BASIC
HEAT AND MASS
TRANSFER

(Chapter 1 Excerpt for BHMT Students - Not for Reproduction)

Third Edition

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To Brigid
For your patience and understanding.

To Kaori
For your loving support.

PREFACE

For this third edition of *Basic Heat and Mass Transfer* Anthony Mills is joined by Carlos Coimbra as a co-author. Professor Coimbra brings to this venture the perspective and skills of a younger generation of heat transfer educators and his own special expertise in areas of heat transfer research. Fifteen years after the second edition was published, a new edition is perhaps overdue, but in a mature field such as heat transfer, it is not at all clear what topics should be introduced, and then what topics should be removed to retain an acceptable length for an introductory text. As a result, our main motivation in publishing a third edition has been a different consideration.

Our concern was the excessive prices of college textbooks, which in recent years have destroyed the established role played by these texts in the education of engineering students. Traditionally, students bought a required textbook, became familiar with it in taking the course, and then retained the book as a tool for subsequent courses and an engineering career. Nowadays the pattern is for a student to sell the textbooks back to the university bookstore at the end of the course in order to obtain funds for buying textbooks for the next term. Alternatively, electronic versions of portions of the text are used during the course, or course readers containing selected material from the text may be used. It is particularly frustrating to instructors of subsequent design and laboratory courses to find that the students no longer have appropriate textbooks. Also, the traditional role formerly played by textbooks as professional manuals for engineering practice has been significantly affected. Basic methodology and data are more easily and reliably obtained from a familiar text than from an internet search.

In an attempt to mitigate these problems and improve the experience of our engineering students we decided to retain creative and publishing rights over the content of this book for this and future editions. A company called Temporal Publishing LLC was created to publish quality engineering textbooks at more reasonable prices.[1]

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[1] Books can be ordered directly at discounted prices at www.temporalpublishing.com

This entailed first converting the previous edition of *Basic Heat and Mass Transfer* to LaTeX, which we could then modify efficiently. Since the conversion proved to be a major project in itself, our objective with this third edition is rather modest. We have focused on corrections, clarifications, minor updates and the production of a dedicated companion website.[2] We envisage this website to be an integral part of the project and hope to make it a really useful adjunct to the text, for both students and instructors. The website contains links to the dedicated software BHMT that automates most of the calculations done in the text, instructor aides (such as complete solutions manual for adoptees of the text, additional examples and exercises, presentations, etc.) and a compilation of answers to odd-numbered exercises to assist self-study by students. We will be continuously adding new technical content to the website while we work on future editions of the textbook. Also, given our closer association with the print-on-demand process, it will be easy for the authors to implement small improvements in subsequent printings of this edition. We certainly welcome input and suggestions from users to improve our product.

In preparing this new edition we have had valuable assistance from:

Marius Andronie

Kuang Chao

Kaori Yoshida Coimbra

We would like to dedicate the collaborative effort of bringing a new edition of *Basic Heat and Mass Transfer* to the memory of Prof. Donald K. Edwards, our teacher.

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[2] www.temporalpublishing.com/bhmt-students

PREFACE TO THE SECOND EDITION

Basic Heat and Mass Transfer has been written for undergraduate students in mechanical engineering programs. Apart from the usual lower-division mathematics and science courses, the preparation required of the student includes introductory courses in fluid mechanics and thermodynamics, and preferably the usual junior-level engineering mathematics course. The ordering of the material and the pace at which it is presented have been carefully chosen so that the beginning student can proceed from the most elementary concepts to those that are more difficult. As a result, the book should prove to be quite versatile. It can be used as the text for an introductory course during the junior or senior year, although the coverage is sufficiently comprehensive for use as a reference work in undergraduate laboratory and design courses, and by the practicing engineer.

Throughout the text, the emphasis is on engineering calculations, and each topic is developed to a point that will provide students with the tools needed to practice the art of design. The worked examples not only illustrate the use of relevant equations but also teach modeling as both an art and science. A supporting feature of *Basic Heat and Mass Transfer* is the fully integrated software available from the author's website[3]. The software is intended to serve primarily as a tool for the student, both at college and after graduation in engineering practice. The programs are designed to reduce the effort required to obtain reliable numerical results and thereby increase the efficiency and effectiveness of the engineer. I have found the impact of the software on the educational process to be encouraging. It is now possible to assign more meaningful and interesting problems, because the students need not get bogged down in lengthy calculations. Parametric studies, which are the essence of engineering design, are relatively easily performed. Of course, computer programs are not a substitute for a proper understanding. The instructor is free to choose the extent to

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[3] http://www.mae.ucla.edu/people/faculty/anthony-mills

which the software is used by students because of the unique exact correspondence between the software and the text material. My practice has been to initially require students to perform various hand calculations, using the computer to give immediate feedback. For example, they do not have to wait a week or two until homework is returned to find that a calculated convective heat transfer coefficient was incorrect because a property table was misread.

The extent to which engineering design should be introduced in a heat transfer course is a controversial subject. It is my experience that students can be best introduced to design methodology through an increased focus on equipment such as heat and mass exchangers: *Basic Heat and Mass Transfer* presents more extensive coverage of exchanger design than do comparable texts. In the context of such equipment one can conveniently introduce topics such as synthesis, parametric studies, tradeoffs, optimization, economics, and material or health constraints. The computer programs HEX2 and CTOWER assist the student to explore the consequences of changing the many parameters involved in the design process. If an appropriate selection of this material is taught, I am confident that Accreditation Board for Engineering and Technology guidelines for design content will be met. More important, I believe that engineering undergraduates are well served by being exposed to this material, even if it means studying somewhat less heat transfer science.

More than 300 new exercises have been added for this edition. They fall into two categories: (1) relatively straightforward exercises designed to help students understand fundamental concepts, and (2) exercises that introduce new technology and that have a practical flavor. The latter play a very important role in motivating students; considerable care has been taken to ensure that they are realistic in terms of parameter values and focus on significant aspects of real engineering problems. The practical exercises are first steps in the engineering design process and many have substantial design content. Since environmental considerations have required the phasing out of CFC refrigerants, R-12 and R-113 property data, worked examples and exercises, have been replaced with corresponding material for R-22 and R-134a.

Basic Heat and Mass Transfer complements *Heat Transfer*, which is published concurrently. *Basic Heat and Mass Transfer* was developed by omitting some of the more advanced heat transfer material from *Heat Transfer* and adding a chapter on mass transfer. As a result, *Basic Heat and Mass Transfer* contains the following chapters and appendixes:

1. Introduction and Elementary Heat Transfer
2. Steady One-Dimensional Heat Conduction
3. Multidimensional and Unsteady Conduction
4. Convection Fundamentals and Correlations
5. Convection Analysis
6. Thermal Radiation
7. Condensation, Evaporation, and Boiling
8. Heat Exchangers

9. Mass Transfer

A. Property Data

B. Units, Conversion Factors, and Mathematics

C. Charts

In a first course, the focus is always on the key topics of conduction, convection, radiation, and heat exchangers. Particular care has been taken to order the material on these topics from simpler to more difficult concepts. In Chapter 2 one-dimensional conduction and fins are treated before deriving the general partial differential heat conduction equation in Chapter 3. In Chapter 4 the student is taught how to use convection correlations before encountering the partial differential equations governing momentum and energy conservation in Chapter 5. In Chapter 6 radiation properties are introduced on a total energy basis and the shape factor is introduced as a geometrical concept to allow engineering problem solving before having to deal with the directional and spectral aspects of radiation. Also, wherever possible, advanced topics are located at the ends of chapters, and thus can be easily omitted in a first course.

Chapter 1 is a brief but self-contained introduction to heat transfer. Students are given an overview of the subject and some material needed in subsequent chapters. Interesting and relevant engineering problems can then be introduced at the earliest opportunity, thereby motivating student interest. All the exercises can be solved without accessing the property data in Appendix A.

Chapters 2 and 3 present a relatively conventional treatment of heat conduction, though the outdated and approximate Heissler and Grober charts are replaced by exact charts and the computer program COND2. The treatment of finite-difference numerical methods for conduction has been kept concise and is based on finite-volume energy balances. Students are encouraged to solve the difference equations by writing their own computer programs, or by using standard mathematics software such as Mathcad or MATLAB.

In keeping with the overall philosophy of the book, the objective of Chapter 4 is to develop the students' ability to calculate convective heat transfer coefficients. The physics of convection is explained in a brief introduction, and the heat transfer coefficient is defined. Dimensional analysis using the Buckingham pi theorem is used to introduce the required dimensional groups and to allow a discussion of the importance of laboratory experiments. A large number of correlation formulas follow; instructors can discuss selected geometrical configurations as class time allows, and students can use the associated computer program CONV to reliably calculate heat transfer coefficients and skin friction coefficients or pressure drop for a wide range of configurations. Being able to do parametric studies with a wide variety of correlations enhances the students' understanding more than can be accomplished by hand calculations. Design alternatives can also be explored using CONV.

Analysis of convection is deferred to Chapter 5: simple laminar flows are considered, and high-speed flows are treated first in Section 5.2, since an understanding of the recovery temperature concept enhances the students' problem-solving capabilities. Mixing length turbulence models are briefly discussed, and the chapter closes with a development of the general conservation equations.

Chapter 6 focuses on thermal radiation. Radiation properties are initially defined on a total energy basis, and the shape factor is introduced as a simple geometrical concept. This approach allows students to immediately begin solving engineering radiation exchange problems. Only subsequently need they tackle the more difficult directional and spectral aspects of radiation. For gas radiation, the ubiquitous Hottel charts have been replaced by the more accurate methods developed by Edwards; the accompanying computer program RAD3 makes their use particularly simple.

The treatment of condensation and evaporation heat transfer in Chapter 7 has novel features, while the treatment of pool boiling is quite conventional. Heatpipes are dealt with in some detail, enabling students to calculate the wicking limit and to analyze the performance of simple gas-controlled heatpipes.

Chapter 8 expands the presentation of the thermal analysis of heat exchangers beyond the customary and includes the calculation of exchanger pressure drop, thermal-hydraulic design, heat transfer surface selection for compact heat exchangers, and economic analysis leading to the calculation of the benefit-cost differential associated with heat recovery operations. The computer program HEX2 serves to introduce students to computer-aided design of heat exchangers.

Chapter 9 is an introduction to mass transfer. The focus is on diffusion in a stationary medium and low mass-transfer rate convection. As was the case with heat convection in Chapter 4, mass convection is introduced using dimensional analysis and the Buckingham pi theorem. Of particular importance to mechanical engineers is simultaneous heat and mass transfer, and this topic is given detailed consideration with a focus on problems involving water evaporation into air.

The author and publisher appreciate the-efforts of all those who provided input that helped develop and improve the text. We remain dedicated to further refining the text in future editions, and encourage you to contact us with any suggestions or comments you might have.

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Some of the material in *Basic Heat and Mass Transfer*, in the form of examples and exercises, has been adapted from an earlier text by my former colleagues at UCLA, D. K. Edwards and V. E. Denny (*Transfer Processes* 1/e, Holt, Rinehart & Winston, 1973; 2/e Hemisphere-McGraw-Hill, 1979). I have also drawn on material in radiation heat transfer from a more recent text by D. K. Edwards (*Radiation Heat Transfer Notes*, Hemisphere, 1981). I gratefully acknowledge the contributions of these gentlemen, both to this book and to my professional career. The late D. N. Bennion provided a chemical engineering perspective to some of the material on mass exchangers. The computer software was ably written by Baek Youn, Hae-Jin Choi, and Benjamin Tan. I would also like to thank former students S. W. Hiebert, R. Tsai, B. Cowan, E. Myhre, B. H. Chang, D. C. Weatherly, A. Gopinath, J. I. Rodriguez, B. P. Dooher, M. A. Friedman, and C. Yuen.

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NOTES TO THE INSTRUCTOR AND STUDENT

These notes have been prepared to assist the instructor and student and should be read before the text is used. Topics covered include conventions for artwork and mathematics, the format for example problems, organization of the exercises, comments on the thermophysical property data in Appendix A, and a guide for use of the accompanying computer software.

ARTWORK

Conventions used in the figures are as follows.

→ Conduction or convection heat flow
⇝ Radiation heat flow
⇾ Fluid flow
→ Species flow

MATHEMATICAL SYMBOLS

Symbols that may need clarification are as follows.

$\simeq$ Nearly equal
$\sim$ Of the same order of magnitude
$\big|_x$ All quantities in the term to the left of the bar are evaluated at x

EXAMPLES

Use of standard format for presenting the solutions of engineering problems is a good practice. The format used for the examples in *Basic Heat and Mass Transfer*, which is but one possible approach, is as follows.

Problem statement

Solution

Given:

Required:

Assumptions: 1.
2. etc.

Sketch (when appropriate)
Analysis (diagrams when appropriate)
Properties evaluation
Calculations
Results (tables or graphs when appropriate)

Comments

1.
2. etc.

It is always assumed that the problem statement precedes the solution (as in the text) or that it is readily available (as in the *Solutions Manual*). Thus, the *Given* and *Required* statements are concise and focus on the essential features of the problem. Under *Assumptions*, the main assumptions required to solve the problem are listed; when appropriate, they are discussed further in the body of the solution. A sketch of the physical system is included when the geometry requires clarification; also, expected temperature and concentration profiles are given when appropriate. (Schematics that simply repeat the information in the problem statements are used sparingly. We know that many instructors always require a schematic. Our view is that students need to develop an appreciation of when a figure or graph is necessary, because artwork is usually an expensive component of engineering reports. For example, we see little use for a schematic that shows a 10 m length of straight 2 cm–O.D. tube.) The analysis may consist simply of listing some formulas from the text, or it may require setting up a differential equation and its solution. Strictly speaking, a property should not be evaluated until its need is identified by the analysis. However, in routine calculations, such as evaluation of convective heat transfer coefficients, it is often convenient to list all the property values taken from an Appendix A table in one place. The calculations then follow with results listed, tabulated, or graphed as appropriate. Under *Comments*, the significance of the results can be discussed, the validity of assumptions further evaluated, or the broader implications of the problem noted.

In presenting calculations for the examples in *Basic Heat and Mass Transfer*, we have rounded off results at each stage of the calculation. If additional figures are retained for the complete calculations, discrepancies in the last figure will be observed. Since many of the example calculations are quite lengthy, we believe our policy will facilitate checking a particular calculation step of concern. As is common practice, we have generally given results to more significant figures than is justified, so that these results can be conveniently used in further calculations. It is safe to say that no engineering heat transfer calculation will be accurate to within 1%, and that most experienced engineers will be pleased with results accurate to within 10% or 20%. Thus, preoccupation with a third or fourth significant figure is misplaced (unless required to prevent error magnification in operations such as subtraction). Fundamental constants are rounded off to no more than five significant figures.

EXERCISES

The diskette logo next to an exercise statement indicates that it can be solved using the *Basic Heat and Mass Transfer* software, and that the sample solution provided to the instructor has been prepared accordingly. There are many additional exercises that can be solved using the software but that do not have the logo designation. These exercises are intended to give the student practice in hand calculations, and thus the sample solutions were also prepared manually.

