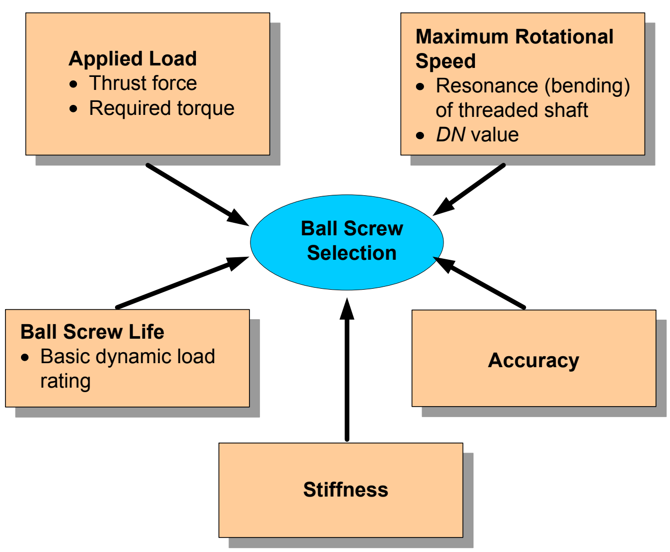
Ball Screw Selection and Calculations

ME EN 7960 – Precision Machine Design
Topic 4

Ball Screw Selection Criteria



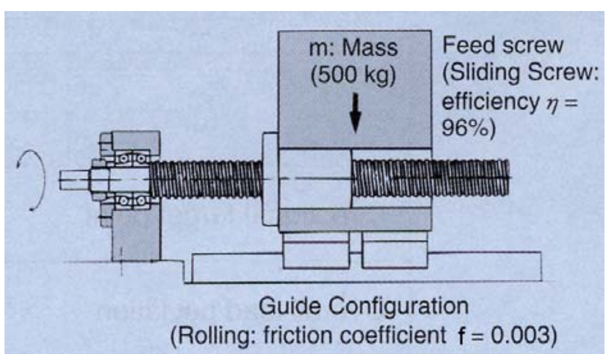
Based on Load

- A ball screw transforms rotational motion into translational motion. As a result, the shaft is subject to loads:
  - Thrust force (the sum of all external forces such as machining load, gravity, friction, inertial forces, etc.).
  - Torque required to generate the thrust force.

Driving Torque to Obtain Thrust

$$T = \frac{F_a l}{2\pi\eta}$$

T: driving torque [Nm]
F_a: thrust force [N]
l: screw lead [m]
η: efficiency



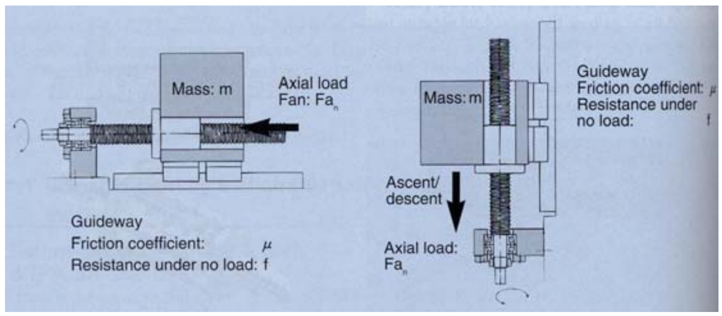
Source: THK Co., Ltd.

Required Thrust

- The thrust is the sum of all forces acting in the axial direction.

$$F_a = F_M + F_f + F_i + F_g$$

F_M: Machining force [N]
F_f: Frictional force [N]
F_i: Inertial force [N]
F_g: Gravitational force [N]



Source: THK Co., Ltd.

