*You can watch a PBS documentary about the Navajo Math Camp at http://www.pbs.org/program/navajo-math-circles/

COMMUNICATION

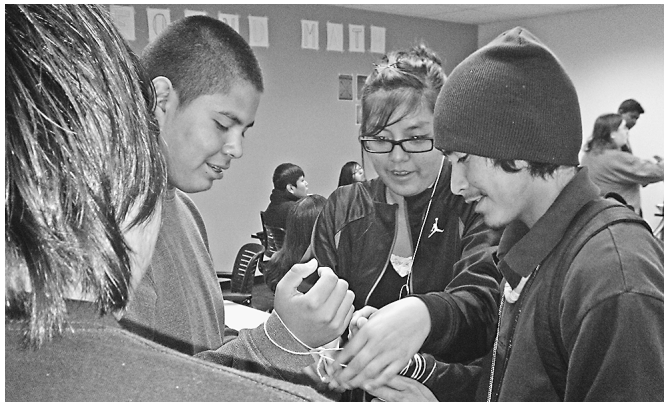
Baa Hózhó Math: Math Circles for Navajo Students and Teachers

Dave Auckly, Bob Klein, Amanda Serenevy, and Tatiana Shubin

The authors describe their mathematics program for the Navajo Nation of Native Americans in the southwest United States.

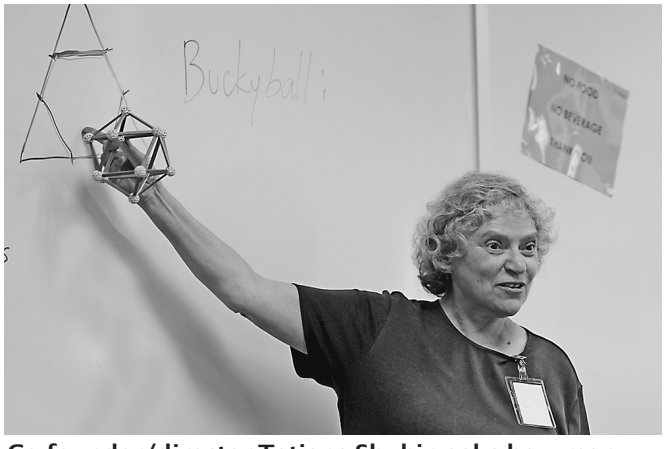
It's 8:00 am on Tuesday morning, and two vans have arrived at Diné College having just completed 6:30 am trips from Many Farms and Fort Defiance, Arizona, to bring thirty middle and high school students to the Navajo Baa Hózhó Math Camp. Baa Hózhó means "balance" or "harmony." The kids race into the building, grab breakfast, and find seats in the classroom to eat and play with Rubik's cubes and other puzzles and to catch up with friends. Shortly they will be busy with two math circle sessions, physical activity, sessions on Navajo culture and language, and lunch—all before the 4:00 pm trip home.

At 9:00 am, the math circle sessions begin, and Tatiana Shubin, a mathematician from San Jose State University and co-founder/director of the Navajo Nation Math Circles (NNMC) project, holds up a stick-and-ball model of a Buckyball, built of twenty hexagons and twelve pentagons. She asks, "How many diagonals are there for this object?" Students stare at the object for a minute, then begin group discussions and calculations that will take more than an hour. Shubin smiles, knowing that this is what she had in mind when she first dreamed of sharing math with the Navajo people.



Navajo students Jamie Tsosie, Mya Chee, and Waylon Begay getting untied before breakfast.

In another room, middle school student Nataani Yazzie is smiling as Dave Auckly, a mathematician from Kansas State University and co-founder/director, is racing around the classroom acting like a pirate. He is delighting the



Co-founder/director Tatiana Shubin asks how many diagonals a Buckyball has.

David Auckly is professor of mathematics at Kansas State University. His email is dav@ksu.edu. *His research interests cover a broad range of geometry/topology and overlap with PDE, mathematical physics, and algebraic geometry. He has also been very involved in many special educational programs.*

Bob Klein is associate professor and undergraduate chair of the Department of Mathematics at Ohio University. His email address is kleinr@ohio.edu. *Bob is also working on math circle projects in SE Ohio, Guatemala, and Nepal (see* mathunbounded.org/ *for details).*

Amanda Serenevy is director of Riverbend Community Math Center in South Bend, Indiana. Her email address is amanda@riverbendmath.org.