The exercises have been ordered to correspond with the order in which the material is presented in the text, rather than in some increasing degree of difficulty. Since the range of difficulty of the exercises is considerable, the instructor is urged to give students guidance in selecting exercises for self-study. Answers to all exercises are listed in the *Solutions Manual* provided to instructors. Odd- and even-numbered exercises are listed separately; answers to odd-numbered exercises are available to students on the book website.

PROPERTY DATA

A considerable quantity of property data has been assembled in Appendix A. Key sources are given as references or are listed in the bibliography. Since *Basic Heat and Mass Transfer* is a textbook, our primary objective in preparing Appendix A was to provide the student with a wide range of data in an easily used form. Whenever possible, we have used the most accurate data that we could obtain, but accuracy was not always the primary concern. For example, the need to have consistent data over a wide range of temperature often dictated the choice of source. All the tables are in SI units, with temperature in kelvins. The computer program UNITS can be used for conversions to other systems of units. Appendix A should serve most needs of the student, as well as of the practicing engineer, for doing routine calculations. If a heat transfer research project requires accurate and reliable thermophysical property data, the prudent researcher should carefully check relevant primary data sources.

SOFTWARE

The *Basic Heat and Mass Transfer* software has a menu that describes the content of each program. The programs are also described at appropriate locations in the text. The input format and program use are demonstrated in example problems in the text. Use of the text index is recommended for locating the program descriptions and examples. There is a one-to-one correspondence between the text and the software. In principle, all numbers generated by the software can be calculated manually from formulas, graphs, and data given in the text. Small discrepancies may be seen when interpolation in graphs or property tables is required, since some of the data are stored in the software as polynomial curve fits.

The software facilitates self-study by the student. Practice hand calculations can be immediately checked using the software. When programs such as CONV, PHASE, and BOIL are used, properties evaluation and intermediate calculation steps can also be checked when the final results do not agree.

Since there is a large thermophysical property database stored in the software package, the programs can also be conveniently used to evaluate these properties for other purposes. For example, in CONV both the wall and fluid temperatures can be set equal to the desired temperature to obtain property values required for convection calculations. We can even go one step further when evaluating a convective heat transfer coefficient from a new correlation not contained in CONV: if a corresponding item is chosen, the values of relevant dimensionless groups can also be obtained from CONV, further simplifying the calculations.

Presently the BHMT software is only available in a DOS version, which runs on both Mac OS X and Windows platforms with DOS emulators. We are preparing a Windows version of the BHMT software and will announce its availability on the website. However, all the heat transfer components (excluding material relevant to Chapter 9 only) are already available in the Windows HT (Heat Transfer) package on the book companion website at www.temporalpublishing.com/bhmt. Some examples in the text show sample inputs from the DOS version of the software. For the Windows version the inputs are essentially the same.

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CHAPTER

1

INTRODUCTION AND ELEMENTARY HEAT TRANSFER

CONTENTS

1.1 INTRODUCTION

The process of heat transfer is familiar to us all. On a cold day we put on more clothing to reduce heat transfer from our warm body to cold surroundings. To make a cup of coffee we may plug in a kettle, inside which heat is transferred from an electrical resistance element to the water, heating the water until it boils. The engineering discipline of **heat transfer** is concerned with methods of calculating **rates** of heat transfer. These methods are used by engineers to design components and systems in which heat transfer occurs. Heat transfer considerations are important in almost all areas of technology. Traditionally, however, the discipline that has been most concerned with heat transfer is mechanical engineering because of the importance of heat transfer in energy conversion systems, from coal-fired power plants to solar water heaters.

Many *thermal design* problems require reducing heat transfer rates by providing suitable *insulation*. The insulation of buildings in extreme climates is a familiar example, but there are many others. The space shuttle has thermal tiles to insulate the vehicle from high-temperature air behind the bow shock wave during reentry into the atmosphere. Cryostats, which maintain the cryogenic temperatures required for the use of superconductors, must be effectively insulated to reduce the cooling load on the refrigeration system. Often, the only way to ensure protection from severe heating is to provide a fluid flow as a heat "sink". Nozzles of liquid-fueled rocket motors are cooled by pumping the cold fuel through passages in the nozzle wall before injection into the combustion chamber. A critical component in a fusion reactor is the "first wall" of the containment vessel, which must withstand intense heating from the hot plasma. Such walls may be cooled by a flow of helium gas or liquid lithium.

A common thermal design problem is the transfer of heat from one fluid to another. Devices for this purpose are called *heat exchangers.* A familiar example is the automobile radiator, in which heat is transferred from the hot engine coolant to cold air blowing through the radiator core. Heat exchangers of many different types are required for power production and by the process industries. A power plant, whether the fuel be fossil or nuclear, has a *boiler* in which water is evaporated to produce steam to drive the turbines, and a *condenser* in which the steam is condensed to provide a low back pressure on the turbines and for water recovery. The condenser patented by James Watt in 1769 more than doubled the efficiency of steam engines then being used and set the Industrial Revolution in motion. The common vapor cycle refrigeration or air-conditioning system has an *evaporator* where heat is absorbed at low temperature and a *condenser* where heat is rejected at a higher temperature. On a domestic refrigerator, the condenser is usually in the form of a tube coil with cooling *fins* to assist transfer of heat to the surroundings. An oil refinery has a great variety of heat transfer equipment, including rectification columns and thermal crackers. Many heat exchangers are used to transfer heat from one process stream to another, to reduce the total energy consumption by the refinery.

Often the design problem is one of *thermal control*, that is, maintaining the operating temperature of temperature-sensitive components within a specified range.

Cooling of all kinds of electronic gear is an example of thermal control. The development of faster computers is now severely constrained by the difficulty of controlling the temperature of very small components, which dissipate large amounts of heat. Thermal control of temperature-sensitive components in a communications satellite orbiting the Earth is a particularly difficult problem. Transistors and diodes must not overheat, batteries must not freeze, telescope optics must not lose alignment due to thermal expansion, and photographs must be processed at the proper temperature to ensure high resolution. Thermal control of space stations present even greater problems, since reliable life-support systems are also necessary.

From the foregoing examples, it is clear that heat transfer involves a great variety of physical phenomena and engineering systems. The phenomena must first be understood and quantified before a methodology for the thermal design of an engineering system can be developed. Chapter 1 is an overview of the subject and introduces key topics at an elementary level. In Section 1.2, the distinction between the subjects of heat transfer and thermodynamics is explained. The first law of thermodynamics is reviewed, and closed- and open-system forms required for heat transfer analysis are developed. Section 1.3 introduces the three important modes of heat transfer: **heat conduction**, **thermal radiation**, and **heat convection**. Some formulas are developed that allow elementary heat transfer calculations to be made. In practical engineering problems, these modes of heat transfer usually occur simultaneously. Thus, in Section 1.4, the analysis of heat transfer by combined modes is introduced. Engineers are concerned with the changes heat transfer processes effect in engineering systems and, in Section 1.5, an example is given in which the first law is applied to a simple model closed system to determine the temperature response of the system with time. In Section 1.6, the subject of mass transfer is briefly introduced and its relation to heat transfer explained. Finally, in Section 1.7, the International System of units (SI) is reviewed, and the units policy that is followed in the text is discussed.

1.2 HEAT TRANSFER AND ITS RELATION TO THERMODYNAMICS

When a hot object is placed in cold surroundings, it cools: the object loses internal energy, while the surroundings gain internal energy. We commonly describe this interaction as a *transfer of heat* from the object to the surrounding region. Since the caloric theory of heat has been long discredited, we do not imagine a “heat substance” flowing from the object to the surroundings. Rather, we understand that internal energy has been transferred by complex interactions on an atomic or subatomic scale. Nevertheless, it remains common practice to describe these interactions as transfer, transport, or flow, of heat. The engineering discipline of heat transfer is concerned with calculation of the rate at which heat flows within a medium, across an interface, or from one surface to another, as well as with the calculation of associated temperatures.

It is important to understand the essential difference between the engineering discipline of heat transfer and what is commonly called thermodynamics. Classical thermodynamics deals with systems in equilibrium. Its methodology may be used to calculate the energy required to change a system from one equilibrium state to another, but it cannot be used to calculate the rate at which the change may occur. For example, if a 1 kg ingot of iron is quenched from 1000°C to 100°C in an oil bath, thermodynamics tells us that the loss in internal energy of the ingot is mass (1 kg) × specific heat capacity (∼450 J/kg K) × temperature change (900 K), or approximately 405 kJ. But thermodynamics cannot tell us how long we will have to wait for the temperature to drop to 100°C. The time depends on the temperature of the oil bath, physical properties of the oil, motion of the oil, and other factors. An appropriate heat transfer analysis will consider all of these.

Analysis of heat transfer processes does require using some thermodynamics concepts. In particular, the **first law of thermodynamics** is used, generally in particularly simple forms since work effects can often be ignored. The first law is a statement of the *principle of conservation of energy*, which is a basic law of physics. This principle can be formulated in many ways by excluding forms of energy that are irrelevant to the problem under consideration, or by simply redefining what is meant by energy. In heat transfer, it is common practice to refer to the first law as the *energy conservation principle* or simply as an *energy* or *heat balance* when no work is done. However, as in thermodynamics, it is essential that the correct form of the first law be used. The student must be able to define an appropriate system, recognize whether the system is *open* or *closed*, and decide whether a steady state can be assumed. Some simple forms of the energy conservation principle, which find frequent use in this text, follow.

A closed system containing a fixed mass of a solid is shown in Fig. 1.1. The system has a volume V[m^3], and the solid has a density ρ [kg/m^3]. There is *net* heat transfer into the system at a rate of $\dot{Q}$ [J/s or W], and heat may be generated within the solid, for example, by nuclear fission or by an electrical current, at a rate $\dot{Q}_v$ [W]. Solids may be taken to be incompressible, so no work is done by or on the system. The principle of conservation of energy requires that over a time interval Δt [s],

$$\begin{array}{c}\text{Change in internal energy}\\ \text{within the system}\end{array} = \begin{array}{c}\text{Net heat transferred}\\ \text{into the system}\end{array} + \begin{array}{c}\text{Heat generated}\\ \text{within the system}\end{array}$$

$$\Delta U = \dot{Q}\Delta t + \dot{Q}_v \Delta t \tag{1.1}$$

Dividing by Δt and letting Δt go to zero gives

$$\frac{dU}{dt} = \dot{Q} + \dot{Q}_v$$

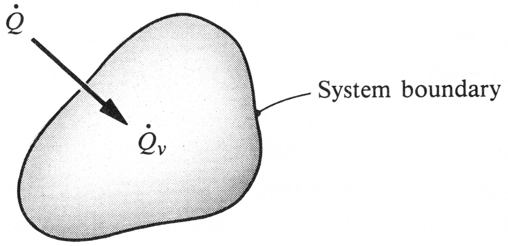

Figure 1.1 Application of the energy conservation principle to a closed system.

The system contains a fixed mass (ρV); thus, we can write $dU = \rho V du$, where u is the specific internal energy [J/kg]. Also, for an incompressible solid, $du = c_v dT$, where c_v is the constant-volume specific heat[1] [J/kg K], and T [K] is temperature. Since the solid has been taken to be incompressible, the constant-volume and constant-pressure specific heats are equal, so we simply write $du = c dT$ to obtain

$$\rho V c \frac{dT}{dt} = \dot{Q} + \dot{Q}_v \tag{1.2}$$

Equation (1.2) is a special form of the first law of thermodynamics that will be used often in this text. It is written on a *rate* basis; that is, it gives the rate of change of temperature with time. For some purposes, however, it will prove convenient to return to Eq. (1.1) as a statement of the first law.

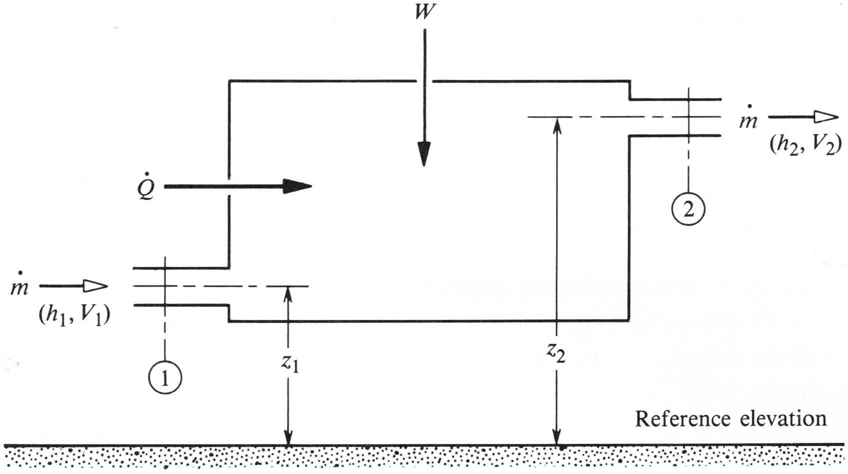

Figure 1.2 Application of the energy conservation principle to a steady-flow open system.

Figure 1.2 shows an *open* system (or *control volume*), for which a useful form of the first law is the **steady-flow energy equation**. It is used widely in the thermodynamic analysis of equipment such as turbines and compressors. Then

$$\dot{m}\Delta\left(h + \frac{V^2}{2} + gz\right) = \dot{Q} + \dot{W} \tag{1.3}$$

where $\dot{m}$ [kg/s] is the mass flow rate, h [J/kg] is the specific enthalpy, V [m/s] is velocity, g [m/s^2] is the gravitational acceleration, z is elevation [m], $\dot{Q}$ [W] is the net rate of heat transfer, as before, and $\dot{W}$ [W] is the rate at which external (shaft) work is done on the system.[2] Notice that the sign convention here is that external work done *on* the system is positive; the opposite sign convention is also widely used. The symbol ΔX means $X_{\text{out}} - X_{\text{in}}$, or the change in X. Equation (1.3) applies to a pure

---

[1] The terms *specific heat capacity* and *specific heat* are equivalent and interchangeable in the heat transfer literature.

[2] Equation (1.3) has been written as if h, V, and z are uniform in the streams crossing the control volume boundary. Often such an assumption can be made; if not, an integration across each stream is required to give appropriate average values.

substance when conditions within the system, such as temperature and velocity, are unchanging over some appropriate time interval. Heat generation within the system has not been included. In many types of heat transfer equipment, no external work is done, and changes in kinetic and potential energy are negligible; Eq. (1.3) then reduces to

$$\dot{m}\Delta h = \dot{Q} \tag{1.4}$$

The specific enthalpy h is related to the specific internal energy u as

$$h = u + Pv \tag{1.5}$$

where P [N/m^2 or Pa] is pressure, and v is specific volume [m^3/kg]. Two limit forms of Δh are useful. If the fluid enters the system at state 1 and leaves at state 2:

1. For ideal gases with $Pv = RT$,

   $$\Delta h = \int_{T_1}^{T_2} c_p dT \tag{1.6a}$$

   where R [J/kg K] is the gas constant and c_p [J/kg K] is the constant-pressure specific heat.
2. For incompressible liquids with $\rho = 1/v =$ constant

   $$\Delta h = \int_{T_1}^{T_2} c dT + \frac{P_2 - P_1}{\rho} \tag{1.6b}$$

   where $c = c_v = c_p$. The second term in Eq. (1.6b) is often negligible as will be assumed throughout this text.