Stresses from Applied Loads

$$\sigma_{axial} = \frac{F_a}{\pi r_{tr}^2} \qquad \tau_{torsional} = \frac{2T}{\pi r_{tr}^3}$$

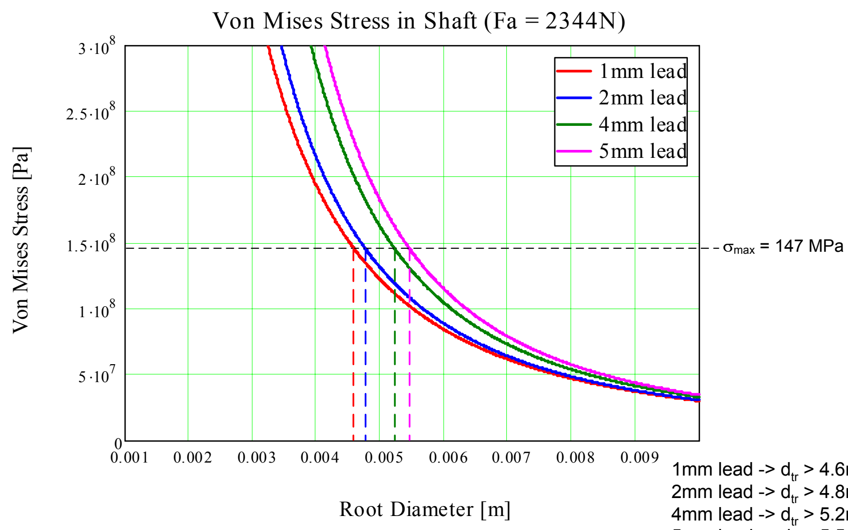
The equivalent (Von Mises) stress:

$$\sigma_{eq} = \sqrt{\sigma_{axial}^2 + 3\tau_{torsional}^2}$$

$$\rightarrow \sigma_{eq} = \frac{4F_a}{\pi d_{tr}^2}\sqrt{1 + \frac{12 l^2}{\pi^2 d_{tr}^2 \eta^2}}$$

σ_{max}: Permissible stress [147 MPa]

Graphic Solution



1mm lead -> d_{tr} > 4.6mm
2mm lead -> d_{tr} > 4.8mm
4mm lead -> d_{tr} > 5.2mm
5mm lead -> d_{tr} > 5.5mm

Permissible Compressive Load

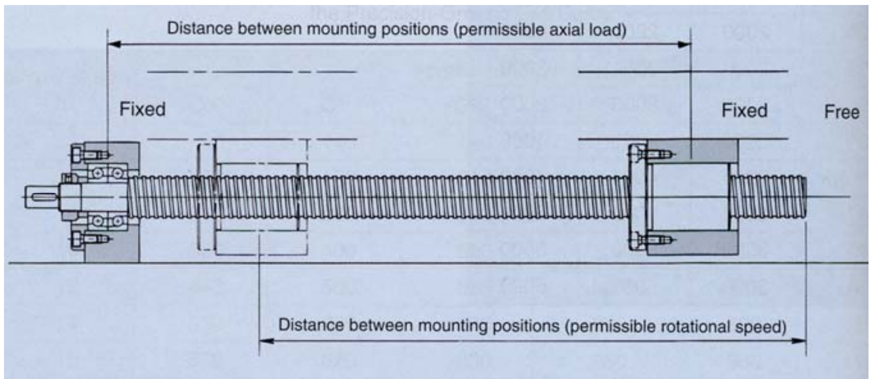
- Buckling Load

$$P_1 = \frac{\lambda \pi^2 E I}{l_b^2}$$

P_1: Buckling load [N]
l_b: Distance between mounting positions [m]
E: Elastic modulus [Pa]
I: Second moment of inertia [m^4]
λ: Support factor

