf < -1

so **<u>NOT</u>** an AVL tree!

What else can BST Delete break?



- Balance factors of ancestors…

Need a new Delete algorithm

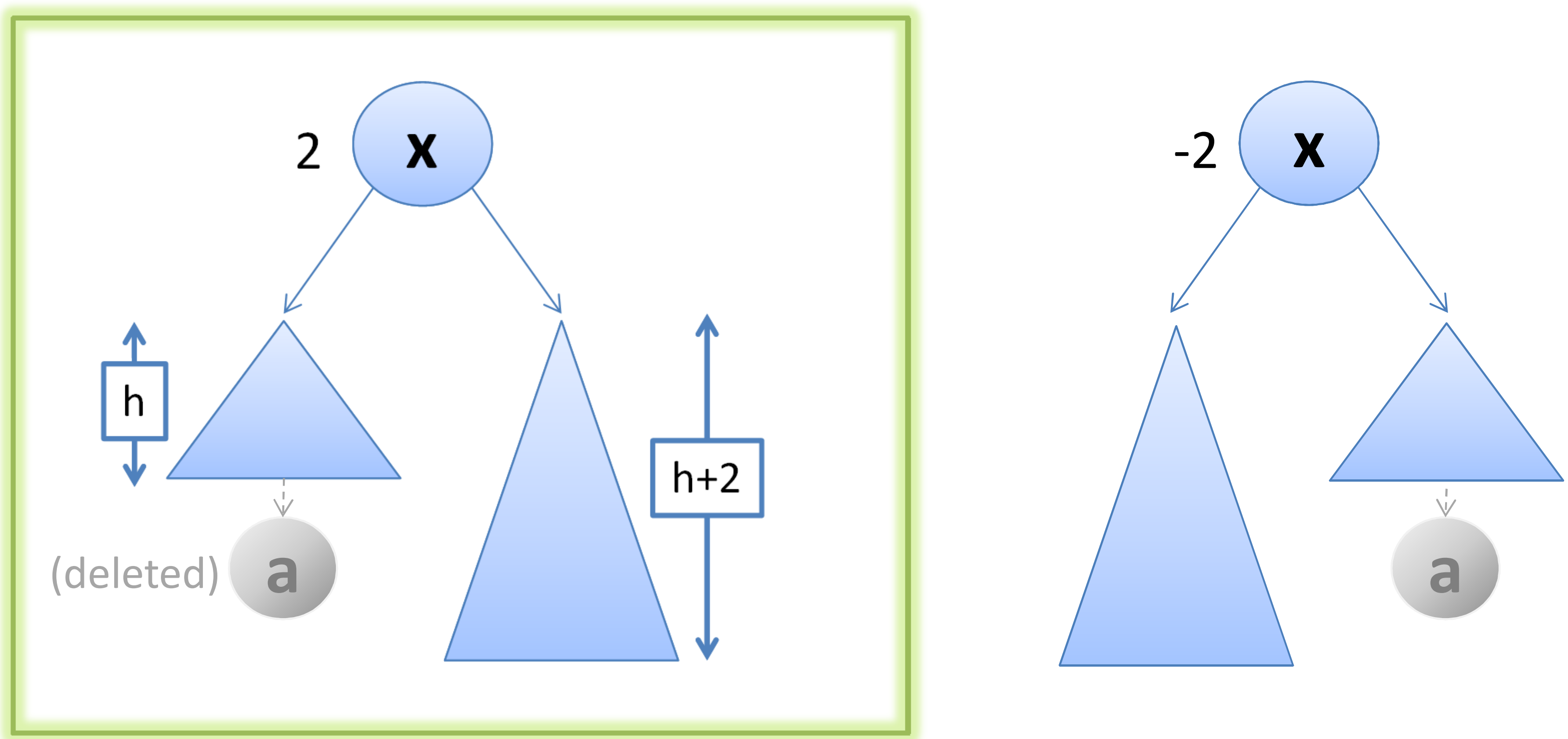
- **Goal:** if tree is AVL before Delete, then tree is AVL after Delete.
- **Step 1:** do BST delete.
  - This maintains the *BST property*, but can cause the balance factors of ancestors to be outdated!
- **Step 2:** fix the height constraint and update balance factors.
  - Update any invalid balance factors affected by delete.
    - After updating them, they can be **< -1 or > 1.**
  - Do rotations to fix any balance factors that are too small or large while maintaining the BST property.
    - Rotations can cause balance factors to be outdated also!