Tatiana Shubin is professor in the Department of Mathematics and Statistics at San Jose State University. Her email address is tatiana.shubin@sjsu.edu. *Among other things, she has been running San Jose Math Circle since 1998.*

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Co-founder/director Dave Auckly works with Tyline Descheny and Michael Begay on a version of the Ham Sandwich theorem.

students with his energy while intriguing them with a combinatorics problem set in the context of pirate treasure: strands of multicolored pearls assembled according to a simple set of rules. Auckly has been with the project since Tatiana asked him if he knew anyone in the Navajo Nation back in 2011 and together they plotted how to create a permanent Math Circles program for students and teachers there. He has been managing many of the administrative details throughout the calendar year to arrange mathematician visits, finances, and the yearly math festivals.

In the two weeks of camp, the students will go on a campout; meet another ten to twelve mathematicians and scientists from institutions around the United States; learn Navajo games, language, and culture; eat; play; and grow in their abilities and confidence with problem solving. In particular, they will interact with project co-founder/director Henry Fowler. Fowler, a mathematician and Navajo scholar from Diné College, shared Tatiana's enthusiasm for the project from the first time she told him about it. He has dedicated his professional life to helping his people realize their potential in mathematics and science.

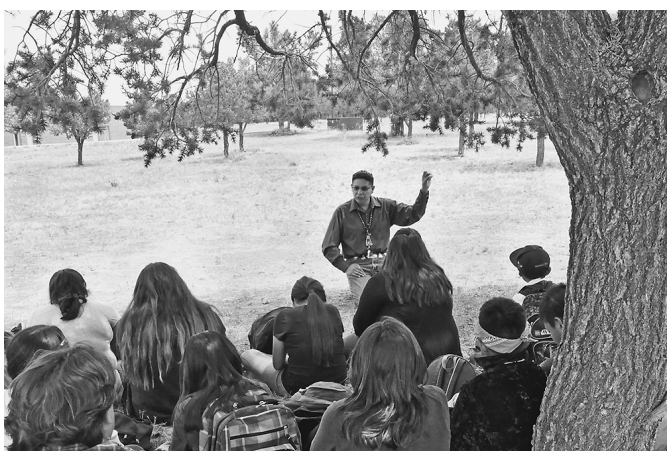
Mathematics is very close to Navajo culture: both are deeply rooted in the love of beauty

But the students at the summer camp are participating in something much bigger than a two-week summer camp. They are part of the Navajo Nation Math Circles project. A group of their math teachers (from kindergarten through high school) will be gathering on Saturday for a Math Teachers' Circle workshop, one of many throughout the year led by mathematicians who also visit schools to lead math circle sessions. The teachers will also participate in a math festival held annually by the NNMC and attended last year by more than seven hundred fifty students and parents.

The goal of the project has always been to bring the spirit of mathematics to the Navajo Nation and to spark interest and joy in problem solving in the students and teachers who participate. Mathematics is very close to Navajo culture: both are deeply rooted in the love of beauty.

Beginnings

Shubin had visited the Navajo Nation before and was deeply moved by the culture, environment, and people. Remembering how the network of math circles in Russia and Kazakhstan had spurred her own interest in mathematics, she wanted to bring them to the Navajo Nation. She dreamed of spending her sabbatical leave on the project. In January 2012 Auckly and Shubin designed a broad-reaching and sustainable program for teachers and students in the Navajo Nation. The program would include mathematical visitors who would lead teacher workshops and special math circles at local schools. Auckly immediately began the work of raising funds to support the project, the beginning of what would be more than a dozen successful grant proposals.



Co-founder/director Henry Fowler (top). Below, Fowler, the son of a Navajo code talker, discusses Navajo culture with students.

Auckly and Shubin needed an invitation to the reservation, however. One can't just show up and announce, "I'm here to do math circles." Jennifer White, an acquaintance of Auckly's, introduced Shubin to Henry Fowler. Fowler introduced Shubin to school principals and math departments and helped to lay the groundwork for establishing math circles in schools across the reservation.

