Basic Equations and Quadratics

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Teacher

1 Solve the following equation: -3(x + 2) - 4(3x - 2) = 2(x + 6) - 12

Teacher

What are the steps used to solve a basic equation with one variable?

Teacher

2 Solve the following inequality: -2(x + 4) - 5(x - 6) < 0

A $x > \frac{22}{7}$

B $x < \frac{22}{7}$

C $x > -\frac{22}{7}$

D $x < -\frac{22}{7}$

Teacher

What are the differences between solving equations and inequalities?

Teacher

Write down all of the different ways that you can write the answer to the inequality you just solved,

$$x > \frac{22}{7}$$

Teacher

Remember the connections between Inequality Notation and Interval Notation:

Inequality Notation:	Interval Notation:
$< or > = \circ$	$< or > = (\ ,\ )$
$\leq or \geq = \bullet$	$\leq or \geq = [\ ,\ ]$

Examples:

$-6 < x \leq 35 \longrightarrow (-6, 35]$

$y \geq 20 \longrightarrow [20, \infty)$

Teacher

3 Which of the following is interval notation for: $-3 \leq x \leq 15$

- A [-3, 15]
- B (-3, 15)
- C [15, 3]
- D (15, -3)

Teacher

4 What is the interval notation for the following inequality? x > -2

- A $[\infty, -2]$
- B $[-2, \infty]$
- C $(\infty, -2)$
- D $(-2, \infty)$

Teacher

5 What is the inequality notation for: $-6 \leq m < 31$

- A $(-6, 31]$
- B $[-6, 31)$
- C $[-6, \infty]$ *and* $[31, \infty]$
- D $[-6, 31]$

Teacher

6 What is the inequality notation for: $x \leq -24$

- A $(-\infty, -24)$
- B $(-\infty, -24]$
- C $[-24, -\infty)$
- D $[-24, \infty)$

Teacher

7 Solve the following equation: $x^2 - 4x = 32$

- A x = $2\sqrt{2}$
- B x = $2\sqrt{7}$
- C x = -4
- D x = 8
- E x = -4 or 8
- F Cannot be solved

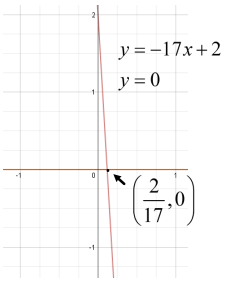
Teacher

What is the difference between a basic equation and a quadratic equation? How can you tell if it is quadratic?

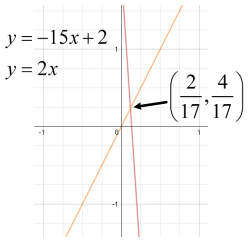
Teacher

Basic Equation

$$-3(x+2)-4(3x-2)=2(x+6)-12$$



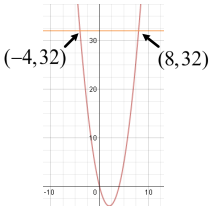
Either way, $x=\frac{2}{17}$.



Teacher

Quadratic Equation

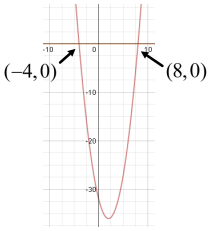
$y=x^2-4x$
$y=32$



$x^2-4x=32$

Either way, $x=-4$ *or* 8

$y=x^2-4x-32$
$y=0$



Teacher

8 Solve the following: $2x^2+5x-12=0$

- A $x=2,-12$
- B $x=2,-3$
- C $x=\frac{3}{2},-4$
- D $x=\frac{2}{3},4$

Teacher

9 Solve the following: $3x^2=-9x$

- A x = 0, -3
- B x = 3, -9
- C x = -3, -9
- D No solution

Teacher

10 Solve the following: $3x^2+x=1$

- A $x=1,3$
- B $x=\frac{1}{3}$
- C $x=-\frac{1}{6}\pm\frac{i\sqrt{10}}{6}$
- D $x=-\frac{1}{6}\pm\frac{\sqrt{13}}{6}$

Basic Equations and Quadratics

11 Solve the following: $3x^3+5x^2-2x=0$

Teacher

A $x=0,-2,\frac{1}{3}$

B $x=3,5,-2$

C $x=0,\frac{1}{2},-3$

D No solution

Basic Equations and Quadratics

12 Solve the following: $2x^2=x-9$

Teacher

A $x=2,9$

B $x=\frac{1}{4},\frac{2}{9}$

C $x=-\frac{1}{4}\pm\frac{\sqrt{73}}{4}$

D $x=\frac{1}{4}\pm\frac{i\sqrt{73}}{4}$

Basic Equations and Quadratics

How would you solve and write an answer for this quadratic inequality?