Bad balance factors

- Start with an AVL tree, then do a BST Delete.
- What bad values can bf(x) take on?
  - Delete can reduce a subtree's height by 1.
  - So, it might increase or decrease h(x.right) – h(x.left) by 1.
  - So, bf(x) might increase or decrease by 1.
  - This means:
    - if bf(x) = 1 before Delete, it might become 2. **BAD.**
    - If bf(x) = -1 before Delete, it might become -2. **BAD.**
    - If bf(x) = 0 before Delete, then it is still -1, 0 or 1. **OK.**

2 cases

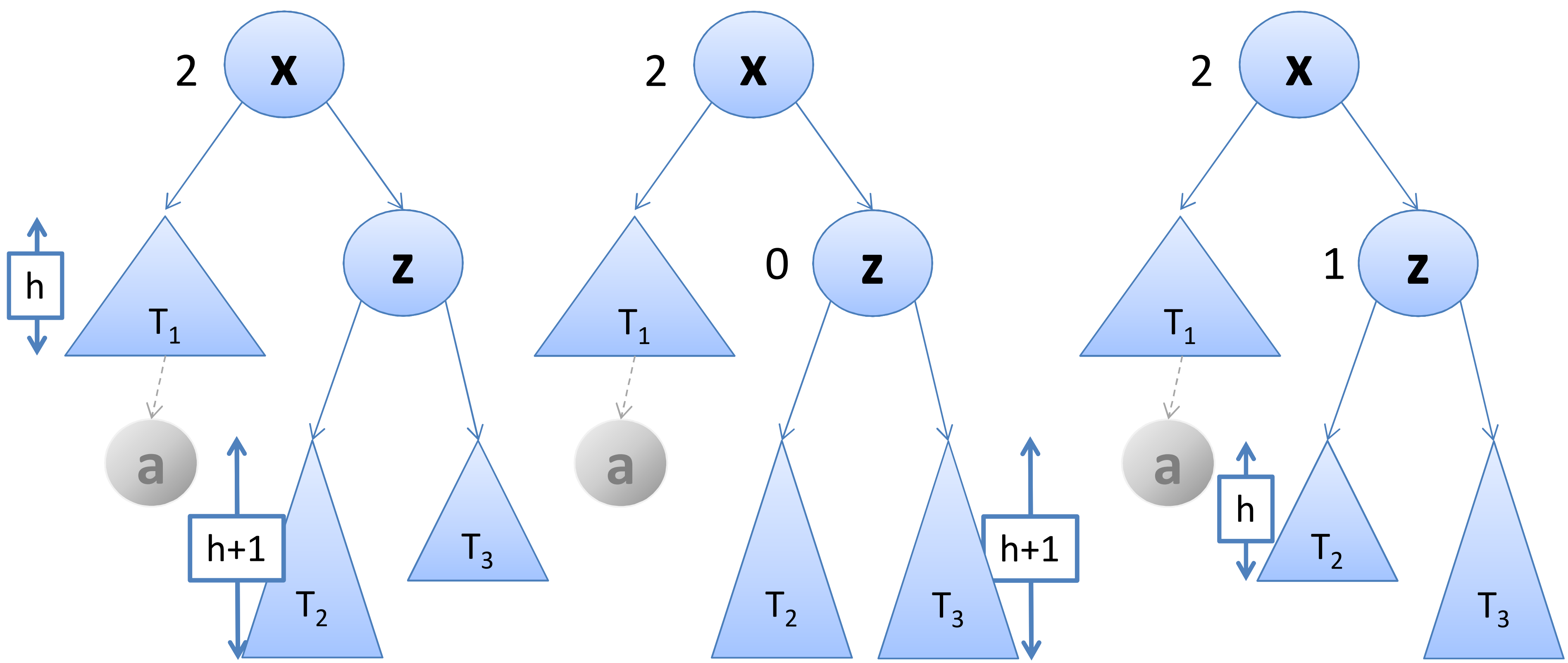
Problematic cases for Delete(a)



- bf(x) = -2 is just **symmetric** to bf(x) = 2.
- So, we just look at bf(x) = 2.