Equation (1.4) is the usual starting point for the heat transfer analysis of steady-state open systems.

The *second law of thermodynamics* tells us that if two objects at temperatures T_1 and T_2 are connected, and if $T_1 > T_2$, then heat will flow spontaneously and irreversibly from object 1 to object 2. Also, there is an entropy increase associated with this heat flow. As T_2 approaches T_1, the process approaches a reversible process, but simultaneously the rate of heat transfer approaches zero, so the process is of little practical interest. All heat transfer processes encountered in engineering are irreversible and generate entropy. With the increasing realization that energy supplies should be conserved, efficient use of available energy is becoming an important consideration in thermal design. Thus, the engineer should be aware of the irreversible processes occurring in the system under development and understand that the optimal design may be one that minimizes entropy generation due to heat transfer and fluid flow. Most often, however, energy conservation is simply a consideration in the overall economic evaluation of the design. Usually there is an important trade-off between energy costs associated with the operation of the system and the capital costs required to construct the equipment.

1.3 MODES OF HEAT TRANSFER

In thermodynamics, *heat* is defined as energy transfer due to temperature gradients or differences. Consistent with this viewpoint, thermodynamics recognizes only two modes of heat transfer: *conduction* and *radiation.* For example, heat transfer across a steel pipe wall is by conduction, whereas heat transfer from the Sun to the Earth or to a spacecraft is by thermal radiation. These modes of heat transfer occur on a molecular or subatomic scale. In air at normal pressure, conduction is by molecules that travel a very short distance ($\sim 0.065 \mu m$) before colliding with another molecule and exchanging energy. On the other hand, radiation is by photons, which travel almost unimpeded through the air from one surface to another. Thus, an important distinction between conduction and radiation is that the energy carriers for conduction have a short *mean free path,* whereas for radiation the carriers have a long mean free path. However, in air at the very low pressures characteristic of high-vacuum equipment, the mean free path of molecules can be much longer than the equipment dimensions, so the molecules travel unimpeded from one surface to another. Then heat transfer by molecules is governed by laws analogous to those for radiation.

A fluid, by virtue of its mass and velocity, can transport momentum. In addition, by virtue of its temperature, it can transport energy. Strictly speaking, *convection* is the transport of energy by bulk motion of a medium (a moving solid can also convect energy in this sense). In the steady-flow energy equation, Eq. (1.3), convection of internal energy is contained in the term $\dot{m}\Delta h$, which is on the left-hand side of the equation, and heat transfer by conduction and radiation is on the right-hand side, as $\dot{Q}$. However, it is common engineering practice to use the term *convection* more broadly and describe heat transfer from a surface to a moving fluid also as convection, or *convective heat transfer,* even though conduction and radiation play a dominant role close to the surface, where the fluid is stationary. In this sense, convection is usually regarded as a distinct mode of heat transfer. Examples of convective heat transfer include heat transfer from the radiator of an automobile or to the skin of a hypersonic vehicle. Convection is often associated with a change of phase, for example, when water boils in a kettle or when steam condenses in a power plant condenser. Owing to the complexity of such processes, boiling and condensation are often regarded as distinct heat transfer processes.

The hot water home heating system shown in Fig. 1.3 illustrates the modes of heat transfer. Hot water from the furnace in the basement flows along pipes to radiators located in individual rooms. Transport of energy by the hot water from the basement is true convection as defined above; we do not call this a heat transfer process. Inside the radiators, there is convective heat transfer from the hot water to the radiator shell, conduction across the radiator shell, and both convective and radiative heat transfer from the hot outer surface of the radiator shell into the room. The convection is *natural* convection: the heated air adjacent to the radiator surface rises due to its buoyancy, and cooler air flows in to take its place. The radiators are heat exchangers. Although commonly used, the term *radiator* is misleading since heat transfer

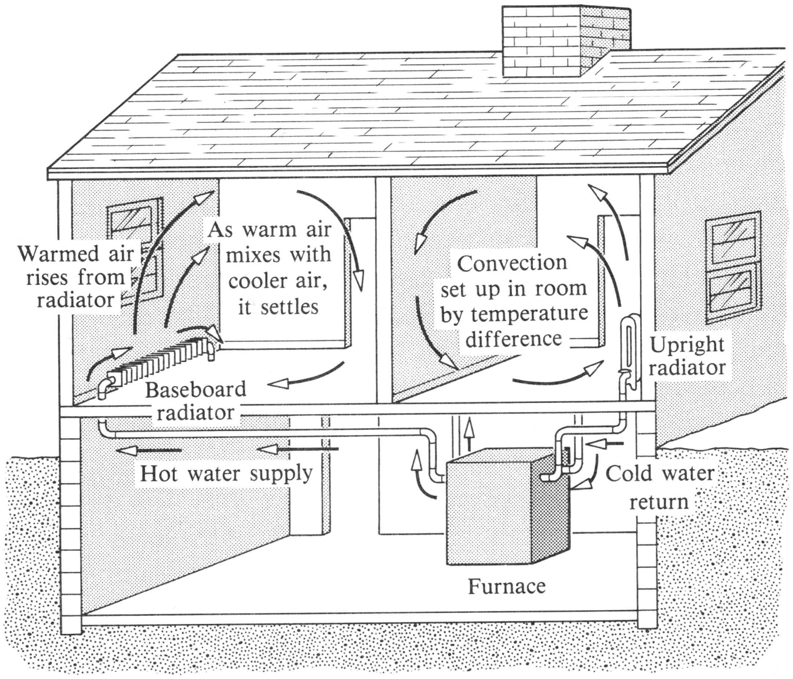

Figure 1.3 A hot-water home heating system illustrating the modes of heat transfer.

from the shell surface can be predominantly by convection rather than by radiation (see Exercise 1–20). Heaters that transfer heat predominantly by radiation are, for example, electrical resistance wire units.

Each of the three important subject areas of heat transfer will now be introduced: conduction, in Section 1.3.1; radiation, in Section 1.3.2; and convection, in Section 1.3.3.

1.3.1 Heat Conduction

On a microscopic level, the physical mechanisms of conduction are complex, encompassing such varied phenomena as molecular collisions in gases, lattice vibrations in crystals, and flow of free electrons in metals. However, if at all possible, the engineer avoids considering processes at the microscopic level, preferring to use *phenomenological laws,* at a macroscopic level. The phenomenological law governing heat conduction was proposed by the French mathematical physicist J. B. Fourier in 1822. This law will be introduced here by considering the simple problem of one-dimensional heat flow across a plane wall—for example, a layer of insulation.[3] Figure 1.4 shows a plane wall of surface area A and thickness L, with its face at $x = 0$ maintained at temperature T_1 and the face at $x = L$ maintained at T_2. The heat flow $\dot{Q}$ through the wall is in the direction of decreasing temperature: if

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[3] In thermodynamics, the term *insulated* is often used to refer to a *perfectly* insulated (zero-heat-flow or adiabatic) surface. In practice, insulation is used to *reduce* heat flow and seldom can be regarded as perfect.

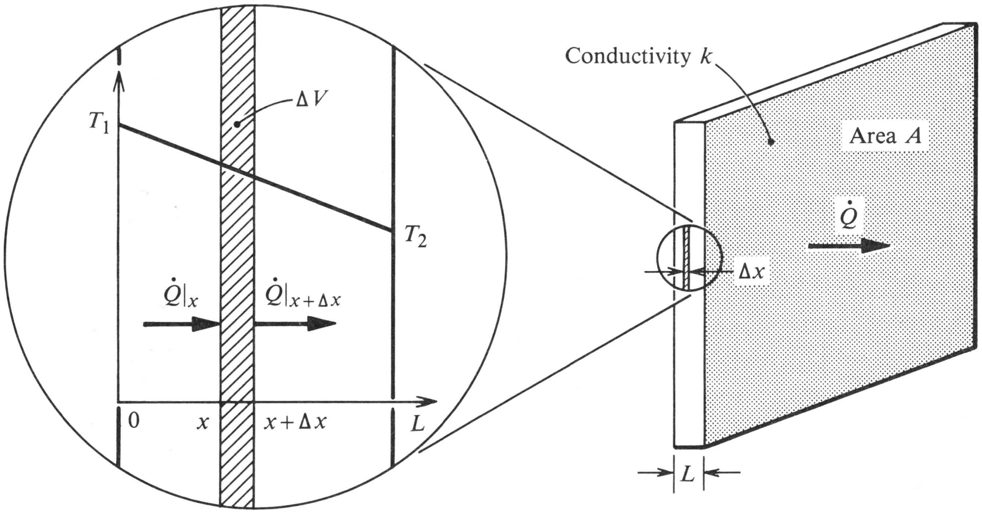


Figure 1.4 Steady one-dimensional conduction across a plane wall, showing the application of the energy conservation principle to an elemental volume Δx thick.

$T_1 > T_2$, $\dot{Q}$ is in the positive x direction.[4] The phenomenological law governing this heat flow is **Fourier's law of heat conduction,** which states that in a homogeneous substance, the local heat flux is proportional to the negative of the local temperature gradient:

$$\frac{\dot{Q}}{A} = q \quad \text{and} \quad q \propto -\frac{dT}{dx} \tag{1.7}$$

where q is the heat flux, or heat flow per unit area perpendicular to the flow direction [W/m^2], T is the local temperature [K or °C], and x is the coordinate in the flow direction [m]. When dT/dx is negative, the minus sign in Eq. (1.7) gives a positive q in the positive x direction. Introducing a constant of proportionality k,

$$q = -k\frac{dT}{dx} \tag{1.8}$$

where k is the **thermal conductivity** of the substance and, by inspection of the equation, must have units [W/m K]. Notice that temperature can be given in kelvins or degrees Celsius in Eq. (1.8): the temperature gradient does not depend on which of these units is used since one kelvin equals one degree Celsius (1 K = 1°C). Thus, the units of thermal conductivity could also be written [W/m°C], but this is not the recommended practice when using the SI system of units. The magnitude of the thermal conductivity k for a given substance very much depends on its microscopic structure and also tends to vary somewhat with temperature; Table 1.1 gives some selected values of k.

---

[4] Notice that this $\dot{Q}$ is the heat flow in the x direction, whereas in the first law, Eqs. (1.1)–(1.4), $\dot{Q} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}$ is the net heat transfer into the whole system. In linking thermodynamics to heat transfer, some ambiguity in notation arises when common practice in both subjects is followed.

Table 1.1 Selected values of thermal conductivity at 300 K (~ 25°C).

Material	k W/m K
Copper	386
Aluminum	204
Brass (70% Cu, 30% Zn)	111
Mild steel	64
Stainless steel, 18–8	15
Mercury	8.4
Concrete	1.4
Pyrex glass	1.09
Water	0.611
Neoprene rubber	0.19
Engine oil, SAE 50	0.145
White pine, perpendicular to grain	0.10
Polyvinyl chloride (PVC)	0.092
Freon 12	0.071
Cork	0.043
Fiberglass (medium density)	0.038
Polystyrene	0.028
Air	0.027

Note: Appendix A contains more comprehensive data.

Figure 1.4 shows an elemental volume ΔV located between x and $x + \Delta x$; ΔV is a closed system, and the energy conservation principle in the form of Eq. (1.2) applies. If we consider a steady state, then temperatures are unchanging in time and $dT/dt = 0$; also, if there is no heat generated within the volume, $\dot{Q}_v = 0$. Then Eq. (1.2) states that the net heat flow into the system is zero. Because the same amount of heat is flowing into ΔV across the face at x, and out of ΔV across the face at $x + \Delta x$,

$$\dot{Q}|_x = \dot{Q}|_{x+\Delta x}$$

Since the rate of heat transfer is constant for all x, we simplify the notation by dropping the $|_x$ and $|_{x+\Delta x}$ subscripts (see the footnote on page 9), and write

$$\dot{Q} = \text{Constant}$$

But from Fourier's law, Eq. (1.8),

$$\dot{Q} = qA = -kA\frac{dT}{dx}$$

The variables are separable: rearranging and integrating across the wall,

$$\frac{\dot{Q}}{A}\int_0^L dx = -\int_{T_1}^{T_2} k\,dT$$

where $\dot{Q}$ and A have been taken outside the integral signs since both are constants. If the small variation of k with temperature is ignored for the present we obtain

$$\dot{Q} = \frac{kA}{L}(T_1 - T_2) = \frac{T_1 - T_2}{L/kA} \tag{1.9}$$

Comparison of Eq. (1.9) with Ohm's law, $I = E/R$, suggests that $\Delta T = T_1 - T_2$ can be viewed as a driving potential for flow of heat, analogous to voltage being the driving potential for current. Then $R \equiv L/kA$ can be viewed as a **thermal resistance** analogous to electrical resistance.

If we have a composite wall of two slabs of material, as shown in Fig. 1.5, the heat flow through each layer is the same:

$$\dot{Q} = \frac{T_1 - T_2}{L_A/k_A A} = \frac{T_2 - T_3}{L_B/k_B A}$$

Rearranging

$$\dot{Q}\left(\frac{L_A}{k_A A}\right) = T_1 - T_2$$

$$\dot{Q}\left(\frac{L_B}{k_B A}\right) = T_2 - T_3$$

Adding eliminates the interface temperature T_2:

$$\dot{Q}\left(\frac{L_A}{k_A A} + \frac{L_B}{k_B A}\right) = T_1 - T_3$$

or

$$\dot{Q} = \frac{T_1 - T_3}{L_A/k_A A + L_B/k_B A} = \frac{\Delta T}{R_A + R_B} \tag{1.10a}$$

Using the electrical resistance analogy, we would view the problem as two resistances in series forming a **thermal circuit**, and immediately write

$$\dot{Q} = \frac{\Delta T}{R_A + R_B} \tag{1.10b}$$

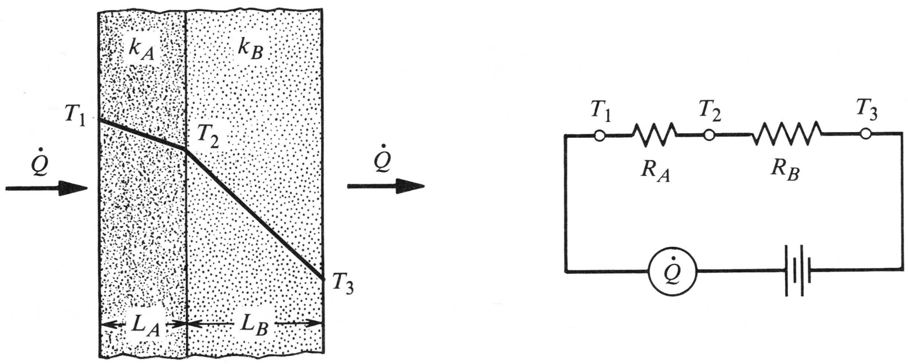


Figure 1.5 The temperature distribution for steady conduction across a composite plane wall and the corresponding thermal circuit.