$12x^2-x-6>0$

Teacher

Basic Equations and Quadratics

Solve: $x^3+2x^2\le 24x$

Teacher

Basic Equations and Quadratics

13 What is the solution to: $3x^2+17x-6\le 0$

Teacher

A $\left[-6,\frac{1}{3}\right]$

C $(-\infty,-6]\cup\left[\frac{1}{3},\infty\right)$

B $\left(\frac{1}{3},-6\right)$

D $(-\infty,-6)\cup\left(\frac{1}{3},\infty\right)$

Basic Equations and Quadratics

14 What is the solution to the following: $4x^3+4x^2-15x>0$

Teacher

A $(-\infty,0)$

C $\left(-\frac{5}{2},0\right)\cup\left(\frac{3}{2},\infty\right)$

B $(-\infty,0)\cup\left(\frac{3}{2},\infty\right)$

D $\left(-\infty,-\frac{5}{2}\right)\cup\left(0,\frac{3}{2}\right)$

15 Solve the following: $x^4 + 5x^3 - 6x^2 < 0$

Teacher

A $(-\infty, -6) \cup (1, \infty)$

B $(-6, 0) \cup (1, \infty)$

C $(-6, 1)$

D $(-6, 0) \cup (0, 1)$

Absolute Value Equations

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Goals and Objectives

Students will be able to solve absolute value equations using the properties of algebra.

Why do we need this?

Many measures in engineering or architecture are never negative. The one that is used most often is distance. Can you run negative three miles? Absolute value allows us to work with such numbers.

Solve: $|3x - 2| = 12$

Teacher

Solve: $-4|2x - 1| + 3 = -21$

Teacher

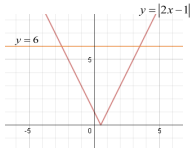
Absolute Value Equations

Could you solve this one graphically?

Isolate the absolute value algebraically and then graph each side of the equation.

$-4|2x-1|+3=-21$

$|2x-1|=6$



Teacher

Absolute Value Equations

16 Solve the following equation: $-2|x+8|-3=-17$

A x = 1, 15

B x = -1, -15

C x = 8, -8

D x = 15 only

Teacher

Absolute Value Equations

17 Solve the following equation: $4|3x+2|-5=95$

A $x=\frac{5}{3},-\frac{13}{3}$

B $x=-4,\frac{29}{3}$

C $x=-9,\frac{25}{3}$

D $x=\frac{23}{3},-9$

Teacher

Absolute Value Equations

18 Solve the following: $-|x-8|+4=-36$

A $x=-24,40$

B $x=-24,-32$

C $x=-36,40$

D $x=-32,48$

Teacher

Absolute Value Equations

19 Solve the following: $|3x+2|-8=-36$

A $x=10,\frac{26}{3}$

B $x=-32,24$

C $x=26,30$

D *no solution*

Teacher

Absolute Value Inequalities

What happens when you have an inequality?

Solve: $2|x-3|\geq 14$

Teacher

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Absolute Value Inequalities

What would this look like if we graphed it to find a solution?

$2|x-3| \geq 14$

Teacher

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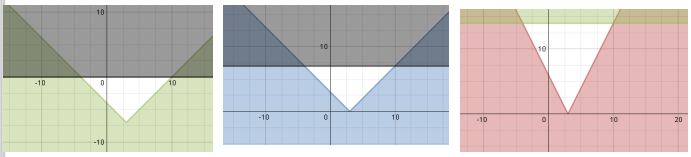
Absolute Value Inequalities

There are several ways to graph this, how were these graphs made?

$2|x-3| \geq 14$



Do they all give you the same solution?

Teacher

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Absolute Value Inequalities

Solve: $-2|x+4|+6 \geq 0$

Teacher

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Absolute Value Inequalities

20 Solve the following inequality: $3|2x-1| \leq 12$

A $\left(-\frac{3}{2},\frac{5}{2}\right)$

B $\left[-\frac{3}{2},\frac{5}{2}\right]$

C $\left(-\infty,-\frac{3}{2}\right)\cup\left(\frac{5}{2},\infty\right)$

D $\left(-\infty,-\frac{3}{2}\right]\cup\left[\frac{5}{2},\infty\right)$

Teacher

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Absolute Value Inequalities

21 Solve: $-2|x-4|+6 \geq -16$

A $(-\infty,-7]\cup[15,\infty)$

B $(-\infty,-7)\cup(15,\infty)$

C $(-7,15)$

D $[-7,15]$

Teacher

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Absolute Value Inequalities

22 Solve: $3|2x-1|-5>10$

A $(-7,8)$

B $[-7,8]$

C $(-\infty,-7)\cup(8,\infty)$

D $(-\infty,-7]\cup[8,\infty)$

Teacher

Equations with Radicals

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Goals and Objectives

Students will be able to solve equations that contain radicals, including checking for extraneous solutions.

