BATHTUB, FAILURE DISTRIBUTION, MTBF, MTTF, AND MORE: THEY ARE RELATED

Thomas Heiser and James P. Hofmeister
Ridgetop Group, Inc.
3580 West Ina Road
Tucson, AZ 85741
520-742-3300
theiser@ridgetopgroup.com

Abstract: This paper describes the relationship between a bathtub curve, failure distributions, and commonly used metrics: mean-time-before failure (MTBF), mean-time-between failure (MTBF), and mean-time-to failure (MTTF) – metrics that are often misunderstood and misused. A bathtub curve is a statistical depiction of the failure rate over the lifetime of a population of products and is related to a failure-distribution curve: they can be combined to form a continuous curve. A bathtub curve graphically relates three types of failure: early, random, and wear out. Manufacturing and material defects typically result in early, rapid failure of products: that region of a bathtub curve is called infant mortality. After the infant-mortality region, there is a constant-failure region during which a low number of failures occur that are often referred to as random failures that are characterized as having a mean-time-between failures (MTBF). In-use products are subjected to stresses and strains that are cyclic in nature, plastic work, and which eventually causes irreversible damage and the onset of degraded operation. Damage accumulates until the product is no longer capable of operating within specifications and is said to have functionally failed – it has worn out. Such functional failures are characterized by a failure distribution having two common metrics: mean-time-before failure (MTBF) and mean-time-to failure (MTTF). Associated with bathtub and failure distribution curves are other metrics, including the following: failure rate, prognostic trigger point, prognostic distance (PD), failures-in-time (FIT), and useful life. Those metrics and how they are related are the focus of this paper.

Keywords: Bathtub; degradation; failure; metrics; prognostics; trigger; useful life

1. Introduction

There is a relationship between a bathtub curve failure distributions and commonly used metrics: mean-time-before failure (MTBF), mean-time-between failure (MTBF), and mean-time-to failure (MTTF). These metrics are often misunderstood and misused. A bathtub curve, illustrated in Figure 1, is a statistical depiction of the failure rate over the lifetime of a population of products; and it is related to a failure-distribution curve (see Figure 2). They can be combined to form a continuous curve [1]-[4].

Prognostics

Prognostics is an ability to accurately detect and report future failures in systems. The purpose of prognostics is to detect degradation and create prognostic information such as estimates of state-of-health (SoH) and remaining-useful-life (RUL) [5]-[7].

Bathtub Curve

A bathtub curve graphically relates three types of failure: early, random (constant failure), and wear out. Manufacturing and material defects typically result in early, rapid failure of objects: that region of a bathtub curve is called infant mortality. After the infant-mortality region, there is a constant-failure region during which a low number of failures occur that are often referred to as random failures. These failures are characterized as having a mean-time-between failures (MTBF). In-use products are subjected to stresses and strains that are cyclic in nature: plastic work. Plastic work eventually causes irreversible damage and the onset of degraded operation. Damage accumulates until the product is no longer capable of operating within specifications and is said to have functionally failed – it has worn out. The random, or constant, failure region is sometimes referred to as the useful life region [7],[8].

Figure 1: Example of a bathtub curve

Failure Distribution Curve

Products fail at different rates, creating a failure distribution (see Figure 2) that may or may not be a normal distribution.

Referring to Figure 3, mean-time-**between** failure (MTBF) is defined as the time between low-rate failures. Such failures are said to be random failures occurring at constant rate of failure: products having an exponential distribution. MTBF is also defined as mean-time-**before** failure (MTBF). More confusing, MTBF (between) has been used in the context of repairable products while MTBF (before) has been used in the context of non-repairable products. Even more confusing: EQ. (1) is used to calculate mean time to failure (MTTF) and EQ. (2) is used to calculate MTBF (before) – the calculations are identical [3], [4], [9]-[12].