EXAMPLE 1.1 Heat Transfer through Insulation

A refrigerated container is in the form of a cube with 2 m sides and has 5 mm-thick aluminum walls insulated with a 10 cm layer of cork. During steady operation, the temperatures on the inner and outer surfaces of the container are measured to be −5°C and 20°C, respectively. Determine the cooling load on the refrigerator.

Solution

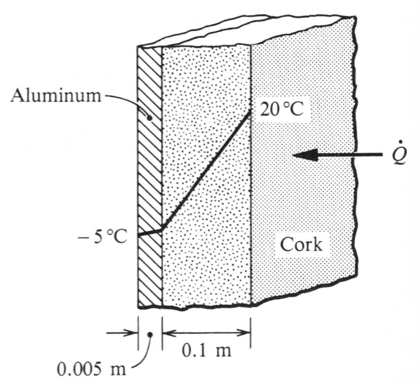
Given: Aluminum container insulated with 10 cm—thick cork.

Required: Rate of heat gain.

Assumptions: 1. Steady state.
2. One-dimensional heat conduction (ignore corner effects).

Equation (1.10) applies:

$$\dot{Q} = \frac{\Delta T}{R_A + R_B} \qquad \text{where } R = \frac{L}{kA}$$



Let subscripts A and B denote the aluminum wall and cork insulation, respectively. Table 1.1 gives $k_A = 204$ W/m K, $k_B = 0.043$ W/m K. We suspect that the thermal resistance of the aluminum wall is negligible, but we will calculate it anyway. For one side of area $A = 4\,\text{m}^2$, the thermal resistances are

$$R_A = \frac{L_A}{k_A A} = \frac{(0.005\text{ m})}{(204\text{ W/m K})(4\text{ m}^2)} = 6.13 \times 10^{-6}\text{ K/W}$$

$$R_B = \frac{L_B}{k_B A} = \frac{(0.10\text{ m})}{(0.043\text{ W/m K})(4\text{ m}^2)} = 0.581\text{ K/W}$$

Since R_A is five orders of magnitude less than R_B, it can be ignored. The heat flow for a temperature difference of $T_1 - T_2 = 20 - (-5) = 25$ K, is

$$\dot{Q} = \frac{\Delta T}{R_B} = \frac{25\text{ K}}{0.581\text{ K/W}} = 43.0\text{ W}$$

For six sides, the total cooling load on the refrigerator is 6.0 × 43.0 = 258 W.

Comments

1. In the future, when it is obvious that a resistance in a series network is negligible, it can be ignored from the outset (no effort should be expended to obtain data for its calculation).
2. The assumption of one-dimensional conduction is good because the 0.1 m insulation thickness is small compared to the 2 m-long sides of the cube.

3. Notice that the temperature difference $T_1 - T_2$ is expressed in kelvins, even though T_1 and T_2 were given in degrees Celsius.
4. We have assumed perfect thermal contact between the aluminum and cork; that is, there is no thermal resistance associated with the interface between the two materials (see Section 2.2.2).

1.3.2 Thermal Radiation

All matter and space contains electromagnetic radiation. A particle, or *quantum,* of electromagnetic energy is a photon, and heat transfer by radiation can be viewed either in terms of electromagnetic waves or in terms of photons. The flux of radiant energy incident on a surface is its **irradiation,** G [W/m^2]; the energy flux leaving a surface due to emission and reflection of electromagnetic radiation is its **radiosity**, J [W/m^2]. A **black surface** (or **blackbody**) is defined as a surface that absorbs all incident radiation, reflecting none. As a consequence, all of the radiation leaving a black surface is emitted by the surface and is given by the **Stefan-Boltzmann law** as

$$J = E_b = \sigma T^4 \tag{1.11}$$

where E_b is the **blackbody emissive power**, T is absolute temperature [K], and σ is the Stefan-Boltzmann constant ($\simeq 5.67 \times 10^{-8}$ W/m^{2}K^4). Table 1.2 shows how $E_b = \sigma T^4$ increases rapidly with temperature.

Table 1.2 Blackbody emissive power σT^4 at various temperatures.

Surface Temperature K	Blackbody Emissive Power W/m^2
300 (room temperature)	459
1000 (cherry-red hot)	56,700
3000 (lamp filament)	4,590,000
5760 (Sun temperature)	62,400,000

Figure 1.6 shows a convex black object of surface area A_1 in an evacuated black isothermal enclosure at temperature T_2. At equilibrium, the object is also at temperature T_2, and the radiant energy incident on the object must equal the radiant energy leaving from the object:

$$G_1 A_1 = J_1 A_1 = \sigma T_2^4 A_1$$

Hence

$$G_1 = \sigma T_2^4 \tag{1.12}$$

and is uniform over the area. If the temperature of the object is now raised to T_1, its radiosity becomes σT_1^4 while its irradiation remains σT_2^4 (because the enclosure reflects no radiation). Then the net radiant heat flux through the surface, q_1, is the

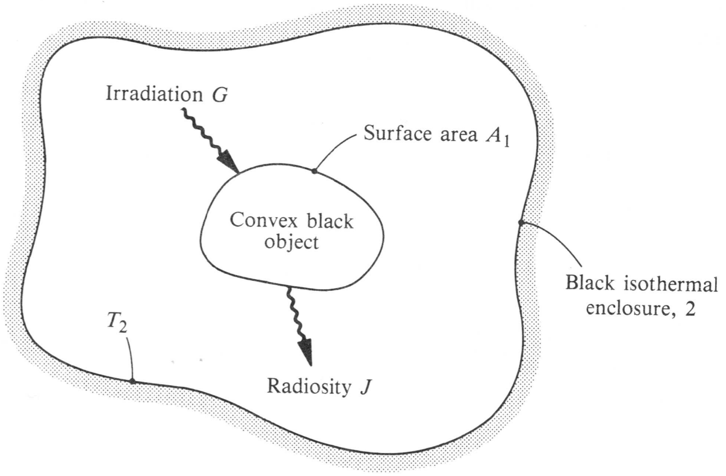


Figure 1.6 A convex black object (surface 1) in a black isothermal enclosure (surface 2).

radiosity minus the irradiation:

$$q_1 = J_1 - G_1 \tag{1.13}$$

or

$$q_1 = \sigma T_1^4 - \sigma T_2^4 \tag{1.14}$$

where the sign convention is such that a net flux away from the surface is positive. Equation (1.14) is also valid for two large black surfaces facing each other, as shown in Fig. 1.7.

The blackbody is an ideal surface. Real surfaces absorb less radiation than do black surfaces. The fraction of incident radiation absorbed is called the absorptance (or absorptivity), α. A widely used model of a real surface is the **gray surface,** which is defined as a surface for which α is a constant, irrespective of the nature of the incident radiation. The fraction of incident radiation reflected is the **reflectance** (or reflectivity), ρ. If the object is opaque, that is, not transparent to electromagnetic radiation, then

$$\rho = 1 - \alpha \tag{1.15}$$

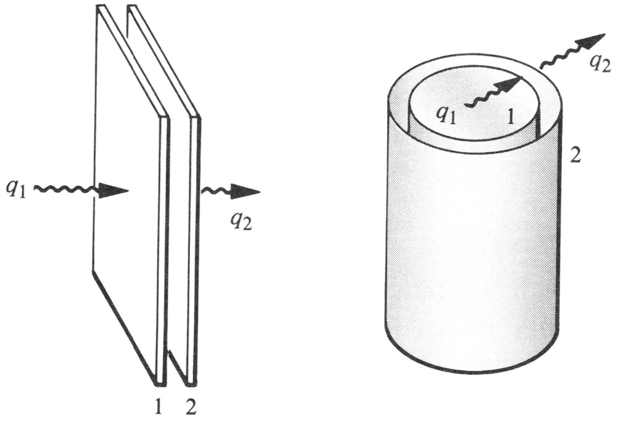


Figure 1.7 Examples of two large surfaces facing each other.

Table 1.3 Selected approximate values of emittance, ε (total hemispherical values at normal temperatures).

Surface	Emittance, ε
Aluminum alloy, unoxidized	0.035
Black anodized aluminum	0.80
Chromium plating	0.16
Stainless steel, type 312, lightly oxidized	0.30
Inconel X, oxidized	0.72
Black enamel paint	0.78
White acrylic paint	0.90
Asphalt	0.88
Concrete	0.90
Soil	0.94
Pyrex glass	0.80

Note: More comprehensive data are given in Appendix A. Emittance is very dependent on surface finish; thus, values obtained from various sources may differ significantly.

Real surfaces also emit less radiation than do black surfaces. The fraction of the blackbody emissive power σT^4 emitted is called the **emittance** (or emissivity), ε.[5] A gray surface also has a constant value of ε, independent of its temperature, and, as will be shown in Chapter 6, the emittance and absorptance of a gray surface are equal:

$$\varepsilon = \alpha \quad \text{(gray surface)} \tag{1.16}$$

Table 1.3 shows some typical values of ε at normal temperatures. Bright metal surfaces tend to have low values, whereas oxidized or painted surfaces tend to have high values. Values of α and ρ can also be obtained from Table 1.3 by using Eqs. (1.15) and (1.16).

If heat is transferred by radiation between two gray surfaces of finite size, as shown in Fig. 1.8, the rate of heat flow will depend on temperatures T_1 and T_2 and emittances ε_1 and ε_2, as well as the geometry. Clearly, some of the radiation leaving surface 1 will not be intercepted by surface 2, and vice versa. Determining the rate of heat flow is usually quite difficult. In general, we may write

$$\dot{Q}_{12} = A_1 \mathscr{F}_{12} (\sigma T_1^4 - \sigma T_2^4) \tag{1.17}$$

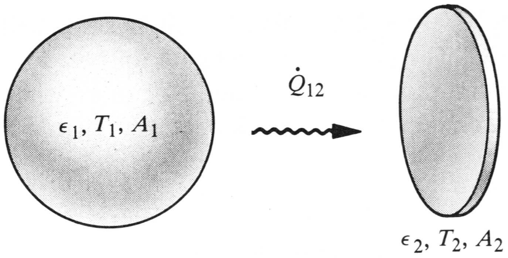


Figure 1.8 Radiation heat transfer between two finite gray surfaces.

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[5] Both the endings *-ance* and *-ivity* are commonly used for radiation properties. In this text, *-ance* will be used for surface radiation properties. In Chapter 6, *-ivity* will be used for gas radiation properties.

where $\dot{Q}_{12}$ is the net radiant energy interchange (heat transfer) from surface 1 to surface 2, and $\mathscr{F}_{12}$ is a **transfer factor,** which depends on emittances and geometry. For the special case of surface 1 surrounded by surface 2, where either area A_1 is small compared to area A_2, or surface 2 is nearly black, $\mathscr{F}_{12} \simeq \varepsilon_1$ and Eq. (1.17) becomes

$$\dot{Q}_{12} = \varepsilon_1 A_1(\sigma T_1^4 - \sigma T_2^4) \tag{1.18}$$

Equation (1.18) will be derived in Chapter 6. It is an important result and is often used for quick engineering estimates.

The T^4 dependence of radiant heat transfer complicates engineering calculations. When T_1 and T_2 are not too different, it is convenient to linearize Eq. (1.18) by factoring the term $(\sigma T_1^4 - \sigma T_2^4)$ to obtain

$$\begin{aligned}\dot{Q}_{12} &= \varepsilon_1 A_1 \sigma (T_1^2 + T_2^2)(T_1 + T_2)(T_1 - T_2)\\ &\simeq \varepsilon_1 A_1 \sigma (4T_m^3)(T_1 - T_2)\end{aligned}$$

for $T_1 \simeq T_2$, where T_m is the mean of T_1 and T_2. This result can be written more concisely as

$$\dot{Q}_{12} \simeq A_1 h_r (T_1 - T_2) \tag{1.19}$$

where $h_r = 4\varepsilon_1 \sigma T_m^3$ is called the **radiation heat transfer coefficient** [W/m^2 K]. At 25°C (= 298 K),

$$h_r = (4)\varepsilon_1(5.67 \times 10^{-8}\ \mathrm{W/m^2\,K^4})(298\ \mathrm{K})^3$$

or

$$h_r \simeq 6\varepsilon_1\ \mathrm{W/m^2\,K}$$

This result can be easily remembered: The radiation heat transfer coefficient at room temperature is about six times the surface emittance. For $T_1 = 320$ K and $T_2 = 300$ K, the error incurred in using the approximation of Eq. (1.19) is only 0.1%; for $T_1 = 400$ K and $T_2 = 300$ K, the error is 2%.

EXAMPLE 1.2 Heat Loss from a Transistor

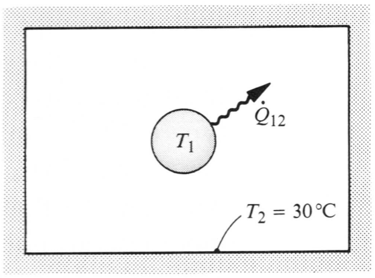
An electronic package for an experiment in outer space contains a transistor capsule, which is approximately spherical in shape with a 2 cm diameter. It is contained in an evacuated case with nearly black walls at 30°C. The only significant path for heat loss from the capsule is radiation to the case walls. If the transistor dissipates 300 mW, what will the capsule temperature be if it is (i) bright aluminum and (ii) black anodized aluminum?

Solution

Given: 2 cm-diameter transistor capsule dissipating 300 mW.

Required: Capsule temperature for (i) bright aluminum and (ii) black anodized aluminum.

Assumptions: Model as a small gray body in large, nearly black surroundings.



Equation (1.18) is applicable with

$$\dot{Q}_{12} = 300 \text{ mW}$$
$$T_2 = 30^\circ\text{C} = 303 \text{ K}$$

and T_1 is the unknown.

$$\dot{Q}_{12} = \varepsilon_1 A_1 (\sigma T_1^4 - \sigma T_2^4)$$

$$0.3 \text{ W} = (\varepsilon_1)(\pi)(0.02\,\text{m})^2[\sigma T_1^4 - (5.67 \times 10^{-8}\,\text{W/m}^2\,\text{K}^4)(303 \text{ K})^4]$$

Solving,

$$\sigma T_1^4 = 478 + \frac{239}{\varepsilon_1}$$

(i) For bright aluminum ($\varepsilon = 0.035$ from Table 1.3),

$$\sigma T_1^4 = 478 + 6828 = 7306\,\text{W/m}^2$$
$$T_1 = 599\,\text{K}\ (326^\circ\text{C})$$

(ii) For black anodized aluminum ($\varepsilon = 0.80$ from Table 1.3),

$$\sigma T_1^4 = 478 + 298 = 776\,\text{W/m}^2$$
$$T_1 = 342\,\text{K}(69^\circ\text{C})$$

Comments

1. The anodized aluminum gives a satisfactory operating temperature, but a bright aluminum capsule could not be used since 326°C is far in excess of allowable operating temperatures for semiconductor devices.
2. Note the use of kelvins for temperature in this radiation heat transfer calculation.