The second year of the project saw the addition of two additional co-directors: Bob Klein, associate professor at Ohio University, and Amanda Serenevy, executive director of the Riverbend Community Math Center in South Bend, Indiana. Klein grew up in Albuquerque, New Mexico, so the project has helped him reconnect with the Southwest, as well as provide him an opportunity to investigate the NNMC model for developing and sustaining math circles globally. This work has informed his development of similar projects in Guatemala (with Javier Ronquillo Rivera) and Nepal (with Rabi and Mohan K.C., and Prabha Shrestha).

Serenevy was one of the first visitors to the project and is an energetic math circle leader who formed the Riverbend Community Math Center in fall 2006. She is one of few people in the world who runs math circles as a full-time job. Serenevy contributes a wealth of experience working with teachers and students on collaborative problem solving and visits regularly throughout the school year and summer.

A Comprehensive Project

The project has eight components that interact to improve mathematics education in the Navajo Nation. The components are:

1. a ***mathematical visitor program*** sending mathematicians to Navajo schools to lead in-school and after-school math activities in math circles;

2. a two-week nonresidential ***Baa Hózhó Summer Camp*** for students in grades 6–12;

3. public ***math festivals*** to introduce engaging mathematical activities to large numbers of Navajo students;

4. a year-long ***student mentoring program*** that pairs Navajo students with professional mathematicians;

5. a series of ***professional development workshops*** for Navajo teachers;

6. a ***teacher mentoring program*** that connects small groups of teachers to mathematicians;

7. a summer ***teacher development program***, and

8. ***math circles for students***, which meet regularly at schools and are led by teachers.

This structure is amazing in its effectiveness: Math festivals advertise the project to students and teachers, demonstrating that math can be fun; the math circles during and after school get kids even more interested—they want to work on more math problems; the teachers' workshops provide an opportunity for local teachers to garner more ideas, share their experiences, and air their concerns; the summer math camp, teacher mentoring program, student mentoring program, and summer teacher development program all provide more in-depth math training for those who truly get hooked.

In the words of students Buddy Joe and Charmayne Seaton and teacher Dana Busenbark:

"I brag about this math camp, because it's really a lot of fun. It's not like regular classrooms where you're reading a textbook and listening to a teacher in front of the class writing on the board. It's a lot of fun; we get involved in the lesson. It's a lot different."

—Buddy Joe, student, Fort Defiance, Arizona

"I thought I was dumb, that I wasn't really smart. But then I actually got to know myself and I realized that I am smart. I can do it if I put my mind to it. And I believe that for everyone. If you have a goal in mind and you really want it, you can do it. You just have to believe in yourself, and you just have to push."

—Charmayne Seaton, student, Ganado, Arizona

"I talk with others about the math camp because it provides [the] Navajo Nation students that we teach with opportunities to meet others that they may not know, learn about jobs that require math skills, and most importantly to think outside the box. So often, our students think in black and white and do not consider other options. The math camp gives them an opportunity to discuss options and open their minds to new ideas."

—Dana Busenbark, teacher, Fort Defiance, Arizona

The impact is already being felt at the postsecondary level. Since the project was formed, Diné College has seen a marked increase in the demand for mathematics. The college has introduced two new mathematics classes and runs more sections of existing math classes. In the fall of 2013 the college had one section of precalculus with twenty-three students. In 2014–15 the college offered precalculus to thirty-five students, calculus to eighty-two students, and linear algebra to six students. The college just introduced its first mathematics degree program. This new program in mathematics education has fifteen students.

Charmayne Seaton has attended many NNMC programs. She calls Shubin her "auntie" as a sign of the bond they share. She says that she longed to escape from her family's home at Ganado, but since attending the math camp at Diné College, her goal is to get into a university, then return home to help her people.

Shubin, Auckly, Klein, and Serenevy have been working for the past five years to bring these programs to the Diné people in the Navajo Nation. The Navajo Nation is an area the size of West Virginia in the Four Corners region of the American Southwest. It is home to around 200,000 Navajos, and it is very isolated. Industry is nearly absent, and there are high levels of poverty. The hope is that the program will grow into a model that may be used to help other underserved populations.