Fixed – free:	**λ = 0.25**
Fixed – supported:	**λ = 2.0**
Fixed – fixed:	λ = 4.0

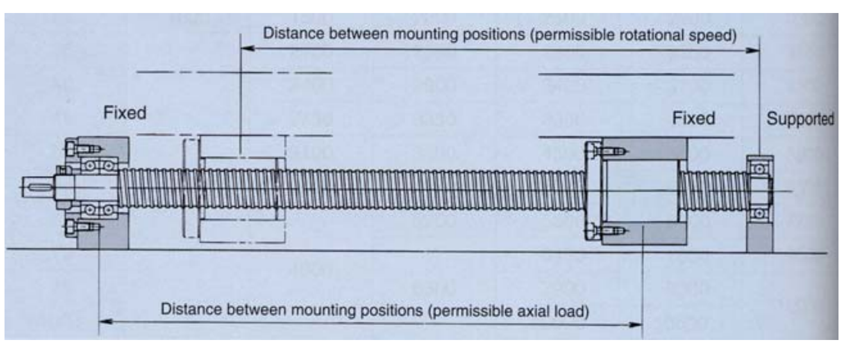
Fixed-Free Mount



Source: THK Co., Ltd.

Inexpensive but only applicable for short ball screws and/or slow speeds.

Fixed-Supported Mount



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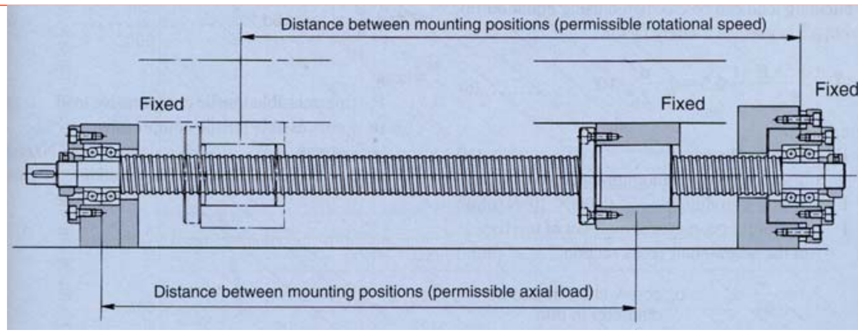
Most commonly used mounting setup.

Fixed-Fixed Mount



Source: THK Co., Ltd.

Overconstrained mounting setup for applications where high stiffness, accuracy, and high shaft speed is required. Ball screw needs to be pre-stretched to avoid buckling in the case of thermal expansion.

Basic Static Load Rating C_{oa}

- When ball screws are subjected to excessive loads in static condition (non rotating shaft), local permanent deformations are caused between the track surface and the steel balls.
- When the amount of this permanent deformation exceeds a certain degree, smooth movement will be impaired.

$$C_{oa} \geq f_s F_a$$

C_{oa}: Basic static load rating [N, kgf, lbf]
f_s: Static safety factor
F_a: Load on shaft in axial direction [N, kgf, lbf]

Use conditions	**f_s (lower limit)**
Normal operation	1.0 – 2.0
Operation with impacts and vibrations	2.0 – 3.0

Permissible Speed

- When the speed of a ball screw increases, the ball screw will approach its natural frequency, causing a resonance and the operation will become impossible.

$$n_c = \frac{60\lambda^2}{2\pi l_b{}^2}\sqrt{\frac{EI}{\rho A}} = \frac{15\lambda^2 d_{tr}}{2\pi l_b^2}\sqrt{\frac{E}{\rho}}$$

n_c:	Critical speed [min^{-1}]	
l_b:	Distance between supports [m]	
E:	Elastic modulus [Pa]	
I:	Second moment of inertia [m^4]	
ρ:	Density [kg/m^3]	
A:	Root cross sectional area [m^2]	
λ:	Support factor	
	Fixed – free:	**$\lambda = 1.875$**
	Supported – supported:	$\lambda = 3.142$
	Fixed – supported:	**$\lambda = 3.927$**
	Fixed – fixed:	$\lambda = 4.730$

Spindle Speed and *DN* Value

- Shaft speed

$$n = \frac{v_a}{l}$$

n:	Revolutions per second [s^{-1}]
v_a:	axial speed [m/s]
l:	lead [m]

- DN Value. Unless specified otherwise:

$$DN \leq 70000$$

D:	Ball circle diameter [mm]
N:	Revolutions per minute [min^{-1}]

Dynamic Load Rating C_a and *Life*

- The basic load rating C_a is the load in the shaft direction with 90% of a group of the same ball screws operating individually will reach a life of 10^6 (1 million) revolutions.

