

Basic capacity calculation methods and benchmarking for MF-TDMA and MF-CDMA communication satellites

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SUMMARY

This paper introduces basic capacity calculation methods for circuit switched, multiple-beam, low earth orbit (LEO) communication satellites that use MF-TDMA and MF-CDMA schemes. Capacity is defined as the number of simultaneous duplex channels that a satellite can support for a given data rate and bit error rate. This paper integrates the bandwidth limit and the power limit on capacity, and derives the equations by which the capacity of a MF-TDMA or a MF-CDMA system can be calculated using key system parameters. Therefore, these methods enable system designers to quickly estimate the communication system capacity when key system parameters are known. The paper uses the Iridium and Globalstar systems for benchmarking by comparing the results obtained from the analytical methods with the 1,100 and 2,5000 simultaneous channels reported by Iridium and Globalstar, respectively. The analytic capacity estimate matches reported single satellite capacities to within 6%, but the predictions are sensitive to various loss assumptions made in the link budget. Copyright © 2004 John Wiley & Sons, Ltd.

KEY WORDS: LEO satellite communications, capacity, MF-TDMA, MF-CDMA, power limit, bandwidth limit

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Contract/grant sponsor: Alfred P. Sloan Foundation; contract/grant number: 2000-10-20

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Nomenclature

α	Expected value of voice activity state
β	filter roll-off factor
B_{ch}	Channel bandwidth
B_g	Guard bandwidth
B_{sat}	Satellite frequency bandwidth
B_T	TDMA carrier bandwidth
ε	Minimum elevation angle
f	Transmission frequency or neighboring cell interference factor
F	MF-TDMA framing bits
G_t	Transmitter gain
G_r	Receiver gain
k	Boltzmann's constant
K	Cluster size
L_{tot}	Total loss in link budget
M	signal modulation level
n	Number of bits per time slot in MF-TDMA
N_c	Number of simultaneous channels a satellite supports in a cell
R_b	TDMA carrier data rate in MF-TDMA or channel information data rate in MF-CDMA (b/s)
T	Number of TDMA carriers in MF-TDMA or CDMA carriers in MF-CDMA
T_f	MF-TDMA time frame duration (second)
T_g	MF-TDMA guard time (second)
T_s	System noise temperature
θ	Half-power beamwidth
Z	Number of cells in a satellite's footprint

1. Introduction

Low earth orbit (LEO) communication satellites are generally defined as communication systems that orbit the Earth at altitudes between 500-1500 km and provide wireless services between terminals on the ground. The system typically consists of multiple satellites forming a polar or Walker constellation. Some of these systems have inter-satellite links and on-board processing that allow transmission between neighboring satellites in the constellation, while other systems act as "bent pipes" that simply "bounce" the transmission between different ground users.

User capacity of a LEO communications system in this paper is defined as the number of simultaneous voice duplex channels that a particular system can support. To maintain multiple simultaneous channels, two physical limits must be overcome: the bandwidth limit

and the power limit, either of which may act as a constraint on capacity. To overcome the bandwidth limit, frequency-division multiple access (FDMA), time-division multiple access (TDMA), or code-division multiple access (CDMA) are typically implemented. Based on these basic schemes, more sophisticated multiple-frequency time-division multiple access (MF-TDMA) and multiple-frequency code-division multiple access (MF-CDMA) are often applied to increase the satellite capacity further, within a limited allocated frequency bandwidth. To overcome the power limit, higher transmitter power and antennas with higher gain are used, as well as more robust coding schemes.

This paper will first explore previous studies on both the bandwidth limit and power limit, which are typically treated separately. This will lead to the integration of the two limits into one single equation that allows an estimation of the system capacity of both MF-TDMA and MF-CDMA satellite communication systems in LEO. At the end, the results obtained from the equations will be benchmarked against reported numbers for both the Iridium system and Globalstar system.

In this paper, in addition to previous work on the subject [1], the authors have refined the calculations, clarified the underlying assumptions, discussed potential sources of calculation errors, and clarified the contribution made by this study.

2. Overview of the Bandwidth Limit

This section will examine the bandwidth limitation of the MF-TDMA and MF-CDMA schemes. The derivation follows the one proposed by Lutz, Werner, and Jahn [2], except for the derivation of the capacity equations for the MF-TDMA scheme under bandwidth constraint in Equations (12), (26), and (27), which have been extended in this paper.

2.1. Frequency Band Utilization

Let R_b denote the bit data rate of a digital signal per channel, and M denote the signal modulation level. The receiver is assumed to use Nyquist filtering to avoid inter-symbol interference (ISI), and β is the filter roll-off factor, normally ranging from 0.2 to 0.5. Then, the channel bandwidth required by the Nyquist-filtered signal is

$$B_{ch} = \frac{(1 + \beta)R_b}{\log_2 M} \quad (1)$$

where B_{ch} is in Hz and R_b in b/s.

2.2. FDMA Capacity

In the U.S., the frequency band for a system is assigned by the Federal Communication Commission (FCC). In FDMA, the assigned frequency band, B_c , is divided into channel bands of width B_{ch} . This division typically happens inside a spot beam and between neighboring spot beams. We will discuss spot beams in Subsection 2.5. These channel bands are separated from each other by guard bands B_g , as illustrated in Figure 1. The number of FDMA channels that can be supported, N , can be calculated based on (1) as

$$N = \frac{B_c}{B_{ch} + B_g} = \frac{B_c}{R_b} \cdot \frac{\log_2 M}{1 + \beta + \log_2 M \cdot B_g/R_b} \quad (2)$$

where R_b is the data rate per FDMA channel, and B_c , B_{ch} , and B_g are in Hz.