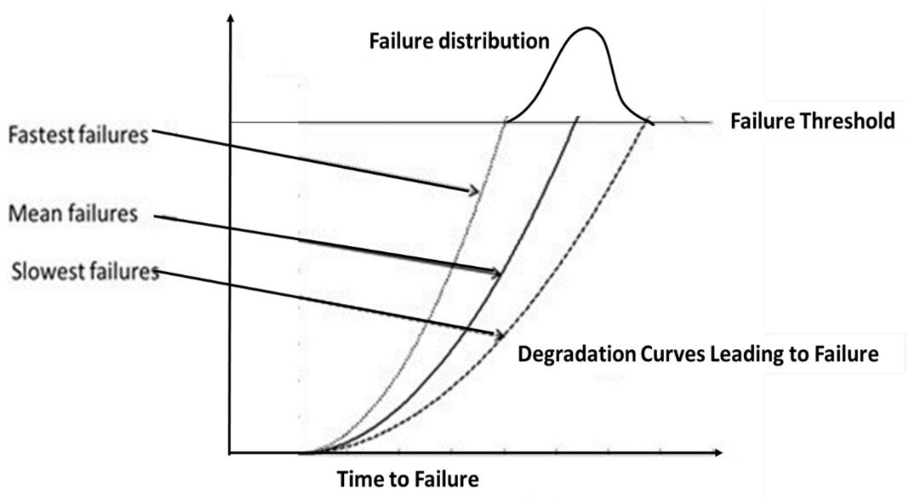

Figure 2: Example of a failure distribution curve due to degradation failures (wear out)

MTBF and MTTF: Relationship to a Bathtub and a Failure-Distribution Curve

$$MTTF = (Total\ Time)/(Number\ of\ Failures) \quad (1)$$

$$MTBF = (Total\ Time)/(Number\ of\ Failures) \quad (2)$$

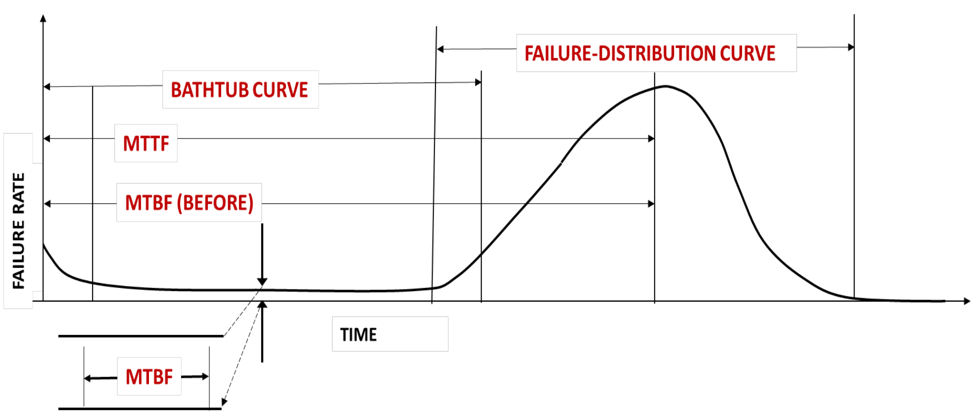

Figure 3: Combined bathtub and failure-distribution curve showing MTBF (between)

2. Bathtub: Other Terms

Other metrics related to failures include prognostic trigger point, prognostic distance (PD), failures-in-time (FIT), and useful life. Related to useful life are the following: (1) remaining useful life (RUL), and (2) end of life (EOL).

Prognostic Trigger, PD, RUL, and EOL

In the early days of IC devices, 1993 for example, it was believed that those devices did not experience wear out: they failed randomly. IC devices were subjected to 'burn in' testing where high stresses were applied to cause devices with manufacturing and/or material defects to fail: the remaining devices would then fail randomly at a low rate of failure,

$$failure\ rate = 1 - \exp[\,-\lambda t^{\beta}] \qquad (3)$$

EQ. (3) is related to a bathtub curve as follows (see Figure 4):

For $\beta < 1$, the failure rate decreases as time increases – infant mortality
For $\beta = 1$, the failure rate is constant – useful life
For $\beta > 1$, the failure rate increases – end of life

One method of prognostic-enabling an IC product, such as a microprocessor, is to include a device that fails earlier than any other device: much like a canary used in mines to detect lethal gases such as methane. Referring to Figure 4, that earlier time of failure is called a *prognostic trigger point* and *prognostic distance (PD)* is defined as

$$PD = useful\ life - (trigger\ time - begin\ time\ of\ useful\ life) \qquad (4)$$