$$L = \left(\frac{C_a}{f_w F_a} \right)^3 \times 10^6$$

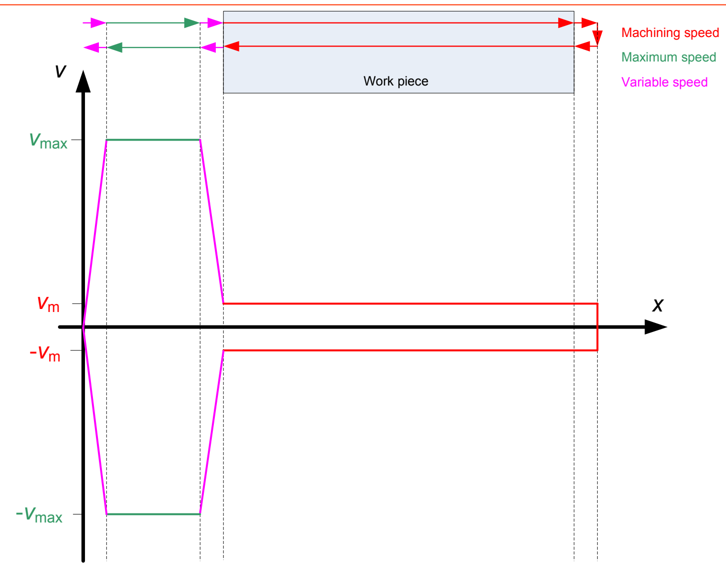
L: Rotation life [rev]
C_a: Basic dynamic load rating [N, kgf, lbf]
f_w: Load factor
F_a: Load in shaft direction [N, kgf, lbf]

Use conditions	f_w
Smooth movement without impacts	1.0 – 1.2
Normal movements	1.2 – 1.5
Movement with impacts and vibrations	1.5 – 2.5

Example

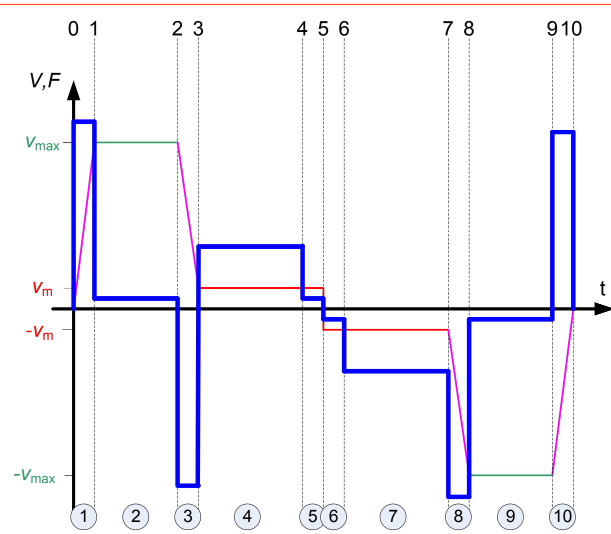
- Mass of axis: 350kg
- Maximum velocity: 20m/min
- Acceleration time: 0.05s
- Bearing friction factor: 0.003
- Machining force: 500N
- Length of work piece: 500mm
- Length of travel at maximum speed: 100mm
- Orientation of axis: horizontal

Machining Profile of Making a Slot



$v_0 = 0$
$v_1 = v_{m\,a\,x}$
$v_2 = v_{m\,a\,x}$
$v_3 = v_m$
$v_4 = v_m$
$v_5 = 0$
$v_6 = -v_m$
$v_7 = -v_m$
$v_8 = -v_{m\,a}$
$v_r = -v_{m\,a}$
$v_{10} = 0$