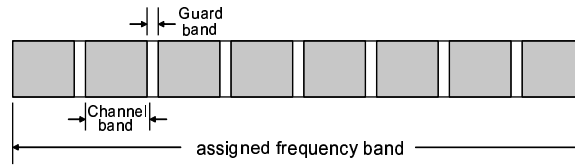


Figure 1. Frequency Division Multiple Access (FDMA)

2.3. TDMA and MF-TDMA Capacity

In TDMA, access time is divided into frames, and frames are again divided into time slots. A basic channel is formed by a particular time slot inside every frame. In the forward link (satellite-to-user downlink) and return link (user-to-satellite uplink), usually the same frame structure is used. In order to avoid simultaneous transmission and reception of a user, the corresponding time slots for the forward and return links are separated in time. The TDMA scheme is illustrated in Figure 2.

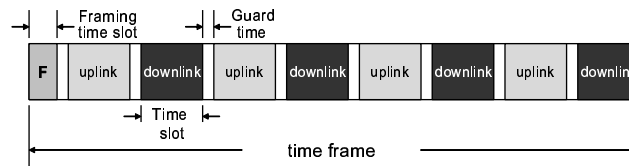


Figure 2. Time Division Multiple Access (TDMA)

To find the number of TDMA channels, N , for a given bandwidth, we first start with the frame duration, T_f , in seconds and the allowed burst rate for each TDMA carrier, R_b , in b/s, then

$$n_f = R_b T_f \quad (3)$$

gives the number of bits within each frame. If the duration of a time slot is T_{slot} in seconds, then n , the number of bits per time slot, is

$$n = R_b T_{slot} \quad (4)$$

If each time slot begins with a header of H bits for the purpose of synchronization, and a guard time of T_g seconds is inserted between every two time slots, then the relationship between R_b and the number of half duplex channels per frame, N_{hd} , is

$$R_b = N_{hd} \frac{H + n}{T_f - N_{hd} T_g} \quad (5)$$

where R_b is in b/s.

If we assume that instead of the header, a certain number of framing bits F are added at the beginning of each frame, then (5) becomes

$$R_b = \frac{F + N_{hd} n}{T_f - N_{hd} T_g} \quad (6)$$

To find the number of TDMA channels for a given burst rate, we solve (5) to get

$$N_{hd} = \frac{R_b T_f}{H + n + R_b T_g} \tag{7}$$

If framing bits are used, we solve (6) to get

$$N_{hd} = \frac{R_b T_f - F}{n + R_b T_g} \tag{8}$$

The number of full duplex TDMA channels is

$$N = N_{hd}/2 \tag{9}$$

In *MF-TDMA*, multiple TDMA carriers at different frequency channels are used to increase the total number of channels. For example, Iridium uses 4 duplex time channels and 120 frequency channels, as illustrated in Figure 3. In the frequency domain, the bandwidth, B_T , occupied by a TDMA carrier can be obtained from (1):

$$B_T = \frac{(1 + \beta)R_b}{\log_2 M} \tag{10}$$

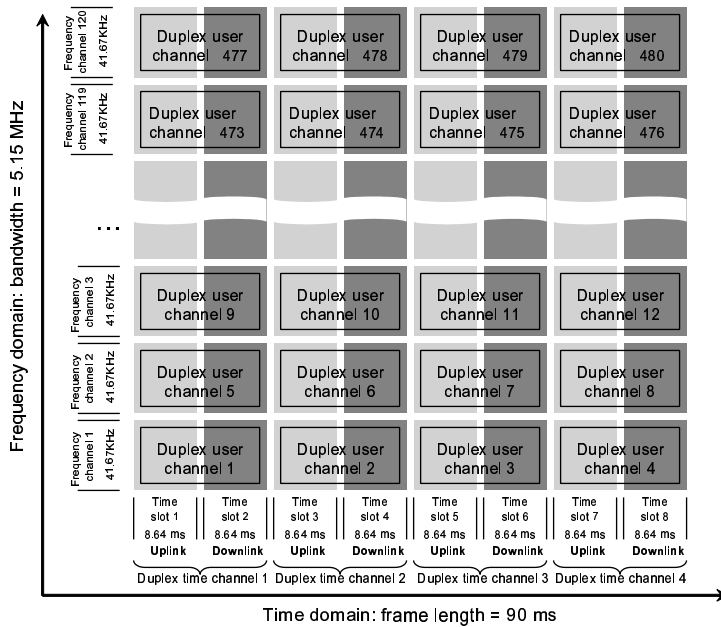


Figure 3. Multiple Frequency - Time Division Multiple Access (MF-TDMA)

The carriers are separated by guard bands B_g . Therefore, the total bandwidth required to support T TDMA channels is

$$B = T(B_T + B_g) \tag{11}$$

The number of active duplex MF-TDMA channels for bandwidth B can then be evaluated using (8), (10), and (11) as

$$N_B = T \cdot N = \frac{1}{2} \frac{B}{B_T + B_g} \frac{R_b T_f - F}{n + R_b T_g} \quad (12)$$

2.4. CDMA and MF-CDMA Capacity

Using a unique pseudorandom noise (PN) code, a CDMA transmitting station spreads the signal in a bandwidth wider than actually needed. Each authorized receiving station must have the identical PN code to retrieve the information. Other channels may operate simultaneously within the same frequency spectrum as long as different, orthogonal codes are used.

The long PN code is called a *chip*. The chip is modulated by the information data stream. The ratio of the chip rate R_c to the information data rate R_b is called the *processing gain* G .

$$G = \frac{R_c}{R_b} \quad (13)$$

Because the CDMA carrier bandwidth is equal to $B_T = R_c$, the processing gain can also be expressed as

$$G = \frac{B_T}{R_b} \quad (14)$$

Then, if E_b is the average bit energy of any channel's signal, the power spectral density of the transmit signal (energy per chip) is

$$E_b R_b / R_c = E_b / G \quad (15)$$

By using the PN code, CDMA overcomes the bandwidth limit. But there is still the problem of interference between channels. To find the maximum number of active channels allowed by the system, we adopt the following derivation.