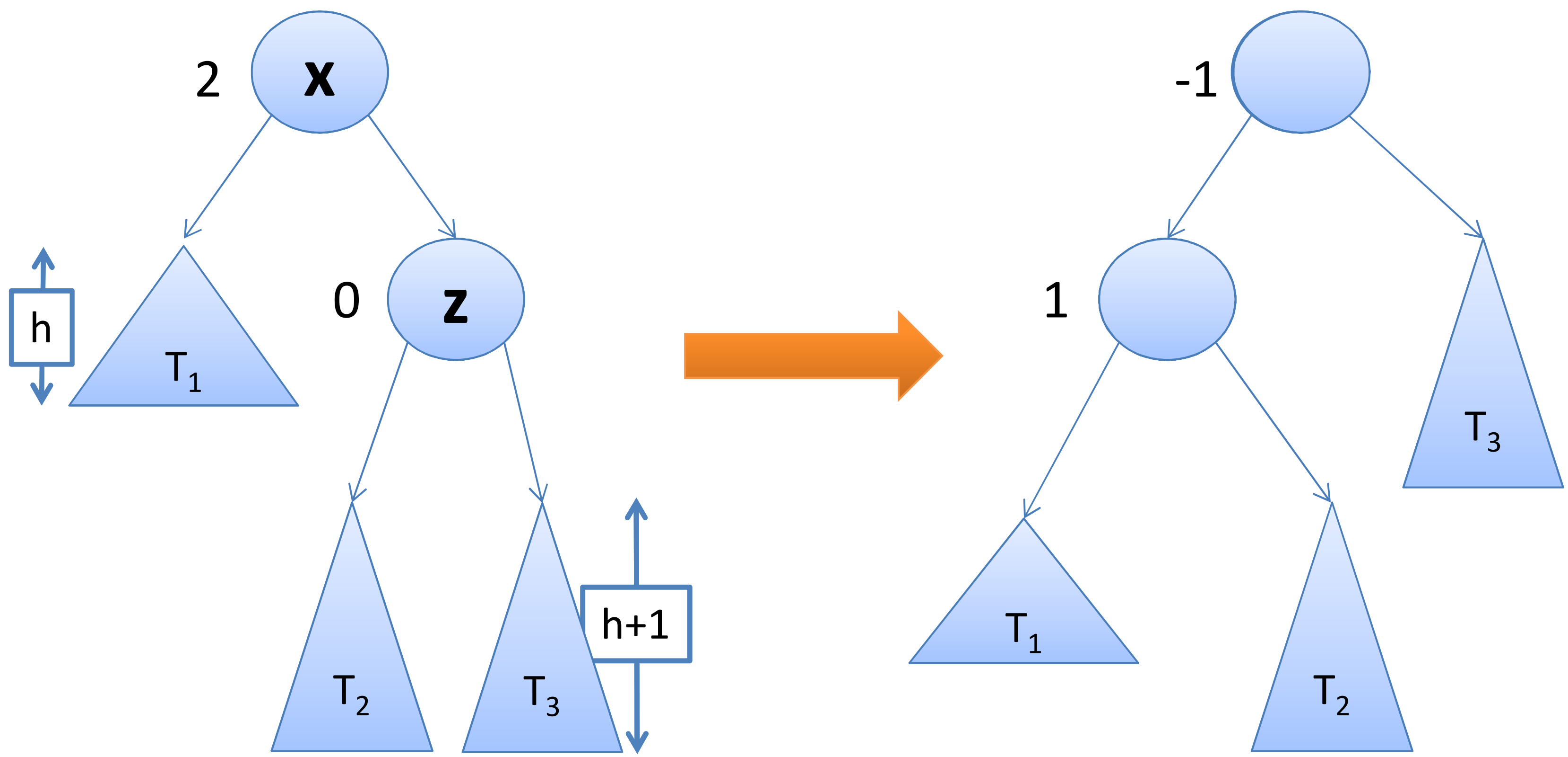
Delete(a): 3 subcases for bf(x)=2

- Since tree was AVL before, **bf(z) = -1, 0 or 1**