1.3.3 Heat Convection

As already explained, *convection* or *convective heat transfer* is the term used to describe heat transfer from a surface to a moving fluid, as shown in Fig. 1.9. The surface may be the inside of a pipe, the skin of a hypersonic aircraft, or a water-air interface in a cooling tower. The flow may be *forced*, as in the case of a liquid pumped through

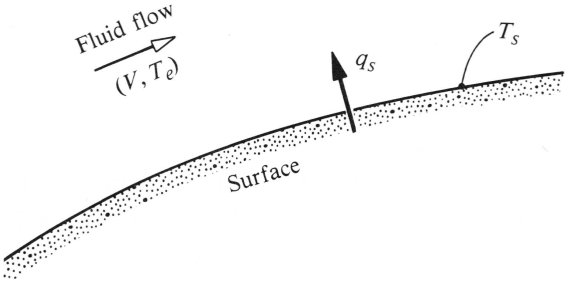


Figure 1.9 Schematic of convective heat transfer to a fluid at temperature T_e flowing at velocity V past a surface at temperature T_s.

the pipe or air on the flight vehicle propelled through the atmosphere. On the other hand, the flow could be *natural* (or *free*), driven by buoyancy forces arising from a density difference, as in the case of a natural-draft cooling tower. Either type of flow can be *internal*, such as the pipe flow, or *external*, such as flow over the vehicle. Also, both forced and natural flows can be either *laminar* or *turbulent*, with laminar flows being predominant at lower velocities, for smaller sizes, and for more viscous fluids. Flow in a pipe may become turbulent when the dimensionless group called the **Reynolds number**, $\mathrm{Re}_D = VD/\nu$, exceeds about 2300, where V is the velocity [m/s], D is the pipe diameter [m], and ν is the kinematic viscosity of the fluid [m^2/s]. Heat transfer rates tend to be much higher in turbulent flows than in laminar flows, owing to the vigorous mixing of the fluid. Figure 1.10 shows some commonly encountered flows.

The rate of heat transfer by convection is usually a complicated function of surface geometry and temperature, the fluid temperature and velocity, and fluid thermophysical properties. In an external forced flow, the rate of heat transfer is approximately proportional to the difference between the surface temperature T_s and the temperature of the free stream fluid T_e. The constant of proportionality is called the **convective heat transfer coefficient** h_c:

$$q_s = h_c \Delta T \tag{1.20}$$

where $\Delta T = T_s - T_e$, q_s is the heat flux from the surface into the fluid [W/m^2], and h_c has units [W/m^2 K]. Equation (1.20) is often called *Newton's law of cooling* but is a definition of h_c rather than a true physical law. For natural convection, the situation is more complicated. If the flow is laminar, q_s varies as $\Delta T^{5/4}$; if the flow is turbulent, it varies as $\Delta T^{4/3}$. However, we still find it convenient to define a heat transfer coefficient by Eq. (1.20); then h_c varies as $\Delta T^{1/4}$ for laminar flows and as $\Delta T^{1/3}$ for turbulent ones.

An important practical problem is convective heat transfer to a fluid flowing in a tube, as may be found in heat exchangers for heating or cooling liquids, in condensers, and in various kinds of boilers. In using Eq. (1.20) for internal flows, $\Delta T = T_s - T_b$, where T_b is a properly averaged fluid temperature called the **bulk temperature** or mixed mean temperature and is defined in Chapter 4. Here it is sufficient to note that enthalpy in the steady-flow energy equation, Eq. (1.4), is also the bulk value, and T_b is the corresponding temperature. If the pipe has a uniform wall temperature T_s along its length, and the flow is laminar ($\mathrm{Re}_D \lesssim 2300$), then sufficiently far from the pipe entrance, the heat transfer coefficient is given by the exact relation

$$h_c = 3.66 \frac{k}{D} \tag{1.21}$$

where k is the fluid thermal conductivity and D is the pipe diameter. Notice that the heat transfer coefficient is directly proportional to thermal conductivity, inversely proportional to pipe diameter, and—perhaps surprisingly —independent of flow velocity. On the other hand, for fully turbulent flow ($\mathrm{Re}_D \gtrsim 10{,}000$), h_c is given

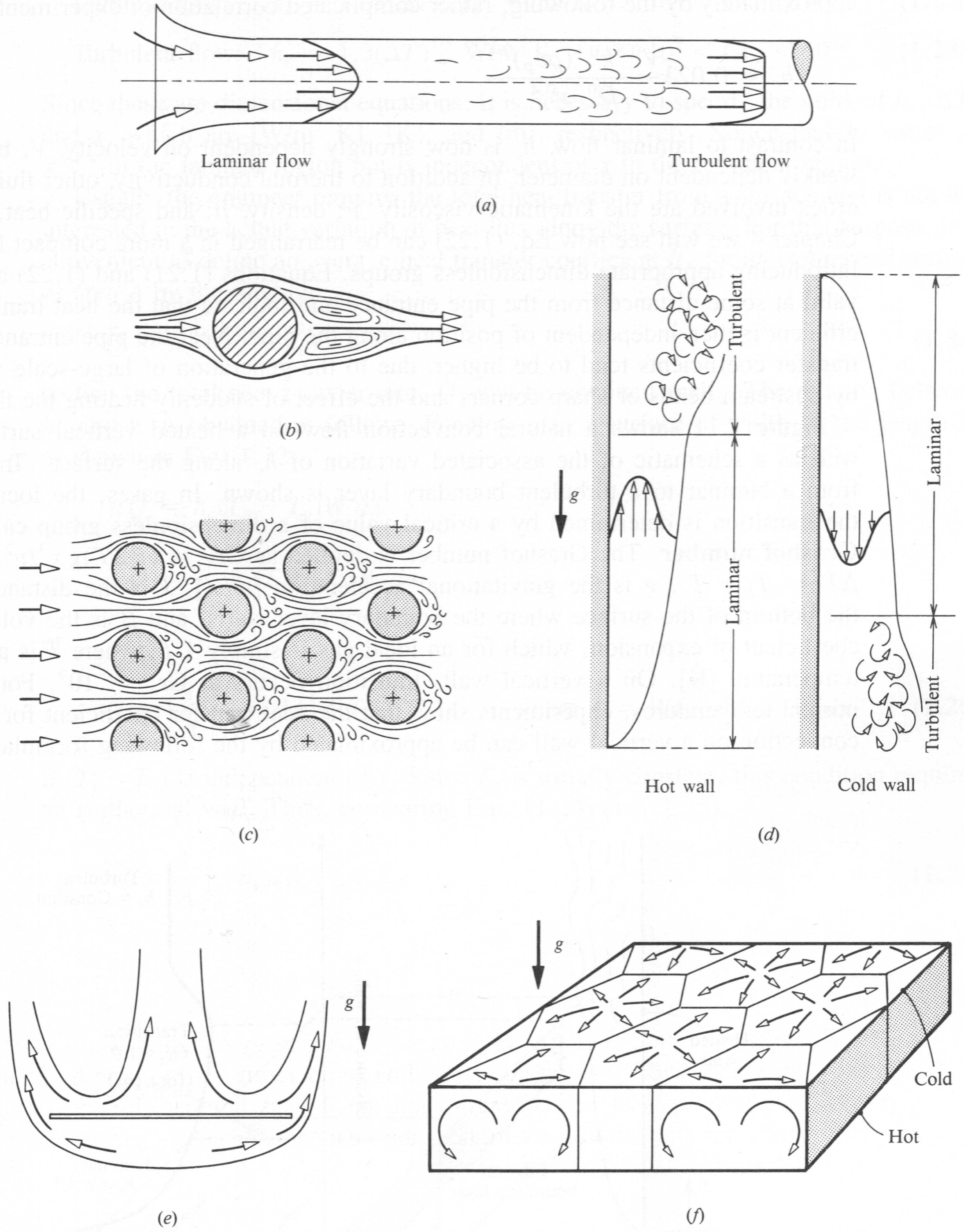


Figure 1.10 Some commonly encountered flows, *(a)* Forced flow in a pipe, $\mathrm{Re}_D \simeq 50,000$. The flow is initially laminar because of the "bell-mouth" entrance but becomes turbulent downstream, *(b)* Laminar forced flow over a cylinder, $\mathrm{Re}_D \simeq 25$. *(c)* Forced flow through a tube bank as found in a shell-and-tube heat exchanger, *(d)* Laminar and turbulent natural convection boundary layers on vertical walls, *(e)* Laminar natural convection about a heated horizontal plate, *(f)* Cellular natural convection in a horizontal enclosed fluid layer.

approximately by the following, rather complicated correlation of experimental data:

$$h_c = 0.023\frac{V^{0.8}k^{0.6}(\rho c_p)^{0.4}}{D^{0.2}\nu^{0.4}} \tag{1.22}$$

In contrast to laminar flow, h_c is now strongly dependent on velocity, V, but only weakly dependent on diameter. In addition to thermal conductivity, other fluid properties involved are the kinematic viscosity, ν; density, ρ; and constant-pressure specific heat, c_p. In Chapter 4 we will see how Eq. (1.22) can be rearranged in a more compact form by introducing appropriate dimensionless groups. Equations (1.21) and (1.22) are only valid at some distance from the pipe entrance and indicate that the heat transfer coefficient is then independent of position along the pipe. Near the pipe entrance, heat transfer coefficients tend to be higher, due to the generation of large-scale vortices by upstream bends or sharp corners and the effect of suddenly heating the fluid.

Figure 1.11 shows a natural convection flow on a heated vertical surface, as well as a schematic of the associated variation of h_c along the surface. Transition from a laminar to a turbulent boundary layer is shown. In gases, the location of the transition is determined by a critical value of a dimensionless group called the **Grashof number**. The Grashof number is defined as $\mathrm{Gr}_x = (\beta\Delta T)gx^3/\nu^2$, where $\Delta T = T_s - T_e$, g is the gravitational acceleration [m/s^2], x is the distance from the bottom of the surface where the boundary layer starts, and β is the volumetric coefficient of expansion, which for an ideal gas is simply $1/T$, where T is absolute temperature [K]. On a vertical wall, transition occurs at $\mathrm{Gr}_x \simeq 10^9$. For air, at normal temperatures, experiments show that the heat transfer coefficient for natural convection on a vertical wall can be approximated by the following formulas:

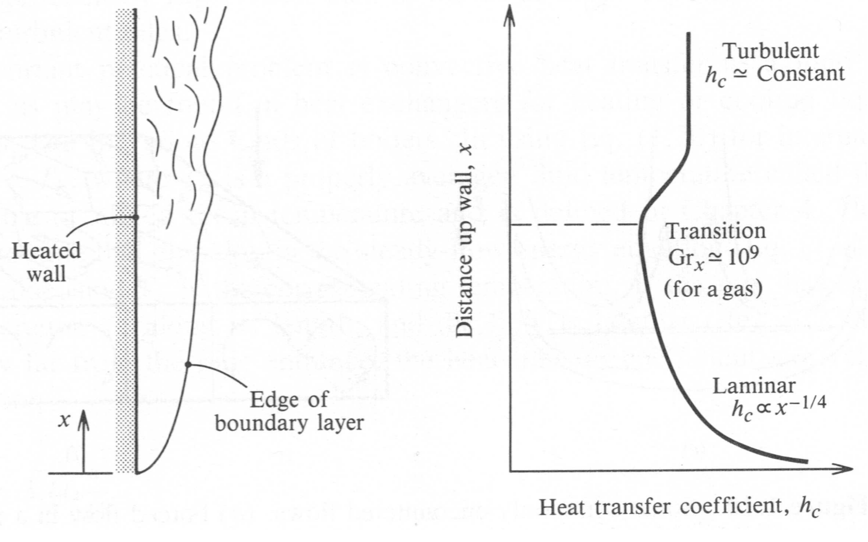


Figure 1.11 A natural-convection boundary layer on a vertical wall, showing the variation of local heat transfer coefficient. For gases, transition from a laminar to turbulent flow occurs at a Grashof number of approximately 10^9; hence $x_{tr} \simeq [10^9\nu^2/\beta\Delta T g]^{1/3}$.

$$\text{Laminar flow:} \quad h_c = 1.07(\Delta T/x)^{1/4}\ \text{W/m}^2\text{K} \qquad 10^4 < \text{Gr}_x < 10^9 \tag{1.23a}$$

$$\text{Turbulent flow:} \quad h_c = 1.3(\Delta T)^{1/3}\ \text{W/m}^2\text{K} \qquad 10^9 < \text{Gr}_x < 10^{12} \tag{1.23b}$$

Since these are dimensional equations, it is necessary to specify the units of h_c, ΔT, and x, which are [W/m^2 K], [K], and [m], respectively. Notice that h_c varies as $x^{-1/4}$ in the laminar region but is independent of x in the turbulent region.

Usually the engineer requires the total heat transfer from a surface and is not too interested in the actual variation of heat flux along the surface. For this purpose, it is convenient to define an average heat transfer coefficient $\overline{h}_c$ for an *isothermal* surface of area A by the relation

$$\dot{Q} = \overline{h}_c A(T_s - T_e) \tag{1.24}$$

so that the total heat transfer rate, $\dot{Q}$, can be obtained easily. The relation between $\overline{h}_c$ and h_c is obtained as follows: For flow over a surface of width W and length L, as shown in Fig. 1.12,

$$d\dot{Q} = h_c(T_s - T_e)W\,dx$$

$$\dot{Q} = \int_0^L h_c(T_s - T_e)W\,dx$$

or

$$\dot{Q} = \left(\frac{1}{A}\int_0^A h_c\,dA\right)A(T_s - T_e), \qquad \text{where } A = WL,\ dA = W\,dx \tag{1.25}$$

if $(T_s - T_e)$ is independent of x. Since T_e is usually constant, this condition requires an isothermal wall. Thus, comparing Eqs. (1.24) and (1.25),

$$\overline{h}_c = \frac{1}{A}\int_0^A h_c\,dA \tag{1.26}$$

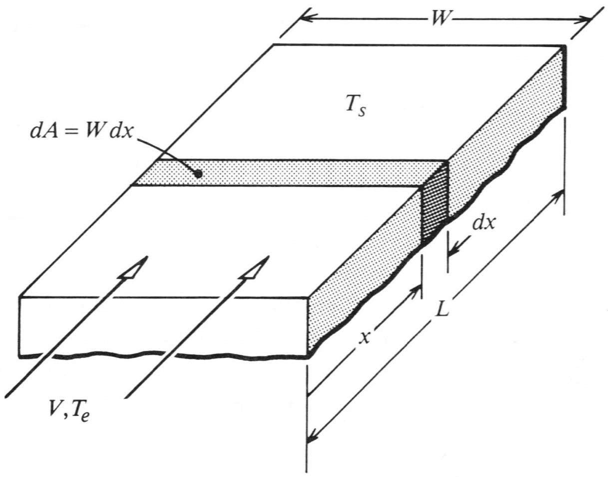


Figure 1.12 An isothermal surface used to define the average convective heat transfer coefficient $\overline{h}_c$.