In typical CDMA technology, when speech pauses, the transmit signal is switched off to save power. The voice activity state of channel n is denoted $\alpha_n \in \{0; 1\}$, representing both the on and off states. The expected value is $E\{\alpha_n\} = \alpha$. Assuming that each user in the duplex channel talks for half of the conversation time, then $\alpha \approx 0.5$ in theory. Instead of switching off the transmit power, real systems transmit at a low power and data rate during voice inactivity. Therefore α can also be considered as the average transmit power divided by the transmit power during talk spurts [2]. Let's define \bar{I}_i to be the mean value of the power spectral density of the interference noise caused by other channels. Because there are N channels in total, there are $N - 1$ channels other than the channel under consideration. Based on (15),

$$\bar{I}_i = \alpha(N - 1)E_b / G \quad (16)$$

If N_0 stands for thermal noise of the system, then the mean total noise power spectral density I_{tot} is the sum of the interference noise and the thermal noise,

$$I_{tot} = \bar{I}_i + N_0 = \alpha(N - 1)E_b / G + N_0 \quad (17)$$

We can then derive the number of total CDMA channels N from (17) to be

$$N = 1 + \frac{G}{\alpha} \left[\left(\frac{E_b}{I_{tot}} \right)^{-1} - \left(\frac{E_b}{N_0} \right)^{-1} \right] \quad (18)$$

If we assume that the modulation used is BPSK or QPSK, and that the noise type is Gaussian additive noise, we can express the ratio of the bit energy to total noise power spectral density E_b/I_{tot} as

$$\frac{E_b}{I_{tot}} = [\text{erfc}^{-1}(2p_b)]^2 \quad (19)$$

where p_b is the required bit error probability.

In real applications, because convolutional coding is used, the actual required value for E_b/I_{tot} is lower than what is given by (19). This will be covered in Subsection 3.2. We can find the available E_b/N_0 from the link budget equation, see Section 3. Combining (14) and (18), we obtain the expression for the number of CDMA channels:

$$N = 1 + \frac{B_T}{R_b} \frac{1}{\alpha} \left[\left(\frac{E_b}{I_{tot}} \right)^{-1} - \left(\frac{E_b}{N_0} \right)^{-1} \right] \quad (20)$$

In *MF-CDMA*, multiple CDMA carriers at different frequencies are used to increase the number of channels. In the frequency spectrum, the carriers are separated by guard bands, B_g . If T CDMA carriers are used, the total required bandwidth is

$$B = T(B_T + B_g) = T(GR_b + B_g) \quad (21)$$

The total number of channels for bandwidth B will be

$$N_B = T \cdot N \quad (22)$$

Combining (20), (21), and (22), we get the following expression that gives the total number of CDMA channels for the given frequency bandwidth B :

$$N_B = T \cdot N = T + \frac{B - TB_g}{R_b} \frac{1}{\alpha} \cdot \left[\left(\frac{E_b}{I_{tot}} \right)^{-1} - \left(\frac{E_b}{N_0} \right)^{-1} \right] \quad (23)$$

2.5. Capacity When Using Spot Beams

A LEO communications satellite typically concentrates its transmission power in multiple spot beams. Each spot beam covers a cell on the ground, and all the cells together form the footprint of the satellite. The spot beam contour is usually defined by a 3dB decrease of antenna gain relative to the peak gain. The usage of spot beams offers two distinct advantages: 1. Focusing transmitted power on a much smaller area than the total coverage area of the satellite. Spot beams increase the transmitter gain and therefore improve the link budget. 2. The reuse of frequency bands in different cells improves bandwidth efficiency. We will focus on the second advantage in this section and leave the first to be discussed in Section 3.

The number of different frequency bands used in the cells of a satellite is called *cluster size*, designated by K . For FDMA and TDMA, typical values of the cluster size are $K = 4$ or 7 . For CDMA, a cluster size of $K = 1$ can be used because all channels can operate simultaneously within the same frequency band. The Iridium system has a total of 48 spot beams with a cluster size of 12, as illustrated in Figure 4, in which each letter corresponds to a different frequency band.

Let A_s be the service area, which is equal to the area of the footprint. Let A_c be the cell area, then the number of cells is

$$Z = A_s/A_c \quad (24)$$

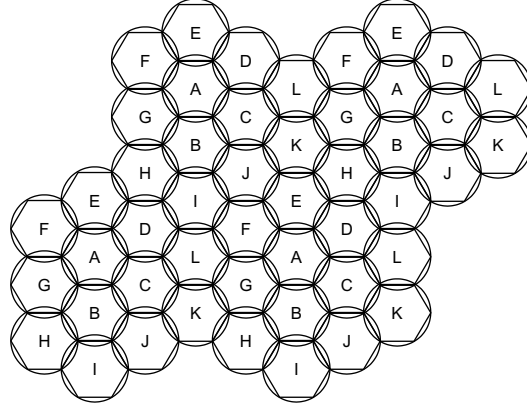


Figure 4. Spot beam pattern of an Iridium satellite

Next, we will calculate how the capacities of MF-TDMA and MF-CDMA systems are affected by the use of spot beams. Spot beams represent a form of spatial multiplexing.