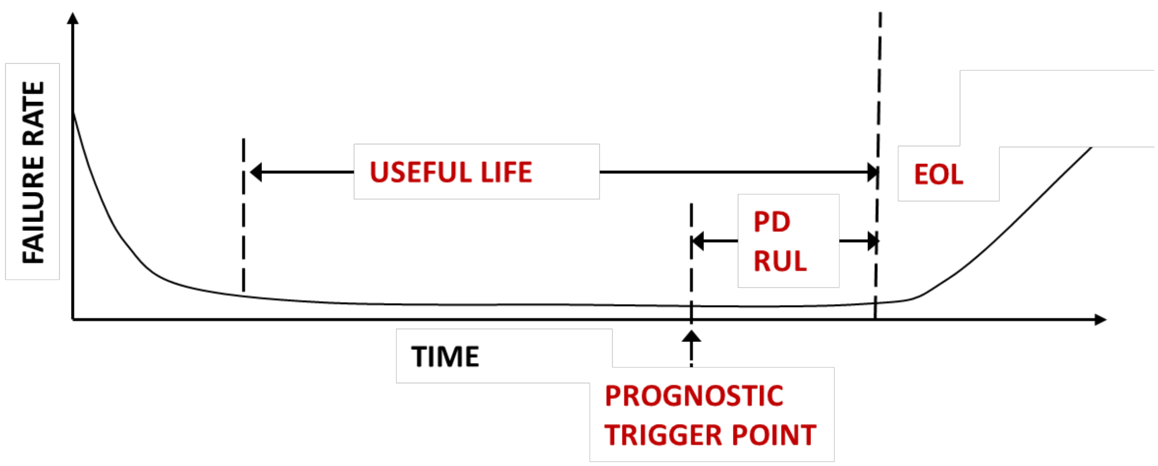

Figure 4: Bathtub showing other metrics

A canary is used as an early warning (alert) that a product needs to be replaced because it is about to enter the (EOL) region of operation. Such use is predicated on a belief that careful design, high-quality manufacturing, and thorough understanding of physics of failure would lead to canary failures that could be used to accurately calculate PD and RUL estimates, where RUL is the amount of time between the trigger point and the beginning of the EOL region [6], [7].

FIT

FIT values are used as a measure of the reliability of a product [13],

$$\text{Failure Rate, Lifetime} = \lambda = \frac{\text{(number of failures)}}{\text{(number of tested parts)(hours of test)}\ AF}\ 10^9\ \text{FIT} \qquad (5)$$

$$\text{FIT} = 1/\lambda \qquad (6)$$

Suppose a particular system has a product that is tested using a regime that calls for 40 units to be tested for 2,500 hours and the AF value for the required test is 10,000 hours using a test cycle of 1 hour. During the test, only 1 of the 40 fails: the failure rate would be calculated as

$$\lambda = \frac{(1)}{(40)(2{,}500\ \text{cycles})(10{,}000\ \text{hours/cycle})}\ 10^9 = 1\ \text{FIT}$$

This does not help you because you typically only know the FIT number. Suppose the product specifications list FIT number of 50,

$$\lambda = 1/(50\ FIT) = 10^9/50 = 20{,}000{,}000 = 20\ million\ hours!$$

Because the calculated rate of failure is so large, you decide to calculate MTBF using EQ. (2),

$$MTBF = (Total\ Time)/(Number\ of\ Failures)$$

But this does not help because you do not know the total time, so you turn to EQ. (7),

$$Total\ Time = (\#\ of\ tested\ units) * (test\ time) \qquad (7)$$

which is no help because you don't know the number of tested units nor the test time. So, you find another expression for MTBF:

$$MTBF = 10^9/FIT \qquad (8)$$

which, for FIT = 50 results in an MTBF (life time) of 20,000,000 hours: over 2,000 years! We highly recommend for prognostics never to use MTBF and MTTF metrics related to a bathtub curve. Similarly, never use FIT values for prognostics.

3. Failure Distribution: Other Terms

Referring back to Figure 3, you see that MTBF (before) is related to failure distribution and is interchangeable with a more modern use of MTTF. Further research leads to the following discoveries:

- MTTF is defined as 'mean time to failure' for a non-repairable product

- A relationship of MTBF to MTTF is defined as follows [13]:

$$MTBF = MTTF + MTTR \tag{9}$$

  Where MTTR is defined as 'mean time to repair'

- MTTR is usually not specified

More importantly, from EQ. (3), an MTTF and/or MTBF value only means there is 63% probability the system will fail prior in time to that value: not very useful in estimating when a particular instantiation of product is going to fail.