The majority of funding for the NNMC project has come from the National Science Foundation and National Security Agency, with additional funding from the American Institute of Mathematics, the Mathematical Sciences

Research Institute, the Mathematical Association of America, a number of colleges and universities, and a few foundations. A stable funding model has been elusive, though the model represents a high impact-to-cost ratio, a cause for optimism. External funding is important because the average family income in the Navajo Nation is less than $16,000 per year, so NNMC provides all programs to members of the community for free. Meals and transportation are included with longer programs.

With a dozen visits by mathematicians to the NNMC project, more than one thousand students from across the Navajo Nation have engaged with working mathematicians and scientists from across the United States. Mathematicians come during the school year and stay about a week, visiting several schools during each visit, running math circle sessions on topics that have included hat color logic problems, RSA encryption, the Collatz conjecture, sum free partitions, mathematical origami, rational tangles, chromatic numbers, distance puzzles, Fibonacci tiling patterns, Liar's Bingo (binary representations), Friends and Enemies (a dynamical systems game), repeating decimals, the orbit-stabilizer theorem, Nim games, and Intersection Math.

Benefits run both ways as visiting mathematicians and scientists have the opportunity to meet and learn more about the Navajo people and the beautiful Diné Bikéyah, or Navajo Country. As Blake Thornton, mathematician at Washington University, St. Louis, wrote, "I worked really hard the entire week, but I had a great time. I would love to go back." Many mathematicians do come back. Matthias Kawski has driven up from Arizona State University many times.

Parents and guardians of the student participants in the project work hard to encourage and support the participation of the students in the summer camp and festival, sometimes driving them several hours each way to participate.

Sample from Thornton's visit: You tell a group of ten students that you will put a party hat with a number 1–10 on the head of each student. (However, not all numbers need to be used. It might be the case that one hat has a 1 on it and the rest have a 7.) When this is done each student will be able to see the hats on all of the other students, but not the hat on his or her own head. Without communicating, the students will each write a guess of the number on their own hat. At the same time, the students will show their guess. If just one student correctly guesses the number on his or her hat it will be considered a win for all. The students may plan before the hats are placed on their heads and agree what they will do. Is there a way the students can be sure to win? If so, how?

Summer Camp

To help participants develop relationships with their peers and therefore the ability to collaborate and to feel comfortable sharing ideas, leaders divide participants into two teams for the duration of the camp. Students quickly identify with those teams in playful camaraderie. The camp consists of whole-group sessions and sessions split by teams (such as the math circle sessions, where this approach means working with smaller groups of about fifteen students).

Participants have a range of mathematics backgrounds and come from grades 6–12. They submit an application and references from teachers and are chosen based not only on the strength of those references but also on the desire and "heart" evident in their answers to questions like "Why do you want to participate in the Baa Hózhó Math Camp?"

Joe Buhler of the Institute for Defense Analyses working with Dawnae Etsitty.

Each Friday of the summer camp, a guest of honor spends part of the day interacting with students and gives an inspirational talk that combines mathematical content with their personal and professional story. Guests of honor have included Robert Megginson, one of very few Native Americans with a PhD in mathematics; John Herrington, the first Native American in space; and Sandra Begay and Julius Yellowhair, both Navajos who work as senior scientists at Sandia Labs.

While the focus is on collaborative problem solving, representing an authentic model for mathematical investigation, the project also has a fun and playful competition structure in place: Math Wrangles, a form of mathematical debate. The process of preparing for the Math Wrangle has greatly increased the confidence of participants and offered project leaders a way to assess overall effectiveness of the camp and to watch the development of participants' abilities.

Teacher Mentoring and Workshops

The teacher mentoring program has been refined by the NNMC directors into what seems for now to be an effective format. The directors initially hoped that by letting teachers watch mathematicians make presentations to their students and by offering teacher workshops, the Navajo teachers would begin leading math circle sessions at their own schools. This has proved difficult, because while teachers see the value of the math circle sessions, they do not feel prepared to run such sessions on their own. The teacher mentoring program explicitly addresses this.

Leegene Barlow and Elene Morris working on a problem.