2.5.1. Capacity of MF-TDMA When Using Spot Beams If the bandwidth available for one satellite is B_{sat} , then the cell bandwidth is

$$B_c = B_{sat}/K \quad (25)$$

Replacing B in (12) with the expression for B_c in (25), we get number of channels per cell to be

$$N_c = T \cdot N = \frac{1}{2K} \frac{B_{sat}}{B_T + B_g} \frac{R_b T_f - F}{n + R_b T_g} \quad (26)$$

The number of channels per satellite is then

$$N_{sat} = Z \cdot N_c = \frac{Z}{2K} \frac{B_{sat}}{B_T + B_g} \frac{R_b T_f - F}{n + R_b T_g} \quad (27)$$

2.5.2. Capacity of MF-CDMA When Using Spot Beams In CDMA, we set the cluster size to $K = 1$ because all the cells have an identical frequency band. Therefore $B_c = B_{sat}$. Based on (23), the number of channels per cell is

$$N_c = T + \frac{B_{sat} - T B_g}{R_b} \frac{1}{\alpha(1+f)} \cdot \left[\left(\frac{E_b}{I_{tot}} \right)^{-1} - \left(\frac{E_b}{N_0} \right)^{-1} \right] \quad (28)$$

And the capacity of the satellite is

$$N_{sat} = Z \cdot N_c \quad (29)$$

where the factor $(1 + f)$ represents the increase in interference caused by the users in neighboring cells. A lower bound value for f is 1.36 [2].

2.6. Capacity of the Entire Constellation

For a global coverage satellite constellation consisting of S satellites, if the constellation is a Walker constellation, then the total number of channels that can be used at any time is simply

$$N_s = SN_{sat} \quad (30)$$

But if the constellation is of polar type, the overlapping cells at the poles will reduce the total capacity to approximately

$$N_s = 0.68SN_{sat} \quad (31)$$

due to overlapping cells at both polar caps [3, 4].

3. Overcoming the Power Limit

The power limit problem has been worked out in numerous references, such as those by Larson and Wertz [3] and by Gordon and Morgan [5]. The authors will reiterate it here for the sake of a complete and consistent derivation for the final capacity equations.

3.1. Link Budget

The link budget is important in estimating the capacity of a LEO communication satellite system. To find physical parameters of the transmission such as data rate or E_b/N_0 , a series of factors must be accounted for along the transmission path, including transmitter power, transmitter gain, space loss, atmosphere attenuation, system noise temperature, receiver gain, and other losses. In this section, we will briefly consider each of these factors.

Because the satellite power needs to be divided among all simultaneous channels in the satellite-user forward link, the satellite-user link is generally considered as the bottleneck of the network. Therefore we consider the satellite-user link budget problem. First, the transmitter power of a satellite needs to be divided among all its cells. In the MF-TDMA scheme, the power of each cell is further divided among all the TDMA carriers. In the MF-CDMA scheme, the power of each cell is divided among all CDMA channels.

If the transmitter operates at frequency f_{GHz} in GHz, and D is either the diameter in meters of a parabolic antenna transmitter or the side dimension of a phased-array antenna, then the transmitter half power beamwidth θ in degree can be calculated as

$$\theta = \frac{21}{f_{GHz}D} \quad (32)$$

If η_t is the transmitter efficiency, and θ_x and θ_y are the beamwidths of a parabolic antenna along the major and minor axes, or for a phased-array antenna along the long side and short side, then the transmitter gain can be estimated in dB as

$$G_t \text{ dB} \approx 46.9 + 10 \log(\eta_t) - 10 \log(\theta_x \theta_y) \quad (33)$$

Let λ be the signal wavelength in m, d be the path length in m, c be the speed of light, and f be the transmission frequency in Hz. Space loss of the transmission L_s is

$$L_s = (\lambda/4\pi d)^2 = (c/4\pi df)^2 \quad (34)$$

In dB, L_s dB is

$$L_s \text{ dB} = 147.55 - 20 \log d - 20 \log f \quad (35)$$

The rain and atmospheric attenuation loss, L_a , can be calculated following a standard, but complex procedure listed in the relevant International Telecommunication Union ITU-R recommendations [6]. Because it is not the purpose of this paper, the procedure is omitted here. We will just approximate the atmospheric loss to be -0.2dB.

The gain of the receiver antenna can be obtained in the same way as the gain of the transmitter antenna. But in practice, the antennas of the user terminals of Iridium and Globalstar are of the omni-directional type, with a theoretical gain of 1 (0 dB).

System noise temperatures in satellite communication links in clear weather have been estimated previously [3]. The results are listed in Figure 5.

	Frequency (GHz)					
	Downlink			Crosslink	Uplink	
	0.2	2-12	20	60	0.2 - 20	40
Noise (dB-K)	25.7	27.4	28.6	32.5	31.1	32.6

Figure 5. Typical System Noise Temperatures, T_s , in Satellite Communication Links in Clear Weather

Next we consider other possible losses along the link. The line loss between transmitter and antenna in dB is represented by L_l dB, with a typical value of -1 dB. The pointing offset loss of the transmitter in dB is estimated as

$$L_{offset} \text{ dB} = -12(e_t/\theta_t)^2 \quad (36)$$

where e_t is the transmitter pointing error, and θ_t is the transmitter antenna beamwidth. For a phased-array antenna utilizing spot beams, the pointing offset loss is negligible.