Failure Distribution: Degradation Signatures and Time-to-Failure

Referring to Figure 2 and Figure 5, let the fastest failure curve be replaced by a degradation signature, CBD1, let the slowest failure curve be replaced by another degradation signature, CBD2, and let each degradation signature begin at and reach failure at different times. Then we define TTF (time-to-failure),

$$TTF = (functional\ failure) - (onset\ of\ degradation) \tag{10}$$

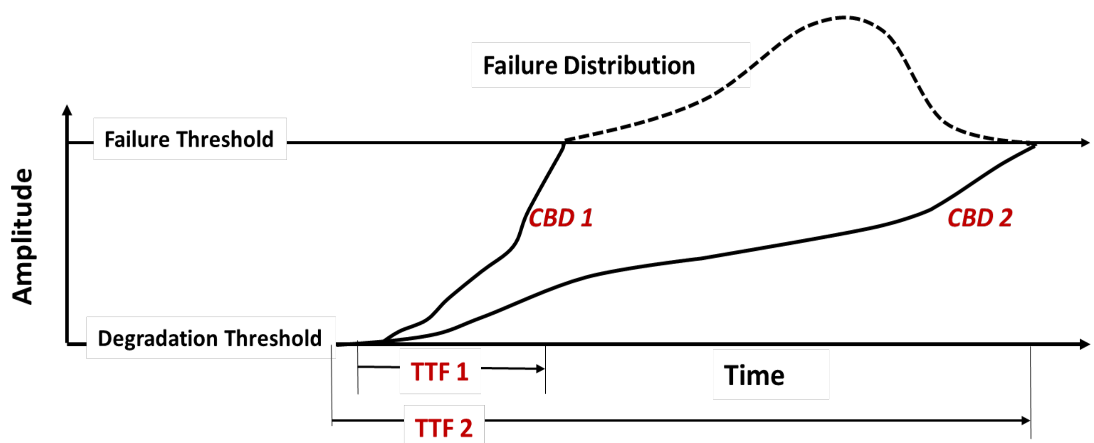

Figure 5: Degradation signatures: TTF relationship to failure distribution

Degradation signature, TTF, sampled data, and RUL

Referring to Figure 6, suppose you have a PHM system with a set of algorithms for conditioning, signature processing, and prediction estimating that accurately detects the onset of degradation, processes input signature data to accurately estimate when functional failure is likely to occur. Then from EQ. (10) for every sampled data at time S,

$$RUL = TTF - (\text{sample time}) \tag{11}$$

To produce RUL estimates, you need to use a PHM system that uses condition-based data (CBD) to produce degradation signatures that are processed to produce accurate estimates of TTF. Then for each sampled data point, that PHM system calculates accurate RUL estimates.

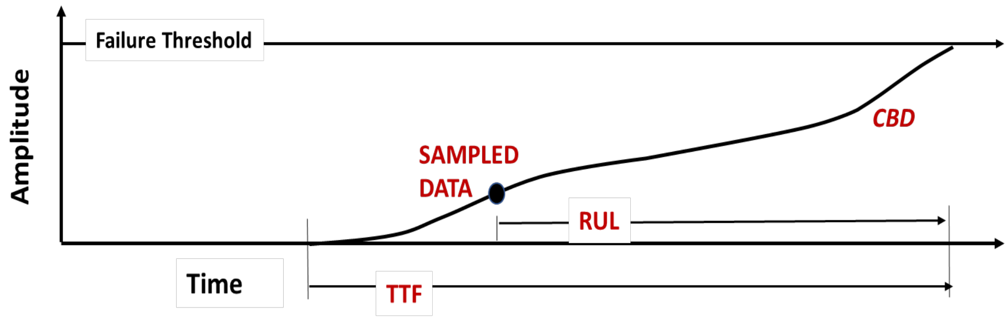

Figure 6: Degradation signature, TTF, sampled data, and RUL

Degradation and Prognostic Information

You cannot use metrics related to a bathtub curve or a failure distribution curve to produce accurate estimates of RUL for a particular product. Instead, you must process CBD, detect the onset of degradation, transform CBD points into degradation signature data, project that signature and estimate a time when the projected signature is likely to reach a defined threshold level for functional failure. You need to use algorithms that (1) mitigate time of functional-failure variability not caused by degradation and (2) minimizes variability caused by degradation.