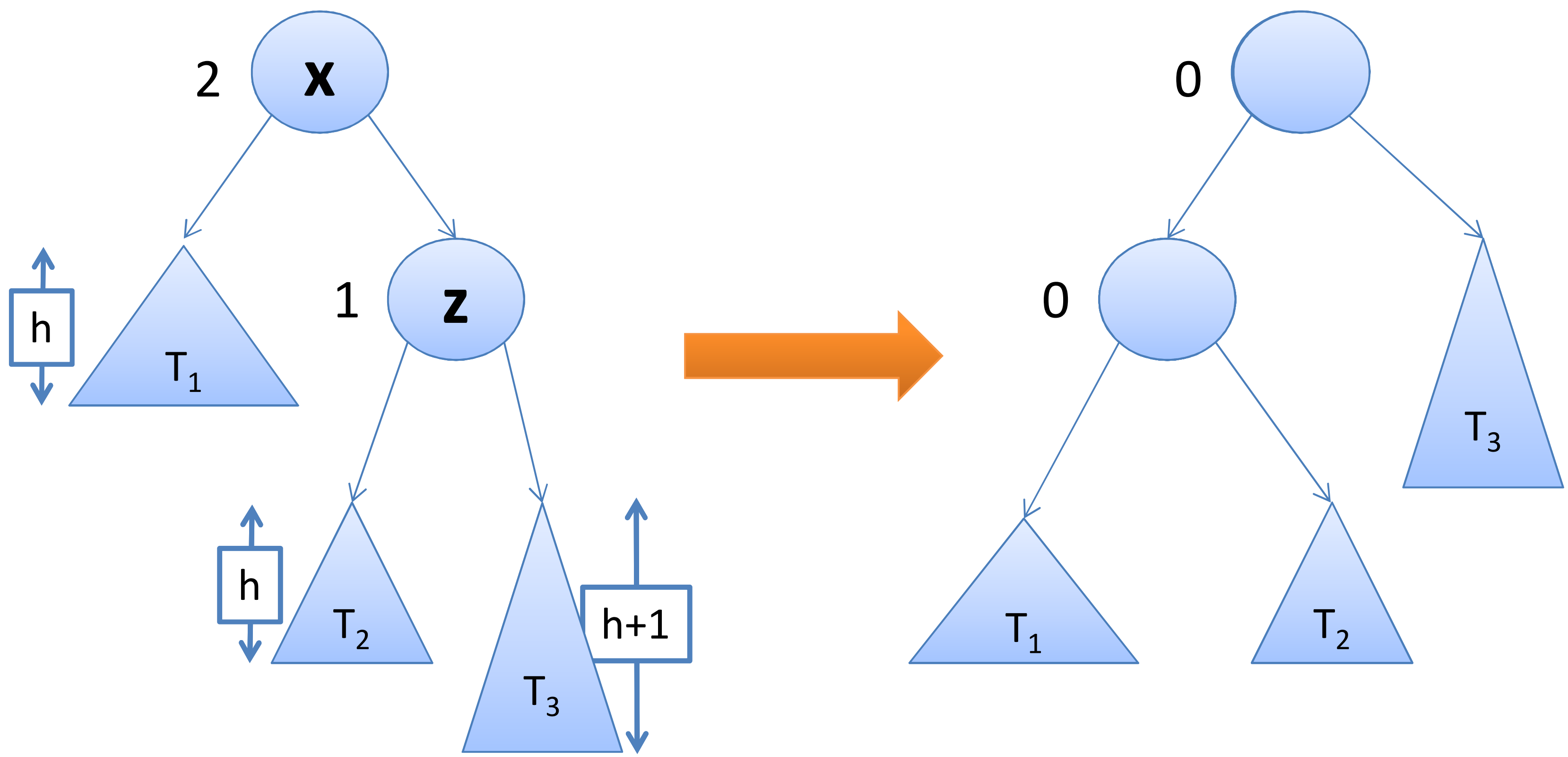
Fixing case bf(x) = 2, bf(z) = 0

- We do a ***single left rotation***
- Preserves the BST property, and fixes bf(x) = 2