Load Profile for Making a Slot



$v_0 = 0$ $F_{a1} = F_i + F_f = 2344N$
$v_1 = v_{max}$ $F_{a2} = F_f = 10N$
$v_2 = v_{max}$ $F_{a3} = -F_i + F_{f\,i} = -2324N$
$v_3 = v_m$ $F_{a4} = F_m + F_f = 510N$
$v_4 = v_m$ $F_{a5} = F_f = 10N$
$v_5 = 0$ $F_{a6} = -F_f = -10N$
$v_6 = -v_m$ $F_{a7} = -F_m - F_f = -510N$
$v_7 = -v_m$ $F_{a8} = -F_i - F_f = -2344N$
$v_8 = -v_{max}$ $F_{a9} = -F_f = -10N$
$v_r = -v_{max}$ $F_{a10} = F_i - F_f = 2324N$
$v_{10} = 0$

Running Lengths Depending on Usage

Running distance during acceleration: $l_{accl} = \frac{(v_1 + v_2)t_{accl}}{2}$

Running distance during deceleration: $l_{decl} = \frac{(v_1 + v_2)t_{decl}}{2}$

Mean Axial Force

Determine mean axial load in the positive direction by collecting all individual, positive axial loads.

$$F_{a,mean+} = \sqrt[3]{\frac{\sum_{i+} F_{ai+}^3 l_{i+}}{\sum_i l_i}}$$

Determine mean axial load in the negative direction by collecting all individual, negative axial loads.

$$F_{a,mean-} = \sqrt[3]{\frac{\sum_{i-} \left|F_{ai-}^3\right| l_{i-}}{\sum_i l_i}}$$

Determine mean axial load: $F_{a,mean} = \frac{F_{a,mean+} + F_{a,mean-}}{2}$

Load Profile Based on Utilization

Mean axial force:

$$F_{a,mean} = \sqrt[3]{\frac{F_m^3 l_m + F_{uni}^3 l_{uni} + F_{accl}^3 l_{accl}}{l_b}}$$

F_m: Machining force
F_{uni}: Force at constant velocity (not machining)
F_{accl}: Maximum force during acceleration and deceleration
l_m: Total travel per cycle during machining
l_{uni}: Total travel per cycle at constant velocity
l_{accl}: Total travel per cycle during acceleration and deceleration
l_b: Length of ball screw

Total travel length:

$$l_b = l_m + l_{uni} + l_{accl}$$

Travel length:

$$l_m = q_m l_b$$

$$l_{uni} = q_{uni} l_b$$

$$l_{accl} = q_{accl} l_b$$

Load Profile Based on Utilization (contd.)

Utilization:

$$q_m + q_{uni} + q_{accl} = 1$$

q_m: Percentage per cycle spent machining (typically 0.5 – 0.9)
q_{uni}: Percentage per cycle spent at constant velocity (typically 0.05 – 0.45)
q_{accl}: Percentage per cycle spent during acceleration and deceleration (typically 0.05 – 0.1)

Dynamic Load Rating C_a and *Life*

- When the rotation life L has been obtained, the life time can be obtained according to the following formula if the stroke length and the operation frequency are constant:

$$L_h = \frac{L}{60 n_m}$$

L: Rotation life [rev]
L_h: Life time [hr]
n_m: mean rotational speed [min^{-1}]

$$n_m = \frac{\sum_i (n_i l_i)}{\sum_i l_i}$$

n_i: rotational speed at phase i [min^{-1}]
l_i: distance traveled at phase i [m]

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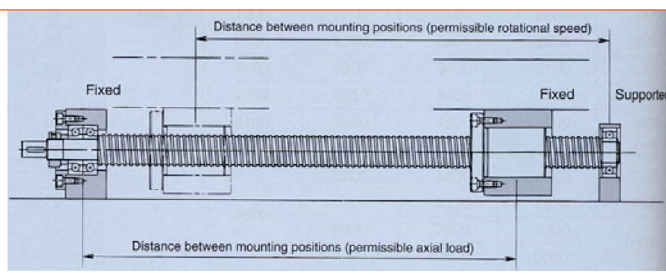
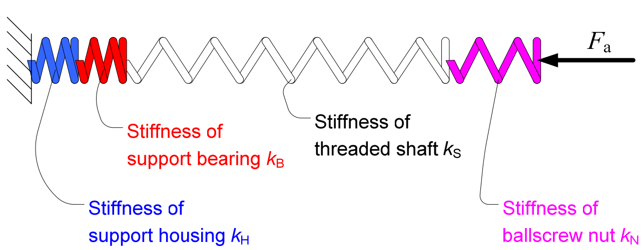
Axial Rigidity