Polarization mismatch, L_{polar} , adds about -0.3 dB to the link budget, and a loss caused by a radome on the ground terminal, L_{rad} , is negligible here (0 dB), since the ground terminals of interest in this paper are user handsets. If we also include an implementation loss, L_{imp} dB, of -1 dB, then the total loss along the path is

$$L_{tot} \text{ dB} = L_l \text{ dB} + L_{offset} \text{ dB} + L_s \text{ dB} + L_a \text{ dB} + L_{polar} \text{ dB} + L_{rad} \text{ dB} + L_{imp} \text{ dB} \quad (37)$$

Besides the losses mentioned above, the signal attenuates also due to two additional reasons: shadowing and multipath fading. Shadowing is caused by obstacles in the path that partially block the transmission, for example, terrain, buildings, and natural vegetation. Multipath fading is caused by the interference of the echoes that the signal reflects off the environment. A complete model of the probability density function of the received signal power S is given in [2] as follows

$$p(S) = (1 - A) \cdot p_{Rice}(S) + A \cdot \int_0^\infty p_{Rayl.}(S | S_0) p_{LN}(S_0) dS_0 \quad (38)$$

In equation (38), $p_{Rice}(S)$ is the Ricean probability density function of S when the signal is under multipath fading only. And $p_{Rayl.}(S | S_0)$ is the Rayleigh probability density function of

S when the signal is under both multipath fading and shadowing. The slow shadowing process results in a time-varying short-term mean received power S_0 for which a lognormal distribution $p_{LN}(S_0)$ is assumed. Variable A represents the percentage of time when the channel is subject to shadowing.

To compensate the losses caused by multipath fading and shadowing, some extra budget must be given to the link, and this extra budget is called the *link margin*. For example, the Iridium downlink has a required link margin of 16dB.

After the power, aperture gains, losses, system noise, and margin are defined, the link budget equation becomes a trade-off between the transmission data rate R_b dB and energy per bit to noise ratio E_b/N_0 . The link budget equation, in defining E_b/N_0 , is

$$E_b/N_{0dB} = Power_{dB} + G_t dB + G_r dB - k_{dB} - T_s dB - R_b dB + L_{tot dB} - margin \quad (39)$$

where $Power_{dB}$ is transmit power in dBW, $E_b/N_{0 dB}$ in dBs^{-1} , R_b dB in dBb/s, and k , the Boltzmann constant, is $1.38 \cdot 10^{-23} \text{ Ws/K} = -288.6 \text{ dBWs/K}$.

3.2. Convolutional Coding

Redundancy coded into transmitted information reduces the transmit power required for a specific bit error probability. This reduction process has two important parameters, code rate R_c and constraint length C . Code rate R_c is defined by the ratio between the original information bits k and the coded content bits n ,

$$R_c = k/n. \quad (40)$$

The bit energy to mean total noise power spectral density ratio E_b/I_{tot} corresponding to several bit error probabilities, p_b , for soft-decision Viterbi decoding are listed in Figure 6 [7].

p_b	E_b/I_{tot} uncoded (dB-b ⁻¹)	$R_c=1/3$		$R_c=1/2$			$R_c=2/3$		$R_c=3/4$	
		C=7	C=8	C=5	C=6	C=7	C=6	C=8	C=6	C=9
10^{-3}	6.8	4.2	4.4	3.3	3.5	3.8	2.9	3.1	2.6	2.6
10^{-5}	9.6	5.7	5.9	4.3	4.6	5.1	4.2	4.6	3.6	4.2
10^{-7}	11.3	6.2	6.5	4.9	5.3	5.8	4.7	5.2	3.9	4.8

Figure 6. Coding Gain $E_b/N_0(\text{dB-b}^{-1})$ for Soft-Decision Viterbi Decoding, QPSK

4. Combining Bandwidth Limit And Power Limit

With the previous two sections paving the way, this section contains the main contribution of the paper by integrating the power limit and bandwidth limit into one single equation. Previously, both limits had been considered separately.

4.1. MF-TDMA

For MF-TDMA, equation (39) can also be written as

$$R_b dB = Power_{dB} + G_t dB + G_r dB - k_{dB} - T_s dB - E_b/N_{0dB} + L_{tot dB} - margin \quad (41)$$

Note that if $Power_{dB}$ is the power per TDMA carrier, then R_b $_{dB}$ is the data rate of each TDMA carrier. Substituting for R_b in (26) with the R_b $_{dB}$ in (41) (after R_b $_{dB}$'s units are converted to b/s), Equation (26) will give the number of channels per cell. For the same reason, Equation (27) will give the number of channels per satellite, taking into account both the available power and bandwidth.

4.2. MF-CDMA

In MF-CDMA, the transmitter power of the satellite is divided among all cells, and the power in a cell is further divided among all CDMA channels in the cell. If $Power_{cell}$ is the transmitter power per cell and N_c is the number of channels per cell, the link budget of a CDMA downlink becomes

$$E_b/N_{0dB} = (Power_{cell}/N_c)_{dB} + G_t \text{ }_{dB} + G_r \text{ }_{dB} - k_{dB} - T_s \text{ }_{dB} - R_b \text{ }_{dB} + L_{tot} \text{ }_{dB} - margin \quad (42)$$

If we use original units instead of decibel units, equation (42) can be re-written as

$$E_b/N_0 = \frac{Power_{cell} \cdot G_t \cdot G_r \cdot L_{tot}}{N_c \cdot k \cdot T_s \cdot R_b \cdot margin} \quad (43)$$

To find the cell capacity, we substitute (43) into (28) and solve for N_c . We get

$$N_c = \frac{T + \frac{B_{sat} - TB_g}{R_b} \frac{1}{\alpha(1+f)} \frac{I_{tot}}{E_b}}{1 + \frac{B_{sat} - TB_g}{R_b} \frac{1}{\alpha(1+f)} \frac{k \cdot T_s \cdot R_b \cdot margin}{Power_{cell} \cdot G_t \cdot G_r \cdot L_{tot}}} \quad (44)$$

The capacity of the satellite follows as $N_{sat} = Z \cdot N_c$.