Fixing case bf(x) = 2, bf(z) = 1

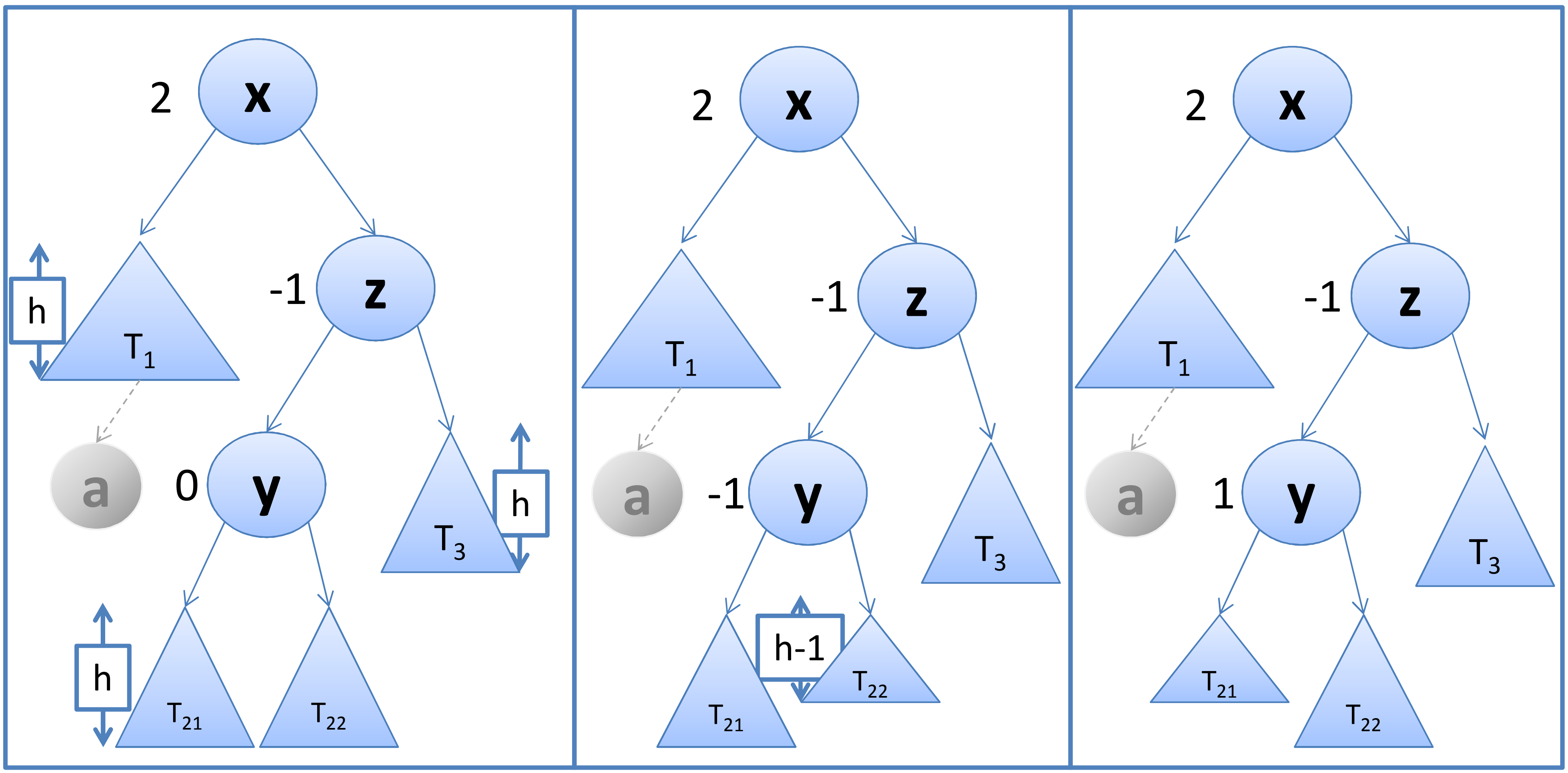
- We do a ***single left rotation*** (same as last case)
- Preserves the BST property, and fixes bf(x) = 2



Delete(a): bf(x)=2, bf(z)=-1 subcases

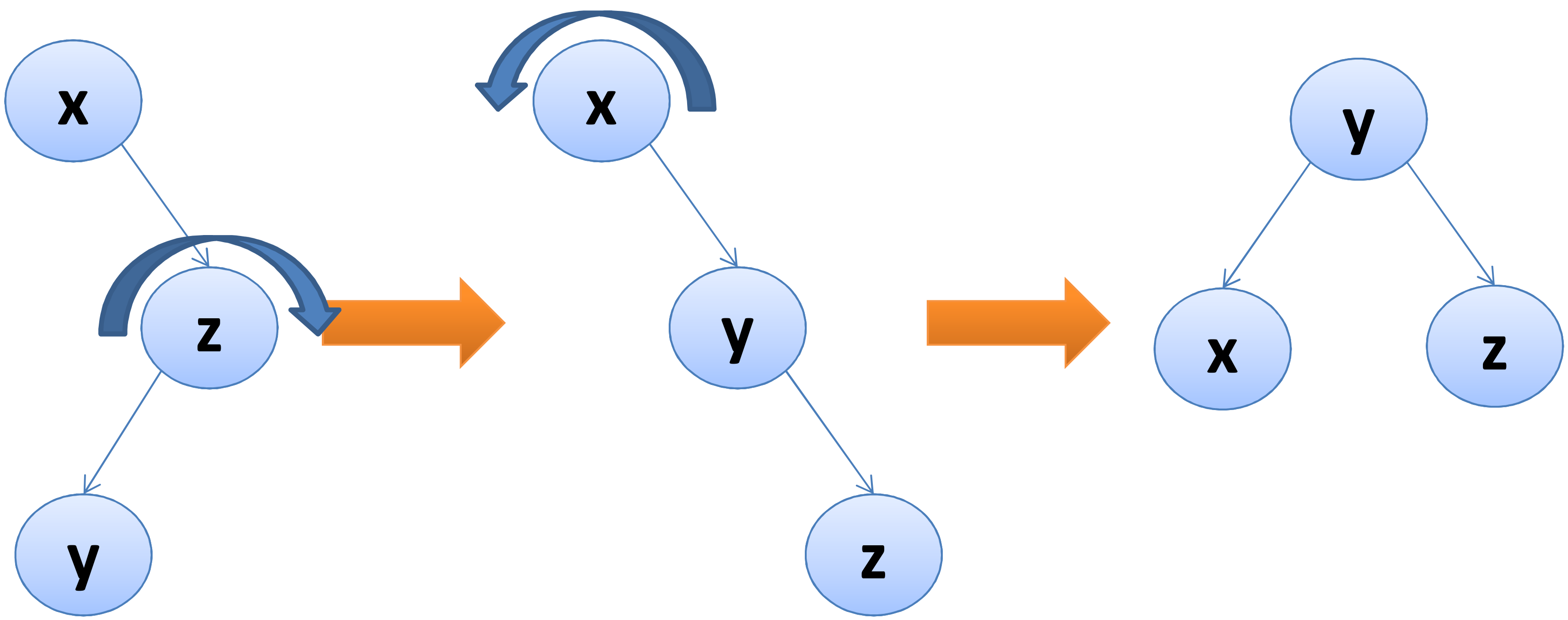
Case bf(z) = -1: we have 3 subcases. (More details)