Table 1.4 Orders of magnitude of average convective heat transfer coefficients.

Flow and Fluid	$\bar{h}_c$ W/m^2 K
Free convection, air	3–25
Free convection, water	15–1000
Forced convection, air	10–200
Forced convection, water	50–10,000
Forced convection, liquid sodium	10,000–100,000
Condensing steam	5000–50,000
Boiling water	3000–100,000

The surface may not be isothermal; for example, the surface may be electrically heated to give a uniform flux q_s along the surface. In this case, defining an average heat transfer coefficient is more difficult and will be dealt with in Chapter 4. Table 1.4 gives some order-of-magnitude values of average heat transfer coefficients for various situations. In general, high heat transfer coefficients are associated with high fluid thermal conductivities, high flow velocities, and small surfaces. The high heat transfer coefficients shown for boiling water and condensing steam are due to another cause: as we will see in Chapter 7, a large enthalpy of phase change (latent heat) is a contributing factor.

The complexity of most situations involving convective heat transfer precludes exact analysis, and *correlations* of experimental data must be used in engineering practice. For a particular situation, a number of correlations from various sources might be available, for example, from research laboratories in different countries. Also, as time goes by, older correlations may be superseded by newer correlations based on more accurate or more extensive experimental data. Heat transfer coefficients calculated from various available correlations usually do not differ by more than about 20%, but in more complex situations, much larger discrepancies may be encountered. Such is the nature of engineering calculations of convective heat transfer, in contrast to the more exact nature of the analysis of heat conduction or of elementary mechanics, for example.

EXAMPLE 1.3 Heat Loss through Glass Doors

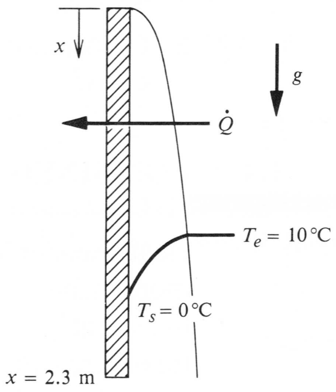
The living room of a ski chalet has a pair of glass doors 2.3 m high and 4.0 m wide. On a cold morning, the air in the room is at 10°C, and frost partially covers the inner surface of the glass. Estimate the convective heat loss to the doors. Would you expect to see the frost form initially near the top or the bottom of the doors? Take $\nu = 14 \times 10^{-6}$ m^2/s for the air.

Solution

Given: Glass doors, width $W = 4$ m, height $L = 2.3$ m.

Required: Estimate of convective heat loss to the doors.

Assumptions:
1. Inner surface isothermal at $T_s \simeq 0°\mathrm{C}$.
2. The laminar to turbulent flow transition occurs at $Gr_x \simeq 10^9$.



Equation (1.24) will be used to estimate the heat loss. The inner surface will be at approximately 0°C since it is only partially covered with frost. If it were warmer, frost couldn't form; and if it were much colder, frost would cover the glass completely. There is a natural convection flow down the door since $T_e = 10°$ C is greater than $T_s = 0°$ C. Transition from a laminar boundary layer to a turbulent boundary layer occurs when the Grashof number is about 10^9. For transition at $x = x_{\mathrm{tr}}$,

$$\mathrm{Gr} = 10^9 = \frac{(\beta \Delta T) g x_{\mathrm{tr}}^3}{\nu^2}; \quad \beta = 1/T \text{ for an ideal gas}$$

$$x_{\mathrm{tr}} = \left[\frac{10^9\,\nu^2}{(\Delta T/T)g}\right]^{1/3} = \left[\frac{(10^9)(14\times 10^{-6}\,\mathrm{m^2/s})^2}{(10/278)(9.81\,\mathrm{m/s^2})}\right]^{1/3} = 0.82\,\mathrm{m}$$

where the average of T_s and T_e has been used to evaluate β. The transition is seen to take place about one third of the way down the door.

We find the average heat transfer coefficient, $\overline{h}_c$, by substituting Eqs. (1.23 a,b) in Eq. (1.26):

$$\begin{aligned}
\overline{h}_c &= \frac{1}{A}\int_0^A h_c\,dA; \quad A = WL, \quad dA = W\,dx \\
&= \frac{1}{L}\int_0^L h_c\,dx \\
&= \frac{1}{L}\left[\int_0^{x_{\mathrm{tr}}} 1.07(\Delta T/x)^{1/4}\,dx + \int_{x_{\mathrm{tr}}}^L 1.3(\Delta T)^{1/3}\,dx\right] \\
&= (1/L)[(1.07)(4/3)\Delta T^{1/4} x_{\mathrm{tr}}^{3/4} + (1.3)(\Delta T)^{1/3}(L - x_{\mathrm{tr}})] \\
&= (1/2.3)[(1.07)(4/3)(10)^{1/4}(0.82)^{3/4} + (1.3)(10)^{1/3}(2.3 - 0.82)] \\
&= (1/2.3)[2.19 + 4.15] \\
&= 2.75\ \mathrm{W/m^2K}
\end{aligned}$$

Then, from Eq. (1.24), the total heat loss to the door is

$$\dot{Q} = \overline{h}_c A \Delta T = (2.75\ \mathrm{W/m^2\,K})(2.3 \times 4.0\ \mathrm{m^2})(10\ \mathrm{K}) = 253\ \mathrm{W}$$

Comments

1. The local heat transfer coefficient is larger near the top of the door, so that the relatively warm room air will tend to cause the glass there to be at a higher temperature than further down the door. Thus, frost should initially form near the bottom of the door.
2. In addition, interior surfaces in the room will lose heat by radiation through the glass doors.

1.4 COMBINED MODES OF HEAT TRANSFER

Heat transfer problems encountered by the design engineer almost always involve more than one mode of heat transfer occurring simultaneously. For example, consider the nighttime heat loss through the roof of the house shown in Fig. 1.3. Heat is transferred to the ceiling by convection from the warm room air, and by radiation from the walls, furniture, and occupants. Heat transfer across the ceiling and its insulation is by conduction, across the attic crawlspace by convection and radiation, and across the roof tile by conduction. Finally, the heat is transferred by convection to the cold ambient air, and by radiation to the nighttime sky. To consider realistic engineering problems, it is necessary at the outset to develop the theory required to handle *combined modes* of heat transfer.

1.4.1 Thermal Circuits

The electrical circuit analogy for conduction through a composite wall was introduced in Section 1.3.1. We now extend this concept to include convection and radiation as well. Figure 1.13 shows a two-layer composite wall of cross-sectional area A with the layers A and B having thickness and conductivity L_A, k_A and L_B, k_B, respectively. Heat is transferred from a hot fluid at temperature T_i to the inside of the wall with a convective heat transfer coefficient $h_{c,i}$, and away from the outside of the wall to a cold fluid at temperature T_o with heat transfer coefficient $h_{c,o}$.

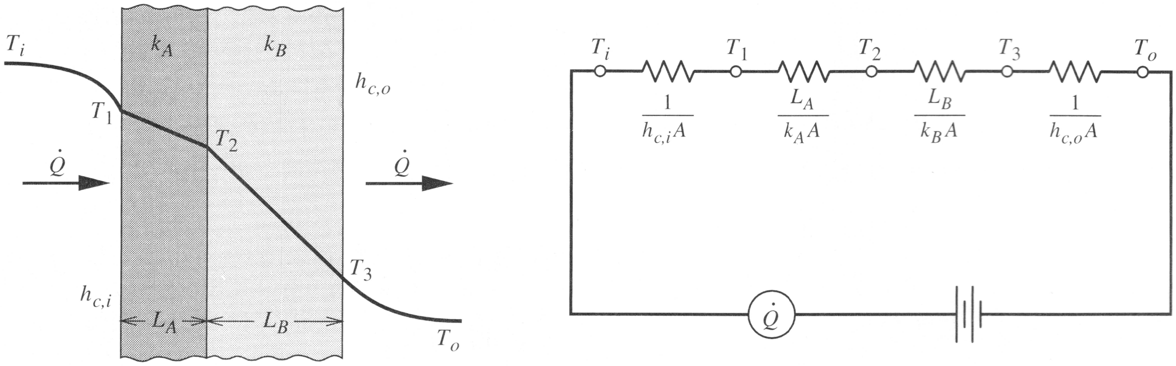


Figure 1.13 The temperature distribution for steady heat transfer across a composite plane wall, and the corresponding thermal circuit.

Newton's law of cooling, Eq. (1.20), can be rewritten as

$$\dot{Q} = \frac{\Delta T}{1/h_c A} \tag{1.27}$$

with $1/h_c A$ identified as a convective thermal resistance. At steady state, the heat flow through the wall is constant. Referring to Fig. 1.13 for the intermediate temperatures,

$$\dot{Q} = \frac{T_i - T_1}{1/h_{c,i} A} = \frac{T_1 - T_2}{L_A/k_A A} = \frac{T_2 - T_3}{L_B/k_B A} = \frac{T_3 - T_o}{1/h_{c,o} A} \tag{1.28}$$

Equation (1.28) is the basis of the thermal circuit shown in Fig. 1.13. The total resistance is the sum of four resistances in series. If we define the **overall heat transfer coefficient** U by the relation

$$\dot{Q} = UA(T_i - T_o) \tag{1.29}$$

then $1/UA$ is an overall resistance given by

$$\frac{1}{UA} = \frac{1}{h_{c,i} A} + \frac{L_A}{k_A A} + \frac{L_B}{k_B A} + \frac{1}{h_{c,o} A} \tag{1.30a}$$

or, since the cross-sectional area A is constant for a plane wall,

$$\frac{1}{U} = \frac{1}{h_{c,i}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{1}{h_{c,o}} \tag{1.30b}$$

Equation (1.29) is simple and convenient for use in engineering calculations. Typical values of U [W/m^2 K] vary over a wide range for different types of walls and convective flows.

Figure 1.14 shows a wall whose outer surface loses heat by both convection and radiation. For simplicity, assume that the fluid is at the same temperature as the surrounding surfaces, T_o. Using the approximate linearized Eq. (1.19),

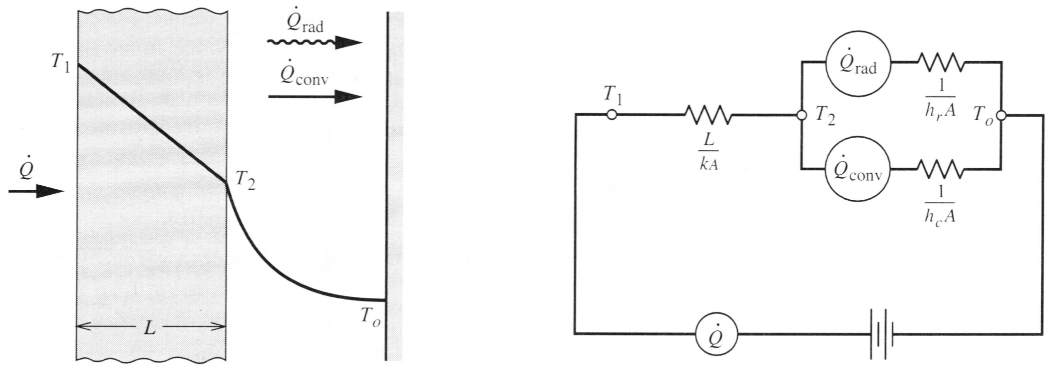


Figure 1.14 A wall that loses heat by both conduction and radiation; the thermal circuit shows resistances in parallel.

$$\dot{Q}_{\text{rad}} = \frac{\Delta T}{1/h_r A} \tag{1.31}$$

with $1/h_r A$ identified as a radiative thermal resistance. We now have two resistances in parallel, as shown in Fig. 1.14. The sum of the resistances is

$$\sum R = \frac{L}{kA} + \frac{1}{h_c A + h_r A}$$

so

$$\frac{1}{UA} = \frac{L}{kA} + \frac{1}{(h_c + h_r)A} \tag{1.32}$$

so that the convective and radiative heat transfer coefficients can simply be added. However, often the fluid and surrounding temperatures are not the same, or the simple linearized representation of radiative transfer [Eq. (1.19)] is invalid, so the thermal circuit is then more complex. When appropriate, we will write $h = h_c + h_r$ to account for combined convection and radiation.[6]

EXAMPLE 1.4 Heat Loss through a Composite Wall

The walls of a sparsely furnished single-room cabin in a forest consist of two layers of pine wood, each 2 cm thick, sandwiching 5 cm of fiberglass insulation. The cabin interior is maintained at 20°C when the ambient air temperature is 2°C. If the interior and exterior convective heat transfer coefficients are 3 and 6 W/m^2 K, respectively, and the exterior surface is finished with a white acrylic paint, estimate the heat flux through the wall.

Solution

Given: Pine wood cabin wall insulated with 5 cm of fiberglass.

Required: Estimate of heat loss through wall.

Assumptions:
1. Forest trees and shrubs are at the ambient air temperature, $T_e = 2$°C.
2. Radiation transfer inside cabin is negligible since inner surfaces of walls, roof, and floor are at approximately the same temperature.

From Eq. (1.29), the heat flux through the wall is

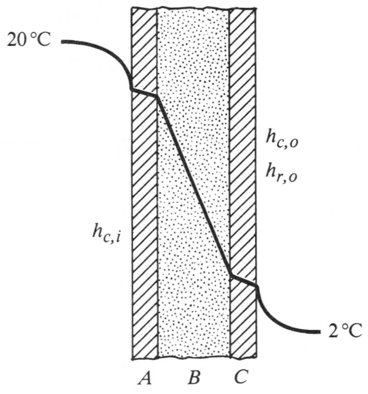
$$q = \frac{\dot{Q}}{A} = U(T_i - T_o)$$

From Eqs. (1.30) and (1.32), the overall heat transfer coefficient is given by

$$\frac{1}{U} = \frac{1}{h_{c,i}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{(h_{c,o} + h_{r,o})}$$

---

[6] Notice that the notation used for this combined heat transfer coefficient, h, is the same as that used for enthalpy. The student must be careful not to confuse these two quantities. Other notation is also in common use, for example. α for the heat transfer coefficient and i for enthalpy.