A design approach is to treat data points as particles having inertia and momentum: particles do not exhibit rapid changes, instead they tend to maintain velocity and direction. A satisfactory design objective is to employ a dampening factor to changes in amplitude and velocity. Another design approach is to develop a random-walk solution for particles that progress from a zero-degradation state (lower-left corner) to a maximum-degradation state (upper-right corner). For example, (1) use the previous data point; (2) calculate the predicted location of the next data point; (3) adapt a signature model to an adjusted location between the predicted and the actual location of the next data point; (4) use dampening factors and coefficients to adjust the signature model; (5) use the adapted model to estimate when functional failure is likely to occur; and (6) calculate prognostic information such as RUL and prognostic horizon (PH), the relative time between the onset of degradation and the time of functional failure. Use EQ. (10) and EQ. (11) to calculate RUL, then calculate PH using EQ. (12),

$$PH = RUL + (sample\ time) - (onset\_of\_degradation\ time) \qquad (12)$$

From the definition of PH and TTF (see Figure 6), PH and TFF are the same. The authors prefer using PH because TTF is sometimes referenced to a time before the onset of degradation: for example, the time when a product is first used or a value of time reference to a calendar time (yy/mm/dd hh: mm: ss).

Predefined RUL Value

Prior to the onset of degradation, you cannot make any accurate estimate of RUL: you must use a predefined value. For example, use a customer-specified value or a vendor-specified value, or a value derived from evaluating historical or experimental data. That initial RUL value is likely to be higher or lower than actual. Figure 7 is an example of a solution applied to an initial-estimate error [14].

In addition to a high-value initial-estimate error, a reliability engineer might specify an initial RUL value that is low compared to that for a prognostic target. Suppose for a prognostic target a reliability engineer specified 100 days as the estimated TTF after the onset of degradation and it actually takes 200 days to become functionally failed: a random-walk with Kalman-like solution will result in curves that resembles those shown in Figure 8 [14].

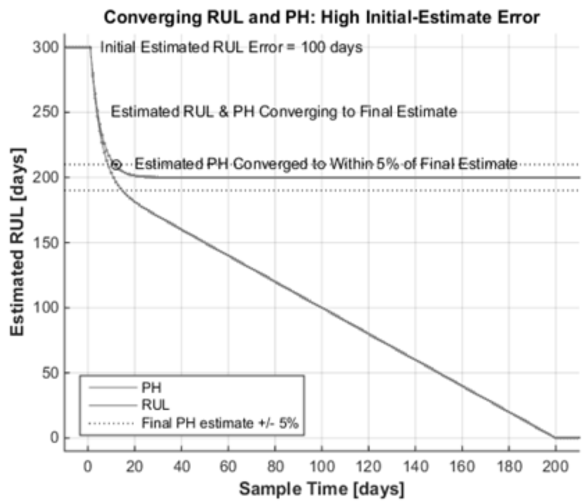

Figure 7: RUL and PH estimates when initial RUL is higher than actual

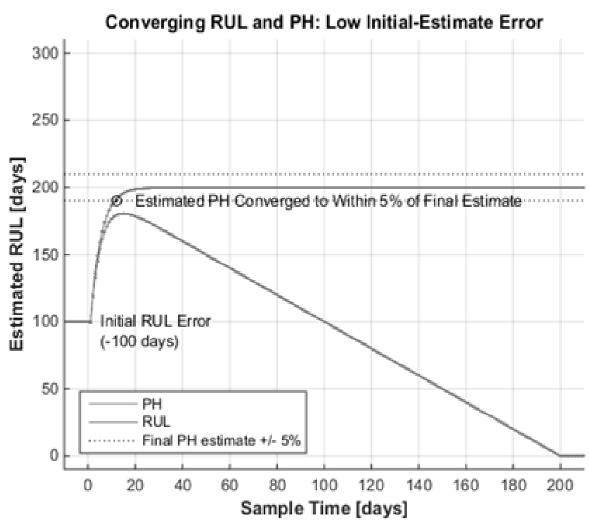

Figure 8: RUL and PH estimates when initial RUL is lower than actual

4. Conclusion

This paper described problems in relating metrics related to bathtub and failure distributions: two MTBF metric and an MTTF metric. Those metrics have multiple definitions that are confusing and, more importantly, are not suitable for use in prognostics for products. Other metrics related to a bathtub curve (prognostic trigger point, RUL, and PD, and FIT) are also not suitable for use in prognostics.