Case bf(y) = 0 **Case bf(y) = -1** **Case bf(y) = 1**



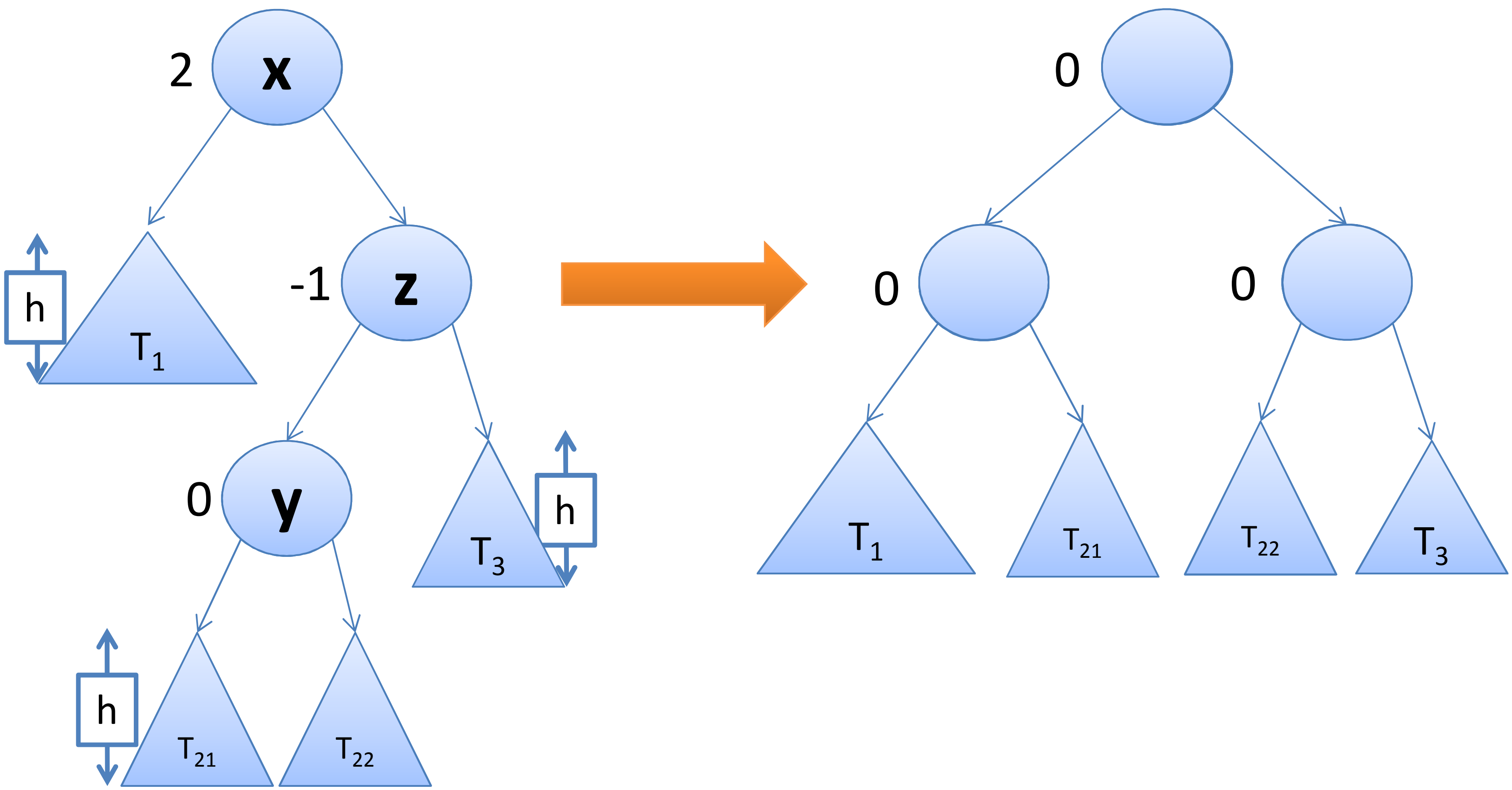
Double right-left rotation

- All three subcases of bf(x)=2, bf(z)=-1 simply perform a double right-left rotation.