5. Benchmarking Against Existing Systems

In order to verify the predictive accuracy of the capacity equations developed above, we benchmark them against the existing systems Iridium and Globalstar. To do so, we find publicly available data on the design parameters of these two systems, and substitute them into the capacity equations we have derived. A comparison between the results from these equations and the capacities claimed by the manufacturers will suggest how accurate these equations are for capacity estimation during early design.

5.1. Benchmarking MF-TDMA against Iridium

Built and launched by Motorola, and now owned and operated by Iridium LLC, Iridium is among the first generation of global LEO satellite systems for telephony. The Iridium constellation consists of 66 satellites orbiting at an altitude of 780km. The constellation type is polar, with 6 orbital planes at an inclination of 86.4°. Iridium is chosen for the benchmarking because it is the most mature LEO communications system that uses MF-TDMA technology. Iridium's key technical specifications are reported by various sources. They are listed in Figure 7. Note that the right column provides more updated information than the left column. The difference is due to the fact that the FCC assigned only half of the requested frequency band to Iridium.

		Initial Design Data ⁹	Final Design Data ²
	Satellite to user frequency (GHz)		1.6239
	Link BER		0.001
	Convolutional code rate R_c		$\frac{3}{4}$
	Convolutional constraint length K		6
T D M A	TDMA frame length T_f (ms)	90	90
	Burst data rate R_b (bps)	50,000	50,000
	Duplex channel data rate (bps)	4,800	4,600
	# of time slots per frame	8	
	# of TDMA duplex channels	4	
	Time slot length (ms)	8.64	
	Time slot bits n (bits)	432	414
	Framing length (ms)	17.28	
	Framing bits F (bits)	864	
	Guard length, GL (ms)	0.36	
F D M A	Satellite bandwidth B_{sat} (MHz)	10.5	5.15
	# of FDMA channels	240	120
	Channel bandwidth B_T (KHz)	41.67	41.67
	Guard bandwidth B_g (KHz)	2 (241 guard bands)	1.236 (121 guard bands)
	# of cells per satellite Z	48	48
	Cluster size K	12	12
	Modulation scheme	QPSK	QPSK
	# of FDMA channels per cell	20	10
	# of MF-TDMA channels per cell N_c	4x20=80	4x10=40
	# of MF-TDMA channels per satellite N_{sat}	80x48=3,840	40x48=1,920

Figure 7. Technical Specification of Iridium from various references [2, 9]

Because the gain of the Iridium antenna in an edge cell is given [8], we will use the edge cells to estimate the channel capacity per cell, and multiply this cell capacity with the number of cells to get the capacity of a satellite. The beamwidth of the edge cell spot beam is about 13.4° , estimated based on the transmitter gain in that cell $G_{t\ edge}$ [2].

$$\theta_{edge} = \frac{35\pi}{\sqrt{G_{t\ edge}}} \quad (45)$$

The derivation of the distance from the satellite to the center of the edge cell, D_{edge} , is illustrated in Figure 8. The satellite nadir angle ϑ is found using the earth radius R_e and minimum elevation angle ε [2].

$$\vartheta = \arcsin\left(\frac{R_e}{r} \cos \varepsilon\right) \quad (46)$$

To find the distance from the satellite to the center of the edge cell, let

$$\alpha = \vartheta - \theta_{edge} \quad (47)$$

and where β is the earth centered angle corresponding to α and $r = R_e + h$.

$$\beta = \arcsin\left(\frac{r}{R_e} \sin \alpha\right) - \alpha \quad (48)$$

The distance is then given as

$$D_{edge} = \sqrt{R_e^2 + r^2 - 2R_e r \cos \beta} \quad (49)$$

The distance for Iridium is calculated to be 1606.9km.

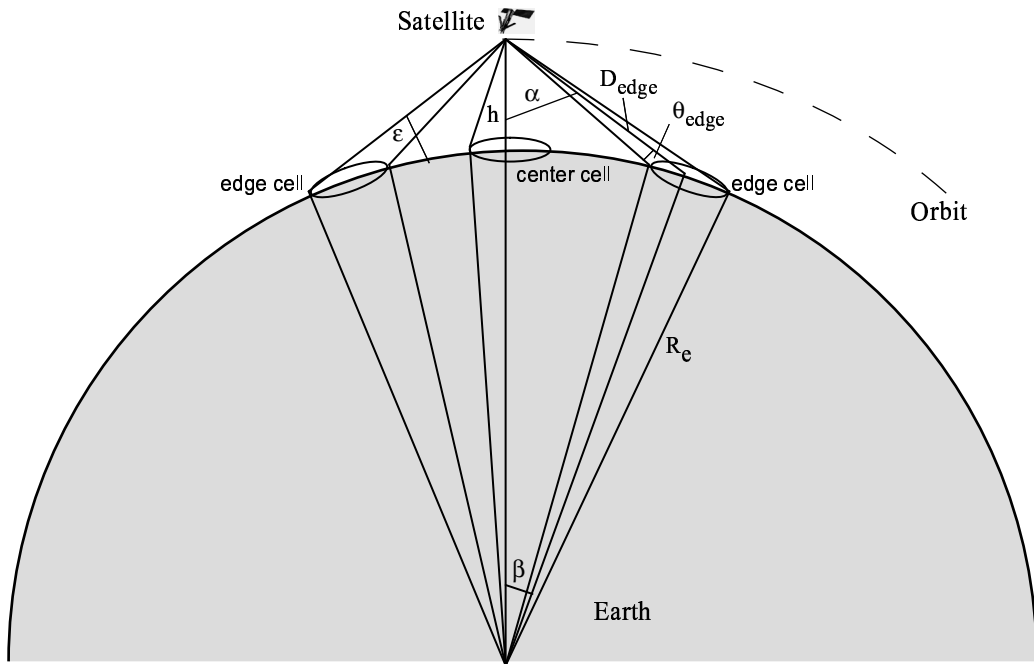


Figure 8. Geometry for the derivation of the distance from the satellite to the center of the edge cell

