Solving Linear Equations

I. Linear Equations

   a. <u>Definition</u>: A linear equation in one unknown is an equation in which the only exponent on the unknown is 1.

   b. The *General Form* of a basic linear equation is: $ax + b = c$.

   c. <u>To Solve</u>: the goal is to write the equation in the form *variable = constant*.

   d. The solution to an equation is the set of all values that *check* in the equation.

II.

> **A STEP BY STEP PROCEDURE FOR SOLVING LINEAR EQUATIONS:**
> 1. Remove any parentheses or grouping symbol.
> 2. Multiply <u>every term</u> on both sides of the equation by the L.C.D. of all fractions appearing in the equation. This will get rid of all fractions.
> 3. Combine similar terms on each side of the equation.
> 4. Add or subtract terms on both sides of the equation to get the unknowns on one side and the number on the other side.
> 5. Divide both sides of the equation by the coefficient of the unknown.
> 6. Simplify the answer if necessary.
> 7. Check your answer!

III. Examples:

1. $3x - 1 = 2(x - 5)$

$3x - 1 = 2x - 10$	(Step 1)
$-2x + 3x - 1 = 2x - 10 - 2x$	
$x - 1 = -10$	
$+1 \quad +1$	(Step 4)
$x = -9$	(Step 4)

<u>Answer</u>

$x = -9$

<u>Check</u>

$3(-9) - 1 = 2(-9 - 5)$

$-27 - 1 = 2(-14)$

$-28 = -28$ (It checks!)

2. $\dfrac{x}{5} - \dfrac{2}{3}x + \dfrac{1}{2} = \dfrac{1}{3}(x-4)$ $\qquad\qquad$ L.C.D. = 30

$\dfrac{x}{5} - \dfrac{2}{3}x + \dfrac{1}{2} = \dfrac{x}{3} - \dfrac{4}{3}$ $\qquad\qquad$ (Step 1)

$(30)\dfrac{x}{5} - (30)\dfrac{2}{3}x + (30)\dfrac{1}{2} = (30)\dfrac{x}{3} - (30)\dfrac{4}{3}$ $\qquad$ (Step 2)

$6x - 20x + 15 = 10x - 40$

$-14x + 15 = 10x - 40$ $\qquad\qquad\qquad$ (Step 3)

$\underline{-10x \qquad\quad -10x}$ $\qquad\qquad\qquad$ (Step 4)

$-24x + 15 = -40$

$\underline{\qquad -15 \quad -15}$ $\qquad\qquad\qquad$ (Step 4)

$-24x = -55$

$x = \dfrac{-55}{-24}$ $\qquad\qquad\qquad\qquad$ (Step 5)

$x = \dfrac{55}{24}$ $\qquad\qquad\qquad\qquad\quad$ (Step 6)

<u>Answer:</u> $x = \dfrac{55}{24}$

3. $\dfrac{1}{2}(x+3) = 1 + (\dfrac{1}{4}x + \dfrac{x}{2})$ $\qquad\qquad$ L.C.D. = 4

$\dfrac{1}{2}x + \dfrac{3}{2} = 1 + \dfrac{1}{4}x + \dfrac{x}{2}$ $\qquad\qquad$ (Step 1)

$(4)\dfrac{1}{2}x + (4)\dfrac{3}{2} = (4)1 + (4)\dfrac{1}{4}x + (4)\dfrac{x}{2}$ $\qquad$ (Step 2)

$2x + 6 = 4 + x + 2x$

$\underline{-2x \qquad\quad -2x}$ $\qquad\qquad\qquad$ (Step 4)

$6 = 4 + x$

$\underline{-4 \quad -4}$ $\qquad\qquad\qquad\qquad$ (Step 4)

$2 = x$

<u>Answer:</u> $x = 2$

4. $3(x+2) = 6(\dfrac{x}{2} + 1)$

$3x + 6 = 3x + 6$ $\qquad\qquad\qquad\qquad$ (Step 1)

$\underline{-3x \quad -3x}$ $\qquad\qquad\qquad\qquad$ (Step 4)

$6 = 6$

This is a true statement which implies x can be any real number we want it to be.