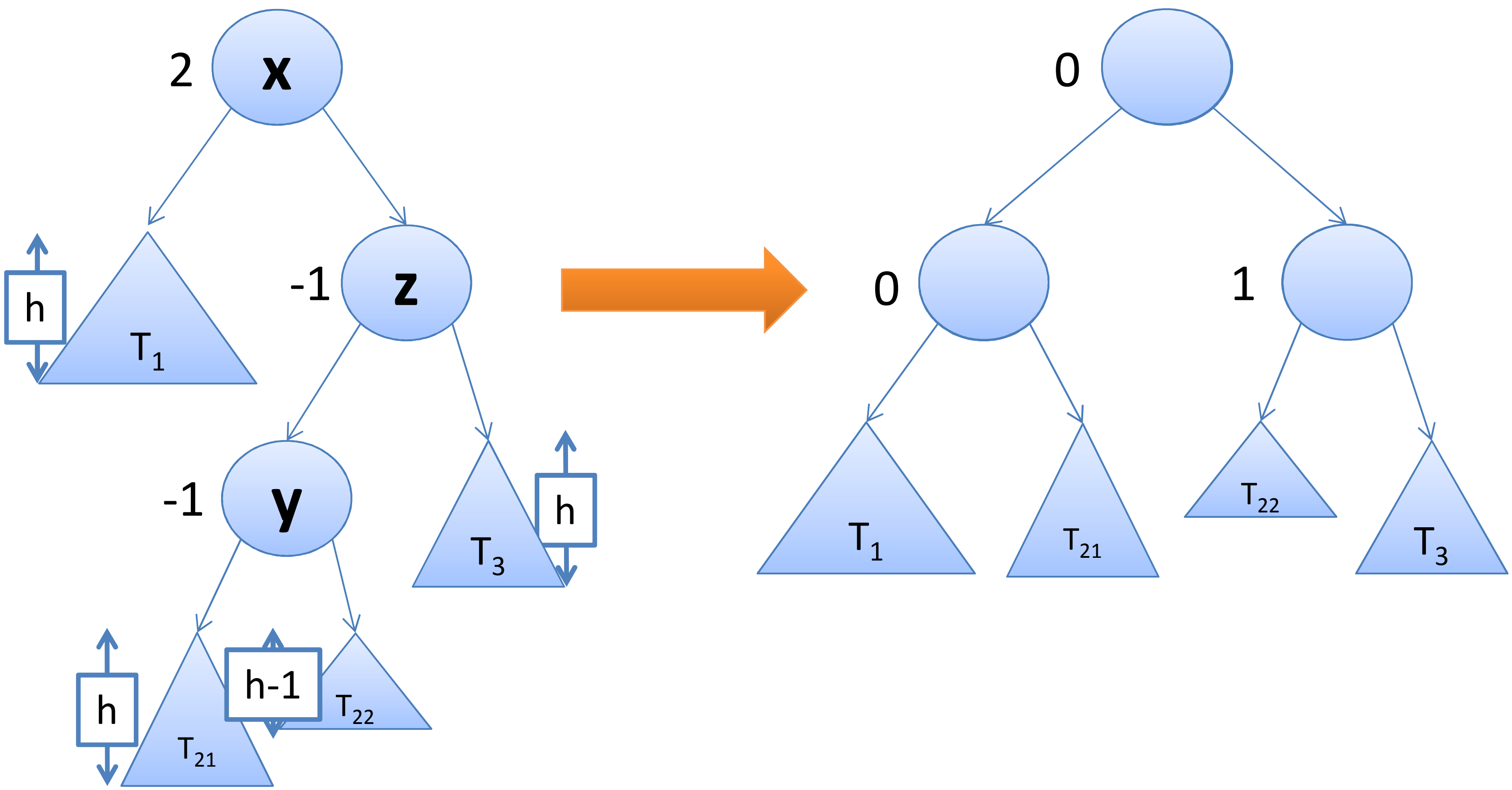
Delete subcases for bf(x)=2, bf(z)=-1

- Case bf(y)=0: double right-left rotation!