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Axial Rigidity

Fixed-free
Fixed-supported

$$\frac{1}{k} = \frac{1}{k_s} + \frac{1}{k_N} + \frac{1}{k_B} + \frac{1}{k_H}$$

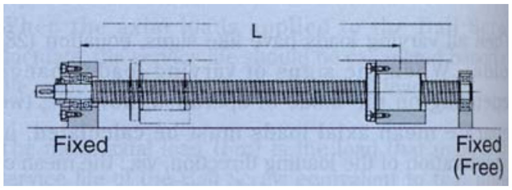
Fixed-fixed

$$\frac{1}{k} = \frac{1}{k_s} + \frac{1}{k_N} + \frac{1}{k_{B1} + k_{B2}} + \frac{1}{k_{H1} + k_{H2}}$$

k:	Axial rigidity of linear motion system [N/m]
k_s:	Axial rigidity of screw shaft [N/m]
k_N:	Axial rigidity of nut [N/m]
k_B:	Axial rigidity of support bearing [N/m]
k_H:	Axial rigidity of support housing [N/m]

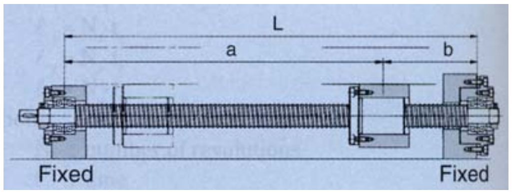
Ball Screw Selection Procedure

- Axial rigidity of shaft



Fixed-free and fixed-supported:

$$k_s = \frac{AE}{L}$$



Fixed-fixed:

$$k_s = \frac{AEL}{ab}$$

$$k_{s,\min} = \frac{4AE}{L}$$

Ball Screw Accuracy

- Applicable if used in combination with rotary encoders.

Table 1 Accumulated reference lead error and variation (tolerance) — Unit: μm

Accuracy grade		Ground products: C3		Ground products: C5		Ground products: C7	Rolled products: C8	Rolled products: C10
Thread part length (mm)		Accumulated reference lead error	Variation	Accumulated reference lead error	Variation	Accumulated reference lead error	Accumulated reference lead error	Accumulated reference lead error
Over	Up to							
—	315	12	8	23	18	±0.05mm/300mm (No other specifications)	±0.1mm/300mm (No other specifications)	±0.21mm/300mm (No other specifications)
315	400	13	10	25	20			
400	500	15	10	27	20			
500	630	16	12	30	23			
630	800	18	13	35	25			
800	1000	21	15	40	27			
1000	1250	24	16	46	30			
1250	1600	29	18	54	35			
1600	2000	35	21	65	40			
2000	2500	41	24	77	46			
2500	3150	50	29	93	54			
3150	4000	—	—	115	65			
4000	5000	—	—	140	77			

Source: THK Co., Ltd.

Overall Error

- The overall error is the summation of a number of individual errors:
  - Non-uniformity of threaded shaft.
  - Resolution/accuracy of rotary encoder.
  - Axial compliance of ball screw assembly.
  - Torsional compliance of threaded shaft.

$$\delta_a = \delta_S + \frac{l}{N_{rot}} + \frac{F_a}{k_{axial}} + \frac{32 T l_b}{\pi d_{tr}^4 G} \cdot \frac{l}{2\pi}$$

δ_a:	overall linear error [m]	k_{axial}:	overall linear stiffness [N/m]
δ_s:	ball screw non-linearity [m]	T:	applied torque [Nm]
l:	lead [m]	l_b:	ball screw length [m]
N_{rot}:	resolution of rotary encoder [pulses/rev]	d_{tr}:	root diameter of shaft [m]
F_a:	thrust force onto system [N]	G:	shear modulus [Pa]