<u>Answer:</u> x = Every real number

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5. $2x - 10 = \dfrac{1}{2}(4x + 12)$

$\quad 2x - 10 = 2x + 6$ (Step 1)

$\quad \underline{-2x \qquad -2x}$ (Step 4)

$\quad -10 = 6$

This is an untrue statement which implies that no value of x will satisfy the equation.

<u>Answer:</u> There is no solution.

6. $\dfrac{2x+6}{2x+1} - 3 = \dfrac{5}{2x+1} - \dfrac{1}{3}$ \qquad L.C.D. $= 3(2x+1)$

$3(2x+1)\dfrac{2x+6}{2x+1} - 3(2x+1)3 = 3(2x+1)\dfrac{5}{2x+1} - 3(2x+1)\dfrac{1}{3}$

$3(\cancel{2x+1})\dfrac{2x+6}{\cancel{2x+1}} - 3(2x+1)3 = 3(\cancel{2x+1})\dfrac{5}{\cancel{2x+1}} - \cancel{3}(2x+1)\dfrac{1}{\cancel{3}}$

$3(2x+6) - 9(2x+1) = 3(5) - (2x+1)$

$6x + 18 - 18x - 9 = 15 - 2x - 1$

$-12x + 9 = 14 - 2x$

$-10x + 9 = 14$

$-10x = 5$

$x = \dfrac{5}{-10}$

$x = -\dfrac{1}{2}$

<u>Check</u>

$\dfrac{2(-\frac{1}{2})+6}{2(-\frac{1}{2})+1} - 3 = \dfrac{5}{2(-\frac{1}{2})+1} - \dfrac{1}{3}$ \qquad **It is crucial to check your answer when x appears in the denominator of a fraction!**

$\dfrac{-1+6}{0} - 3 = \dfrac{5}{0} - \dfrac{1}{3}$

undefined

<u>Answer:</u> Since $x = -\dfrac{1}{2}$ does not check the answer is: "There is no solution."

Practice Problems:

1. $x + 7 = 11$

2. $n - 3 = 10$

3. $y + 8 = 4$

4. $8y = 48$

5. $1 = \dfrac{x}{-4}$

6. $15 = \dfrac{a}{3}$

7. $-10y = 40$

8. $\dfrac{4}{9}a = \dfrac{3}{2}$

9. $-\dfrac{3}{4}y = -\dfrac{5}{11}$

10. $3s - 7 = -1$

11. $7 = -11z + 7$

12. $\dfrac{d}{3} - 1 = -7$

13. $-3d + 20 = d$

14. $5y - 3 = 4y + 2$

15. $5 - b = 8b - 13$

16. $2(w + 3) = -2$

17. $2(m - 5) = m - 3$

18. $\dfrac{s}{2} + \dfrac{3s + 1}{5} = \dfrac{s + 3}{10}$

19. $\dfrac{8k + 3}{6} - \dfrac{7k - 1}{4} = -\dfrac{1}{2}$

20. $\dfrac{3(w - 5)}{4} - \dfrac{2(w - 2)}{6} = w + 1$

21. Solve for r: $D = rt$

22. Solve for W: $P = 2L + 2W$

23. Solve for x: $3ax - 1 = 4$

24. Solve for x: $3ax - 4a = 7 + 3ax$

25. Solve for x: $3ax - 4ax = 7 + 3a$

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Answers to Linear Equations:

1. $x = 4$

2. $n = 13$

3. $y = -4$

4. $y = 6$

5. $x = -4$

6. $a = 45$

7. $y = -4$

8. $a = \dfrac{27}{8}$

9. $y = \dfrac{20}{33}$

10. $s = 2$

11. $z = 0$

12. $d = -18$

13. $d = 5$

14. $y = 5$

15. $b = 2$

16. $w = -4$

17. $m = 7$

18. $s = \dfrac{1}{10}$

19. $k = 3$

20. $w = -7$

21. $r = \dfrac{D}{t}$

22. $W = \dfrac{P - 2L}{2}$

23. $x = \dfrac{5}{3a}$

24. No Solution.

25. $x = \dfrac{7 + 3a}{3a - 4}$

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