Mentoring takes place via teleconference with a group of teachers who work through a math circle session together with a project director. When they work through a problem for the first time, the teachers act in the role of students. Teachers get written support materials, including handouts, notes on the relevant mathematical background and big ideas, presentation tips, and evaluations for teachers and mentors to use in examining the efficacy of the sessions. A second (and possibly third) teleconference with the teachers ensures that they are comfortable leading the session. Then a team of two to four teachers leads the session with a group of students. Last year the project included twelve Navajo-led math circle sessions.

When Navajo teachers lead math circles, students see a member of their tribe modeling mathematical inquiry, serving to counter stereotypes and promote positive attitudes toward mathematics.

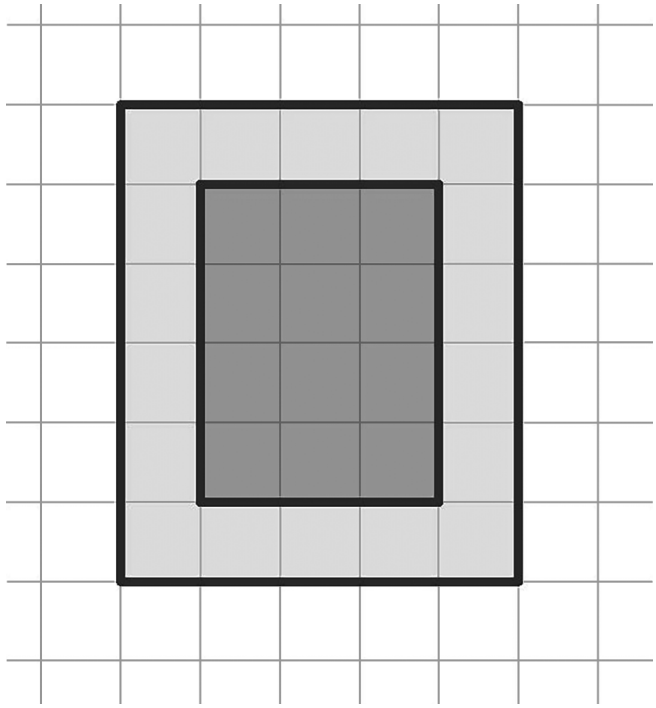
Keeping mathematics and culture mutually reinforcing is an important part of the project approach. Dawnlei Ben is largely responsible for creating the Navajo cultural component of the math camp. Ben grew up in Canyon de Chelly in a house with no running water or heating. "I'm okay with that. We have cornfields, apples, plums, peaches, apricots on our land," she says. Ben was raised with traditional teachings and wants to impart what she learned to children so the culture can survive. At the camp she has taught Navajo remedies, toolmaking, and games.

The Documentary

When George Csicsery heard of the Navajo Nation Math Circles project he envisioned a documentary film about the meeting of two worlds: that of some of the country's most accomplished mathematicians and math educators with the children and teachers in underserved Navajo schools. The resulting *Navajo Math Circles* film premiered in Seattle, Washington, on January 6, 2016, at the Joint Mathematics Meetings. Nine of the Navajo participants attended this screening, including three students, two teachers, two parents, as well as many of the mathematicians involved in the project. The DVD contains the 58-minute film as well as five extra features totaling another 34 minutes.

Screenings of *Navajo Math Circles* are being scheduled throughout the United States, and the film is being promoted for broadcast on public television stations.

Sample Wrangle Problem: Using a sheet of grid paper, Dana marked a rectangle and drew a picture inside the rectangle (in the example below it is dark gray). Then she surrounded the rectangle by a one-square-wide border consisting of grid squares (in the example below it is light gray). It turned out that the area of the border is exactly the same as the area of the inside rectangle. What size is the rectangle? List all possible sizes and prove that other sizes won't work. The example below doesn't work.



At least once per semester, visiting mathematicians lead teacher training workshops for groups of teachers from across the reservation. At one of these workshops, Shubin, Auckly, Serenevy, and Klein facilitated sessions designed to help the more than forty teachers use problem-solving prompts in their classrooms and to deepen their understanding of the math behind the problems. In Exploding Dots, for instance, Shubin helped teachers think about how to explain place value, addition, multiplication, subtraction, and division, and standard manipulations of polynomials to their students, but in a new and thought-provoking way. Auckly led Toilet Paper Math, a session for elementary school teachers in which he demonstrated several ways to use toilet paper as a playful, yet useful, manipulative to help the teachers describe various number concepts to their students. Serenevy presented sessions on introducing serious mathematical play in lower elementary classrooms, conceptual approaches to operations with numbers, and mathematical modeling using spreadsheets.