The thermal conductivities of pine wood, perpendicular to the grain, and of fiberglass are given in Table 1.1 as 0.10 and 0.038 W/m K, respectively. The exterior radiation heat transfer coefficient is given by Eq. (1.19) as

$$h_{r,o} = 4\varepsilon\sigma T_m^3$$

where $\varepsilon = 0.9$ for white acrylic paint, from Table 1.3, and $T_m \simeq 2°\text{C} = 275$ K (since we expect the exterior resistance to be small). Thus,

$$h_{ro} = 4(0.9)(5.67 \times 10^{-8}\ \text{W/m}^2\ \text{K}^4)(275\text{K})^3$$

$$= 4.2\ \text{W/m}^2\ \text{K}$$

$$\frac{1}{U} = \frac{1}{3} + \frac{0.02}{0.10} + \frac{0.05}{0.038} + \frac{0.02}{0.10} + \frac{1}{6+4.2}$$

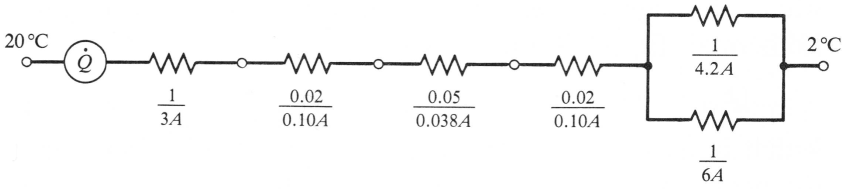
$$= 0.333 + 0.200 + 1.316 + 0.200 + 0.098$$

$$= 2.15\ (\text{W/m}^2\ \text{K})^{-1}$$

$$U = 0.466\ \text{W/m}^2\ \text{K}$$

Then the heat flux $q = U(T_i - T_o) = 0.466(20 - 2) = 8.38\ \text{W/m}^2$.

The thermal circuit is shown below.



Comments

1. The outside resistance is seen to be $0.098/2.15 \simeq 5\%$ of the total resistance; hence, the outside wall of the cabin is only about 1 K above the ambient air, and our assumption of $T_m = 275$ K for the evaluation of $h_{r,o}$ is adequate.
2. The dominant resistance is that of the fiberglass insulation; therefore, an accurate calculation of q depends mainly on having accurate values for the fiberglass thickness and thermal conductivity. Poor data or poor assumptions for the other resistances have little impact on the result.

1.4.2 Surface Energy Balances

Section 1.4.1 assumed that the energy flow $\dot{Q}$ across the wall surfaces is continuous. In fact, we used a procedure commonly called a *surface energy balance*, which is used in various ways. Some examples follow. Figure 1.15 shows an opaque solid that is losing heat by convection and radiation to its surroundings. Two imaginary surfaces are located on each side of the real solid-fluid interface: an s-surface in

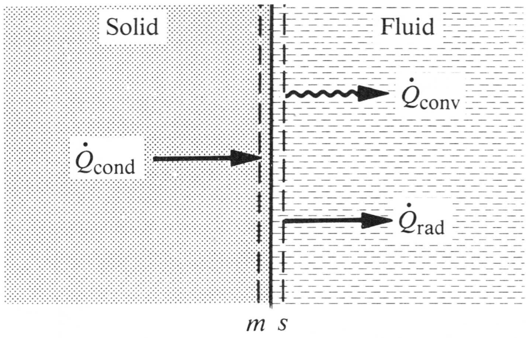


Figure 1.15 Schematic of a surface energy balance, showing the *m*- and *s*-surface in the solid and fluid, respectively.

the fluid just adjacent to the interface, and an *m*-surface in the solid located such that all radiation is emitted or absorbed between the *m*-surface and the interface. Thus, energy is transferred across the *m*-surface by conduction only. (The choice of *s* and *m* to designate these surfaces follows an established practice. In particular, the use of the *s* prefix is consistent with the use of the subscript *s* to denote a surface temperature T_s, in convection analysis.) The first law as applied to the closed system located between *m*- and *s*-surfaces requires that $\sum \dot{Q} = 0$; thus,

$$\dot{Q}_{\text{cond}} - \dot{Q}_{\text{conv}} - \dot{Q}_{\text{rad}} = 0 \tag{1.33}$$

or, for a unit area,

$$q_{\text{cond}} - q_{\text{conv}} - q_{\text{rad}} = 0 \tag{1.34}$$

where the sign convention for the fluxes is shown in Fig. 1.15. If the solid is isothermal, Eq. (1.33) reduces to

$$\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 0 \tag{1.35}$$

which is a simple energy balance on the solid. Notice that these surface energy balances remain valid for unsteady conditions, in which temperatures change with time, provided the mass contained between the *s*- and *m*-surfaces is negligible and cannot store energy.

EXAMPLE 1.5 Air Temperature Measurement

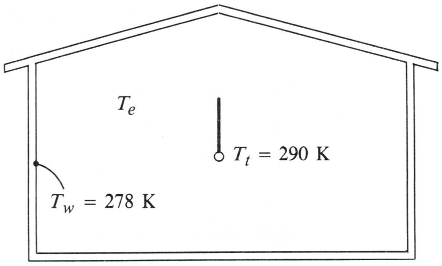
A machine operator in a workshop complains that the air-heating system is not keeping the air at the required minimum temperature of 20°C. To support his claim, he shows that a mercury-in-glass thermometer suspended from a roof truss reads only 17°C. The roof and walls of the workshop are made of corrugated iron and are not insulated; when the thermometer is held against the wall, it reads only 5°C. If the average convective heat transfer coefficient for the suspended thermometer is estimated to be 10 W/m^2 K, what is the true air temperature?

Solution

Given: Thermometer reading a temperature of 17°C.

Required: True air temperature.

Assumptions: Thermometer can be modeled as a small gray body in large, nearly black surroundings at 5°C.



Let T_t be the thermometer reading, T_e the air temperature, and T_w the wall temperature. Equation (1.35) applies,

$$\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 0$$

since at steady state there is no conduction within the thermometer. Substituting from Eqs. (1.24) and (1.18),

$$\overline{h}_c A(T_t - T_e) + \varepsilon\sigma A(T_t^4 - T_w^4) = 0$$

From Table 1.3, $\varepsilon = 0.8$ for pyrex glass. Canceling A,

$$10(290 - T_e) + (0.8)(5.67)(2.90^4 - 2.78^4) = 0$$

Solving,

$$T_e = 295 \text{ K} \simeq 22°\text{C}$$

Comments

1. Since $T_e > 20°$C, the air-heating system appears to be working satisfactorily.
2. Our model assumes that the thermometer is completely surrounded by a surface at 5°C: actually, the thermometer also receives radiation from machines, workers, and other sources at temperatures higher than 5°C, so that our calculated value of $T_e = 22°$C is somewhat high.

1.5 TRANSIENT THERMAL RESPONSE

The heat transfer problems described in Examples 1.1 through 1.5 were *steady-state* problems; that is, temperatures were not changing in time. In Example 1.2, the transistor temperature was steady with the resistance (I^2R) heating balanced by the radiation heat loss. *Unsteady-state* or *transient* problems occur when temperatures change with time. Such problems are often encountered in engineering practice, and the engineer may be required to predict the temperature-time response of a system involved in a heat transfer process. If the system, or a component of the system, can be assumed to have a spatially uniform temperature, analysis involves a relatively simple application of the energy conservation principle, as will now be demonstrated.

1.5.1 The Lumped Thermal Capacity Model

If a system undergoing a transient thermal response to a heat transfer process has a nearly uniform temperature, we may ignore small differences of temperature within the system. Changes in internal energy of the system can then be specified in terms of changes of the assumed uniform (or average) temperature of the system. This approximation is called the **lumped thermal capacity** model.[7] The system might

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[7] The term *capacitance* is also used, in analogy to an equivalent electrical circuit.

be a small solid component of high thermal conductivity that loses heat slowly to its surroundings via a large external thermal resistance. Since the thermal resistance to conduction in the solid is small compared to the external resistance, the assumption of a uniform temperature is justified. Alternatively, the system might be a well-stirred liquid in an insulated tank losing heat to its surroundings, in which case it is the mixing of the liquid by the stirrer that ensures a nearly uniform temperature. In either case, once we have assumed uniformity of temperature, we have no further need for details of the heat transfer within the system—that is, of the conduction in the solid component or the convection in the stirred liquid. Instead, the heat transfer process of concern is the interaction of the system with the surroundings, which might be by conduction, radiation, or convection.

Governing Equation and Initial Condition

For purposes of analysis, consider a metal forging removed from a furnace at temperature T_0 and suddenly immersed in an oil bath at temperature T_e, as shown in Fig. 1.16. The forging is a closed system, so the energy conservation principle in the form of Eq. (1.2) applies. Heat is transferred out of the system by convection. Using Eq. (1.24) the rate of heat transfer is $\overline{h}_c A(T - T_e)$, where $\overline{h}_c$ is the heat transfer coefficient averaged over the forging surface area A, and T is the forging temperature. There is no heat generated within the forging, so that $\dot{Q}_v = 0$. Substituting in Eq. (1.2):

$$\rho V c \frac{dT}{dt} = -\overline{h}_c A(T - T_e)$$

$$\frac{dT}{dt} = -\frac{\overline{h}_c A}{\rho V c}(T - T_e) \tag{1.36}$$

which is a first-order ordinary differential equation for the forging temperature, T, as a function of time, t. One initial condition is required:

$$t = 0: \quad T = T_0 \tag{1.37}$$

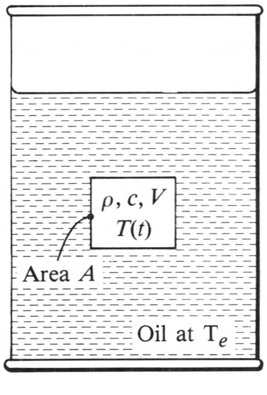

Figure 1.16 A forging immersed in an oil bath for quenching.

Solution for the Temperature Response

A simple analytical solution can be obtained provided we assume that the bath is large, so T_e is independent of time, and that $\overline{h}_cA/\rho Vc$ is approximated by a constant value independent of temperature. The variables in Eq. (1.36) can then be separated:

$$\frac{dT}{T-T_e} = -\frac{\overline{h}_cA}{\rho Vc}dt$$

Writing $dT = d(T-T_e)$, since T_e is constant, and integrating with $T = T_0$ at $t = 0$,

$$\int_{T_0}^{T} \frac{d(T-T_e)}{T-T_e} = -\frac{\overline{h}_cA}{\rho Vc}\int_0^t dt$$

$$\ln\frac{T-T_e}{T_0-T_e} = -\frac{\overline{h}_cA}{\rho Vc}t$$

$$\frac{T-T_e}{T_0-T_e} = e^{-(\overline{h}_cA/\rho Vc)t} = e^{-t/t_c} \tag{1.38}$$

where $t_c = \rho Vc/\overline{h}_cA$ [s] is called the **time constant** of the process. When $t = t_c$, the temperature difference $(T-T_e)$ has dropped to be 36.8% of the initial difference (T_0-T_e). Our result, Eq. (1.38), is a relation between two dimensionless parameters: a dimensionless temperature, $T^* = (T-T_e)/(T_0-T_e)$, which varies from 1 to 0; and a dimensionless time, $t^* = t/t_c = \overline{h}_cAt/\rho Vc$, which varies from 0 to ∞. Equation (1.38) can be written simply as

$$T^* = e^{-t^*} \tag{1.39}$$

and a graph of T^* versus t^* is a single curve, as illustrated in Fig. 1.17.

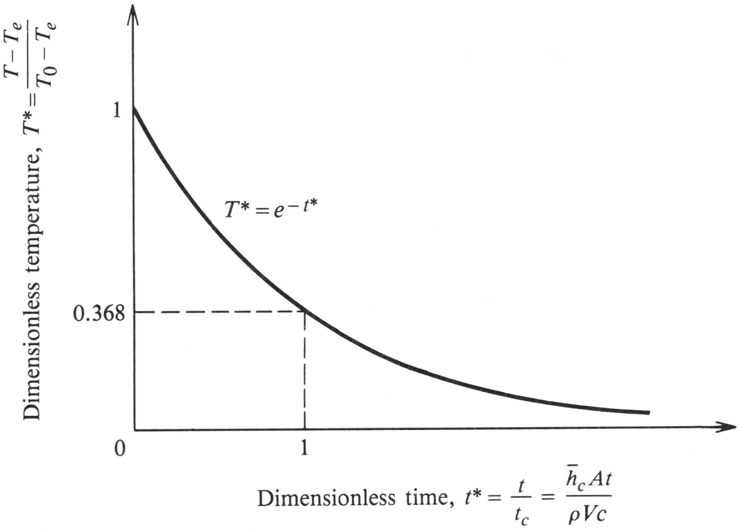

Figure 1.17 Lumped thermal capacity capacity temperature response in terms of dimensionless variables $T^*$and t^*.

Methods introduced in Chapter 2 can be used to deduce directly from Eqs. (1.36) and (1.37) that T^* must be a function of t^* alone [i.e., $T^* = f(t^*)$] without solving the equation. Of course, the solution also gives us the form of the function. Thus, the various parameters, $\overline{h}_c$, c, ρ, and so on, only affect the temperature response in the combination t^*, and not independently. If both $\overline{h}_c$ and c are doubled, the temperature at time t is unchanged. This dimensionless parameter t^* is a dimensionless group in the same sense as the Reynolds number, but it does not have a commonly used name.

Validity of the Model

We would expect our assumption of negligible temperature gradients within the system to be valid when the internal resistance to heat transfer is small compared with the external resistance. If L is some appropriate characteristic length of a solid body, for example, V/A (which for a plate is half its thickness), then

$$\frac{\text{Internal conduction resistance}}{\text{External convection resistance}} \simeq \frac{L/k_sA}{1/\overline{h}_cA} = \frac{\overline{h}_cL}{k_s} \simeq \frac{\overline{h}_cV}{k_sA} \tag{1.40}$$

where k_s is the thermal conductivity of the solid material. The quantity $\overline{h}_cL/k_s$ [W/m^2 K][m]/[W/m K] is a dimensionless group called a **Biot number**, Bi.[8] More exact analyses of transient thermal response of solids indicate that, for bodies resembling a plate, cylinder, or sphere, $\text{Bi}_{LTC} = \overline{h}_cV/k_sA < 0.1$ ensures that the temperature given by the lumped thermal capacity (LTC) model will not differ from the exact volume averaged value by more than 5%, and that our assumption of uniform temperature is adequate. Nonetheless, the choice of both the length scale L and the threshold (e.g., $\text{Bi}_{LTC} < 0.1$) used to determine the validity of the lumped thermal capacity model should be done carefully if accurate calculations are critical (see Chapter 3). If the heat transfer is by radiation, the convective heat transfer coefficient in Eq. (1.40) can be replaced by the approximate radiation heat transfer coefficient h_r defined in Eq. (1.19).

In the case of the well-stirred liquid in an insulated tank, it will be necessary to evaluate the ratio

$$\frac{\text{Internal convection resistance}}{\text{External resistance}} \simeq \frac{1/h_{c,i}A}{1/UA} = \frac{U}{h_{c,i}} \tag{1.41}$$

where U is the overall heat transfer coefficient, for heat transfer from the inner surface of the tank, across the tank wall and insulation, and into the surroundings. If this ratio is small relative to unity, the assumption of a uniform temperature in the liquid is justified.

The approximation or model used in the preceding analysis is called a lumped thermal capacity approximation since the thermal capacity is associated with a single temperature. There is an electrical analogy to the lumped thermal capacity model, owing to the mathematical equivalence of Eq. (1.36) to the equation governing the voltage in the simple resistance-capacitance electrical circuit shown

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[8] To avoid confusion with the Biot number used in Chapter 3, we will denote the Biot number based on $L = V/A$ as Bi_{LTC} for use with the lumped thermal capacity model.