According to (35), the space loss is

$$\begin{aligned} L_{s \text{ dB}} &= 147.55 - 20 \log d - 20 \log f \\ &= 147.55 \text{ dB} - 20 \log(1.6069 \cdot 10^6 \text{ m}) - 20 \log(1.6239 \cdot 10^9 \text{ Hz}) \\ &= -160.78 \text{ dB} \end{aligned} \quad (50)$$

The total loss along the path, as in (36), is

$$\begin{aligned}
 L_{tot \text{ dB}} &= L_{ldB} + L_{offset \text{ dB}} + L_s \text{ dB} + L_a \text{ dB} + L_{polar \text{ dB}} + L_{rad \text{ dB}} + L_{imp \text{ dB}} \\
 &= -1\text{dB} - 0\text{dB} - 160.78\text{dB} - 0.2\text{dB} - 0.3\text{dB} - 0\text{dB} - 1\text{dB} \\
 &= -163.28\text{dB}
 \end{aligned} \tag{51}$$

As listed in Figure 7 [2, 9], for the Iridium system the convolutional coding code rate is $R_c=3/4$, and the constraint length is $K=6$. In order to achieve a bit error probability of $p_b = 10^{-3}$, E_b/I_{tot} must be $2.6\text{dB}\cdot\text{b}^{-1}$ according to Figure 6.

As mentioned above, the gain of Iridium and Globalstar's omni-directional user terminal antenna is 0dB. The transmitter power is first distributed among 48 cells, and then distributed again among the 10 TDMA carriers in each cell. $T_{s\text{dB}} = 25.7\text{dBK}$ according to Figure 5. With all the other variables known, equation (41) gives the TDMA carrier data rate as

$$\begin{aligned}
 R_b \text{ dB} &= Power_{\text{dB TDMA-carrier}} + G_t \text{ dB} + G_r \text{ dB} - k_{\text{dB}} \\
 &\quad - T_s \text{ dB} - E_b/N_{0\text{dB}} + L_{tot \text{ dB}} - margin \\
 &= 10 \cdot \log(400/48/10)W + 24.3\text{dB} + 0\text{dB} + 228.6\text{dBW}(Hz \cdot K) \\
 &\quad - 25.7\text{dBK} - 2.6\text{dB} - b^{-1} - 163.28\text{dB} - 16\text{dB} \\
 &= 44.53\text{dB} - b/s
 \end{aligned} \tag{52}$$

Then $R_b = 28.379 \text{ kb/s}$. Substitute this value in (27),

$$\begin{aligned}
 N_{sat} &= Z \cdot N_c \\
 &= \frac{Z}{2K} \frac{B_{sat}}{B_T + B_g} \frac{R_b T_f - F}{n + R_b T_g} \\
 &= \frac{48}{2 \times 12} \cdot \frac{5150\text{kHz}}{41.67\text{kHz} + 1.236\text{kHz}} \cdot \frac{28379\text{b/s} \cdot 90 \times 10^{-3}\text{s} \cdot \left(1 - \frac{17.28\text{ms}}{90\text{ms}}\right)}{414\text{b} \cdot \left(1 + \frac{3.5}{90 - 17.28 - 3.6} \frac{\text{ms}}{\text{ms}}\right)} \\
 &= 1139
 \end{aligned} \tag{53}$$

The actual Iridium system's number of channels is quoted as being power-limited to 1,100 [2]. The estimated capacity is 1,139 channels, or about 3.55% higher than the reported capacity.

5.2. Benchmarking MF-CDMA against Globalstar

A joint effort of Loral Space Systems, Alcatel, and QUALCOMM, Globalstar is another of the first generation global LEO systems for mobile communications. The system consists of 48 satellites spread over eight orbital planes with six satellites per plane. The constellation type is a 48/8/1 Walker constellation. The orbits are at an altitude of 1,389km and inclination of 52° . Globalstar is probably the most mature commercial MF-CDMA LEO satellite communication system at present time. Its communication parameters are listed in Figure 9 [2]. The right column has more updated information than the left column as the FCC assigned only a portion of the requested frequency band to Globalstar. However, because the 2,500 reported capacity is based on the original frequency band, for the purpose of benchmarking, we will use the data in the left column for our calculations.

Similar as in the Iridium case, the distance from the satellite to the center of the edge cell

	Initial Design Data ²	Final Design Data ²
Channel data rate R_b (bps)	9,600	9,600
Full user link bandwidth B_{sat} (MHz)	16.5	11.35
CDMA carrier bandwidth B_T (MHz)	1.23	1.23
Bit error probability p_b	0.01	0.01
E_b/N_0 (1/bit)	4.1072	4.1072
# of CDMA carriers T	13	9
# of cells per satellite Z	16	16
Modulation scheme	QPSK	QPSK
# of channels per satellite N_{sat}	2,500	

Figure 9. Technical Specification of Globalstar[2]

for Globalstar is calculated to be 1943.9km. The space loss can be obtained from (35) as

$$\begin{aligned}
 L_s \text{ dB} &= 147.55 - 20 \log d - 20 \log f \\
 &= 147.55 \text{ dB} - 20 \log(1.9439 \cdot 10^6 \text{ m}) - 20 \log(2.4918 \cdot 10^9 \text{ Hz}) \\
 &= -166.15 \text{ dB}
 \end{aligned} \tag{54}$$

The total loss along the path is

$$\begin{aligned}
 L_{tot} \text{ dB} &= L_{ldB} + L_{offset \text{ dB}} + L_s \text{ dB} + L_a \text{ dB} + L_{polar \text{ dB}} + L_{rad \text{ dB}} + L_{imp \text{ dB}} \\
 &= -1 \text{ dB} - 0 \text{ dB} - 166.15 \text{ dB} - 0.2 \text{ dB} - 0.3 \text{ dB} - 0 \text{ dB} - 1 \text{ dB} \\
 &= -168.65 \text{ dB}
 \end{aligned} \tag{55}$$

After converting from decibel, $L_{tot} = 1.3646 \times 10^{-17}$. The E_b/I_{tot} required can be obtained from the rules of convolutional coding. With $R_c = 1/2$ and $K = 9$, the satellite to user transmission with a bit error rate of 0.01 requires an E_b/I_{tot} of $1.18 \text{ dB} - b^{-1}$, or $1.3122b^{-1}$.