Interviews with the teachers and the visiting mathematicians made clear the value of this training, a sense of conditions at schools on the reservation, and participants' and facilitators' hopes for bringing improvements to their classrooms using fun and interesting mathematical investigations.

Project leaders hope that this model and these experiences will be shared by more mathematicians, teachers, and students in many other places in this country and beyond.

A day at the festival, in pictures. From the top down: 1. Participants; 2. Counting tree rings; 3. Ballast puzzles; 4. Life-size counting problems.

When saying goodbye the Navajo wish for others to "Walk in beauty," and the directors of the Navajo Nation Math Circle project similarly wish that the story of this project inspires others to use math circles to help everyone to walk in beauty and to do so with mathematics.

For more information about the Navajo Nation Math Circles project or to contribute, please visit: navajomathcircles.org

For more information about the production and a detailed synopsis or for updates on upcoming screenings of *Navajo Math Circles*, please visit navajomathcirclesfilm.com, where a sample video covering the summer activities of 2014 can be viewed.

Credits

Page 784 photo of Navajo students, courtesy of Tatiana Shubin.
Page 784 photo of Tatiana Shubin, courtesy of George Csicsery.
Page 785 photo of Dave Auckly, courtesy of George Csicsery.
Page 785 photos of Henry Fowler, courtesy of George Csicsery.
Page 787 photo of Joe Buhler and students, courtesy of George Csicsery.
Page 788 photo of Leegene Barlow and Elene Morris, courtesy of George Csicsery.
Page 789 festival photos, courtesy of Barsine Benaly.

Grid Wrangle Solution

Problem: Using a sheet of grid paper, Dana marked a rectangle and drew a picture inside the rectangle. Then she surrounded the rectangle by a one-square-wide border consisting of grid squares. It turned out that the area of the border is exactly the same as the area of the inside rectangle. What size is the rectangle? List all possible sizes and prove that other sizes won't work.

Solution: Let w be the width of the rectangle and h be the height of the rectangle. Then the area of the rectangle will be wh. There will be a row of w grid squares on top and one row of w on the bottom for a total of $2w$ squres. The sides will contribute $2h$ additional squares to the border. Finally, there will be one square for each corner. Thus the border will have $2w + 2h + 4$ grid squares. For the border area to equal the total area one must have

$$wh = 2w + 2h + 4\,.$$

to find all integer solutions of this re-write it as a factorization problem, i.e.,

$$\begin{aligned}(w-2)(h-2) &= wh - 2w - 2h + 4\\ &= 4 + 4 = 8\,.\end{aligned}$$

The only ways to factor 8 into positive integers are as $1 \cdot 8$, $2 \cdot 4$, $4 \cdot 2$ and $8 \cdot 1$. These correspond to the four possible solutions $w = 3$, $h = 10$; $w = 4$, $h = 6$; $w = 6$, $h = 4$; and $w = 10$, $h = 3$.

Commentary: The solution we just described uses algebra that middle school students might find a bit sophisticated. Once phrased in the language of algebra, one sees that it is what is known as a *diophantine problem*. That means an algebraic problem where one looks for integer solutions. In the specific case given the equation is quadratic. Such problems are fairly well-understood. For example, you may read more about this type of problem at:

http://www.numbertheory.org/php/main_pell.html.

Such problems with specific numbers turn into nice puzzles for students in grades 4 - 12. With smaller numbers the problems will be easier, and the problems will get more difficult with larger numbers. Great progress may be made on such problems via trial-and-error. Such explorations are much more important than explicit methods to solve the problems.

Posing the problem in a grid allows students to explore the problem geometrically. Giving them graph paper will further encourage exploration.

Ther are many interesting problems that can be given to students. Here is one we learned from Gordon Hamilton at Math Pickle. http://mathpickle.com/

New Problem: Pick two small positive integers, say a and p. Now find all connected grid regions with area a and perimeter p.

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