Why do we need this?

Not every equation that we solve is linear or quadratic. It is important to solve equations with radicals and to recognize why extraneous solutions exist.

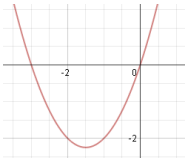
Solve: $3\sqrt{x}-8=2$

Teacher

Solve: $-\sqrt{x+4}+2=x+4$

Teacher

Why do we get extraneous solutions when we solve these equations? Think graphically...



$-\sqrt{x+4}+2=x+4$

Can be turned into...

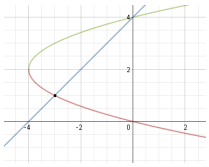
$x^2+3x=0$

But squaring both sides returns a value that is not part of the solution.

Teacher

Equations with Radicals

Also, think about completing the parabola on the original graph. Doesn't it also cross at x = 0? But, x = 0 is extraneous because it is not in the initial problem.



Equations with Radicals

Teacher

23 Solve: $4-\sqrt{x+6}=3$

A x = 1

B x = 7

C x = -5

D x = -7

Equations with Radicals

Teacher

24 Solve: $\sqrt{8x+1}+5=x-3$

A x = 3

B x = 21

C x = 3 and x = 21

D No solution

Equations with Radicals

Teacher

25 Solve: $\sqrt{x-4}-2=x+3$

A $x=\frac{-9\pm\sqrt{197}}{2}$

B $x=\frac{-9\pm\sqrt{35}}{2}$

C x = 4.5

D No Solution

Equations with Radicals

Teacher

26 Solve $\sqrt{6x+5}-2=5x+1$

A $x=-\frac{19}{25},-\frac{1}{5}$

B $x=-4,-24$

C $x=\frac{-12\pm2\sqrt{11}}{25}$

D No Solution

Rational Equations

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Rational Equations

Definition: A Rational Equation is an equation that has one or more terms as fractions.

Solve: $\frac{x}{5}+\frac{3}{4}=\frac{x}{6}-\frac{3}{5}$

Teacher

Rational Equations

Solve: $\frac{2x+5}{3x-1}=\frac{5}{2}$

Teacher

Rational Equations

Solve: $\frac{4}{x}+\frac{3}{x+2}=\frac{1}{x}+3$

Teacher

Rational Equations

The methods most commonly used when solving rational equations are:

1) Removing fractions by multiplying each term by the common denominator.
2) Making common denominators and combining fractions.
3) Getting two fractions equal to each other and cross-multiplying.

Which ones did you use?

Teacher

Rational Equations

What about extraneous solutions?

Teacher

Rational Equations

Solve: $\frac{x^2+2}{18x}-\frac{1}{9x}=\frac{1}{x+3}$

Teacher

Solve: $\frac{-3}{x^2-5x+6}-\frac{2}{x^2-9}=-\frac{1}{x-2}$

Teacher

27 Solve the equation. Remember to check for extraneous solutions.

$$\frac{2}{x}+\frac{3}{5}=\frac{1}{5x}$$

Teacher

28 Solve the equation. Remember to check for extraneous solutions!

$$\frac{2}{3m}+\frac{3m}{m+1}=3$$

Teacher

30 Solve the equation: $\frac{3k+22}{k^2+5k-3}-\frac{2}{2k+1}=\frac{6}{k+3}$

Teacher

Rational Inequalities

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Rational Inequalities

Consider the following inequality. What is the question asking you to find? How can we do this?