Metrics related to degradation were introduced and shown to be useful in prognostics: TTF, sample time, RUL, and PH. In the absence of detected degradation, TTF cannot be calculated: instead, a predefined value must be used, which results in an initial RUL value that is higher or lower in value than actual.

To produce accurate RUL estimates, it is necessary to use prediction algorithms that do not rely on statistical-based and reliability-based metrics such MTBF, MTTF, and FIT.

Acknowledgments

The authors thank Naval Air, Naval Sea, U.S. Army, U.S. Air Force, and NASA research centers for their support and funding of multiple projects that led to the solutions and the results described and shown in this paper.

References

[1] NIST (2018), "Assessing Product Reliability," Engineering Statistics Handbook, 8.1, 8.3, and 8.4. [Online], accessed 2018, https://www.itl.nist.gov/div898/handbook.

[2] Seastrunk, B. (2016), "Reliability, Warranty, and Why Nothing Lasts as Long as it Used To," Blog post. [Online], accessed 2018, http://opinionbypen.com/reliability-warranty-and-why-nothing-lasts-as-long-as-it-used-to/

[3] Speaks, S. (2005), Reliability and MTBF Overview. VICORP. Andover, MD. [Online], accessed 2018, www.vicorpower.com/documents/quality/Rel_MTBF.PDF.

[4] Wilkins, D.J. (2002), "The Bathtub Curve and Product Failure Behavior, Part Two – Normal Life and Wear-Out," HotWire Issue 22, 2002. [Online], accessed 2018, https://www.microsemi.com/document-portal/doc_view/124041-calculating-reliability-using-fit-mttf-arrhenius-htol-model.

[5] Kumar, S. and Pecht, M. (2010) "Modeling Approaches for Prognostics and Health Management of Electronics," *International Journal of Performability Engineering*, Vol. 6, No. 5, pp. 467-476.

[6] Pecht, M. (2008) *Prognostics and Health Management of Electronics*. Hoboken, NJ: John Wiley & Sons.

[7] O'Connor, P. and Kleyner, A. (2012) *Practical Reliability Engineering*. Chichester, UK: John Wiley & Sons.

[8] Hofmeister, James P.; Goodman, Douglas, L.; and Szidarovszky, Ferenc (2018); "Transforming Condition-based Data Signatures into Functional Failure Signatures," IEEE 2018 Aerospace Conference, Big Sky, MT, 3-9 March, 2018.

[9] Abernethy, R.B. (2006), The New Weibull Handbook 5th edition, Berringer and Associates, 2006.

[10] RAC (2005), Reliability Toolkit: Commercial Practices Editions, Reliability Analysis Center, Rome Laboratory [Online]. accessed 2018,

[11] https://www.dsiac.org/sites/default/files/journals/2Q2005.pdf.

[12] Seymour, B. (1993), "MTTF, FAILRATE, RELIABILITY AND LIFE TESTING," Application Bulletin, AB-059, Burr-Brown Corporation, Tucson, AZ

[13] Weibull (2008), Reliability Engineering, "MTTF, MTBF, Mean Time Between Replacements and MTBF with Scheduled Replacements," HotWire Issue 94, 2008. [Online], accessed 2018, https://www.weibull.com/hotwire/issue94.

[14] Ellerman, P. (2012), "Calculating Reliability using FIT & MTTF: Arrhenius HTOL Model," MicroNote™ 1002, Rev 0, MicroSemi Corp., 1/9/2012.

[15] Goodman, Douglas L.; Hofmeister, James P.; and Szidarovszky, Ferenc; 2019 (Pending), *Prognostics and Health Management: A Practical Approach Using Conditioned-Based Data,* 1st Edition, John Wiley & Sons, Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, United Kingdom, *ISBN: 9781119356653,* 2019, pp. 44-46.