According to the data in Figure 9, Globalstar's power per cell is

$$Power_{cell} = 380W/16 = 23.75W \tag{56}$$

To find some of the other values in (44), we have

$$\begin{aligned}
 &\frac{B_{sat} - TB_g}{R_b} \frac{1}{\alpha(1+f)} \\
 = &\frac{B_{channel}T}{R_b} \frac{1}{\alpha(1+f)} \\
 = &\frac{1.23 \times 10^6 \text{ Hz} \cdot 13}{2400b/s} \frac{1}{0.5(1+1.36)} \\
 = &5646.19b^{-1}
 \end{aligned} \tag{57}$$

Because the downlink frequency of Globalstar is at about 2.5GHz, the system noise T_{sdB} is at 27.4 dB-K as shown in Figure 5, or T_s at 549.54K. The link margin of Globalstar is about 6dB on average, therefore

$$\begin{aligned} & \frac{k \cdot T_s \cdot R_b \cdot margin}{Power_{cell} \cdot G_t \cdot G_r \cdot L_{tot}} \\ = & \frac{1.38 \times 10^{-23} Ws/K \times 549.54K \times 2400b/s \times 10^{0.6}}{23.75W \times 50.12 \times 1 \times 1.3646 \times 10^{-17}} \\ = & 0.004461b \end{aligned} \quad (58)$$

Plugging the values into (44), we obtain

$$\begin{aligned} N_c &= \frac{T + \frac{B_{sat} - TB_g}{R_b} \frac{1}{\alpha(1+f)} \frac{I_{tot}}{E_b}}{1 + \frac{B_{sat} - TB_g}{R_b} \frac{1}{\alpha(1+f)} \cdot \frac{k \cdot T_s \cdot R_b \cdot margin}{Power_{cell} \cdot G_t \cdot G_r \cdot L_{tot}}} \\ &= \frac{13 + 5646.19b^{-1}/1.3122b^{-1}}{1 + 5646.19b^{-1} \cdot 0.004461b} \\ &= 164.80 \approx 165 \end{aligned} \quad (59)$$

$$N_{sat} = Z \cdot N_c = 16 \times 165 = 2640 \quad (60)$$

The Globalstar system claims to have a capacity of 2,500 simultaneous channels at the original design point [2]. The value estimated here is 2,640 channels, or 5.60% higher than the reported value.

5.3. Comparison with Previous Work and Error Analysis

The other works mentioned in this paper, including the ones by Lutz, Werner, and Jahn [2], by Larson and Wertz [3], and by Gordon and Morgan [5] have not included a combination of bandwidth limit and power limit. Also, we have focused specifically on predicting the capacity of a LEO communications constellation, based on simplified analytical equations and important system parameters. Simplicity should not be confused with inaccuracy.

The capacities obtained from the calculation for both systems are within 3 ~ 6% compared to the reported capacities. There are several sources that may have caused this discrepancy, as listed:

- The assumption of Gaussian additive noise for system noise type might not perfectly represent reality.
- A series of assumptions made in the link budget calculation, such as -0.2dB for atmosphere loss, 0dB gain for user terminal, the assumption of clear weather, the neglected pointing offset loss, and the approximations for the values of polarization mismatch and implementation loss all contribute to the error in the final results.
- The assumption that the edge cell of a satellite footprint has the same capacity as the center cell approximates reality. Therefore the calculation can only provide an approximation for the system capacity.

6. Conclusion

The method proposed above for capacity estimation of MF-TDMA and MF-CDMA satellite communication systems is accurate within 3 ~ 6% of reported actual capacities. The merit of this method lies in its simplicity. With the key design parameters known, both the bandwidth limit and power limit are integrated into a single equation that yields the number of simultaneous channels the system can support. A program coded from the equations can be a useful tool for doing system-level studies. Typically, this kind of study requires the exploration of a large number of conceptually similar, but parametrically different designs. A quick way to estimate the capacity of these designs is necessary, otherwise the effort required would inhibit searching a broad design space in a limited time.

Most often a LEO satellite constellation will have to be designed to satisfy an expected global capacity in terms of the total number of duplex channels supported. Such a number may be obtained from market forecasts or may be mandated by institutional customers of such a system. Alternatively, the constellation may be driven by the maximum peak capacity required in any cell or single satellite footprint. Either way, capacity is often enforced as a requirement, i.e. as an equality constraint on the system designer. With the left side of Equations (27) or (29) constrained for MF-TDMA and MF-CDMA satellites, and Equations (30) and (31) constrained for polar or Walker constellations, high level system parameters can be traded against each other. Depending on the bandwidth allocation and technologies available, system designers must be able to quickly understand the performance, cost and risk implications of various power, coding, antenna and resource allocation choices within the system.

The method proposed in this paper answers this need. A LEO satellite communication simulator incorporating this method has been developed by the authors in Matlab. This framework has been used for comparative system-level studies and assessments of technology infusion in satellite communication constellation architectures [10].

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