$$\frac{2x+1}{x-5} \geq 0$$

Teacher

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Rational Inequalities

Solve: $\frac{3x+1}{x^2+2x+1} < 0$

Teacher

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Rational Inequalities

Solve: $\frac{2x^2+7x-4}{x^2-x-6} \geq 0$

Teacher

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Rational Inequalities

Solve: $\frac{3x^2+9x}{3x^2+5x-2} \leq 0$

Teacher

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Rational Inequalities

31 Solve: $\frac{x(x+3)}{(x-4)(x+1)} > 0$

Teacher

A $(-\infty,-1)\cup(0,4)$

C $(-\infty,-3]\cup[0,\infty)$

B $(-\infty,-3)\cup(-1,0)\cup(4,\infty)$

D $(-\infty,-3]\cup(-1,0]\cup(4,\infty)$

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Rational Inequalities

32 Solve: $\frac{3x^2-10x+3}{x^2+2} < 0$

Teacher

A $\left(\frac{1}{3},3\right)$

C $\left[\frac{1}{3},3\right]$

B $\left(-\infty,\frac{1}{3}\right)\cup(3,\infty)$

D $(-\infty,0)\cup\left(0,\frac{1}{3}\right)\cup(3,\infty)$

Rational Inequalities

33 Solve: $\frac{5x^2+17x+6}{6x^2+12x} \geq 0$

Teacher

A $(-\infty,-3]\cup(0,\infty)$

C $(-\infty,-3]\cup\left(-2,-\frac{2}{5}\right]$

B $(-\infty,-3]\cup\left(-2,-\frac{2}{5}\right]\cup(0,\infty)$

D $(-\infty,-3)\cup\left(-2,-\frac{2}{5}\right)\cup(0,\infty)$

Rational Inequalities

34 Solve: $\frac{2x^2+4}{5x^2+25} \geq 0$

Teacher

A $[2,5]$

C $(-\infty,-5)\cup(-2,\infty)$

B $(-\infty,-5]\cup[-2,\infty)$

D $(-\infty,\infty)$

Exponential and Logarithmic Equations

Solving Exponential and Logarithmic Equations

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Exponential and Logarithmic Equations

Goals and Objectives

Students will be able to use properties of exponential functions and logarithmic functions to solve equations.

Exponential and Logarithmic Equations

Why do we need this?

Most situations that people study do not have linear relationships. Population growth is now heavily studied around the world. Is this a linear function? Why do we study population growth?

Exponential and Logarithmic Equations

Teacher

Pull for Graph

Algebraically, how would you solve: $3^x = 10$

Is this equation exponential or logarithmic?

You could use a graph, but it is sometimes tough to estimate.

Exponential and Logarithmic Equations

Teacher

Convert the following to the opposite form to solve:

$4^x = 25$

$\log_5 x = 4$

Exponential and Logarithmic Equations

Teacher

Convert the following to the opposite form to solve:

$\log_x 40 = 3$

$3^m = 0.003$

Exponential and Logarithmic Equations

Teacher

35 Solve: $\log_5 x = 3$

Exponential and Logarithmic Equations

Teacher

36 Solve: $5^m = 50$

Exponential and Logarithmic Equations

Teacher

37 Solve: $\log_{25} a = -0.2$

Exponential and Logarithmic Equations

38 Solve: $6^x = 27$

Teacher

Exponential and Logarithmic Equations

Sometimes to solve a logarithmic equation, it needs to be put into one of the following forms:

$\log_b a = c$

*After the equation is in this form, you may need to convert to exponential form.

$\log_b a = \log_b c$

*After the equation is in this form, a and c must be equal. Therefore, you may remove the logarithms and solve.

Exponential and Logarithmic Equations

In order to simplify logarithmic equations to one of these forms, you need to use the properties of logarithms. Remember...

$\log_b(mn) = \log_b m + \log_b n$

$\log_b 1 = 0$

$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b b = 1$

$\log_b\left(m^n\right) = n\log_b m$

Exponential and Logarithmic Equations

Example: Use the properties of logarithms to simplify this equation and then solve.

$\log_3(x-3) + \log_3(4) = 4$

Teacher

Exponential and Logarithmic Equations

Solve: $\log_9(r+3) - \log_9(r) = \log_9(r-1)$

Teacher

Exponential and Logarithmic Equations

Solve: $2\log_4 m = 6$

Teacher

Exponential and Logarithmic Equations

39 Solve: $3\log_2(x) = 24$

Teacher

Exponential and Logarithmic Equations

40 Solve: $\log_6(m) + \log_6(m-5) = 2$

Teacher

Exponential and Logarithmic Equations

41 Solve: $\log_m(18) + \log_m(6) = 4$

Teacher

Exponential and Logarithmic Equations

42 Solve: $\ln(x^2 + x) - \ln(x) = \ln(3x - 1)$

Teacher

Exponential and Logarithmic Equations

43 Solve: $\log_8(27) - 2\log_8(p) = \log_8(p)$

Teacher

Exponential and Logarithmic Equations

44 Solve: $\log_5(k) - \frac{1}{3}\log_5(27) + \log_5(4k) = 0$

Teacher

Exponential and Logarithmic Equations

How can we use these concepts to solve:

$$3^{m^2} = 60^m$$

Teacher

Exponential and Logarithmic Equations

Try: $5^{3b-2} = 36^{b+3}$

Teacher

Exponential and Logarithmic Equations

45 Solve: $2^{3m-2} = 16^{m+3}$

Teacher

Exponential and Logarithmic Equations

46 Solve: $4^{3x+1} = 7^{2x}$

Teacher

Exponential and Logarithmic Equations

47 Solve: $6^{x^2-x} = 3^x$

Teacher

This is the end of Solving Equations and Inequalities