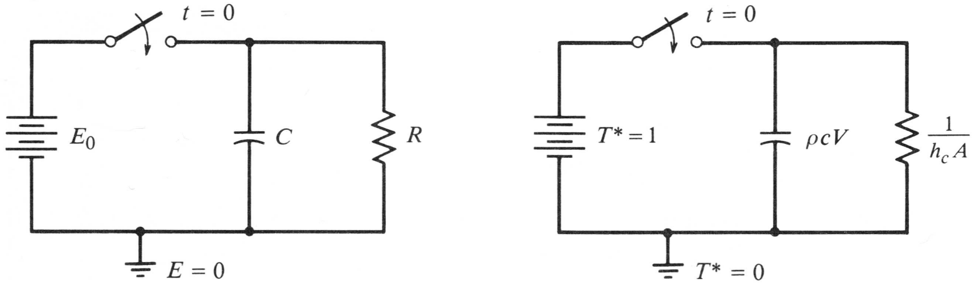


Figure 1.18 Equivalent electrical and thermal circuits for the lumped thermal capacity model of temperature response.

in Fig. 1.18,

$$\frac{dE}{dt} = -\frac{E}{RC} \tag{1.42}$$

with the initial condition $E = E_0$ at $t = 0$ if the capacitor is initially charged to a voltage E_0. The solution is identical in form to Eq. (1.38),

$$\frac{E}{E_0} = e^{-t/RC}$$

and the time constant is RC, the product of the resistance and capacitance [or $C/(1/R)$, the ratio of capacitance to conductance, to be exactly analogous to Eq. (1.38)].

EXAMPLE 1.6 Quenching of a Steel Plate

A steel plate 1 cm thick is taken from a furnace at 600°C and quenched in a bath of oil at 30°C. If the heat transfer coefficient is estimated to be 400 W/m^2 K, how long will it take for the plate to cool to 100°C? Take k, ρ, and c for the steel as 50 W/m K, 7800 kg/m^3 and 450 J/kg K, respectively.

Solution

Given: Steel plate quenched in an oil bath.

Required: Time to cool from 600°C to 100°C.

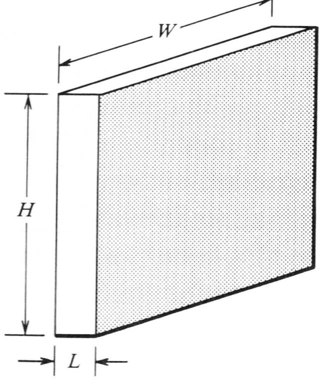
Assumptions: Lumped thermal capacity model valid.

First the Biot number will be checked to see if the lumped thermal capacity approximation is valid. For a plate of width W, height H, and thickness L,

$$\frac{V}{A} \simeq \frac{WHL}{2WH} = \frac{L}{2}$$

where the surface area of the edges has been neglected.

$$\begin{aligned}\text{Bi}_{LTC} &= \frac{\overline{h}_c(L/2)}{k_s}\\ &= \frac{(400\,\text{W/m}^2\,\text{K})(0.005\,\text{m})}{50\,\text{W/m K}}\\ &= 0.04 < 0.1\end{aligned}$$



so the lumped thermal capacity model is applicable. The time constant t_c is

$$t_c = \frac{\rho V c}{\overline{h}_c A} = \frac{\rho(L/2)c}{\overline{h}_c} = \frac{(7800\,\text{kg/m}^3)(0.005\,\text{m})(450\,\text{J/kg K})}{(400\,\text{W/m}^2\,\text{K})} = 43.9\,\text{s}$$

Substituting $T_e = 30°\text{C}$, $T_0 = 600°\text{C}$, $T = 100°\text{C}$ in Eq. (1.38) gives

$$\frac{100-30}{600-30} = e^{-t/43.9}$$

Solving,

$$t = 92\ \text{s}$$

Comments

The use of a constant value of h_c may be inappropriate for heat transfer by natural convection or radiation.

1.6 MASS TRANSFER AND ITS RELATION TO HEAT TRANSFER

The process of mass transfer is not as familiar as heat transfer, even though we encounter many mass transfer phenomena in everyday life. Mass transfer is the movement of a chemical species in a mixture or solution, usually due to the presence of a concentration gradient of the species. Figure 1.19 shows a sugar lump dissolving in a cup of coffee. The concentration of dissolved sugar adjacent to the lump is higher than in the bulk coffee, and the dissolved sugar moves down its concentration gradient by the process known as **ordinary diffusion.** Ordinary diffusion is analogous to heat conduction, which may be viewed as diffusion of thermal energy down its temperature gradient. If the coffee is stirred, the fluid motion transports dissolved sugar away from the lump by the process known as **mass convection.** Mass convection is exactly analogous to heat convection: the fluid can transport both energy and chemical species by virtue of its motion. The transport of perfume vapor or noxious odors in the air surrounding us similarly involves the processes of mass diffusion and mass convection.

We often encounter processes involving the evaporation of water into air, for example, from a hot tub or swimming pool, or when we sweat while trudging up

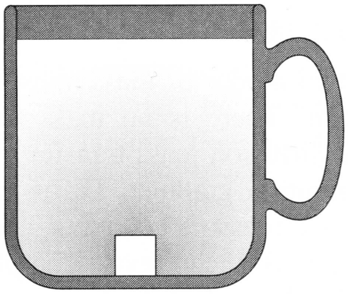


Figure 1.19 A sugar lump dissolving in a cup of coffee: the dissolved sugar moves away from the lump by diffusion in the direction of decreasing sugar concentration.

a steep mountain trail. The air adjacent to the water surface is saturated with water vapor. The corresponding water vapor concentration is usually higher than that in the surrounding air: water vapor diffuses away from the surface and is replenished by evaporation of the liquid water. The enthalpy of vaporization (latent heat) required to evaporate the water is supplied from the bulk water or human body and the surrounding air, causing the familiar cooling effect. Indeed, *sweat cooling,* in which a porous surface is protected from a high-temperature gas stream by supplying water to keep the surface wet, is a technological adaptation of the natural sweat cooling process. We always welcome a breeze when sweating; the mass convection associated with the air motion increases the rate of evaporation and the cooling effect. Sweat cooling involves **simultaneous heat and mass transfer,** as do many other transfer processes of engineering concern. A wet cooling tower cools water from the condenser of a power plant or refrigeration system by evaporating a small portion of the water into an air stream. All combustion processes involve simultaneous mass transfer of the reactants and products, and heat transfer associated with release of the heat of combustion. Examples include combustion of gasoline vapor in a spark ignition automobile engine, of kerosene in an aircraft gas turbine, and of fuel-oil droplets or pulverized coal in a power plant furnace.

Mass transfer occurs in a variety of equipment. Of increasing concern to mechanical engineers is the equipment required to control pollution of the environment by exhaust gases from combustion processes, for example, the exhaust from automobiles or stack gases from power plants. A catalytic converter on an automobile is a **mass exchanger** that removes carbon monoxide, unburnt hydrocarbons, and nitrogen oxides from the engine exhaust. The tuning of a modern automobile engine is dictated by emissions control requirements and the operating characteristics of catalytic converters. The United States is very dependent on coal as a power plant fuel and, unfortunately, our coal has a rather high sulfur content. The sulfur oxides produced in the power plant furnace are the cause of the acid rain problem that plagues the Northeast. Thus, coal-fired power plants are now required to have mass exchangers that remove sulfur oxides (as well as nitrogen oxides and particulate matter) from the furnace exhaust. Such equipment requires a substantial portion of the capital and operating costs of a modern power plant, and the mechanical engineer is concerned with its proper operation, as well as with the development of more effective exchangers.

1.6.1 Modes of Mass Transfer

In this text we focus our attention to the two modes of mass transfer just discussed, namely, ordinary diffusion and convection. There is no mass transfer analog to radiation heat transfer. However, there are diffusion mass transfer modes in addition to ordinary diffusion caused by a concentration gradient. Diffusion of a chemical species can also be caused by temperature and pressure gradients, and by an electrical field; however, these modes are left to more advanced texts. The phenomenological law governing ordinary diffusion is **Fick's law of diffusion** and is analogous to Fourier's law of conduction. It states that the local mass flux of a chemical species is proportional to the negative of the local concentration gradient. A number of measures of concentration are in common use; in this text, we will most often use the **mass fraction,** which for species i is defined as

$$\text{Mass fraction of a species } i = \frac{\text{Partial density of species } i}{\text{Density of the mixture}} = \frac{\rho_i}{\rho} = m_i \quad \textbf{(1.43)}$$

where $\rho = \sum_{i=1}^{n} \rho_i$ for a mixture of n species. Fick's law then gives the *diffusion mass flux* j_1 [kg/m^{2}s] of species 1 in a binary mixture of species 1 and 2 as

$$j_1 \propto -\frac{dm_1}{dx} \quad \textbf{(1.44)}$$

for one-dimensional diffusion in the x-direction. For convenience, we write the constant of proportionality as $\rho\mathscr{D}_{12}$, where ρ [kg/m^3] is the local *mixture* density and $\mathscr{D}_{12}$ [m^2/s] is the *binary diffusion coefficient* (or mass diffusivity); thus

$$j_1 = -\rho\mathscr{D}_{12}\frac{dm_1}{dx} \text{ kg/m}^2\text{ s} \quad \textbf{(1.45)}$$

Data for $\mathscr{D}_{12}$ will be given in Chapter 9 and Appendix A.

Mass convection is essentially identical to heat convection, and similar considerations apply. The flow may be forced or natural, internal or external, and laminar or turbulent. Referring to Fig. 1.20, and analogous to Newton's law of cooling, Eq. (1.20), we may write

$$j_{1,s} = \mathscr{g}_{m_1}\Delta m_1; \quad \Delta m_1 = m_{1,s} - m_{1,e} \quad \textbf{(1.46)}$$

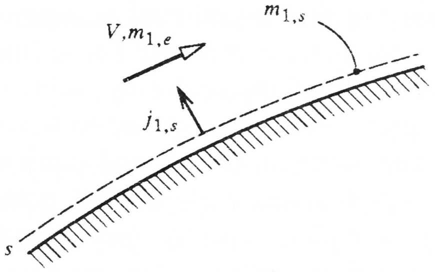


Figure 1.20 Notation for convective mass transfer in an external flow.

where $\mathcal{g}_{m_1}$ [kg/m^{2}s] is the **mass transfer conductance.** The mass transfer conductance and convective heat transfer coefficient play similar but not exactly analogous roles for mass and heat convection. For example, the mass transfer analog to Eq. (1.21) for laminar flow in a tube is

$$\mathcal{g}_{m_1} = 3.66\frac{\rho \mathscr{D}_{12}}{D} \tag{1.47}$$

and to Eq. (1.26) is

$$\overline{\mathcal{g}}_{m_1} = \frac{1}{A}\int_0^A \mathcal{g}_{m_1}\,dA \tag{1.48}$$

which defines an average mass transfer conductance. As for natural heat convection, natural mass convection is driven by buoyancy forces arising from a density difference. The density difference can be due to a concentration difference, for example, in the case of salt dissolving in water. Or the density difference can be due to both concentration and temperature differences, for example, when warm, moist air rises upward from the surface of a heated swimming pool or hot tub. The Grashof number introduced in Section 1.3.3 will be generalized to apply to such situations involving mass transfer or simultaneous heat and mass transfer.

1.6.2 A Strategy for Mass Transfer

From the preceding discussion it is clear that mass transfer is a subject of some importance to mechanical engineers. In particular, processes involving simultaneous heat and mass transfer are frequently encountered. The close analogy between mass and heat transfer suggests two possible strategies for incorporating mass transfer in a heat transfer text. One is to develop the subjects in parallel, and the other is to develop the subjects sequentially. In *Basic Heat and Mass Transfer* the latter strategy is followed. Since heat transfer is the primary focus of the text, and since heat transfer phenomena do not have some of the complexities of mass transfer phenomena, the subject of heat transfer is developed first, in Chapters 1 through 8. The subsequent development of mass transfer in Chapter 9 takes advantage of the experience gained by the student in the earlier chapters, and extensively exploits the analogies between heat and mass transfer phenomena and processes. There is special focus on examples of simultaneous heat and mass transfer, and of pollution control.

1.7 DIMENSIONS AND UNITS

Dimensions are physical properties that are measurable — for example, length, time, mass, and temperature. A system of *units* is used to give numerical values to dimensions. The system most widely used throughout the world in science and industry is the International System of units (SI), from the French name

Système International d' Unités. This system was recommended at the General Conference on Weights and Measures of the International Academy of Sciences in 1960 and was adopted by the U.S. National Bureau of Standards in 1964. In the United States, the transition from the older English system of units to the SI system has been slow and is not complete. The SI system is used in science education, in government contracts, by engineering professional societies, and by many industries. However, engineers in some more mature industries still prefer to use English units, and, of course, commerce and trade in the United States remains dominated by the English system. We buy pounds of vegetables, quarts of milk, drive miles to work, and say that it is a hot day when the temperature exceeds 80°F. (Wine is now sold in 750 ml bottles, though, which is a modest step forward!)

In this text, we will use the SI system, with which the student has become familiar from physics courses. For convenience, this system is summarized in the tables of Appendix B. Base and supplementary units, such as length, time, and plane angle, are given in Table B.1a; and derived units, such as force and energy, are given in Table B.1b. Recognized non-SI units (e.g., hour, bar) that are acceptable for use with the SI system are listed in Table B.1c. Multiples of SI units (e.g., kilo, micro) are defined in Table B.1d. Accordingly, the property data given in the tables of Appendix A are in SI units. The student should review this material and is urged to be careful when writing down units. For example, notice that the unit of temperature is a kelvin (not Kelvin) and has the symbol K (not °K). Likewise, the unit of power is the watt (not Watt). The symbol for a kilogram is kg (not KG). An issue that often confuses the student is the correct use of Celsius temperature. Celsius temperature is defined as $(T - 273.15)$ where T is in kelvins. However, the unit "degree Celsius" is equal to the unit "kelvin" ($1^\circ\mathrm{C} = 1$ K).

Notwithstanding the wide acceptance of the SI system of units, there remains a need to communicate with those engineers (or lawyers!) who are still using English units. Also, component dimensions, or data for physical properties, may be available only in English or cgs units. For example, most pipes and tubes used in the United States conform to standard sizes originally specified in English units. A 1 inch nominal-size tube has an outside diameter of 1 in. For convenience, selected dimensions of U.S. commercial standard pipes and tubes are given in SI units in Appendix A as Tables A.14a and A.14b, respectively. The engineer must be able to convert dimensions from one system of units to another. Table B.2 in Appendix B gives the conversion factors required for most heat transfer applications. The program UNITS is based on Table B.2 and contains all the conversion factors in the table. With the input of a quantity in one system of units, the output is the same quantity in the alternative units listed in Table B.2. It is recommended that the student or engineer perform all problem solving using the SI system so as to efficiently use the Appendix A property data and the computer software. If a problem is stated in English units, the data should be converted to SI units using UNITS; if a customer requires results in units other than SI, UNITS will give the required values.