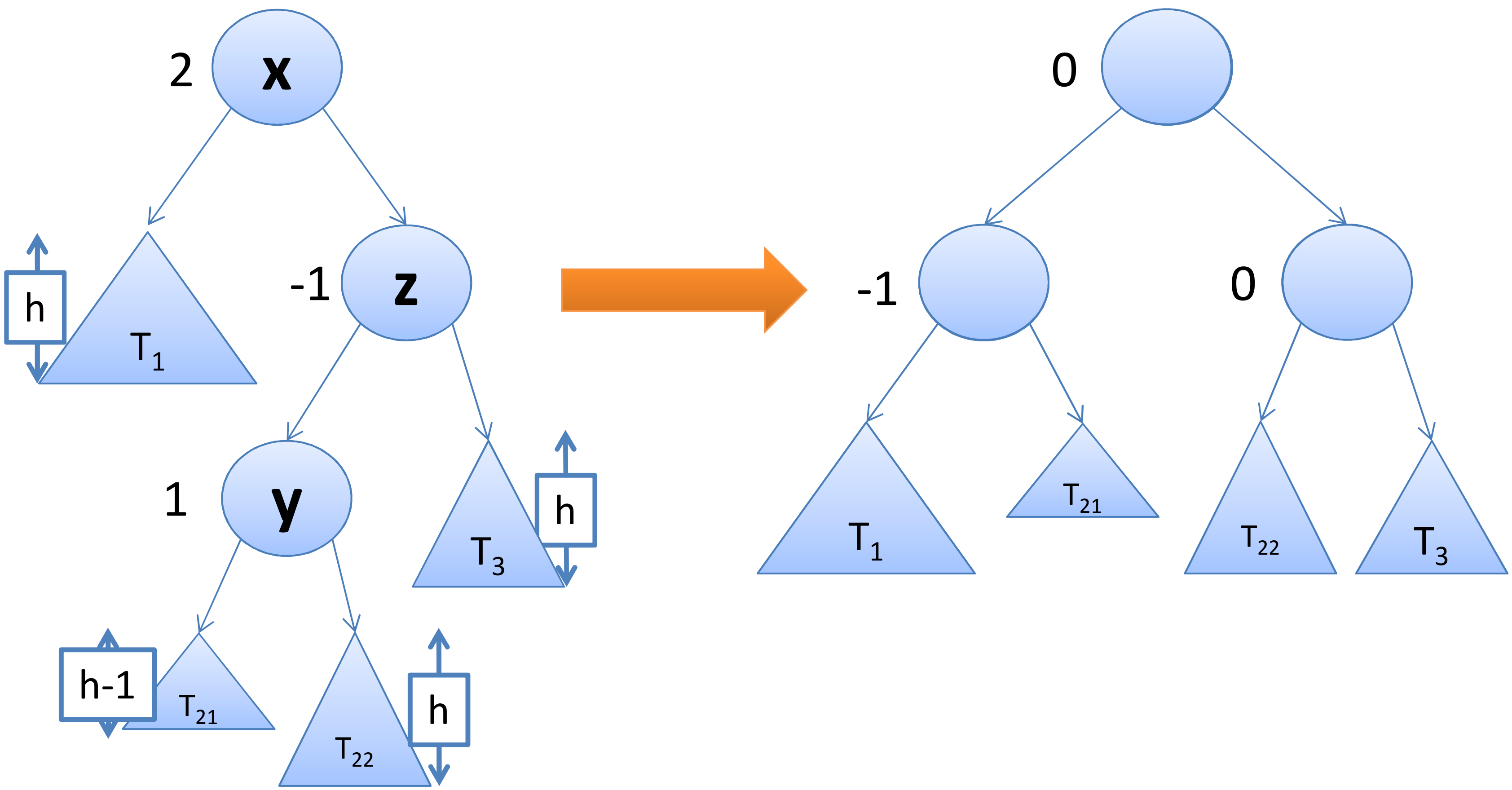
Delete subcases for bf(x)=2, bf(z)=-1

- Case bf(y)=-1: double right-left rotation!



Delete subcases for bf(x)=2, bf(z)=-1

- Case bf(y)=1: double right-left rotation!



Recursively fixing balance factors

- *Idea: start at the node we deleted, fix a problem, then recurse up the tree to the root.*
- At each node x, we update the balance factor: bf(x) := h(bf.right) - h(bf.left).
- If bf(x) = -2 or +2, we perform a rotation.
- Then, we update the balance factors of every node that was changed by the rotation.
- Finally, we recurse one node higher up.

Interactive AVL Deletes

- [Interactive web applet]